Name: _____ **Class:** _____ **Date:** _____ **ID: A**

Logarithms Practice Test

Multiple Choice
Identify the choice that best completes the statement or answers the question.

____ 1. Which of the following statements is true?
a. The domain of a transformed logarithmic function is always $\{x \in \mathbf{R}\}$.
b. Vertical and horizontal translations must be performed before horizontal and vertical stretches/compressions.
c. A transformed logarithmic function always has a horizontal asymptote.
d. The vertical asymptote changes when a horizontal translation is applied.

____ 2. Express $27^{\frac{1}{3}} = 3$ in logarithmic form.
a. $\log_3 27 = 3$
b. $\log_{\frac{1}{3}} 3 = 27$
c. $\log_{27} 3 = \frac{1}{3}$
d. $\log_3 3 = 27$

____ 3. Solve $\log_x 81 = 4$ for x.
a. 3
b. 9
c. 20.25
d. 324

____ 4. Evaluate $\log_m m^{2n}$.
a. n
b. n^2
c. mn
d. $2n$

____ 5. The function $S(d) = 300 \log d + 65$ relates $S(d)$, the speed of the wind near the centre of a tornado in miles per hour, to d, the distance that the tornado travels, in miles. If winds near the centre of tornado reach speeds of 400 mph, estimate the distance it can travel.
a. 130 miles
b. 13 miles
c. 13000 miles
d. 1.1666 miles

____ 6. Evaluate $\log_2 4^5$.
a. 4
b. 5
c. 7
d. 10

____ 7. Which of the following statements will NOT be true regarding the graphs of
$f(x) = \log_3(3x)$, $f(x) = \log_3(9x)$, and $f(x) = \log_3\left(\frac{x}{3}\right)$?
a. They will all have the same vertical asymptote
b. The will all have the same x-intercept
c. They will all curve in the same direction
d. They will all have the same domain

____ 8. Evaluate $\log_2 \sqrt[3]{64}$.
a. 2
b. 3
c. 8
d. 16

____ 9. Which does not help to explain why you cannot use the laws of logarithms to expand or simplify $\log_4(3y-4)$?

a. The expression $3y - 4$ cannot be factored.
b. The expression $3y - 4$ is not raised to a power.
c. $3y$ and 4 are neither multiplied together, nor are they divided into each other
d. Each term in the expression does not have the same variable.

____ 10. Solve $5^{2-x} = \frac{1}{125}$ for x.

a. $\frac{5}{3}$
b. -1
c. 5
d. $\frac{7}{3}$

____ 11. Solve $\log(3x+1) = 5$.

a. $\frac{4}{3}$
b. 8
c. 300
d. 33 333

____ 12. Which of the following is NOT a strategy that is often used to solve logarithmic equations?

a. Express the equation in exponential form and solve the resulting exponential equation.
b. Simplify the expressions in the equation by using the laws of logarithms.
c. Represent the sums or differences of logs as single logarithms.
d. Square all logarithmic expressions and solve the resulting quadratic equation.

____ 13. Solve $\log_x 8 = -\frac{1}{2}$.

a. -64
b. -16
c. $\frac{1}{64}$
d. 4

____ 14. Describe the strategy you would use to solve $\log_6 x = \log_6 4 + \log_6 8$.

a. Use the product rule to turn the right side of the equation into a single logarithm. Recognize that the resulting value is equal to x.
b. Express the equation in exponential form, set the exponents equal to each other and solve.
c. Use the fact that the logs have the same base to add the expressions on the right side of the equation together. Express the results in exponential form, set the exponents equal to each other and solve.
d. Use the fact that since both sides of the equations have logarithms with the same base to set the expressions equal to each other and solve.

____ 15. Given the formula for magnitude of an earthquake, $R = \log\left(\frac{a}{T}\right) + B$, determine the how many times larger the amplitude a is in an earthquake with $R = 6.9,\ B = 3.2, \text{and } T = 1.9s$ compared to one with $R = 5.7,\ B = 2.9, \text{and } T = 1.6s$

a. 1.2 times as large
b. 1.6 times as large
c. 9.4 times as large
d. 15.8 times as large

____ 16. Solve $\log(x+3)+\log(x)=1$.

a. $-5, 2$
b. 10
c. 2
d. 7

____ 17. Which of the following does not describe the use of logarithmic scales?

a. When the range of values vary greatly, using a logarithmic scale with powers of 10 makes comparisons between values more manageable.
b. Scales that measure a wide range of values, such as the pH scale, the Richter scale and decibel scales are logarithmic scales.
c. Logarithmic scales more effectively describe and compare vast or large quantities than they do small or microscopic quantities.
d. To compare concentrations modelled with logarithmic scales, determine the quotient of the values being compared.

____ 18. A radioactive substance has a half-life of 7 h. If a sample of the substance has an initial mass of 2000 g, estimate the instantaneous rate of change in mass 1.5 days later.

a. –5.6 g/ h
b. –56 g/h
c. –707 g/h
d. –0.845 g/h

____ 19. Which of the following statements regarding rates of change of exponential and logarithmic functions is NOT true?

a. The average rate of change is not constant for exponential and logarithmic functions.
b. The methods for finding the instantaneous rate of change at a particular point for logarithmic functions are different than those used for finding the instantaneous rate of change at a point for a rational function.
c. The graph of an exponential or logarithmic function can be used to determine when the average rate of change is the least or greatest.
d. The graph of an exponential or logarithmic function can be used to predict the greatest and least instantaneous rates of change and when they occur.

____ 20. Suppose the population of a given town is increasing for a given period of time. What can you tell about its instantaneous rate of change of the population during that period?

a. The instantaneous rate of change continues to get larger during the entire interval.
b. The instantaneous rate of change will be positive at each point in the interval.
c. The instantaneous rate of change may be zero, but cannot be negative.
d. The instantaneous rate of change at any point in the interval will be larger than the average rate of change for the interval.

Short Answer

21. State the domain and range of the transformed function $f(x)=6\log_{10}-2(x-5)$.

22. The parent function $f(x)=\log_{10}x$ is vertically stretched by a factor of 3, reflected in the *y*-axis, horizontally transformed 4 units to the left and vertically transformed 2.5 units up. What is the equation of the vertical asymptote of the transformed function?

23. State which of the values in the transformed function $f(x)=2\log_{10}\left[-\frac{1}{4}(x-1.5)\right]+5$ must be changed, and what they must be changed to, so that the resulting function has an asymptote at $x = 6$ with the curve of the graph to left of the vertical asymptote.

24. Estimate the value of $\log_3 91$ to two decimals places.

25. Simplify $4^{\log_4 64}+10^{\log 100}$.

26. Evaluate $\log_5 625+\log_2 32$.

27. Put the following in order from smallest to largest:
$\log_2 16, \log 100, \log_3 30, \log_5 40, \log_{20} 200$

28. State the product law of logarithms and the exponent law it is related to.

29. Write $4\log 2+\log 6-\log 3$ as a single logarithm.

30. Rewrite $x=\log_2\left(\frac{1}{\sqrt{8}}\right)$ in exponential form.

31. If you invested money into an account that pays 9%/a compounded weekly, how many years would it take for your deposit to double?

32. Solve $10^{x+2}-10^x=9900$ for *x*.

33. Solve $3^{2x}=7^{3x-1}$ for *x*. Round your answer to two decimal places.

34. Solve $2^{4x}=\frac{1}{32}$ for *x*.

35. What are the restrictions on the variable in the equation $\log(3x-5)-\log(x-2)=\log(x^2-5)$?

36. Solve $2\log x-\log 4=3\log 4$.

37. Solve $\log_2 x+\log(x-7)=3$.

38. The population of a town is increasing at a rate of 6.2% per year. The city council believes they will have to add another elementary school when the population reaches 100 000. If there are currently 76 000 people living in the town, how long do they have before the new school will be needed?

39. If $f(x)=a(b+1)^x$ models an exponential growth situation, write an equation that models an exponential decay situation.

40. If the annual cost of a given good rises 2.3% per year for the next 20 years, write an equation to model the approximate cost of the good during any year in the next 20.

Problem

41. Describe two characteristics of the graph of the function $f(x) = \log_{10} x$ that are changed and two that remain the same under the following transformation: a horizontal compression by a factor of 2, a reflection in the *y*-axis and a vertical translation 3 units up.

42. Without graphing, compare the vertical asymptotes and domains of the functions $f(x) = 3\log_{10}(x-5)+2$ and $f(x) = 3\log_{10}[-(x+5)]+2$.

43. The half-life of radium is 1620 years. If a laboratory has 12 grams of radium, how long will it take before it has 8 grams of radium left?

44. Describe the transformations that take the graph of $f(x) = \log_4 x$ to the graph of $g(x) = \log_4 x^3 - \log_4 8$. Justify your response algebraically

45. Write $\frac{1}{3}\log_a x + \frac{1}{2}\log_a 2y - \frac{1}{6}\log_a 4z$ as a single logarithm. Assume that all variables represent positive numbers.

46. Explain the difference in the process of solving exponential equations where both sides are written as powers of the same base and solving exponential equations where both sides are not written as powers of the same base.

47. If $\log\left(\frac{x-y}{3}\right) = \frac{1}{2}(\log x + \log y)$, show that $x^2 + y^2 = 11xy$.

48. How many years will it take for a \$400 investment to grow to \$1000 with a interest rate of 12%/a compounded monthly?

49. The function $S(d) = 86\log d + 112$ relates the speed of the wind, *S*, in miles per hour, near the centre of a tornado to the distance the tornado travels, *d*, in miles. Estimate the rate at which the speed of the wind at the centre of the tornado is changing the moment it has travelled its 50th mile.

50. Discuss why exponential equations of the form $f(x) = ab^x$ always have positive instantaneous rates of change when *a* is positive and *b* is greater than one, and why they always have negative instantaneous rates of change when *a* is positive and *b* is between 0 and 1.

Logarithms Practice Test
Answer Section

MULTIPLE CHOICE

1. ANS: D PTS: 1 REF: Communication
   OBJ: 8.2 - Transformations of Logarithmic Functions
2. ANS: C PTS: 1 REF: Knowledge and Understanding
   OBJ: 8.3 - Evaluating Logarithms
3. ANS: A PTS: 1 REF: Knowledge and Understanding
   OBJ: 8.3 - Evaluating Logarithms
4. ANS: D PTS: 1 REF: Knowledge and Understanding
   OBJ: 8.3 - Evaluating Logarithms
5. ANS: B PTS: 1 REF: Application OBJ: 8.3 - Evaluating Logarithms
6. ANS: D PTS: 1 REF: Knowledge and Understanding
   OBJ: 8.3 - Evaluating Logarithms
7. ANS: B PTS: 1 REF: Thinking OBJ: 8.4 - Laws of Logarithms
8. ANS: A PTS: 1 REF: Knowledge and Understanding
   OBJ: 8.4 - Laws of Logarithms
9. ANS: D PTS: 1 REF: Communication
   OBJ: 8.4 - Laws of Logarithms
10. ANS: C PTS: 1 REF: Knowledge and Understanding
    OBJ: 8.5 - Solving Exponential Equations
11. ANS: D PTS: 1 REF: Knowledge and Understanding
    OBJ: 8.6 - Solving Logarithmic Equations
12. ANS: D PTS: 1 REF: Communication
    OBJ: 8.6 - Solving Logarithmic Equations
13. ANS: C PTS: 1 REF: Knowledge and Understanding
    OBJ: 8.6 - Solving Logarithmic Equations
14. ANS: A PTS: 1 REF: Thinking
    OBJ: 8.6 - Solving Logarithmic Equations
15. ANS: C PTS: 1 REF: Application
    OBJ: 8.6 - Solving Logarithmic Equations
16. ANS: C PTS: 1 REF: Knowledge and Understanding
    OBJ: 8.6 - Solving Logarithmic Equations
17. ANS: C PTS: 1 REF: Knowledge and Understanding
    OBJ: 8.7 - Solving Problems with Exponential and Logarithmic Functions
18. ANS: A PTS: 1 REF: Knowledge and Understanding
    OBJ: 8.8 - Rates of Change in Exponential and Logarithmic Functions
19. ANS: B PTS: 1 REF: Communication
    OBJ: 8.8 - Rates of Change in Exponential and Logarithmic Functions
20. ANS: B PTS: 1 REF: Thinking
    OBJ: 8.8 - Rates of Change in Exponential and Logarithmic Functions

SHORT ANSWER

21. ANS:
Domain: $\{x \in \mathbf{R} \mid x < 5\}$
Range: $\{x \in \mathbf{R}\}$

PTS: 1 REF: Application OBJ: 8.2 - Transformations of Logarithmic Functions

22. ANS:
$x = -4$

PTS: 1 REF: Thinking OBJ: 8.2 - Transformations of Logarithmic Functions

23. ANS:
Change 1.5 to 6. The curve is already to the left of the vertical asymptote.

PTS: 1 REF: Application OBJ: 8.2 - Transformations of Logarithmic Functions

24. ANS:
4.11

PTS: 1 REF: Knowledge and Understanding OBJ: 8.3 - Evaluating Logarithms

25. ANS:
164

PTS: 1 REF: Thinking OBJ: 8.3 - Evaluating Logarithms

26. ANS:
9

PTS: 1 REF: Knowledge and Understanding OBJ: 8.3 - Evaluating Logarithms

27. ANS:
$\log_{20} 200,\ \log 100,\ \log_5 40,\ \log_3 30,\ \log_2 16$

PTS: 1 REF: Communication OBJ: 8.3 - Evaluating Logarithms

28. ANS:
$\log_a (mn) = \log_a m + \log_a n$
$a^x \times a^y = a^{x+y}$

PTS: 1 REF: Knowledge and Understanding OBJ: 8.4 - Laws of Logarithms

29. ANS:
log32

PTS: 1 REF: Knowledge and Understanding OBJ: 8.4 - Laws of Logarithms

30. ANS:
$2^x = \frac{1}{\sqrt{8}}$

PTS: 1 REF: Knowledge and Understanding OBJ: 8.5 - Solving Exponential Equations

31. ANS:
7.7 years

PTS: 1 REF: Application OBJ: 8.5 - Solving Exponential Equations

32. ANS:
2

PTS: 1 REF: Knowledge and Understanding OBJ: 8.5 - Solving Exponential Equations

33. ANS:
0.53

PTS: 1 REF: Thinking OBJ: 8.5 - Solving Exponential Equations

34. ANS:
$-\frac{5}{4}$

PTS: 1 REF: Knowledge and Understanding OBJ: 8.5 - Solving Exponential Equations

35. ANS:
$x \geq \sqrt{5}$

PTS: 1 REF: Knowledge and Understanding
OBJ: 8.6 - Solving Logarithmic Equations

36. ANS:
16

PTS: 1 REF: Knowledge and Understanding
OBJ: 8.6 - Solving Logarithmic Equations

37. ANS:
8

PTS: 1 REF: Thinking OBJ: 8.6 - Solving Logarithmic Equations

38. ANS:
4.6 years

PTS: 1 REF: Application
OBJ: 8.7 - Solving Problems with Exponential and Logarithmic Functions

39. ANS:
$f(x) = a(b-1)^x$, $1 < b < 2$

PTS: 1 REF: Knowledge and Understanding
OBJ: 8.7 - Solving Problems with Exponential and Logarithmic Functions

40. ANS:
$C_f = C_p(1.023)^t$ C_f = Future Cost, C_p = Current Cost

PTS: 1 REF: Communication
OBJ: 8.7 - Solving Problems with Exponential and Logarithmic Functions

PROBLEM

41. ANS:
The transformed function, $f(x) = \log_{10} - 2x + 3$, has the same range as the parent function, since the range of all transformed logarithmic functions have a range of all real numbers. The *y*-intercept is the vertical asymptote of both the parent and transformed functions.

The transformed function curves to the left, the original function curve to the right. The two functions will have different *x*-intercepts, the intercepts being reflected over the *y*-axis.

PTS: 1 REF: Communication
OBJ: 8.2 - Transformations of Logarithmic Functions

42. ANS:
The vertical asymptote helps define the domain of a function. The vertical asymptote changes when a horizontal translation is applied.
The vertical asymptote of $f(x) = 3\log_{10}(x-5) + 2$ is $x = 5$.
The vertical asymptote of $f(x) = 3\log_{10}[-(x+5)] + 2$ is $x = -5$.
The graph of the first function curves to the right of the asymptote. The domain of $f(x) = 3\log_{10}(x-5) + 2$ is $\{x \in \mathbf{R} \mid x > 5\}$.
Since the expression $(x + 5)$ is multiplied by -1, the graph is reflected in the *y*-axis and curves to the left of the asymptote. The domain of $f(x) = 3\log_{10}[-(x+5)] + 2$ is $\{x \in \mathbf{R} \mid x < -5\}$.

PTS: 1 REF: Communication
OBJ: 8.2 - Transformations of Logarithmic Functions

43. ANS:
The equation for relating the amount of radium, *r*, in grams and the amount of time, *t*, in years is
$r = 12 \times \left(\frac{1}{2}\right)^{(t \div 1620)}$.

Substituting 8 in for *r* gives $8 = 12 \times (\frac{1}{2})^{(t \div 1620)}$

$\frac{2}{3} = \left(\frac{1}{2}\right)^{(t \div 1620)}$

Using guess and check gives $\frac{t}{1620} = 0.59$

$t = 956$ years

PTS: 1 REF: Application OBJ: 8.3 - Evaluating Logarithms

44. ANS:
Using the laws of logarithms, $\log_4 x^3 - \log_4 8$ can be rewritten as the single logarithm $3\log_4\left(\frac{1}{2}x\right)$ by first applying the quotient law and then the product law of logarithms. Comparing the new form of $g(x)$ to $f(x)$ produces a vertical stretch by a factor of 3 and a horizontal stretch by a factor of 2.

PTS: 1 REF: Communication OBJ: 8.4 - Laws of Logarithms

45. ANS:

$$\frac{1}{3}\log_a x+\frac{1}{2}\log_a 2y-\frac{1}{6}\log_a 4z$$
$$=\log_a \sqrt[3]{x}+\log_a \sqrt{2y}-\log_a \sqrt[6]{4z}$$
$$=\log_a \frac{\sqrt[3]{x}\sqrt{2y}}{\sqrt[6]{4z}}$$

PTS: 1 REF: Knowledge and Understanding OBJ: 8.4 - Laws of Logarithms

46. ANS:

We use the fact that when two exponential expressions with the same base are equal their exponents are equal to set the exponents equal to one another and solve. If $a^m=a^n$, then $m=n$.
When we have two exponential expressions with different bases set equal to each other, we use the fact that taking the log of equal expressions maintains their equality to start the solution process. If $M=N$, then $\log M=\log N$. Given that M and N are powers, we use the power rule to continue the solution process.

PTS: 1 REF: Communication OBJ: 8.5 - Solving Exponential Equations

47. ANS:

$$\log\left(\frac{x-y}{3}\right)=\frac{1}{2}(\log x+\log y)$$
$$2\log\left(\frac{x-y}{3}\right)=\log x+\log y$$
$$2\log\left(\frac{x-y}{3}\right)=\log xy$$
$$\log\left(\frac{x-y}{3}\right)^2=\log xy$$
$$\left(\frac{x-y}{3}\right)^2=xy$$
$$\frac{x^2-2xy+y^2}{9}=xy$$
$$x^2-2xy+y^2=9xy$$
$$x^2+y^2=11xy$$

PTS: 1 REF: Thinking OBJ: 8.6 - Solving Logarithmic Equations

48. ANS:

$1000=400(1.01)^{12t}$
$2.5=(1.01)^{12t}$
$\log 2.5=\log(1.01)^{12t}$
$\log 2.5=12t\log 1.01$
$0.3979=12t(0.00432)$
$t=7.7$ years

PTS: 1 REF: Application OBJ: 8.6 - Solving Logarithmic Equations

49. ANS:
$S(d) = 86\log d + 112$
$S(d) = 86\log 49.9 + 112$ $\qquad$ $S(d) = 86\log 50.1 + 112$
$= 258.0366$ $\qquad$ $= 258.1860$
Instantaneous Rate of Change

$$\frac{258.1860 - 259.0366}{50.1 - 49.9} = 0.747 \text{ mph/mi}$$

PTS: 1 REF: Application
OBJ: 8.8 - Rates of Change in Exponential and Logarithmic Functions

50. ANS:
The graph of $f(x) = ab^x$ is constantly increasing when a is positive and b is greater than 1. The graph rises slowly and then more rapidly, but at no point does its direction change. Similarly, the graph of $f(x) = ab^x$ is constantly decreasing when a is positive and b is between 0 and 1. The graph first decreases rapidly and then much more slowly but, again, at no point does its direction change.

PTS: 1 REF: Communication
OBJ: 8.8 - Rates of Change in Exponential and Logarithmic Functions