Formule trigonometrice

1. $\sin\alpha = \frac{a}{c}$; $\cos\alpha = \frac{b}{c}$; $\operatorname{tg}\alpha = \frac{a}{b}$; $\operatorname{ctg}\alpha = \frac{b}{a}$;
   (a, b - catetele, c - ipotenuza triunghiului dreptunghic, α - unghiul, opus catetei a).

2. $\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha}$; $\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha}$.

3. $\operatorname{tg}\alpha \operatorname{ctg}\alpha = 1$.

4. $\sin\left(\frac{\pi}{2} \pm \alpha\right) = \cos\alpha$; $\sin(\pi \pm \alpha) = \mp\sin\alpha$.

5. $\cos\left(\frac{\pi}{2} \pm \alpha\right) = \mp\sin\alpha$; $\cos(\pi \pm \alpha) = -\cos\alpha$.

6. $\operatorname{tg}\left(\frac{\pi}{2} \pm \alpha\right) = \mp\operatorname{ctg}\alpha$; $\operatorname{ctg}\left(\frac{\pi}{2} \pm \alpha\right) = \mp\operatorname{tg}\alpha$.

7. $\sec\left(\frac{\pi}{2} \pm \alpha\right) = \mp\operatorname{cosec}\alpha$; $\operatorname{cosec}\left(\frac{\pi}{2} \pm \alpha\right) = \sec\alpha$.

8. $\sin^2\alpha + \cos^2\alpha = 1$.

9. $1 + \operatorname{tg}^2\alpha = \sec^2\alpha$.

10. $1 + \operatorname{ctg}^2\alpha = \operatorname{cosec}^2\alpha$.

11. $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$.

12. $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$.

13. $\operatorname{tg}(\alpha \pm \beta) = \frac{\operatorname{tg}\alpha \pm \operatorname{tg}\beta}{1 \mp \operatorname{tg}\alpha\operatorname{tg}\beta}$.

14. $\operatorname{ctg}(\alpha \pm \beta) = \frac{\operatorname{ctg}\alpha\operatorname{ctg}\beta \mp 1}{\operatorname{ctg}\beta \pm \operatorname{ctg}\alpha}$.

15. $\sin 2\alpha = 2\sin\alpha\cos\alpha$.

16. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$.

17. $\operatorname{tg} 2\alpha = \frac{2\operatorname{tg}\alpha}{1 - \operatorname{tg}^2\alpha}$.

18. $\operatorname{ctg} 2\alpha = \frac{\operatorname{ctg}^2\alpha - 1}{2\operatorname{ctg}\alpha}$.

19. $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$.

20. $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$.

21. $\left|\sin\dfrac{\alpha}{2}\right| = \sqrt{\dfrac{1-\cos\alpha}{2}}$.

22. $\left|\cos\dfrac{\alpha}{2}\right| = \sqrt{\dfrac{1+\cos\alpha}{2}}$.

23. $\left|\operatorname{tg}\dfrac{\alpha}{2}\right| = \sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$.

24. $\operatorname{tg}\dfrac{\alpha}{2} = \dfrac{\sin\alpha}{1+\cos\alpha} = \dfrac{1-\cos\alpha}{\sin\alpha}$.

25. $\left|\operatorname{ctg}\dfrac{\alpha}{2}\right| = \sqrt{\dfrac{1+\cos\alpha}{1-\cos\alpha}}$.

26. $\operatorname{ctg}\dfrac{\alpha}{2} = \dfrac{\sin\alpha}{1-\cos\alpha} = \dfrac{1+\cos\alpha}{\sin\alpha}$.

27. $1+\cos\alpha = 2\cos^2\dfrac{\alpha}{2}$.

28. $1-\cos\alpha = 2\sin^2\dfrac{\alpha}{2}$.

29. $\sin\alpha \pm \sin\beta = 2\sin\dfrac{\alpha\pm\beta}{2}\cos\dfrac{\alpha\mp\beta}{2}$.

30. $\cos\alpha + \cos\beta = 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2}$.

31. $\cos\alpha - \cos\beta = -2\sin\dfrac{\alpha+\beta}{2}\sin\dfrac{\alpha-\beta}{2}$.

32. $\operatorname{tg}\alpha \pm \operatorname{tg}\beta = \dfrac{\sin(\alpha\pm\beta)}{\cos\alpha\cos\beta}$.

33. $\operatorname{ctg}\alpha \pm \operatorname{ctg}\beta = \dfrac{\sin(\beta\pm\alpha)}{\sin\alpha\sin\beta}$.

34. $\sin\alpha\sin\beta = \dfrac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$.

35. $\sin\alpha\cos\beta = \dfrac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$.

36. $\cos\alpha\cos\beta = \dfrac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$.

37. Ecuatii trigonometrice elementare:

$$\left.\begin{array}{l} \sin x = a,\ |a| \le 1;\ x = (-1)^n \arcsin a + \pi n; \\ \cos x = a,\ |a| \le 1;\ x = \pm \arccos a + 2\pi n; \\ \operatorname{tg} x = a,\ x = \operatorname{arctg} a + \pi n; \\ \operatorname{ctg} x = a,\ x = \operatorname{arcctg} a + \pi n \end{array}\right\} n \in \mathbf{Z}.$$

38. $\arcsin x + \arccos x = \frac{\pi}{2}, \quad |x| \le 1.$

39. $\operatorname{arctg} x + \operatorname{arcctg} x = \frac{\pi}{2}.$

40. $\operatorname{arcsec} x + \operatorname{arccosec} x = \frac{\pi}{2}, \quad |x| \ge 1.$

41. $\sin(\arcsin x) = x, \quad x \in [-1; +1].$

42. $\arcsin(\sin x) = x, \quad x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right].$

43. $\cos(\arccos x) = x, \quad x \in [-1; +1].$

44. $\arccos(\cos x) = x, \quad x \in [0; \pi].$

45. $\operatorname{tg}(\operatorname{arctg} x) = x, \quad x \in \mathbf{R}.$

46. $\operatorname{arctg}(\operatorname{tg} x) = x, \quad x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right).$

47. $\operatorname{ctg}(\operatorname{arcctg} x) = x, \quad x \in \mathbf{R}.$

48. $\operatorname{arcctg}(\operatorname{ctg} x) = x, \quad x \in (0; \pi).$

49. $\arcsin x = \arccos \sqrt{1-x^2} = \operatorname{arctg} \frac{x}{\sqrt{1-x^2}} = \operatorname{arcctg} \frac{\sqrt{1-x^2}}{x}, \quad 0 < x < 1.$

50. $\arccos x = \arcsin \sqrt{1-x^2} = \operatorname{arctg} \frac{\sqrt{1-x^2}}{x} = \operatorname{arcctg} \frac{x}{\sqrt{1-x^2}}, \quad 0 < x < 1.$

51. $\operatorname{arctg} x = \arcsin \frac{x}{\sqrt{1+x^2}} = \arccos \frac{1}{\sqrt{1+x^2}} = \operatorname{arcctg} \frac{1}{x}, \quad 0 < x < +\infty.$

52. $\operatorname{arcctg} x = \arcsin \frac{1}{\sqrt{1+x^2}} = \arccos \frac{x}{\sqrt{1+x^2}} = \operatorname{arctg} \frac{1}{x}, \quad 0 < x < +\infty.$

53. $$\arcsin x + \arcsin y = \left[\begin{array}{ll} \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{daca } xy \le 0 \text{ sau } x^2+y^2 \le 1; \\ \pi - \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{daca } x > 0,\ y > 0 \text{ si } x^2+y^2 > 1; \\ -\pi - \arcsin(x\sqrt{1-y^2} + y\sqrt{1-x^2}), & \text{daca } x < 0, y < 0 \text{ si } x^2+y^2 > 1. \end{array}\right.$$

54. $$\arcsin x - \arcsin y = \begin{cases} \arcsin(x\sqrt{1-y^2} - y\sqrt{1-x^2}), & \text{daca } xy \geq 0 \text{ sau } x^2+y^2 \leq 1; \\ \pi - \arcsin(x\sqrt{1-y^2} - y\sqrt{1-x^2}), & \text{daca } x > 0, y < 0 \text{ si } x^2+y^2 > 1; \\ -\pi - \arcsin(x\sqrt{1-y^2} - y\sqrt{1-x^2}), & \text{daca } x < 0, y > 0 \text{ si } x^2+y^2 > 1. \end{cases}$$

55. $$\arccos x + \arccos y = \begin{cases} \arccos(xy - \sqrt{(1-x^2)(1-y^2)}), & \text{daca } x+y \geq 0; \\ 2\pi - \arccos(xy - \sqrt{(1-x^2)(1-y^2)}), & \text{daca } x+y < 0. \end{cases}$$

56. $$\arccos x - \arccos y = \begin{cases} -\arccos(xy + \sqrt{(1-x^2)(1-y^2)}), & \text{daca } x \geq y; \\ \arccos(xy + \sqrt{(1-x^2)(1-y^2)}), & \text{daca } x < y. \end{cases}$$

57. $$\operatorname{arctg} x + \operatorname{arctg} y = \begin{cases} \operatorname{arctg} \dfrac{x+y}{1-xy}, & \text{daca } xy < 1; \\ \pi + \operatorname{arctg} \dfrac{x+y}{1-xy}, & \text{daca } x > 0 \text{ si } xy > 1; \\ -\pi + \operatorname{arctg} \dfrac{x+y}{1-xy}, & \text{daca } x < 0 \text{ si } xy > 1. \end{cases}$$

58. $$\operatorname{arctg} x - \operatorname{arctg} y = \begin{cases} \operatorname{arctg} \dfrac{x-y}{1+xy}, & \text{daca } xy > -1; \\ \pi + \operatorname{arctg} \dfrac{x-y}{1+xy}, & \text{daca } x > 0 \text{ si } xy < -1; \\ -\pi + \operatorname{arctg} \dfrac{x-y}{1+xy}, & \text{daca } x < 0 \text{ si } xy < -1. \end{cases}$$

59. $$2\arcsin x = \begin{cases} \arcsin(2x\sqrt{1-x^2}), & \text{daca } |x| \leq \dfrac{\sqrt{2}}{2}; \\ \pi - \arcsin(2x\sqrt{1-x^2}), & \text{daca } \dfrac{\sqrt{2}}{2} < x \leq 1; \\ -\pi - \arcsin(2x\sqrt{1-x^2}), & \text{daca } -1 \leq x < -\dfrac{\sqrt{2}}{2}. \end{cases}$$

60. $$2\arccos x = \begin{cases} \arccos(2x^2-1) & \text{cand } 0 \leq x \leq 1; \\ 2\pi - \arccos(2x^2-1) & \text{cand } -1 \leq x < 0. \end{cases}$$

61. $$2\operatorname{arctg} x = \begin{cases} \operatorname{arctg} \dfrac{2x}{1-x^2}, & \text{daca } |x| < 1; \\ \pi + \operatorname{arctg} \dfrac{2x}{1-x^2}, & \text{daca } x > 1; \\ -\pi + \operatorname{arctg} \dfrac{2x}{1-x^2}, & \text{daca } x < -1. \end{cases}$$

62. $\frac{1}{2}\arcsin x = \left[\begin{array}{ll} \arcsin\sqrt{\frac{1-\sqrt{1-x^2}}{2}}, & \text{daca } 0 \le x \le 1; \\ -\arcsin\sqrt{\frac{1-\sqrt{1-x^2}}{2}}, & \text{daca } -1 \le x < 0. \end{array}\right.$

63. $\frac{1}{2}\arccos x = \arccos\sqrt{\frac{1+x}{2}}$, daca $-1 \le x \le 1$.

64. $\frac{1}{2}\operatorname{arctg} x = \left[\begin{array}{l} \operatorname{arctg}\frac{\sqrt{1+x^2}-1}{x}, \ \text{daca } x \ne 0; \\ 0, \ \text{daca } x = 0. \end{array}\right.$