Exercise 8 / page 16

How many different letter arrangements can be made from the letters:
a) FLUKE
b) PROPOSE
c) MISSISSIPPI
d) ARRANGE

Solution:

a) all the letters are different so we can make 5! = 120 arrangements
b) We have 7 letters that can be permuted in 7! ways but because some of the letters repeat themselves we counted some of the arrangements more than once. So the actual number of dictinct ways to arrange the letters is 7!/ (2!*2!) = 1260 (we have 2 P's which were counted 2! Times and 2O's permuted again 2! Times)
c) The letters which are repeating are I – 4 times ,S – 4 times and P - 2 times so the answer should be (following the same reasoning as before) 11!/ (4!*4!*2!) = 34,650
d) We have 2 A's and 2 R's so , 7 ! / (2!*2!) =1260 ways of arranging the letters

Exercise 10 / page 16

In how many ways can 8 people be seated in a row if
a) there are no restrictions on the seating arrangement
b) persons A and B must sit next to each other
c) there are 4 men and 4 women and no 2 men or 2 women can sit next to each other
d) there are 5 men and they must sit next to each other
e) there are 4 married couples and each couple must sit together

Solution :

a) if there're no restrictions then we count all possible permutations which are 8! = 40,320
b) if A and B sit together then they constitute a "block" which can be permuted along with the remaining 6 people in 7! different ways .Inside this "block" there are 2!=2 possibilities of permuting A and B so due to the basic counting principle we get a total number of 2*7! = 10,080 ways
c) the restrictions impose having persons of opposite sex next to each other .We shall apply here the generalized principle of counting in the following way :
on the first position we can have any of the 8 persons ,then for one choice of the person for the first position we shall have 4 possible choices for the second position among the people of opposite sex , then for any particular choice of the first two positions we have 3 choices for the third position among the remaining people of the same sex with the person on the first position(and consequently of different sex with the one on the second position) and so on .. So multiplying all these numbers we get 8*4*3*3*2*2*1*1 = 1152 ways of arranging the people
d) the men can be considered again a "block" and permuted along with the remaining 3 women in 4! ways and as we can also permute the men within the block in 5! Ways then the total amounts to 5!*4!=2,880 ways
e) within each couple there are 2! =2 possibilities of siting the people so a total of 2*2*2*2=16 ways .As we can also permute the couples among themselves in 4! ways then we have 16*4!=384 ways of arranging the people

Exercise 13 / page 16

Consider a group of 20 people. If everyone shakes hands with everyone else , how many handshakes take place ?

Solution : The question is "in how many ways can we select two people from 20 ?" .The answer is ,from the definition, $C(20,2)= 20!/(18!*2!)=18!*19*20/(18!*2)=19*20/2=190$ .I denoted by $C(n,k)$ the number of combinations of n objects taken by k .

Exercise 18 / page 17

A committee of 7 , consisting of 2 Republicans, 2 Democrats, and 3 Independents , is to be selected from a group of 5 Republicans, 6 Democrats , and 4 Independents . How many committees are possible ?

Solution:

This is an application of the generalized principle of counting. The 2 Republicans can be chosen from the group of 5 Republicans in $C(5,2)$ ways .Similarly we can chose 2 Democrats from a total of 6 in $C(6,2)$ and 3 Independents from 4 in $C(4,3)$ ways .So there are $C(5,2)*C(6,2)*C(4,3)=10*15*4=600$ ways of choosing the committee

Exercise 21 / Page 17

If A is the starting point and B the ending point on a rectangular grid with length 5 and height 3 ,how many different paths from A to B are possible ? Suppose that from any point of the grid you can go just one step to the right or one step up.

Solution:

Each path is a linear arrangement of 4 *r*'s and 3 *u*'s (r for right and u for up).For instance the arrangement *r ,r ,u, u, r, r, u* specifies the path whose first 2 steps are to the right ,next 2 steps are up, next 2 are to the right, and final step is up .
Then the answer follows easily : $7!/(3!*4!)= 35$ different paths

Exercise 28 / page 18

If 8 new teachers are to be divided among 4 schools , how many divisions are possible ? What if each school must receive 2 teachers?

Solution:

a) each teacher can be assigned to 4 schools so the answer is $4*4*4*4*4*4*4*4=4^8$
b) the situation is the same with putting 8 different objects in 4 different boxes in such a way that in each box be 2 objects so we deal with multiple combinations and the answer is $C(8 ; 2,2,2,2) = 8!/(2!*2!*2!*2!)=2520$,where I denoted by $C(n;k1,k2,k3,k4)$ the number of ways in which we can put n objects in 4 boxes such that to have k1 objects in the first box, k2 in the second box, k3 in the third and k4 in the last box

Exercise 30 / page 18

Delegates from 10 countries , including Russia, France, England and the United States ,are to be seated in a row. How many different arrangements are possible if the French and English delegates are to be seated next to each other, and the Russian and The U.S. delegates are not to be next to each other?

Answer::

$2*9!-2*2*8!$ since $2*9!$ is the number in which the French and English are next to each other and $2*2*8!$ the number in which the French and English are next to each other and the U.S. and Russian are next to each other . So, the result is 564,480

Theoretical exercise 7 / page 19

Give an analytical proof of the formula : $C(n,r)=C(n-1,r-1)+C(n-1,r)$

Proof:

$C(n-1,r-1)= (n-1)!/[(n-1-r+1)!*(r-1)!]=(n-1)!/[(n-r)!*(r-1)!]$
$C(n-1,r)=(n-1)!/[(n-1-r)!*r!]$
The common denominator for the two ratios will be $(n-r)!*r!$ So we have to multiply the first ratio by r and the second by (n-r)

We get:

$C(n-1,r-1)+C(n-1,r)= [r*(n-1)!+(n-r)*(n-1)!]/[(n-r)!*r!]=(r+ n -r)*(n-1)!/[(n-r)!*r!]=n*(n-1)! /[(n-r)!*r!]=$
$=n!/[(n-r)!*r!]=C(n, r)$ q.e.d