Duration: Formulas and Calculations

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1. Definition

$$D=\frac{\sum_{t=1}^{n}\frac{C_t}{(1+r)^t}(t)}{\sum_{t=1}^{n}\frac{C_t}{(1+r)^t}}$$

2. Explicit Sample Calculations

(a) For an 8% coupon (annual pay) four-year bond with a yield to maturity of 10%, we have:

$$D \;=\; \frac{\frac{80}{1.10}(1)+\frac{80}{(1.10)^2}(2)+\frac{80}{(1.10)^3}(3)+\frac{1080}{(1.10)^4}(4)}{\frac{80}{1.10}+\frac{80}{(1.10)^2}+\frac{80}{(1.10)^3}+\frac{1080}{(1.10)^4}}$$

$$D=3.56$$

(b) If the coupon were 4% rather than 8%, the formula would be:

$$D \;=\; \frac{\frac{40}{1.10}(1)+\frac{40}{(1.10)^2}(2)+\frac{40}{(1.10)^3}(3)+\frac{1040}{(1.10)^4}(4)}{\frac{40}{1.10}+\frac{40}{(1.10)^2}+\frac{40}{(1.10)^3}+\frac{1040}{(1.10)^4}}$$

$$D=3.75$$

(c) Finally, for a zero coupon bond with four years to maturity we have:

$$D = \frac{\frac{1080}{(1.10)^4}(4)}{\frac{1080}{(1.10)^4}} = 4$$

3. Duration Table for an 11.75% Coupon Bond

(1)	(2)	(3)	(3a)	(4)	(4a)
Coupon	MAT	YTM	DUR	YTM	DUR
11.75	3YR	11.75	2.70	6.75	2.71
11.75	7	11.75	5.14	6.75	5.36
11.75	10	11.75	6.38	6.75	6.90
11.75	20	11.75	8.48	6.75	10.43
11.75	30	11.75	9.17	6.75	12.54

Notes:

(1) Column 3a shows duration increasing with maturity, but less than proportionately

(2) Column 4a compared with 3a shows that a decline in yield to maturity (from 11.75% to 6.75%) increases duration, especially for the longer maturities.