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# Lab 1 Projectile Motion

## Pre-lab

Prediction 1:

When you shoot a projectile that lands at its starting height, and if air drag is not a factor, what launch angle should give it the longest range? Why?

Prediction 2:

When you double the launch velocity of a projectile that lands at its starting height, and if air drag is not a factor, will it land at twice the range? Why or why not?

Prediction 3:

When you shoot a projectile from a height above its landing height, and air drag is not a factor, should the same launch angle still give the longest range? Why or why not?

Prediction 4:

If air drag is a factor, how will it affect a projectile's trajectory qualitatively? Its velocity? Its acceleration?



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## Purpose of Experiment:

We study projectile motion to get a deeper understanding of two-dimensional motion.

When air drag can be neglected, projectile motion is well described by the formulas

$$x = v_0 \cos \alpha_0 * t$$

$$v_x = v_0 \cos \alpha_0$$

$$y = v_0 \sin \alpha_0 * t - \frac{1}{2} g t^2$$

$$v_y = v_0 \sin \alpha_0 - g t$$

where the x direction refers to the propagation of the projectile, y to the height of the projectile, and  $\alpha_0$  to the angle from the horizontal at which the projectile was launched.

Our objective is to evaluate a number of aspects of projectile motion by using a launcher, for which we can control  $v_0$  and  $\alpha_0$ .

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## Procedure:

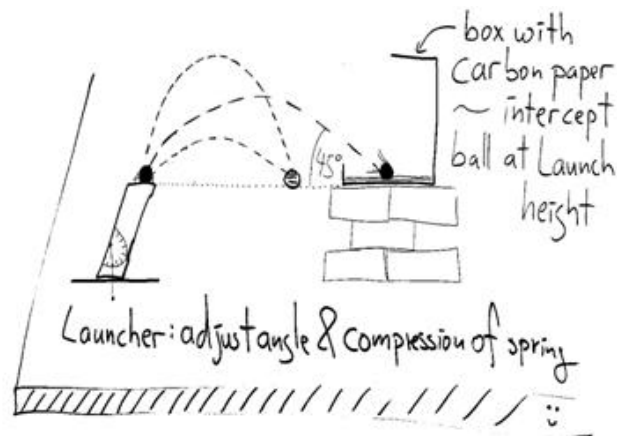
### A Setup and Experimental Preparation

The launcher is set up and a steel ball is fed into the opening. The trigger mechanism is set to one of the three available settings (three compressions of an internal spring) to provide a specific and reproducible  $v_0$ . The pendulum to the side of the launcher is used to read the launching angle  $\alpha_0$ .

Before you start experimentation, consider to use books or bricks to adjust the landing height to the launch height. The launch height may change as function of launch angle.

#### **Safety comment:**

*Be careful that the steel ball cannot hit you or others when it is being launched.*



**Projectile launcher on table with receiving box padded with carbon paper to take imprint of landing position**

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## ***B – Experiment***

### *Investigation 1: Quantitative Analysis of Projectile Motion Trajectories*

#### *Activity 1: Angle of largest range*

Place the launcher at the end of a table, set trigger to first (lowest) level and let the steel ball land on carbon paper in a box near the other end of the table. Make sure the outlet of the launcher is at the same height as the table surface.

Do a preliminary test to see where about the steel ball will land at your first angle setting ( $\alpha_0 = 20^\circ$ ). This helps you to decide where to place the box. You may have to move the box as you vary the angle.

Calculate the launch speed based on the following formula:

$$\underline{\hspace{2cm}}$$
$$\underline{\hspace{2cm}}$$

Determine the ranges to which the ball is shot when you vary  $\alpha_0$  in 5 degree steps from  $20^\circ$  to  $75^\circ$ . Measure the distance between the launcher and the box with the carbon paper. Tick off every mark on the carbon paper after landing so that you won't confuse marks.

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20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°

*Activity 2: Angle of largest range when landing below start level.*

Launch the steel ball from the table in such a direction that the ball will land on the floor level. Set trigger to the lowest level. Vary launching angle from 20° to 75°. Repeat twice for each angle and measure distances to the box and to the carbon paper.

Distance	20°	25°	30°	35°	40°	45°	50°	55°	60°	65°	70°	75°
Attempt1												
Attempt2												
Attempt3												
Mean												
$\sigma$												

Describe the method how you determine the distance between launcher and box when they are on two different height levels:

Find the arithmetic average (see webpage) for the distances, based on the three attempts for each angle. Calculate the standard deviations. Plot the dependence of landing distance vs. launching angle  $\alpha_0$  and indicate the error bars. Draw a best fit curve to determine the angle of maximum range.

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*Extension 1: 'Siege simulation'*

*Varying Angle to Maximize Impact Height on a Wall.*

The purpose of this experiment is to find the launch angle on level ground, which will maximize the height at which a projectile strikes a vertical wall, for launches at a fixed horizontal distance from the wall.

*You may think of the problem as one encountered by ancient armies, who besieged city walls.*

When the ball is launched at an angle from a fixed distance,  $x$ , it hits the vertical wall at a height,  $y$ , given by:

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where  $y_0$  is the initial height of the projectile,  $v_0$  the initial speed as it leaves the muzzle,  $\alpha$  the inclination of the launcher above the horizontal,  $g$  the acceleration due to gravity, and  $t$  the time of flight.

**Task 1:** Place the launcher at a fixed distance in such a way that the ball comfortably reaches the wall when it is past its  $y_{\max}$  when shot at a moderate angle.

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Vary the angle from  $20^\circ$  to  $70^\circ$  and measure the height at which the ball impacts the wall. For each angle, repeat twice before changing to the new angle. Then change your launcher distance to 20% closer to the wall and repeat the measurements for the same  $v_0$ .

Launch angle	Height [m] for Distance 1: [m]			Height [m] for Distance 2: [m]		
	Trial 1	2	3	Trial 1	2	3
$20^\circ$						
$25^\circ$						
$30^\circ$						
$35^\circ$						
$40^\circ$						
$45^\circ$						
$50^\circ$						
$55^\circ$						
$60^\circ$						
$65^\circ$						
$70^\circ$						



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## Post-lab :

*Question 1:* How is the horizontal component  $v_x$  of a projectile's velocity qualitatively different from the vertical component  $v_y$ ?

*Question 2:* If one doubles the value of  $v_0$  at a given launch angle  $\theta$ , what happens to the projectile's initial horizontal and vertical speeds  $v_{0x}$  and  $v_{0y}$ , landing distance  $x_f$ , and maximum height  $y_{\max}$ ?

*Question 3:* Why does a negative second derivative indicate a maximum? Sketch a function with a local maximum and its first and second derivatives. Explain what they tell you, and indicate how they apply to position, velocity, and acceleration.

*Question 4:* Was air drag a significant factor in your experiment? Explain how your data support your answer.

Task: Extension 1 analysis

For each of the two launch distances from the wall, plot the impact heights  $y$  vs. launch angle  $\alpha$ .

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The distance from the wall is the horizontal distance  $x$ . Invert the  $x$ -equation (page 22) to obtain a symbolic expression for the time of flight  $t$ . Substitute this expression into the  $y$ -equation and simplify. You should have a formula for  $y$ , the height of impact, as a function of  $\alpha$ ,  $g$ ,  $x$ , and  $v_0$ .

Using the average launch speed  $v_0$  you calculated in activity 1, overlay a smooth curve for calculated  $y$  onto each of your earlier experimental  $y$ - $\alpha$  plots.

If you are sure your formula is correct but the data and the theoretical curve do not agree, explain what might be wrong with your model. (Is a prominent force neglected? Might the values of your parameters  $v_0$ ,  $g$ , or  $x$  be incorrect?)

*Question 5: Go back to your predictions in the pre-lab and, if it turns out that any of them were wrong, explain where your initial thinking went wrong and give the correct answer.*