PROBLEMS AND PROJECTS

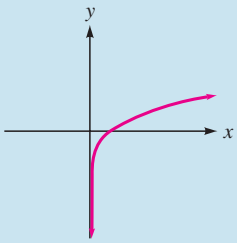
1. Match each equation with its graph.

__ **a.** $y = 3^x$ __ **b.** $y = 3^x - 1$

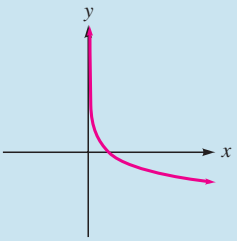
__ **c.** $y = 3^{x-1}$ __ **d.** $y = \log_3 x$

__ **e.** $y = \log_3(x - 1)$ __ **f.** $y = \log_{1/3} x$

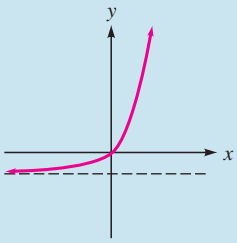
s.



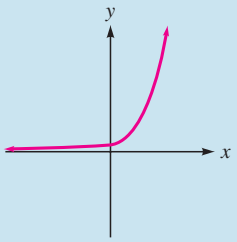
t.



u.



v.



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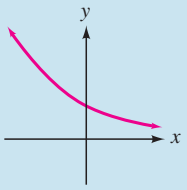
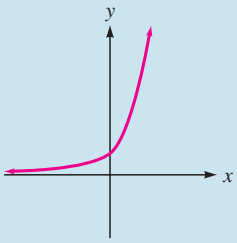
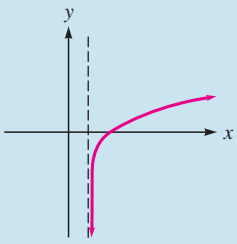


x.



y.



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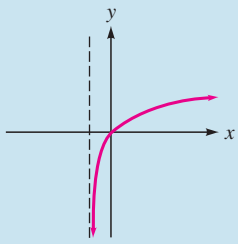


2. Use a graphing calculator to graph the function $y = \ln(e^x)$. Explain why the graph is a line. What is a simpler form of the equation of that line?

3. Use a graphing calculator to graph the function $y = e^{\ln x}$. Is its graph a line, or just part of a line? Explain.

4. The tank in the illustration initially contains 20 gallons of pure water. A brine solution containing 0.5 pounds of salt per gallon is pumped into the tank, and the well-stirred mixture leaves at the same rate. The amount A of salt in the tank after t minutes is given by

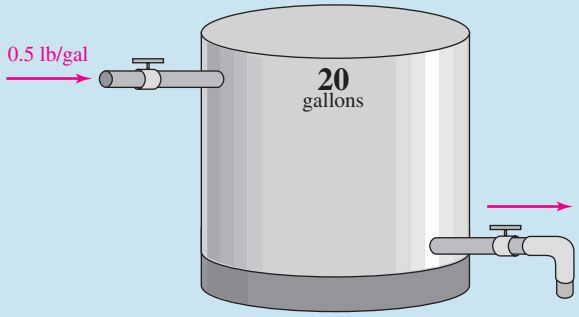
$$A = 10(1 - e^{-0.03t})$$

a. Graph this function.

b. What is A when $t = 0$? Explain why that value is expected.

c. What is A after 2 minutes? After 10 minutes?

d. What value does A approach after a long time (as t becomes large)? Explain why this is the value you would expect.



Project 1

Graphing calculators graph base-e and base-10 logarithmic functions easily, because logarithms to these bases are built-in. Find a way of graphing the function $y = \log_3 x$, even though there is no $\log_3$ key.

Project 2

For positive values of x, the graphs of the polynomial function $y = x^3$ and the exponential function $y = e^x$ are both increasing, as shown in Illustration 1. Which is increasing faster? At $x = 2$, 3, and 4, the graph of the polynomial is winning, but when $x = 5$, the graph of the exponential has caught up to and passed the polynomial graph.

The higher a polynomial's degree, the faster its graph rises. Illustration 2 shows the graphs of $y = x^5$ and $y = e^x$. It looks as if the graph of the polynomial is rising more rapidly. Will the exponential graph ever catch up? In a race between a polynomial and an exponential function, which function will eventually win?

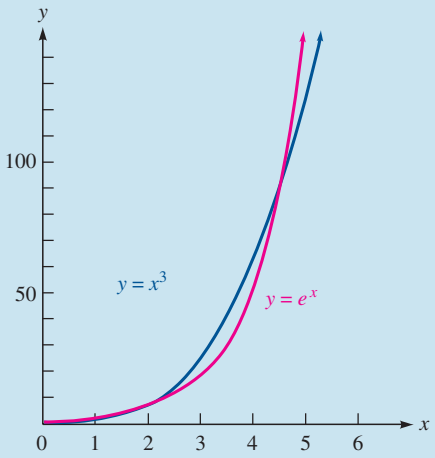


Illustration 1

Use a graphing calculator to graph $y = x^5$ and $y = e^x$. Then try a race between $y = x^{20}$ and $y = e^x$. (*Hint:* Useful viewing windows are difficult to find. For $y = x^{20}$, try $80 \le x \le 100$ and $0 \le Y \le 5 \times 10^{39}$.) Write a brief report of your conclusions.

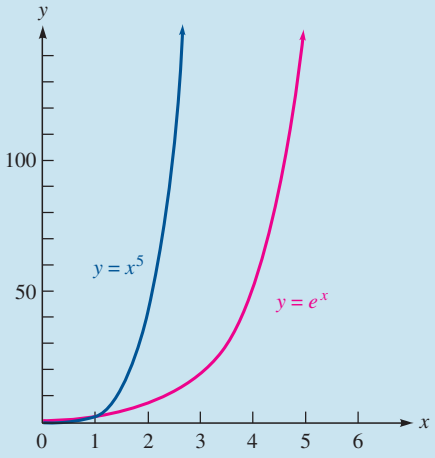


Illustration 2

Project 3

When graphing a function $y = f(x)$, various numbers x are used to produce corresponding values of y, and many pairs (x, y) are plotted to produce the graph. In **parametric equations,** various values of a third variable, called a **parameter,** are used to generate both x- and y-values. Set your graphing calculator for parametric equations (try the MODE key, or consult the owner's manual). Set the range of the parameter, t:

Tmin = 0
Tmax = 10
Tstep = 0.1

Then graph the parametric equations

$$X_{1_T} = \ln t$$
$$Y_{1_T} = t$$

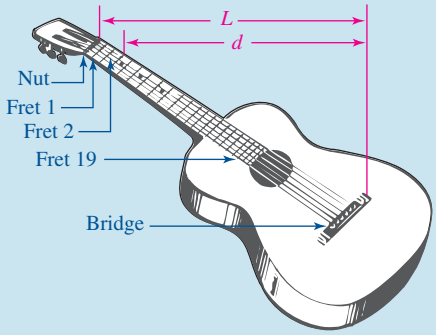
The resulting graph is that of $y = e^x$, which is the inverse of $y = \ln x$. Explain why.

Project 4

The 19 frets on the neck of the classical guitar shown in the illustration are positioned according to the exponential function

$$d = L\left(\frac{1}{2}\right)^{n/12}$$

where L is the distance between the nut and the bridge and d is the distance from the bridge to fret number n.



- Find a classical guitar and measure its nut-to-bridge distance L. Use this value in the equation and calculate the distance to frets 1 and 19. Do the calculated values agree with the measured distances?
- Calculate and measure the distance to fret 12, which produces a tone one octave above that of the open string. What fractional part of L is this distance?
- Calculate the distance to fret zero. From the value calculated, where is fret zero?
- Does this formula work for an electric guitar?

Project 5

Determine the current interest rate for Government EE savings bonds by calling a bank or visiting the Web site www.savingsbonds.gov. Then calculate how long it will take an EE bond to double in value.

Determine the current interest rate for Government I savings bonds by calling a bank or visiting www.savingsbonds.gov. Then calculate how long it will take an I bond to double in value.

Call a bank and determine the interest rate on a 5-year certificate of deposit. Then calculate how long it will take the value of the CD to double.

Write a paper comparing these three investments. Tell which one you think is the better investment for you at this time.