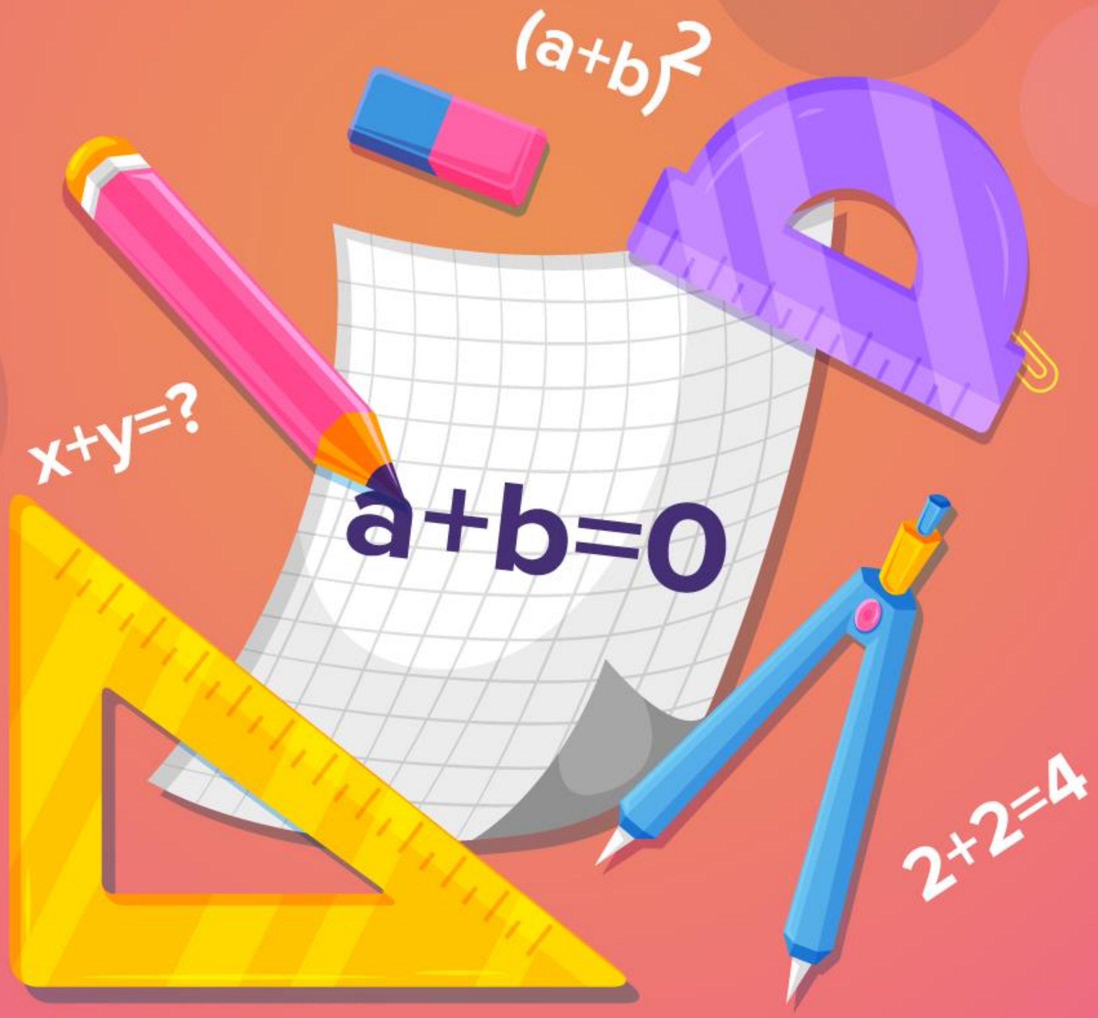
Mathematics | Class 12th

CBSE Board Paper 2019



CBSE Board Paper 2019
Set - 3

Time allowed: 3 Hours **Max Marks: 100**

General Instructions:

1. All questions are compulsory.
2. This question paper contains 29 questions divided into four sections A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 1 question of Section A, 3 questions of Section B, 3 questions of Section C and 3 questions of Section D. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators is not permitted. You may ask logarithmic tables, if required.

Section A

1. If $3A - B = \begin{bmatrix} 5 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$, then find the matrix A. **1**

2. Write the order and the degree of the following differential equation: **1**

$$x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$$

3. If f(x) = x + 1, find $\frac{d}{dx}(fof)(x)$. **1**

4. If a line makes angles 90°, 135°, 45° with x, y and z axes respectively, find its direction cosines. **1**

OR

Find the vector equation of the line which passes through the point (3,4,5) and is parallel to the vector $2\hat{i} + 2\hat{j} - 3\hat{k}$.

Section B

5. Find: $\int \sin x \log_e \cos x dx$ **2**

6. Evaluate: $\int_{-\pi}^{\pi}(1-x^2)\sin x \cos^2 x \, dx$ **2**

OR

Evaluate: $\int_{-1}^{2}\frac{|x|}{x}dx$

7. Examine whether the operation * defined on R by a*b = ab + 1 is **2**

(i) a binary or not.

(ii) if a binary operation, is it associative or not?

8. Find a matrix A such that 2A - 3B + 5C = O, where $B=\begin{bmatrix}-2 & 2 & 0\\ 3 & 1 & 4\end{bmatrix}$ and $C=\begin{bmatrix}2 & 0 & -2\\ 7 & 1 & 6\end{bmatrix}$. **2**

9. A die marked 1,2,3 in red and 4,5,6 in green is tossed. Let A be the event that “number is even” and B be the event that “number is marked red”. Find whether the events A and B are independent or not. **2**

10. Form the differential equation representing the family of curves $y = e^{2x}(a + bx)$, where ‘a’ and ‘b’ are arbitrary constants. **2**

11. A die is thrown 6 times. If "getting an odd number" is considered success, what is the probability of (i) 5 successes? (ii) at most 5 successes? **2**

OR

The random variable X has a probability distribution P(X) of the following form, where 'k' is some number.

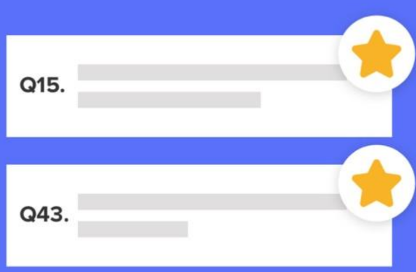
$$P(X=x)=\begin{cases} k, \text{if } x=0 \\ 2k, \text{if } x=1 \\ 3k, \text{if } x=2 \\ 0, \text{otherwise} \end{cases}$$

Determine the value of 'k'.

12. If the sum of two - unit vectors is a unit vector, prove that the magnitude of their differences is $\sqrt{3}$. **2**

OR

If $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$, $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$ and $\vec{c}=-3\hat{i}+\hat{j}+2\hat{k}$, Find $\left[\vec{a}\,\vec{b}\,\vec{c}\right]$.



Section C

13. Using properties of determinant, prove the following: **4**

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

14. Solve: $\tan^{-1} 4x + \tan^{-1} 6x = \frac{\pi}{4}$ **4**

15. Show that the relation R on ℝ defined as R = {(a , b):a ≤ b}, is reflexive, and transitive but not symmetric. **4**

OR

Prove that the function f: ℕ→ℕ, defined by f(x) = x^2 + x + 1 is one - one but not onto. Find inverse of f: ℕ→S, where S is range of f.

16. Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line 4x - 2y + 5 = 0. Also, write the equation of normal to the curve at the point of contact. **4**

17. If $\log_e\left(x^2 + y^2\right) = 2\tan^{-1}\left(\frac{y}{x}\right)$, show that $\frac{dy}{dx} = \frac{x+y}{x-y}$. **4**

OR

If $x^{y} - y^{x} = a^{b}$, find $\frac{dy}{dx}$.

18. If y = $(\sin^{-1}x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-2=0$. **4**

19. Prove that $\int_0^a f(x)dx=\int_0^a f(a-x)dx$, hence evaluate **4**

$\int_0^\pi \frac{x\sin x}{1+\cos^2 x}dx$.

20. Find: $\int \frac{\cos x}{(1+\sin x)(2+\sin x)}dx$. **4**

21. Solve the differential equation: $\frac{dy}{dx}-\frac{2x}{1+x^2}y=x^2+2$. **4**

OR

Solve the differential equation: $(x+1)\frac{dy}{dx}=2e^{-y}-1$;

y (0) = 0.

22. If $\hat{i}+\hat{j}+\hat{k}, 2\hat{i}+5\hat{j}, 3\hat{i}+2\hat{j}-3\hat{k}$ and $\hat{i}-6\hat{j}-\hat{k}$ respectively are the position vectors of points A, B, C and D, then find the angle between the straight lines $\overrightarrow{AB}$ and $\overrightarrow{CD}$. Find whether $\overrightarrow{AB}$ and $\overrightarrow{CD}$ are collinear or not. **4**

23. Find the value of λ, so that the lines $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{3}$ and $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. Also, find whether the lines are intersecting or not. **4**

Section D

24. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that it's depth is 2 m and volume is 8 m^3. If building of tank costs ₹ 70 per square metre for the base and ₹ 45 per square metre for the sides, what is the cost of least expensive tank? **6**

25. If $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 0 & 2\\ 3 & 1 & 1\end{bmatrix}$, Find A^{-1}. Hence, solve the system of equations x + y + z = 6, x + 2z = 7, 3x + y + z = 12. **6**

OR

Find the inverse of the following matrix using elementary operations. $A=\begin{bmatrix}1 & 2 & -2\\ -1 & 3 & 0\\ 0 & -2 & 1\end{bmatrix}$

26. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divide the area of the square bound by x = 0, x = 4, y = 4 and y = 0 into three equal parts. **6**

OR

Using Integration, find the area of the triangle whose vertices are (2,3), (3,5) and (4,4).

27. A manufacturer has employed 5 skilled men and 10 semi - skilled men and makes two models A and B of an article. The making of one item of model A requires 2 hours work by a skilled man and 2 hours work by a semi - skilled man. One item of model B requires 1 hour by a skilled man and 3 hours by a semi - skilled man. No man is expected to work more than 8 hours per day. The manufacturer's profit on an item of model A is ₹15 and on an item of model B is ₹10. How many of items of each model should be made per day in order to maximize profit? Formulate the above LPP and solve it graphically and find the **6**

28. Find the vector and cartesian equations of a plane passing through the points (2,2, - 1), (3,4,2) and (7,0,6). Also, find the vector equation of a plane passing through (4,3,1) and parallel to the plane mentioned above. **6**

OR

Find the vector equation of the plane that contains the line $\vec{r} = \hat{i} + \hat{j} + \lambda(\hat{i} + 2\hat{j} - \hat{k})$ and the point (- 1,3, - 4). Also, find the length of the perpendicular drawn from the point (2,1,4) to the plane thus obtained.

29. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean and variance of number of kings. **6**

Set - 2

1. Find the order and the degree of the differential equation $x^2\frac{d^2y}{dx^2}=\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^4$. **1**

2. If f(x) = x + 7 and g(x) = x – 7, x ϵ R, then find $\frac{d}{dx}(fog)(x)$. **1**

3. Find the value of x – y, if **1**

$$2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

6. If $A=\begin{bmatrix}2 & 0 & 1\\2 & 1 & 3\\1 & -1 & 0\end{bmatrix}$, then find ($A^2$ – 5A). **2**

12. Find: $\int\frac{\tan^2 x.\sec^2 x}{1-\tan^6 x}dx$. **2**

13. Solve for x: $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$ **4**

15. Find: $\int\frac{3x+5}{x^2+3x-18}dx$ **4**

18. Using properties of determinants, prove that **4**

$$\begin{vmatrix}a+b+c & -c & -b\\-c & a+b+c & -a\\-b & -a & a+b+c\end{vmatrix}=2(a+b)(b+c)(c+a)$$

19. If $x = \cos t + \log\tan\left(\frac{t}{2}\right), y = \sin t$, then find the values of $\frac{d^2y}{dt^2}$ and $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$. **4**

22. Solve the differential equation: $xdy - ydx = \sqrt{x^2+y^2}dx$, given that y = 0 when x = 1. **4**

OR

Solve the differential equation: $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$, subject to the initial condition y(0) = 0.

25. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Hence solve the following system of equations. **6**

2x – 3y + 5z = 11

3x + 2y – 4z = - 5

x + y – 2z = - 3

OR

Obtain the inverse of the following matrix using elementary operations:

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

26. A manufacturer has three machine operators A, B and C. The first operator A produces 1% of defective items, whereas the other two operators B and C produces 5% and 7% defective items respectively. A is on the job for 50% of the time, B on the job 30% of the time and C on the job for 20% of the time. All the items are put into one stockpile, and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by A ? **6**

28. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7), and C(6, 2). **6**

OR

Find the area of the region lying above the x - axis and included between the circle $x^2 + y^2 = 8x$ and inside of the parabola $y^2 = 4x$.

Set - 1

1. If A and B are square matrices of the same order 3, such that |A| = 2 and AB = 2I, write the value of |B|. **1**

3. Find the order and degree of the differential equation $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$. **1**

7. Find: $\int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$ **2**

8. Find: $\int \sqrt{1 - \sin 2x}\, dx$, $\frac{\pi}{4} < x < \frac{\pi}{2}$ **2**

OR

$\int \sin^{-1} 2x\, dx$

15. Using the properties of determinants, prove that **4**

$$\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3 .$$

19. Find: $\int \frac{3x+5}{x^2+3x-18} dx$ **4**

21. Solve the differential equation: $xdy - ydx = \sqrt{x^2 + y^2}dx$, given that y = 0 when x = 1. **4**

OR

Solve the differential equation: $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$, subject to the initial condition y(0) = 0.

26. Using integration, find the area of triangle ABC, whose vertices are A(2, 5), B(4, 7), and C(6, 2). **6**

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Solutions (Set - 3)

Section A (Solutions)

1. Given: $3A - B = \begin{bmatrix} 5 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

Adding the given equations, we get,

$3A - B + B = \begin{bmatrix} 5 & 1 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$

$3\ A = \begin{bmatrix} 9 & 4 \\ 2 & 6 \end{bmatrix}$

Then $A = \frac{1}{3} \times \begin{bmatrix} 9 & 4 \\ 2 & 6 \end{bmatrix}$

$$= \begin{bmatrix} 3 & \frac{4}{3} \\ \frac{2}{3} & 2 \end{bmatrix}$$

2. The differential equation is $x^3 \left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$.

As the degree of a differential equation, of which the differential coefficients are free from radicals and fractions, is the positive integral index of the highest power of the highest order derivatives involved.

The highest order derivative term is $\frac{d^2y}{dx^2}$, hence the order of this differential equation is 2.

The power to which $\frac{d^2y}{dx^2}$ is raised is 2, hence the degree of this polynomial is 2.

Hence, the order and degree of the given differential equation are 2.

3. Given,

f(x) = x + 1

Therefore fof(x) = f[f(x)]

= f(x) + 1

= (x + 1) + 1

= x + 2

Differentiating with respect to x, we get,

$$\frac{d}{dx}(fof)(x) = \frac{d}{dx}(x+2)$$

$$= \frac{d}{dx}(x) + \frac{d}{dx}(2)$$

As we know,

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$= \frac{d}{dx}(x^1) + \frac{d}{dx}(x^0)$$

= $1x^{1-1} + 0x^{0-1}$

= $1x^0 + 0x^{-1}$

= 1 + 0

= 1

4. Direction cosines of a line making angle α with x - axis, β with y - axis and γ with z - axis are l, m ,n.

l = cos α , m = cos β , n = cos γ

Here, α = 90° , β = 135° and γ = 45°

l = cos 90° = 0

m = cos 135°

= cos(180° - 45°) = - cos 45°

$$= -\frac{1}{\sqrt{2}}$$

n = cos 45°

$$= \frac{1}{\sqrt{2}}$$

OR

If the coordinates of a point A = (x_1, y_1, z_1), then the position vector is

$$\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$$

Hence, the position vector of point P = (3,4,5) is

$$\vec{p} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$$

If the position vector ($\vec{a}$) of a point on the line and a vector ($\vec{r}$) parallel to the line is given, then the vector equation of a line is given by

$$\vec{l} = \vec{a} + \lambda\vec{r}$$

Here, $\vec{a} = \vec{p} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$ and $\vec{r} = 2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, then

$$\vec{l} = 3\hat{\imath} + 4\hat{\jmath} + 5\hat{k} + \lambda(2\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$$

$$= (3 + 2\lambda)\hat{\imath} + (4 + 2\lambda)\hat{\jmath} + (5 - 3\lambda)\hat{k}$$

Section B (Solutions)

5. Let $I = \int sinx \log_e cosx\, dx$

let cos x = t

Differentiating both sides, we get,

⇒ - sin x dx = dt

$I = -\int 1 \times \log_e t\, dt$

By integartion by parts

∫u v dx = u∫v dx −∫u' (∫v dx) dx

where **u** is the function u(x), **v** is the function v(x) and **u'** is the derivative of the function u(x).

Here u(x) = $\log_e$ t and v(x) = 1

$I = -\log_e t \int 1\, dt + \int \frac{1}{t}\left(\int 1\, dt\right) dt$

$I = -t \log_e t + \int 1\, dt = t - t \log_e t + c$

$= cosx - cosx \log_e cosx + c$

6. Let $I = \int_{-\pi}^{\pi} (1 - x^2) sinx cos^2 x dx$

Let f(x) = (1 - x^2) sin x $\cos^2$x

f(- x) = (1 - (- x)^2) sin(- x) $\cos^2$(- x)

= - (1 - x^2) sin x $\cos^2$x

= - f(x)

Hence f(x) is an odd function.

Using $\int_{-a}^{a} f(x) dx = 0$ if f(x) is an odd function, we get I = 0.

OR

Consider $\int_{-1}^{2} \frac{|x|}{x} dx$,

Let $I = \int_{-1}^{2} \frac{|x|}{x} dx = \int_{-1}^{0} \frac{|x|}{x} dx + \int_{0}^{2} \frac{|x|}{x} dx$

|x| = - x, if x < 0

And |x| = x, if x ≥ 0

$$= \int_{-1}^{0} \frac{-x}{x} dx + \int_{0}^{2} \frac{x}{x} dx$$

$$= \int_{-1}^{0} -1\, dx + \int_{0}^{2} 1\, dx$$

Hence, $I = \int_{-1}^{0} - dx + \int_{0}^{2} dx$

$$I = [-x]_{-1}^{0} + [x]_{0}^{2}$$

= [- 0 - (- 1)] + [2 - 0]

= [0 + 1] + [2 - 0]

= 1 + 2

= 3

7. *: a*b = ab + 1

(i) Given operation is a*b = ab + 1

If any operation is a binary operation, it must follow closure property.

Let a∈R and b∈R

Then ab∈R

Also ab + 1 ∈R

So, a*b ∈R

So * satisfies the closure property.

Since * is defined for all a, b ∈ R, therefore * is a binary operation.

(ii) For * to be associative, (a*b) *c = a*(b*c)

(a*b) *c = (ab + 1) *c

= (ab + 1)c + 1

= abc + c + 1

a*(b*c) = a*(bc + 1)

= a(bc + 1) + 1

= abc + a + 1

Since (a*b) *c ≠ a*(b*c), therefore * is not associative.

8. Given that 2A - 3B + 5C = O,

$B = \begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$

Since, 2A - 3B + 5C = O

⇒ 2A = 3B - 5C

$$A = \frac{1}{2}(3B - 5C)$$

$$= \frac{1}{2}(3)\begin{bmatrix} -2 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix} - \frac{1}{2}(5)\begin{bmatrix} 2 & 0 & -2 \\ 7 & 1 & 6 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -6 & 6 & 0 \\ 9 & 3 & 12 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 10 & 0 & -10 \\ 35 & 5 & 30 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -6-10 & 6-0 & 0+10 \\ 9-35 & 3-5 & 12-30 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -16 & 6 & 10 \\ -26 & -2 & -18 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 3 & 5 \\ -13 & -1 & -9 \end{bmatrix}$$

9. Let A: number is even

B: number is marked red

For A and B to be independent events, P(A∩B) = P(A)P(B)

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B) = \frac{3}{6} = \frac{1}{2}$

$P(A)P(B) = \frac{1}{4}$

$P(A \cap B) = \frac{1}{6}$

Since here, P(A∩B) ≠ P(A)P(B), therefore A and B are not independent events.

10. $y = e^{2x}(a + bx) = ae^{2x} + bxe^{2x}$

Differentiating with respect to x,

$y' = 2ae^{2x} + be^{2x} + 2bxe^{2x}$

$y' = 2y + be^{2x}$ ....(1)

Differentiating with respect to x,

$y'' = 2y' + 2be^{2x}$

$\Rightarrow be^{2x} = \frac{1}{2}(y'' - 2y')$.........(2)

Substituting (2) in (1), we get,

$y' = 2y + \frac{1}{2}(y'' - 2y')$

$\Rightarrow 2y' = 4y + y'' - 2y'$

$\Rightarrow y'' - 4y' + 4y = 0$

11. This is a binomial distribution with n = 6, $p = \frac{3}{6} = \frac{1}{2}$

(i) We know,

$P(r=x)=\binom{n}{x}p^x(1-p)^{n-x}$

$$P(r=5)=\binom{6}{5}\left(\frac{1}{2}\right)^5\left(1-\frac{1}{2}\right)^{6-5}$$

$$=\frac{6!}{5!1!}\left(\frac{1}{32}\right)\left(\frac{1}{2}\right)^1$$

$$=6\times\frac{1}{32}\times\frac{1}{2}$$

$$=\frac{3}{32}$$

(ii)

P(r ≤ 5) = 1 - P(r = 6)

$$=1-\binom{6}{6}\left(\frac{1}{2}\right)^6\left(1-\frac{1}{2}\right)^{6-6}$$

$$=1-\left(1\times\frac{1}{64}\times1\right)$$

$$=1-\frac{1}{64}$$

$$=\frac{63}{64}$$

OR

Given that $P(X=x)=\begin{cases}k, if\ x=0\\2k, if\ x=1\\3k, if\ x=2\\0, otherwise\end{cases}$

The sum of all probabilities of a random variable is equal to 1.

⇒ k + 2k + 3k + 0 = 1

⇒ 6k = 1

Hence, $k=\frac{1}{6}$

12. Let the vectors be $\vec{x}$ and $\vec{y}$. Given that $|\vec{x}| = 1, |\vec{y}| = 1$ and $|\vec{x} + \vec{y}| = 1$

Since ,

$$\left|\vec{x} + \vec{y}\right|^2 = \left|\vec{x}\right|^2 + \left|\vec{y}\right|^2 + 2\left|\vec{x}\right|\left|\vec{y}\right| \cos\,\theta$$

$1^2 = 1^2 + 1^2 + 2(1)(1) \cos \theta$

$\Rightarrow 1 + 1 + 2\cos\theta = 1$

or $2\cos\theta = -1$

$$\left|\vec{x} - \vec{y}\right|^2 = \left|\vec{x}\right|^2 + \left|\vec{y}\right|^2 - 2\left|\vec{x}\right|\left|\vec{y}\right| \cos\,\theta$$

$$\left|\vec{x} - \vec{y}\right| = \sqrt{\left|\vec{x}\right|^2 + \left|\vec{y}\right|^2 - 2\left|\vec{x}\right|\left|\vec{y}\right| \cos\,\theta}$$

$$= \sqrt{1 + 1 - (1)(1)(-1)}$$

$$|\vec{x} - \vec{y}| = \sqrt{1 + 1 + 1}$$

$= \sqrt{3}$

OR

If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$, $\vec{b} = b_1\hat{\imath} + b_2 2\hat{\jmath} + b_3\hat{k}$ and $\vec{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$, then we define $[\vec{a}\,\vec{b}\,\vec{c}]$ as

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

Given that: $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{c} = -3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$$[\vec{a}\,\vec{b}\,\vec{c}] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$$

$= 2(-4 - 1) - 3(2 + 3) + 1(1 - 6)$

$= -30$

Section C (Solutions)

13. Consider $\Delta = \begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix}$

Using the property that if the equalities of corresponding elements of other rows (or columns) are added to every element of any row (or column) of a determinant, then the value of determinant remains the same

Using column transformation, $C_1 \rightarrow C_1 + C_2 + C_3$

we get, $\Delta = \begin{vmatrix} a+b+c & b & c \\ 0 & b-c & c-a \\ 2(a+b+c) & c+a & a+b \end{vmatrix}$

Using the property that if each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k.

Taking out factor (a + b + c) from C_1,

we get, $\Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 2 & c+a & a+b \end{vmatrix}$

Using row transformation, $R_3 \rightarrow R_3 - 2R_1$

we get, $\Delta = (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & b-c & c-a \\ 0 & c+a-2b & a+b-2c \end{vmatrix}$

Expanding along C_1, we get

Δ = (a + b + c)[(b - c)(a + b - 2c) - (c - a)(c + a - 2b)]

= $(a + b + c)[ab + b^2 - 2bc - ac - bc + 2c^2 - c^2 - ac + 2bc + ac + a^2 - 2ab]$

= $(a + b + c)[a^2 + b^2 + c^2 - ab - bc - ac]$

= $a^3 + b^3 + c^3 - 3abc$

14. Given that $tan^{-1}4x + tan^{-1}6x = \frac{\pi}{4}$

Taking tan both sides, we get

$\tan(tan^{-1}4x + tan^{-1}6x) = tan\frac{\pi}{4}$

$(Using\ \ tan(A+B) = \frac{tanA + tanB}{1 - tanAtanB})$

$\frac{\tan(tan^{-1}4x) + \tan(tan^{-1}6x)}{1 - tan(tan^{-1}4x)\tan(tan^{-1}6x)} = 1$

$\Rightarrow \frac{4x+6x}{1-4x\times 6x} = 1$

(Since $\tan(\tan^{-1}A) = A$)

$\Rightarrow 4x + 6x = 1 - 24x^2$

$\Rightarrow 24x^2 + 10x - 1 = 0$

$\Rightarrow 24x^2 + 12x - 2x - 1 = 0$

$\Rightarrow 12x(2x + 1) - 1(2x + 1) = 0$

$\Rightarrow (12x - 1)(2x + 1) = 0$

Hence, $x = \frac{1}{12}\ or\ x = -\frac{1}{2}$

15. Given that R = {(a ,b):a ≤ b}

For Reflexive:

Since $a = a\ \forall a \in \mathbb{R}$,

therefore $a \leq a$ always.

Hence (a, a) always belongs to R $\forall a \in \mathbb{R}$. Therefore, R is reflexive.

For Symmetric:

If $a \leq b$

then $b \geq a \nRightarrow b \leq a$.

Example:

$(2,4) \in \mathbb{R}$ as $2 \leq 4$.

But $(4,2) \notin \mathbb{R}$ as 4 is greater than 2.

Hence if (a, b) belongs to R, then (b, a) does not always belong to R.

Hence R is not symmetric.

For Transitive:

If $a \leq b$ - - - - - (1) and

$b \leq c$ - - - - - (2)

Add (1) and (2) to get,

$a + b \leq b + c$

Hence $a \leq c$.

Hence if $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R\ \forall a, b, c \in \mathbb{R}$.

Hence, R is transitive.

OR

$f(x) = x^2 + x + 1$, $f: \mathbb{N} \rightarrow \mathbb{N}$

A function is one - one if f(a) = f(b)

$\Rightarrow a = b$

$f(a) = f(b)$

$\Rightarrow a^2 + a + 1 = b^2 + b + 1$

$\Rightarrow a^2 - b^2 + a - b = 0$ or $(a - b)(a + b + 1) = 0$

Hence, a = b or a + b = - 1

Since a, b ∈ℕ, therefore a + b = - 1 is not possible.

Hence a = b.

Since f(a) = f(b)

⇒a = b

Therefore, f(x) is one - one.

Let $y = x^2 + x + 1$

differentiating with respect to x, we get,

y' = 2x + 1 > 0 ∀x ∈ℕ, hence f is an increasing function.

The range of y = {3,7,13,21…} which is not equal to ℕ.

Since the range is not equal to codomain, therefore, f is not onto.

Let S be the range of f.

Then $f(x) = x^2 + x + 1$,

f:ℕ→S

$y = x^2 + x + 1$

$\Rightarrow x^2 + x + 1 - y = 0$

Using the quadratic formula, we get,

$$x = \frac{-1 \pm \sqrt{1 - 4(1 - y)}}{2}$$

$$= \frac{-1 \pm \sqrt{4y - 3}}{2}$$

There are two possibilities for $f^{-1}(x)$.

$$f^{-1}(x) = \frac{-1 + \sqrt{4x - 3}}{2}, \frac{-1 - \sqrt{4x - 3}}{2}$$

As f(1) = 3

So, $f^{-1}(3) = 1$

Hence, $f^{-1}: S \to \mathbb{N}$

$$f^{-1}(x) = \frac{-1+\sqrt{4x-3}}{2}$$

16. Given that $y = \sqrt{3x-2}$, we have to find tangent to the curve which is parallel to the line 4x - 2y + 5 = 0.

Differentiating y with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{3x-2}} \times 3$$

$$= \frac{3}{2\sqrt{3x-2}}$$

Now at (x_1,y_1) the equation will be:

$$y_1 = \sqrt{3x_1 - 2}$$

The slope of the tangent at (x_1,y_1) is:

$$= \frac{3}{2\sqrt{3x_1-2}}$$

Now,

The slope of the tangent = slope of the line

$$\Rightarrow \frac{3}{2\sqrt{3x_1-2}} = 2$$

$$\Rightarrow 3 = 4\left(\sqrt{3x_1-2}\right)$$

Squaring both sides we get,

$9 = 16(3x_1 - 2)$

$$\Rightarrow 3x_1 - 2 = \frac{9}{16}$$

$$\Rightarrow 3x_1 = 2 + \frac{9}{16}$$

$$\Rightarrow 3x_1 = \frac{41}{16}$$

$$\Rightarrow x_1 = \frac{41}{48}$$

Now,

$$y_1 = \sqrt{3\left(\frac{41}{48}\right) - 2}$$

$$y_1 = \sqrt{\frac{123}{48} - 2}$$

$$= \sqrt{\frac{123 - 96}{48}}$$

$$= \sqrt{\frac{27}{48}}$$

$$= \sqrt{\frac{9}{16}}$$

$$= \frac{3}{4}$$

Equation of tangent is:

$y - y_1 = m (x - x_1)$

$$\Rightarrow y - \frac{3}{4} = 2\left(x - \frac{41}{48}\right)$$

$$\Rightarrow \frac{4y - 3}{4} = 2\left(\frac{48x - 41}{48}\right)$$

⇒ 6(4y - 3) = 48x - 41

⇒ 24y - 18 = 48x - 41

⇒ 48x - 24y - 23 = 0

Equation of normal is:

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$y - \frac{3}{4} = \frac{-1}{2}\left(x - \frac{41}{48}\right)$$

$$\frac{4y - 3}{4} = \frac{-1}{2}\left(\frac{48x - 41}{48}\right)$$

$$\frac{4y-3}{2}=\frac{41-48x}{48}$$

$$4y-3=\frac{41-48x}{24}$$

96y - 72 = 41 - 48x

⇒ 48x + 96y - 113 = 0

17. Given that $\log_e(x^2+y^2)=2\tan^{-1}\left(\frac{y}{x}\right)$

Differentiating with respect to itself, we get

$$\frac{1}{x^2+y^2}\times(2xdx+2ydy)=2\times\frac{1}{1+\left(\frac{y}{x}\right)^2}\times\frac{xdy-ydx}{x^2}$$

$$(Using\ \frac{d}{dx}log_e x=\frac{1}{x}\ and\ \frac{d}{dx}tan^{-1}x=\frac{1}{1+x^2})$$

$$\Rightarrow\frac{2(xdx+ydy)}{x^2+y^2}=\frac{2x^2}{x^2+y^2}\times\frac{xdy-ydx}{x^2}$$

Simplifying, we get

x dx + y dy = x dy – y dx

or

dx(x + y) = dy(x - y)

Hence, $\frac{dy}{dx}=\frac{x+y}{x-y}$

OR

Given that $x^y - y^x = a^b$

Let $u = x^y$ and $v = y^x$, then we get

$u - v = a^b$

Differentiating with respect to x, we get

$\frac{du}{dx}-\frac{dv}{dx}=0$ (Since a^b is constant) …. (1)

$u = x^y$

Taking log both sides

$\log_e u = y \log_e x$

Differentiating with respect to x, we get

$$\frac{1}{u} \times \frac{du}{dx} = \frac{y}{x} + \log_e x \times \frac{dy}{dx}$$

$$\Rightarrow \frac{du}{dx} = u\left(\frac{y}{x} + \log_e x \times \frac{dy}{dx}\right)$$

Hence, $\frac{du}{dx} = yx^{y-1} + x^y \log_e x \frac{dy}{dx}$ ..... (2)

v = y^x

Taking log both sides

$\log_e v = x \log_e y$

Differentiating with respect to x, we get

$$\frac{1}{v} \times \frac{dv}{dx} = \log_e y + \frac{x}{y} \times \frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = v\left(\log_e y + \frac{x}{y} \times \frac{dy}{dx}\right)$$

Hence, $\frac{dv}{dx} = y^x \log_e y + xy^{x-1} \frac{dy}{dx}$ ......(3)

Substituting (2) and (3) in (1), we get

$$yx^{y-1} + x^y \log_e x \frac{dy}{dx} - y^x \log_e y - xy^{x-1} \frac{dy}{dx} = 0$$

Simplifying, we get

$$\frac{dy}{dx} = \frac{yx^{y-1} - y^x \log_e y}{xy^{x-1} - x^y \log_e x}$$

18. Given that y = $(\sin^{-1}x)^2$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 2\sin^{-1}x \times 1/\sqrt{1-x^2} \; \ldots.(1)$$

Differentiating with respect to x, we get

$$\frac{d^2y}{dx^2} = 2sin^{-1}x \times \frac{-1}{2(1-x^2)^{\frac{3}{2}}} \times -2x + 2 \times \frac{1}{\sqrt{1-x^2}} \times \frac{1}{\sqrt{1-x^2}}$$

Hence, $(1-x^2)\frac{d^2y}{dx^2} = \frac{2xsin^{-1}x}{\sqrt{1-x^2}} + 2 \ldots .(2)$

Substituting the value of (1) in (2), we get

$$(1-x^2)\frac{d^2y}{dx^2} = x\frac{dy}{dx} + 2$$

Or

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

19. Let $I = \int_0^a f(x)dx$

Substitute t = a - x

⇒ dt = - dx.

Also when x = 0, t = a and when x = a, t = 0.

Hence, $I = -\int_a^0 f(a-t)dt = \int_0^a f(a-t)dt$

$$(Since\ -\int_a^b f(x)dx = \int_b^a f(x)dx)$$

Replacing t with x, we get

$$I = \int_0^a f(a-x)dx$$

= RHS

Hence, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.

Let $I = \int_0^\pi \frac{xsinx}{1+cos^2x}dx \ \ldots..(1)$

Using $\int_0^a f(x)dx = \int_0^a f(a-x)dx$, we get

$$I = \int_0^\pi \frac{(\pi-x)\sin(\pi-x)}{1+cos^2(\pi-x)}dx$$

$$= \int_0^\pi \frac{(\pi-x)sinx}{1+cos^2x}dx \ldots .(2)$$

Adding (1) and (2), we get

$$2I = \int_0^{\pi} \frac{\pi sinx}{1 + cos^2 x} dx$$

Let cos x = t

⇒ - sin x dx = dt.

When x = 0, t = 1 and

when x = π, t = - 1

$$2I = -\int_1^{-1} \frac{\pi}{1+t^2} dt = \int_{-1}^{1} \frac{\pi}{1+t^2} dt$$

$$(Since\ -\int_a^b f(x)dx = \int_b^a f(x)dx)$$

$$2I = \pi tan^{-1}t \Big|_{-1}^{1} = \pi(tan^{-1}1 - tan^{-1}(-1))$$

$$(Using\ \int \frac{1}{1+x^2} dx = tan^{-1}x)$$

$$2I = \pi \times \left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) = \pi \times \frac{\pi}{2} = \frac{\pi^2}{2}$$

Hence, $I = \frac{\pi^2}{4}$

20. Let $I = \int \frac{cosx}{(1+sinx)(2+sinx)} dx$

Substitute sin x = t

⇒ cos dx = dt

$$I = \int \frac{1}{(1+t)(2+t)} dt$$

$$= \int \frac{(2+t)-(1+t)}{(1+t)(2+t)} dt$$

Hence, $I = \int \frac{(2+t)}{(1+t)(2+t)} dt - \int \frac{(1+t)}{(1+t)(2+t)} dt$

$$= \int \frac{1}{1+t} dt - \int \frac{1}{2+t} dt$$

$$I = \log_e(1+t) - \log_e(2+t) + c$$

$(Using \int \frac{1}{x} dx = log_e x)$

$I = \log_e \frac{(1+t)}{(2+t)} + c$

$(Using\ log_c a - log_c b = log_c \frac{a}{b})$

Since t = sin x, therefore:

$I = \log_e \frac{(1+sinx)}{(2+sinx)} + c$

21. Given that $\frac{dy}{dx} - \frac{2x}{1+x^2} y = x^2 + 2 \ \ldots..(1)$

This differential equation is of the form y' + P(x)y = Q(x).

To solve such an equation, we multiply the entire equation with the integration factor $e^{\int P dx}$.

Here $P(x) = -\frac{2x}{1+x^2}$

Then the integration factor will be $e^{\int P dx} = e^{\int -\frac{2x}{1+x^2} dx}$.

Let $1 + x^2 = t$

$\Rightarrow 2x\, dx = dt.$

$e^{\int -\frac{1}{t} dt} = e^{-\log_e t}$

$= e^{\log_e \frac{1}{t}}$

$= \frac{1}{t}$

$= \frac{1}{1+x^2}$

Multiplying (1) with $\frac{1}{1+x^2}$, we get

$\frac{1}{1+x^2} \times \frac{dy}{dx} - \frac{2x}{(1+x^2)^2} y$

$= \frac{1}{1+x^2} \times\ x^2 + 2$

$\Rightarrow \frac{d}{dx}\left(\frac{y}{1+x^2}\right) = \frac{x^2+2}{1+x^2}$

Integrating both sides

$$\frac{y}{1+x^2} = \int \frac{x^2+2}{1+x^2} dx$$

$$= \int \frac{x^2+1+1}{1+x^2} dx$$

$$= \int \frac{1}{1+x^2} dx + \int 1\, dx$$

$= \tan^{-1} x + x + c$

$y = (1 + x^2)(\tan^{-1} x + x + c)$

OR

Given that $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$

$$\Rightarrow \frac{dy}{2e^{-y}-1} = \frac{dx}{x+1}$$

Integrating both sides

$$\int \frac{dy}{2e^{-y}-1} = \int \frac{dx}{x+1}$$

$$\Rightarrow \int \frac{e^y dy}{2-e^y} = \int \frac{dx}{x+1}$$

Let $2 - e^{y} = t$

$\Rightarrow e^y dy = - dt$

$$\Rightarrow -\int \frac{dt}{t} = \int \frac{dx}{x+1}$$

or $-\log_e t = \log_e(x+1) + c$

$(Using \int \frac{1}{x} dx = log_e x + c)$

Hence $\log_e \frac{1}{t} = \log_e(x+1) + \log_e k$,

where $c = \log_e k$

$(Using\ -log_a t = log_a \frac{1}{t})$

Or $\log_e \frac{1}{t} = \log_e k(x+1)$

$(Since\ log_a b + log_a c = log_a bc)$

Since bases of both logs are same, therefore the argument must be equal too

$\frac{1}{t} = k(x+1)\ or \frac{1}{2-e^y} = k(x+1)$

Simplifying this equation, we get

$e\hat{}y = 2 - \frac{1}{k(x+1)}\ or\ y = log_e [\![\left(2 - \frac{1}{k(x+1)}\right]\!]\!)$

Since when x = 0, y = 0, therefore

$1 = 2 - \frac{1}{k}$

or k = 1

Hence, $y = f(x) = \log_e\left(2 - \frac{1}{x+1}\right)$

$= \log_e \frac{2x+1}{x+1}$

22. If the position vectors of two points on a line, $\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$ and $b = x_2\hat{\imath} + y_2\hat{\jmath} + z_2\hat{k}$, are given, then the direction ratios of the line are given by $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$.

Given that the position vectors of points A, B, C and D respectively are $\hat{\imath} + \hat{\jmath} + \hat{k}, 2\hat{\imath} + 5\hat{\jmath}, 3\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\hat{\imath} - 6\hat{\jmath} - \hat{k}$. Then the direction ratios of line $\overrightarrow{AB}$ are 1, 4, - 1 and the direction ratios of line $\overrightarrow{CD}$are - 2, - 8, 2.

If the direction ratios of two lines are given, a_1, a_2, a_3 and b_1, b_2, b_3, then the angle between the lines, θ, is given by:

$$\cos\ \theta = \frac{\overrightarrow{AB}.\overrightarrow{CD}}{|\overrightarrow{AB}||\overrightarrow{CD}|}$$

$$cos\ \theta = \frac{(1\times -2)+(4\times -8)+(-1\times 2)}{\sqrt{1^2+4^2+(-1)^2}\times\sqrt{(-2)^2+(-8)^2+(2)^2}}$$

$$= \frac{-2-32-2}{\sqrt{1+16+1}\sqrt{4+64+4}}$$

$$= \frac{-36}{\sqrt{18}\sqrt{72}}$$

$$= \frac{-36}{3\sqrt{2}\,.6\sqrt{2}}$$

$$= \frac{-36}{36}$$

= - 1

hence θ = 180°.

$\overrightarrow{AB}$ **and** $\overrightarrow{CD}$ **are collinear.**

23. The direction ratios of a line, if the cartesian equation of the line is given $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$, is a,b,c.

Hence, the direction ratios of line L: $\frac{1-x}{3}=\frac{7y-14}{\lambda}=\frac{z-3}{3}$

$$\Rightarrow \frac{x-1}{-3}=\frac{y-2}{\frac{\lambda}{7}}=\frac{z-3}{3}\ are\ -3,\frac{\lambda}{7},3$$

and the direction ratios of line M: $\frac{7-7x}{3\lambda}=\frac{y-5}{1}=\frac{6-z}{5}$

$$\Rightarrow \frac{x-1}{\frac{-3\lambda}{7}}=\frac{y-5}{1}=\frac{z-6}{-5}\ are\ -\frac{3\lambda}{7},1,-5.$$

two lines with direction ratios, a_1, a_2, a_3 and b_1, b_2, b_3, are perpendicular if and only if $a_1b_1+a_2b_2+a_3b_3=0$

Hence, $-3\times -\frac{3\lambda}{7}+\frac{\lambda}{7}\times 1+3\times -5=0$

Solving this equation, we get $\lambda=\frac{21}{2}$

Hence, the equation of lines are:

$$\frac{x-1}{-3} = \frac{2y-4}{3} = \frac{z-3}{3} = k(let)$$

$$and\ \frac{2x-2}{-9} = \frac{y-5}{1} = \frac{z-6}{-5} = m(let)$$

Any general point on line $\frac{x-1}{-3} = \frac{2y-4}{3} = \frac{z-3}{3} = k$ will be $\left(-3k+1, \frac{3k}{2}+2, 3k+3\right)$

and on the line $\frac{2x-2}{-9} = \frac{y-5}{1} = \frac{z-6}{-5} = m$ will be $\left(\frac{-9m}{2}+1, m+5, -5m+6\right)$.

For the lines to intersect, $\left(-3k+1, \frac{3k}{2}+2, 3k+3\right) \equiv \left(\frac{-9m}{2}+1, m+5, -5m+6\right)$. Solving the equations for x and y - coordinates,

$$-3k+1 = \frac{-9m}{2}+1\ and\ \frac{3k}{2}+2 = m+5,$$

we get $k = \frac{3}{2}$ and m = 1.

Equating equations for z - coordinates, we get

3k + 3 = 5m + 6

Substituting $k = \frac{3}{2}$ and m = 1, we get

$\frac{9}{2}+3 = 5+6$ or 15 = 22 which is not true.

Hence, the lines do not intersect in real space.

Section D (Solutions)

24. Let the length and width of base of tank be l and b. Given that the height of tank is 2m and it's volume is 8 m^3.

The volume of a cuboid, of length l, width b and height h, is defined by

V(l,b,h) = lbh

Hence, we get 8 = lb × 2 or lb = $4m^2$

or $l = \frac{4}{b}m$

Let the total cost of building tank be T.

Then T(l,b,h) = 70lb + 45(lh + bh + lh + bh)

= 70lb + 90lh + 90bh

Substituting values of h and lb, we get

T = 70×4 + 180(l + b)

= 280 + 180(l + b)

Since $l = \frac{4}{b}m$, we get

$$T = 280 + 180\left(b + \frac{4}{b}\right)$$

Differentiating with respect to b, we get

$$\frac{dT}{db} = 180\left(1 - \frac{4}{b^2}\right)$$

$$= 180 - \frac{720}{b^2}$$

Differentiating with respect to b, we get

$$\frac{d^2T}{db^2} = \frac{1440}{b^3}$$

For minima, $\frac{dT}{db} = 0$ and $\frac{d^2T}{db^2} > 0$

$$\frac{dT}{db} = 0$$

$\Rightarrow b^2 = 4$ or b = 2m

At b = 2 m,

T = 280 + 180(2 + 2)

= 280 + 180(4)

= 280 + 720

= 1000

$$\left(\frac{d^2T}{db^2}\right)_{b=2} = 180 > 0$$
,

hence b = 2m is a point of minima for function T and T(2) = ₹1000 is the least expensive tank.

25. Given that $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}$

Then $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{vmatrix} = 1(0-2) - 1(1-6) + 1(1) = 4$ (Expanding along R_1)

Calculating the Cofactors of matrix A, we get

$C_{11} = -2$, $C_{12} = 5$, $C_{13} = 1$

$C_{21} = 0$, $C_{22} = -2$, $C_{23} = 2$

$C_{31} = 2$, $C_{32} = -1$, $C_{33} = -1$

Adjoint of a matrix A is defined as the transpose of Cofactor matrix of matrix A.

Hence, $adj(A) = \left(\begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}\right)^T$

$$= \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

Inverse of a matrix A is defined as a matrix A^{-1} such that

$AA^{-1} = I = A^{-1}A$.

A^{-1} is also equal to $\frac{1}{|A|}(adj(A))$, $|A| \neq 0$.

Hence, $A^{-1} = \frac{1}{4} \times \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{4} & -\frac{1}{2} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{bmatrix}$

Given that

x + y + z = 6

x + 2z = 7

3x + y + z = 12

Hence $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

Pre - multiplying with A^{-1} both sides, we get

$$\begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{4} & -\frac{1}{2} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{4} & -\frac{1}{2} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{bmatrix} \times \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$ or $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$

Hence x = 3, y = 1 and z = 2.

OR

Given that $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

We know that A = IA

Hence, $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

Using $R_2 \rightarrow R_2 + R_1$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 5 & -2 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Using $R_2 \rightarrow \frac{1}{5} R_2$

$$\begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -\frac{2}{5} \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Using $R_1 \rightarrow R_1 - 2R_2$ and $R_3 \rightarrow R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & -\frac{6}{5} \\ 0 & 1 & -\frac{2}{5} \\ 0 & 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & -\frac{2}{5} & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{2}{5} & 1 \end{bmatrix} A$$

Using $R_3 \rightarrow 5R_3$

$$\begin{bmatrix} 1 & 0 & -\frac{6}{5} \\ 0 & 1 & -\frac{2}{5} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & -\frac{2}{5} & 0 \\ \frac{1}{5} & \frac{1}{5} & 0 \\ 2 & 2 & 5 \end{bmatrix} A$$

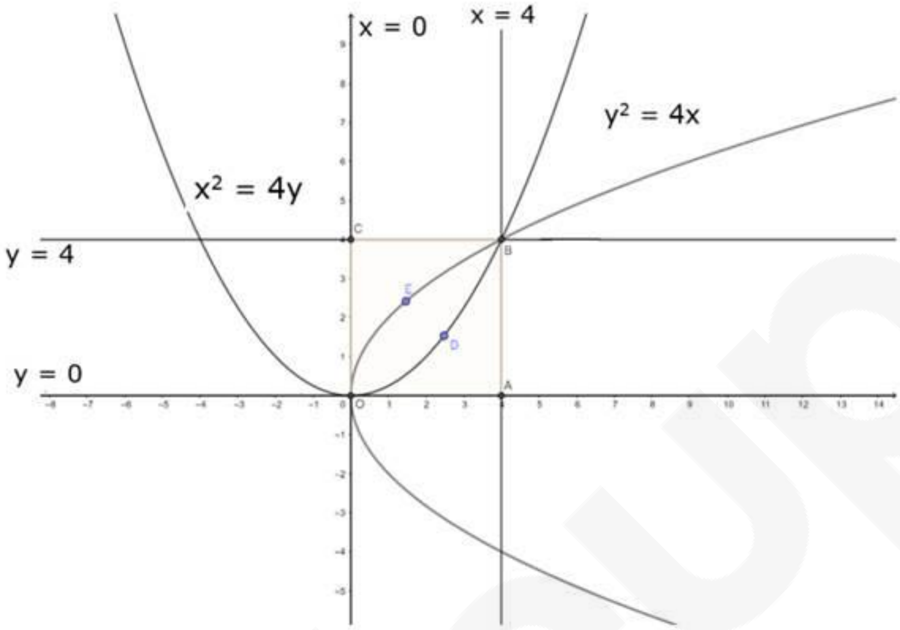
Using $R_1 \rightarrow R_1 + \frac{6}{5} R_3$ and $R_2 \rightarrow R_2 + \frac{2}{5} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A$$

Here $A^{-1}A = I$ and hence

$$A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

26. $y^2 = 4x$



The area of the square bounded by x = 0, x = 4, y = 4 and y = 0 will be 4×4 = 16 square units.

Let f: $y^2 = 4x$ and g: $x^2 = 4y$

Let I_1 be the area bounded by the curve f and y - axis and I_2 be the area bounded by the curve g and x - axis and I_3 be the area bound in between f and g - curve.

Then, $I_1 + I_2 + I_3 = 16$ –(1)

$$I_1 = \int_0^4 x dy = \int_0^4 \frac{y^2}{4} dy = \frac{y^3}{12}\Big|_0^4$$

Hence, $I_1 = \frac{16}{3}$ - (2)

$$I_2 = \int_0^4 y dx = \int_0^4 \frac{x^2}{4} dy = \frac{x^3}{12}\Big|_0^4$$

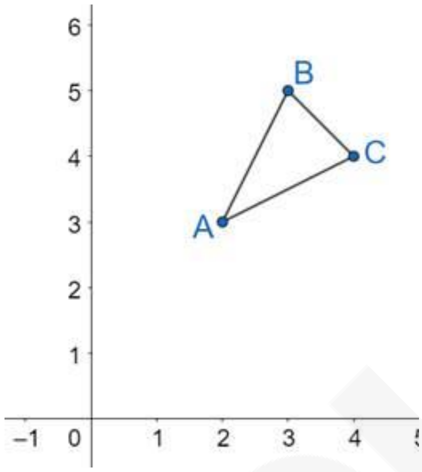
Hence, $I_2 = \frac{16}{3}$ –(3)

Substituting values of (2) and (3) in (1), we get

$\frac{16}{3} + \frac{16}{3} + I_3 = 16$ or $I_3 = \frac{16}{3}$

Hence, $I_1 = I_2 = I_3 = \frac{16}{3}$ and therefore it divides the square in 3 equal parts.

OR



The area of the triangle will be equal to

ΔABC = (area under line segment AB) + (area under line segment BC) - (area under line segment AC)

Let line AB, BC and AC be l, m and n respectively.

Equation of l, by two - point form, will be $y-3=\frac{5-3}{3-2}(x-2)$ or y = 2x - 1.

Equation of m, by two - point form, will be $y-5=\frac{4-5}{4-3}(x-3)$ or y = 8 - x.

Equation of n, by two - point form, will be $y-4=\frac{4-3}{4-2}(x-4)$ or $y=\frac{x}{2}+2$.

$$\Delta ABC=\int_2^3 l\,dx+\int_3^4 m\,dx-\int_2^4 n\,dx$$

Hence, $\Delta ABC=\int_2^3 (2x-1)\,dx+\int_3^4 (8-x)\,dx-\int_2^4 \left(\frac{x}{2}+2\right)dx$

$$\Delta ABC=(x^2-x)\Big|_2^3+\left(8x-\frac{x^2}{2}\right)\Big|_3^4-\left(\frac{x^2}{4}+2x\right)\Big|_2^4$$

$$\Delta ABC=4+\frac{9}{2}-7$$

$= \frac{3}{2}$

Hence, the area of triangle whose vertices are (2,3), (3,5) and (4,4) is $\frac{3}{2}$ square units.

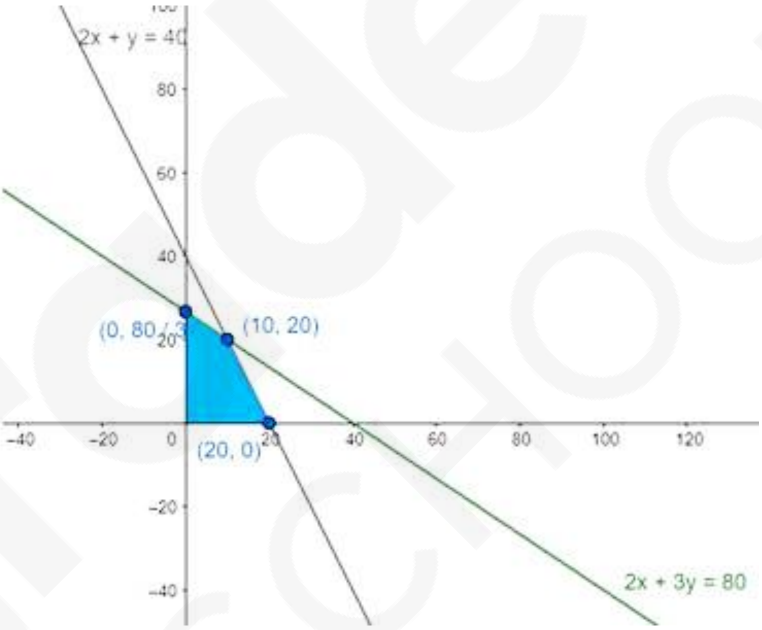
27. Let x and y be the number of items produced per day of model A and model B respectively.

Then, 2x + y≤40

2x + 3y≤80

x≥0, y≥0

We have to maximize Z:15x + 10y



Since the area is bounded, therefore the maximum value will occur on corner points.

When x = 0 and y = 0, Z = 0

When x = 20 and y = 0, Z = 300

When x = 0 and $y = \frac{80}{3}$, $Z = \frac{800}{3}$

When x = 10 and y = 20, Z = 350

Hence the maximum profit that the manufacturer can make in a day is ₹350, when the number of items of model A will be 10 and of model B will be 20.

28. Let A ≡ (2,2, - 1), B ≡ (3,4,2) and C ≡ (7,0,6).

If the co - ordinates of points A ≡ (x_1, y_1, z_1) and B ≡ (x_2, y_2, z_2), then the vector $\overrightarrow{AB}$ is

$$\overrightarrow{AB} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$$

Hence, $\overrightarrow{AB} = (3-2)\hat{\imath} + (4-2)\hat{\jmath} + (2-(-1))\hat{k} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\overrightarrow{AC} = (7-2)\hat{\imath} + (0-2)\hat{\jmath} + (6-(-1))\hat{k} = 5\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$

We have to figure out the normal vector of the required plane. Since, $\overrightarrow{AB}$ and $\overrightarrow{AC}$ lie on the plane, therefore $\overrightarrow{AB} \times \overrightarrow{AC}$ will be perpendicular to the plane and hence be the normal vector of the plane.

If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2 2\hat{\jmath} + b_3\hat{k}$, then we define $\vec{a} \times \vec{b}$ as

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Hence, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ 5 & -2 & 7 \end{vmatrix} = 20\hat{\imath} + 8\hat{\jmath} - 12\hat{k} = \vec{n}$

The vector equation of a plane is given by, $(\vec{r} - \vec{a}).\vec{n} = 0$, where $\vec{a}$ is the position vector of a point on plane and $\vec{n}$ is the normal vector.

If the co - ordinates of a point A ≡ (x_1, y_1, z_1), then the position vector of A($\vec{a}$) is

$$\vec{a} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k}$$

Since B($\vec{b}$) lies on plane therefore $\vec{b} = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$, Hence, the vector equation is $(\vec{r}-3\hat{\imath}-4\hat{\jmath}-2\hat{k}).(20\hat{\imath}+8\hat{\jmath}-12\hat{k})=0$ or $\vec{r}.(20\hat{\imath}+8\hat{\jmath}-12\hat{k})=68$ or $\vec{r}.(5\hat{\imath}+2\hat{\jmath}-3\hat{k})=17$.

The cartesian equation will be $(x\hat{\imath}+y\hat{\jmath}+z\hat{k}).(5\hat{\imath}+2\hat{\jmath}-3\hat{k})=17$ or 5x + 2y - 3z = 17

The new plane is parallel to the original plane and passes through point D ≡ (4,3,1).

Since both planes are parallel, therefore their normal vectors are same. Also, it passes through point D($\vec{d}$) therefore $\vec{d}=4\hat{\imath}+3\hat{\jmath}+\hat{k}$.

Hence, the vector equation is $(\vec{r}-4\hat{\imath}-3\hat{\jmath}-\hat{k}).(20\hat{\imath}+8\hat{\jmath}-12\hat{k})=0$ or $\vec{r}.(20\hat{\imath}+8\hat{\jmath}-12\hat{k})=92$ or $\vec{r}.(5\hat{\imath}+2\hat{\jmath}-3\hat{k})=23$.

OR

Given that the plane contains line $\vec{r}=\hat{\imath}+\hat{\jmath}+\lambda(\hat{\imath}+2\hat{\jmath}-\hat{k})$ and point A ≡ (- 1,3, - 4).

Hence, the point $\vec{b}=\hat{\imath}+\hat{\jmath}$ and the vector $\vec{c}=\hat{\imath}+2\hat{\jmath}-\hat{k}$ lies on the plane.

If the co - ordinates of a point A ≡ (x_1, y_1, z_1), then the position vector of A($\vec{a}$) is

$\vec{a}=x_1\hat{\imath}+y_1\hat{\jmath}+z_1\hat{k}$

Hence, the position vector of point A($\vec{a}$) is

$\vec{a}=-\hat{\imath}+3\hat{\jmath}-4\hat{k}$

If the position vector of point A($\vec{a}$) and B($\vec{b}$)is given then the vector $\overrightarrow{AB}=\vec{b}-\vec{a}$.

Hence, $\overrightarrow{AB}=(\hat{\imath}+\hat{\jmath})-(-\hat{\imath}+3\hat{\jmath}-4\hat{k})=2\hat{\imath}-2\hat{\jmath}+4\hat{k}$

We have to find out the normal vector to plane. Since both $\overrightarrow{AB}$ and $\vec{c}$ lies on plane, $\overrightarrow{AB}\times\vec{c}$ will be perpendicular to the plane and hence be the normal vector of the plane.

If $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ and $\vec{b} = b_1\hat{\imath} + b_2 2\hat{\jmath} + b_3\hat{k}$, then we define $\vec{a} \times \vec{b}$ as

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

Hence, $\overrightarrow{AB} \times \vec{c} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -2 & 4 \\ 1 & 2 & -1 \end{vmatrix} = -6\hat{\imath} + 6\hat{\jmath} + 6\hat{k} = \vec{n}$

The vector equation of a plane is given by, $(\vec{r} - \vec{a}).\vec{n} = 0$, where $\vec{a}$ is the position vector of a point on plane and $\vec{n}$ is the normal vector.

Hence, the vector equation is $(\vec{r} - \hat{\imath} - \hat{\jmath}).(-6\hat{\imath} + 6\hat{\jmath} + 6\hat{k}) = 0$ or $\vec{r}.(-6\hat{\imath} + 6\hat{\jmath} + 6\hat{k}) = 0$ or $\vec{r}.(-\hat{\imath} + \hat{\jmath} + \hat{k}) = 0$.

We have to figure out the intersection point of a line passing through point (2,1,4) and is perpendicular to the plane $\vec{r}.(-\hat{\imath} + \hat{\jmath} + \hat{k}) = 0$. Since, the line is perpendicular to plane, therefore it is parallel to it's normal vector.

If a point on a line $\vec{a}$ and a vector parallel to it $\vec{q}$ is known, then the equation of line is $\vec{r} = \vec{a} + \lambda\vec{q}$.

Hence, the equation of the required line, passing through point $2\hat{\imath} + \hat{\jmath} + 4\hat{k}$ and parallel to $-\hat{\imath} + \hat{\jmath} + \hat{k}$, is $\vec{r} = (2\hat{\imath} + \hat{\jmath} + 4\hat{k}) + \lambda(-\hat{\imath} + \hat{\jmath} + \hat{k}) = (2 - \lambda)\hat{\imath} + (1 + \lambda)\hat{\jmath} + (4 + \lambda)\hat{k}$

For intersection point, substitute $\vec{r} = (2 - \lambda)\hat{\imath} + (1 + \lambda)\hat{\jmath} + (4 + \lambda)\hat{k}$ in the equation of the plane $\vec{r}.(-\hat{\imath} + \hat{\jmath} + \hat{k}) = 0$, hence $((2 - \lambda)\hat{\imath} + (1 + \lambda)\hat{\jmath} + (4 + \lambda)\hat{k}).(-\hat{\imath} + \hat{\jmath} + \hat{k}) = 0$ or λ - 2 + 1 + λ + 4 + λ = 0

Solving, we get λ = - 1

Hence, the point is $3\hat{\imath} + 3\hat{k}$ or (3,0,3)

The distance between point (2,1,4) and (3,0,3) is $\sqrt{(3-2)^2 + (0-1)^2 + (3-4)^2} = \sqrt{1+1+1} = \sqrt{3}$. Hence the perpendicular distance from point (2,1,4) to plane $\vec{r}.(-\hat{\imath} + \hat{\jmath} + \hat{k}) = 0$ is $\sqrt{3}$ units.

29. Let X be the number of kings drawn. Then X can be 0,1 or 2.

$$P(X=0) = P(A\ king\ is\ not\ drawn\ in\ both\ draws)\frac{48}{52}\times\frac{47}{51}=\frac{188}{221}$$

$$P(X=1) = P(A\ king\ is\ drawn\ in\ 1st\ draw\ and\ not\ drawn\ in\ 2nd) + P(A\ king\ is\ not\ drawn\ in\ 1st\ draw\ and\ is\ drawn\ in\ 2nd)$$

$$P(X=1)=\frac{4}{52}\times\frac{48}{51}+\frac{48}{52}\times\frac{4}{51}=\frac{32}{221}$$

$$P(X=2) = P(A\ king\ is\ drawn\ in\ both\ draws) = \frac{4}{52}\times\frac{3}{51}=\frac{1}{221}$$

Sum of probabilities of all possible values of X is equal to 1.

Hence X is a random variable.

The mean of X is defined as, $E(X)=\sum_{i=1}^{n} x_i p_i$, where x_i is the value of random variable and p_i is the probability of that value of random variable.

Hence, $E(X)=0\times\frac{188}{221}+1\times\frac{32}{221}+2\times\frac{1}{221}=\frac{34}{221}=\frac{2}{13}$

The variance of X is defined by the formula $Var(X)={\sigma_X}^2=E(X^2)-(E(X))^2$, where $E(X^2)=\sum_{i=1}^{n} {x_i}^2 p_i$ and $E(X)=\sum_{i=1}^{n} x_i p_i$.

$$E(X^2)=\sum_{i=1}^{n} {x_i}^2 p_i = 0^2\times\frac{188}{221}+1^2\times\frac{32}{221}+2^2\times\frac{1}{221}=\frac{36}{221}$$

Hence, ${\sigma_X}^2=\frac{36}{221}-\left(\frac{2}{13}\right)^2=\frac{36}{221}-\frac{4}{169}=\frac{400}{2873}$

Solutions (Set - 2)

1. Given equation: $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$

The **order** is the highest numbered derivative in the **equation**, while the degree is the highest power to which a derivative is raised.

The highest numbered derivative: $\frac{d^2y}{dx^2}$

Therefore, order = 2

Also,

Highest power to which $\frac{d^2y}{dx^2}$ is raised = 1

Degree = 1

Hence, order is 2 and degree is 1 of the given differential equation.

2. Given: f(x) = x + 7, g(x) = x – 7

(fog)(x) = f(g(x))

= g(x) + 7

= x – 7 + 7

= x

Therefore,

$$\frac{d}{dx}(fog)(x) = \frac{d}{dx}(x) = 1$$

3. **Given:** $2\begin{bmatrix}1 & 3\\ 0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

$\begin{bmatrix}2 & 6\\ 0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\ 1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

$\begin{bmatrix}2+y & 6+0\\ 0+1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\ 1 & 8\end{bmatrix}$

Therefore, we have,

2 + y = 5

y = 3

And, 2x + 2 = 8

2x = 6

x = 3

x – y = 3 – 3

Hence, x – y = 0.

6. **Given:** $A=\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

$A^2 = A.A = \begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}.\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

$=\begin{bmatrix}2\times 2+0\times 2+1\times 1 & 2\times 0+0\times 1+1\times -1 & 2\times 1+0\times 3+1\times 0\\ 2\times 2+1\times 2+3\times 1 & 2\times 0+1\times 1+3\times -1 & 2\times 1+1\times 3+3\times 0\\ 1\times 2+\ -1\times 2+0\times 1 & 1\times 0+\ -1\times 1+0\times -1 & 1\times 1+\ -1\times 3+0\times 0\end{bmatrix}$

$=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}$

$A^2-5A=\begin{bmatrix}5 & -1 & 2\\ 9 & -2 & 5\\ 0 & -1 & -2\end{bmatrix}-5.\begin{bmatrix}2 & 0 & 1\\ 2 & 1 & 3\\ 1 & -1 & 0\end{bmatrix}$

$A^2-5A=\begin{bmatrix}5-10 & -1-0 & 2-5\\ 9-10 & -2-5 & 5-15\\ 0-5 & -1+5 & -2-0\end{bmatrix}$

$\boldsymbol{A^2-5A=\begin{bmatrix}-5 & -1 & -3\\ -1 & -7 & -10\\ -5 & 4 & -2\end{bmatrix}}$

12. Let $I = \int \frac{\tan^2 x.\sec^2 x}{1-\tan^6 x} dx$

Let $\tan^3 x = t$

Differentiating both sides, we get,

$3.\tan^2 x.\sec^2 x \, dx = dt$

Therefore,

$$I = \int \frac{1}{3} \times \frac{1}{1-t^2} dt$$

We know that, $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + C$

Therefore,

$$I = \frac{1}{3} \times \frac{1}{2} \times \log\left|\frac{1+t}{1-t}\right| + C$$

$$I = \frac{1}{6} \log\left|\frac{1+t}{1-t}\right|$$

Putting the value of t,

$$I = \frac{1}{6} \log\left|\frac{1+\tan^3 x}{1-\tan^3 x}\right| + C$$

13. Given: $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

Formula Used: $\tan^{-1} A + \tan^{-1} B = \tan^{-1}\left[\frac{A+B}{1-AB}\right]$

$$\tan^{-1}\left[\frac{2x+3x}{1-6x^2}\right] = \frac{\pi}{4}$$

Taking tan on both the sides, we get,

$$\frac{5x}{1-6x^2} = \tan\left(\frac{\pi}{4}\right)$$

$5x = 1 - 6x^2$

$6x^2 + 5x - 1 = 0 \Rightarrow 6x^2 + 6x - x - 1 = 0$

$6x(x + 1) - 1(x + 1) = 0$

$(6x - 1)(x + 1) = 0$

$$x = \frac{1}{6} \; or \; x = -1$$

15. Let $I = \int \frac{3x+5}{x^2+3x-18} dx$

$$I = \int \frac{3x+5}{x^2+(6x-3x)-18} dx$$

$$I = \int \frac{3x+5}{x(x+6)-3(x+6)} dx$$

$$I = \int \frac{3x+5}{(x-3)(x+6)} dx$$

We will use integration by parts for solving this integral,

Therefore,

$$\frac{3x+5}{(x-3)(x+6)} = \frac{A}{x-3} + \frac{B}{x+6}$$

$$\frac{3x+5}{(x-3)(x+6)} = \frac{[A(x+6)+B(x-3)]}{(x-3)(x+6)}$$

Therefore,

3x + 5 = Ax + Bx + 6A – 3B

Comparing the coefficients, we get,

A + B = 3……(1)

6A – 3B = 5……(2)

Multiplying equation 1 by 6 and subtracting equation 2 from it, we get,

6A + 6B – 6A + 3B = 18 – 5

9B = 13

$$B = \frac{13}{9}$$

Putting the value of B in equation 1, we get,

$$A + \frac{13}{9} = 3$$

$$A = 3 - \frac{13}{9} = \frac{14}{9}$$

Now,

$$I = \int \frac{14}{9} \times \frac{1}{x-3} dx + \int \frac{13}{9} \times \frac{1}{x+6} dx$$

$$I = \frac{14}{9}\log|x-3| + \frac{13}{9}\log|x+6| + C$$

Hence,

$$\int \frac{3x+5}{x^2+3x-18} dx = \frac{14}{9}\log|x-3| + \frac{13}{9}\log|x+6| + C$$

18. To Prove:

$$\begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix} = 2(a+b)(b+c)(c+a)$$

Proof:

$$L.H.S = \begin{vmatrix} a+b+c & -c & -b \\ -c & a+b+c & -a \\ -b & -a & a+b+c \end{vmatrix}$$

Applying $R_1 \to R_1 + R_3$ and $R_2 \to R_2 + R_3$

$$= \begin{vmatrix} a+c & -c-a & a+c \\ -c-b & b+c & b+c \\ -b & -a & a+b+c \end{vmatrix}$$

Now, taking (a + c) common from the first row and (b + c) common from the second row, we have,

$$= (a+c)(b+c) \begin{vmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ -b & -a & a+b+c \end{vmatrix}$$

Expanding, we get,

= (a + c)(b + c)[a + b + c + a + (- a – b – c + b) + 1(a + b)]

= (a + c)(b + c)[2a + 2b] = 2(a + b)(b + c)(c + a)

= R.H.S

Hence, Proved.

19. Given: $x = \cos t + \log\tan\left(\frac{t}{2}\right), y = \sin t$

Differentiating x with respect to t, we get,

$$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times \frac{d}{dt}\left(\tan\frac{t}{2}\right)$$

$$\frac{dx}{dt} = -\sin t + \frac{1}{\tan\frac{t}{2}} \times \sec^2\frac{t}{2} \times \frac{1}{2}$$

Now, differentiating y with respect to t, we get,

$$\frac{dy}{dt} = \cos t$$

$$\frac{d^2y}{dt^2} = -\sin t$$

At $t = \frac{\pi}{4}$

$$\boldsymbol{\frac{d^2y}{dt^2} = -\sin\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}}$$

Now, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

Therefore,

$$\frac{dy}{dx} = \frac{\cos t}{-\sin t + \frac{1}{2} \times \sec^2\frac{t}{2} \times \cot\frac{t}{2}} = \frac{cost}{-\sin t + \frac{1}{\sin t}} = \tan t$$

Now,

$$\frac{d^2y}{dx^2} = \left[\left(-\sin t + \frac{1}{2}.\sec^2\frac{t}{2}.\cot\frac{t}{2} \times \frac{d}{dt}(\cos t)\right.\right.$$
$$\left.\left. - \cos t\frac{d}{dt}\left(-\sin t + \frac{1}{2}.\sec^2\frac{t}{2}.\cot\frac{t}{2}\right)\right] \times \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = \left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}(-\sin t)\right.$$
$$\left. - \cos t\,(-\cos t + -cosec\, t \cot t)\right]\left[\frac{1}{-sint + \frac{1}{\sin t}}\right]$$

$$\frac{d^2y}{dx^2}\left(at\ x = \frac{\pi}{4}\right) = \left[-\frac{1}{\sqrt{2}} - 1\right]\left[\frac{1}{-\frac{1}{\sqrt{2}} + \sqrt{2}}\right]$$

$$\boldsymbol{\frac{d^2y}{dx^2}\left(at\ x = \frac{\pi}{4}\right) = 1}$$

22. Given equation: $xdy - ydx = \sqrt{x^2 + y^2}dx$

On - rearranging the term, we get,

$$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

$$\frac{dy}{dx} - \frac{y}{x} = \sqrt{1 + \frac{y^2}{x^2}}$$

$$\frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}$$

Now, this is a homogenous differential equation of order 1.

Let y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Therefore,

$$v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$$

$$x\frac{dv}{dx} = \sqrt{1 + v^2}$$

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

Integrating both sides, we get,

$$\int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$$

Now, we know that,

$\int \frac{dx}{\sqrt{1+x^2}} = \log\left|x + \sqrt{1 + x^2}\right| + C$ and $\int \frac{dx}{x} = \log x$

Therefore,

$$\log\left|v + \sqrt{1 + v^2}\right| = logx + logC$$

$$\left|v + \sqrt{1 + v^2}\right| = Cx$$

Putting the value of y, we get,

$$\left|\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right| = Cx$$

At x = 1, y = 0,

$$|0+1| = C$$

C = 1

Hence,

$$\left|\frac{y}{x}+\sqrt{1+\frac{y^2}{x^2}}\right| = x$$

$$\boldsymbol{y+\sqrt{x^2+y^2}=x^2}$$

OR

Given Equation: $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$

Dividing the whole equation by $(1 + x^2)$, we get,

$$\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{4x^2}{1+x^2}$$

Now, this is a linear equation of the form,

$$\frac{dy}{dx}+Py=Q(x)$$

We know that the solution of this equation is given by,

$$y\times IF = \int (Q\times IF)dx + C$$

Where $IF = e^{\int pdx}$

Therefore, for a given equation,

$$IF = e^{\int \frac{2x}{1+x^2}dx}$$

Let $1 + x^2 = t$

Differentiating both sides, we get,

2x dx = dt

Therefore,

$$IF = e^{\int \frac{dt}{t}} = e^{\log t}$$

$$IF = e^{\log|1+x^2|} = 1 + x^2$$

The solution of the equation:

$$y \times (1 + x^2) = \int \frac{4x^2}{1 + x^2} \times (1 + x^2) dx + C$$

$$y \times (1 + x^2) = \int 4x^2 dx + C$$

$$y \times (1 + x^2) = \frac{4x^3}{3} + C$$

At x = 0, y = 0

Therefore,

$$0 \times 1 = \frac{4}{3} + C$$

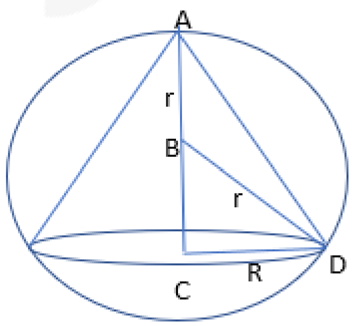
$$C = -\frac{4}{3}$$

Hence,

$$y \times (1 + x^2) = \frac{4}{3}x^3 - \frac{4}{3}$$

$$\boldsymbol{y = \frac{4}{3}\frac{x^3 - 1}{(1 + x^2)}}$$

24. Let R and h be the radius and the height of the cone respectively.



The volume (V) of the cone is given by;

V = $\frac{1}{3}\pi r^2 h$

Now, from the right triangle BCD, we get,

BC = $\sqrt{r^2 - R^2}$

$\therefore h = r + \sqrt{r^2 - R^2}$

V = $\frac{1}{3}\pi R^2\left(r + \sqrt{r^2 - R^2}\right) = \frac{1}{3}\pi R^2 r + \frac{1}{3}\pi R^2\sqrt{r^2 - R^2}$

$$\frac{dV}{dR} = \frac{2}{3}\pi Rr + \frac{2}{3}\pi R\sqrt{r^2 - R^2} + \frac{R^2}{3}.\frac{(-2R)}{2\sqrt{r^2 - R^2}}$$

$$= \frac{2}{3}\pi Rr + \frac{2}{3}\pi R\sqrt{r^2 - R^2} + \frac{R^3}{3\sqrt{r^2 - R^2}}$$

$$= \frac{2}{3}\pi Rr + \frac{2\pi R(r^2 - R^2) - \pi R^3}{3\sqrt{r^2 - R^2}}$$

$$= \frac{2}{3}\pi Rr + \frac{2\pi Rr^2 - 3\pi R^3}{3\sqrt{r^2 - R^2}}$$

Now, if $\frac{dV}{dR} = 0$, then,

$$\frac{2}{3}\pi Rr = -\frac{2\pi Rr^2 - 3\pi R^3}{3\sqrt{r^2 - R^2}}$$

$$\Rightarrow 2r\sqrt{r^2 - R^2} = 3R^2 - 2r^2$$

$$\Rightarrow 4r^2(r^2 - R^2) = (3R^2 - 2r^2)^2$$

$$\Rightarrow 4r^4 - 4r^2R^2 = 9R^4 + 4r^4 - 12R^2r^2$$

$$\Rightarrow 9R^4 - 8r^2R^2 = 0$$

$$\Rightarrow 9R^2 = 8r^2$$

$$\Rightarrow R^2 = \frac{8r^2}{9}$$

Now, $\frac{d^2V}{dR^2} = \frac{2\pi r}{3} + \frac{3\sqrt{r^2-R^2}(2\pi r^2-9\pi R^2)-(2\pi Rr^2-3\pi R^3)(-6R)\frac{1}{2\sqrt{r^2-R^2}}}{9(r^2-R^2)}$

Now, when $R^2 = \frac{8r^2}{9}$, it can be shown that $\frac{d^2V}{dR^2} < 0$.

Therefore, the volume is the maximum when $R^2 = \frac{8r^2}{9}$.

When $R^2 = \frac{8r^2}{9}$,

Height of the cone = r + $\sqrt{r^2 - \frac{8r^2}{9}} = r + \sqrt{\frac{r^2}{9}} = r + \frac{r}{3} = \frac{4r}{3}$.

Therefore, it can be seen that the altitude of the circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

25. We are given with a matrix A = $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ and we have to find the inverse of this matrix which will be

A^{-1}, so the formula to find the inverse of a matrix is,

$A^{-1} = \frac{1}{|A|}\text{Adjoint}(A)$

First we will find the $|A|$ which is,

$|A| = 2[2(-2) - 1(-4)] - (-3)[3(-2) - 1(-4)] + 5[3(1) - 2(1)]$

$|A| = 0 - 6 + 5 = -1$

Now we have to find the adjoint matrix, and for that, we have to first find the cofactor matrix and take its transpose.

To find the cofactor matrix, we have to take determinant similar method, like this,

For finding the cofactor of the a_{11} element, we will take the determinant of $\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$ and a sign of positive will be there when the sum of row the number and column number is even and, the sign will be negative when the sum is odd. In this case, the sum is even, so a positive sign will come.

Therefore the cofactor matrix is, $\begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}$

Taking transpose of the matrix, we get,

$\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ which is the Adjoint(A),

Hence $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

To solve linear equations with the help of the matrix method first write the coefficients in the form,

A X = B,

In A matrix write all the coefficients and in the X matrix write the variables involved and in the B matrix write the constants.

2x - 3y + 5z = 11

3x + 2y - 4z = - 5

x + y - 2z = - 3

The A matrix obtained from these equations is the same as given at the starting of the question, Hence pre - multiplying the general equation by A^{-1}, we get,

$X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

By multiplying these matrices, we will get the values of the variables,

As the column number of the first matrix is equal to the row number of the second matrix, they are feasible to be multiplied.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence x = 1, y = 2, z = 3

OR

Given: - 3 x 3 square matrix

Tip: - Algorithm to find Inverse of a square matrix of ‘n’ order by elementary row transformation

(i) Obtain the square matrix, say A

(ii) Write $A = I_n A$

(iii) Perform a sequence of elementary row operation successively on A on the LHS and pre - factor I_n on the RHS till we obtain the result

$I_n = BA$

(iv) Write $A^{-1} = B$

Now,

We have,

A = I_3A

Where I_3 is 3 x 3 elementary matrix

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to -1r_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to r_2 - r_1 \; and \; r_3 \to r_3 - 3r_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $r_2 \to \frac{1}{3} r_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & \frac{5}{3} \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $r_1 \to r_1 + r_2$ and $r_3 \to r_3 - 4r_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{5}{3} & -\frac{4}{3} & 1 \end{bmatrix} A$$

Applying $r_3 \to 3r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -4 & 3 \end{bmatrix} A$$

Applying $r_1 \to r_1 + \frac{1}{3} r_3 \; and \; r_2 \to r_2 - \frac{5}{3} r_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

Hence , it is of the form

I = BA

So, as we know that

$I = A^{-1}A$

Therefore

$A^{-1} = B$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$ inverse of A

26. Let E_1 be the event that machine A is defective. $P(E_1) = 1\%$

Let E_2 be the event that machine B is defective. $P(E_2) = 5\%$

Let E_3 be the event that machine C is defective. $P(E_3) = 7\%$

Let A be the event that the item chosen is defective.

P(the defective item is manufactured by A) = $P(A|E_1) = 50\%$

P(the defective item is manufactured by B) = $P(A|E_2) = 30\%$

P(the defective item is manufactured by C) = $P(A|E_3) = 20\%$

Now, According to the Bayes' theorem,

Given E_1, E_2, E_3.......E_n are mutually exclusive and exhaustive events, we can find the conditional probability $P(E_i|A)$ for any event A associated with E_i is given by:

$$P\left(\frac{E_i}{A}\right) = \frac{P\left(\frac{A}{E_i}\right)P(E_i)}{P\left(\frac{A}{E_1}\right)P(E_1) + P\left(\frac{A}{E_2}\right)P(E_2) + \cdots \dots \dots . + P\left(\frac{A}{E_n}\right)P(E_n)}$$

Therefore,

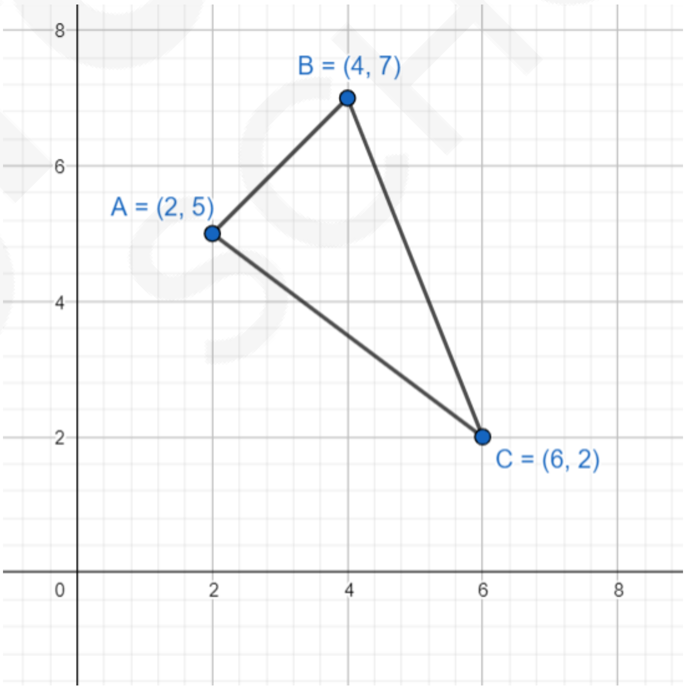
The probability that defective item is produced by A,

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{100} \times \frac{50}{100}}{\frac{1}{100} \times \frac{50}{100} + \frac{5}{100} \times \frac{30}{100} + \frac{7}{100} \times \frac{20}{100}}$$

$$P\left(\frac{E_1}{100}\right) = \frac{\frac{1}{200}}{\frac{1}{200} + \frac{3}{200} + \frac{7}{500}} = 1000 \times \frac{1}{200} \times \frac{1}{34} = \frac{5}{34}$$

Hence, the probability that the defective item is produced by machine A is $\frac{5}{34}$.

28. Plotting the points, we get,



Let us first find the equation of sides of the triangle, Equation of line passing from points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Therefore,

Equation of AB: $y - 5 = \frac{7-5}{4-2}(x - 2)$

y – 5 = (x – 2)

y = x + 3

Equation of BC: $y - 7 = \frac{2-7}{6-4}(x - 4)$

$$y - 7 = -\frac{5}{2}(x - 4)$$

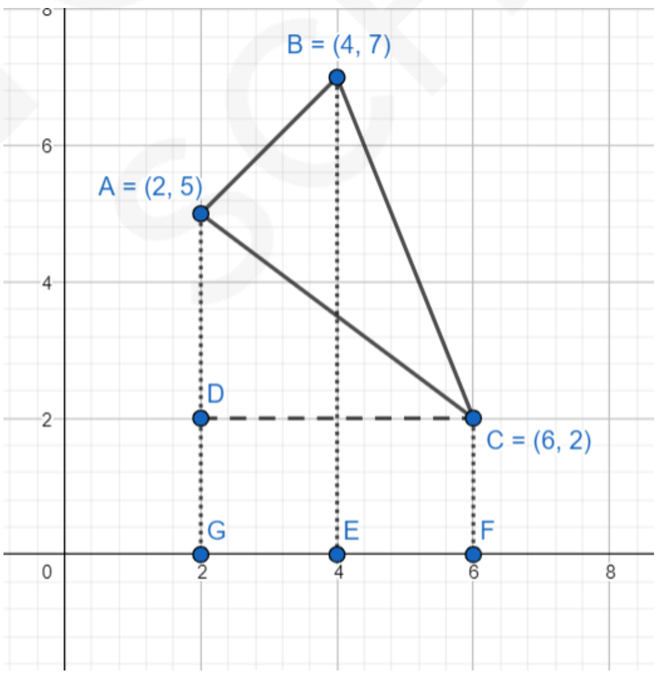
$$y = -\frac{5}{2}x + 17$$

Equation of AC: $y - 2 = \frac{5-2}{2-6}(x - 6)$

$$y - 2 = -\frac{3}{4}(x - 6)$$

$$y = -\frac{3}{4}x + \frac{9}{2}$$

Now, let us look at the limits that we have to take,



Therefore,

$$Area = \int_2^4 (x+3)dx + \int_4^6 \left(-\frac{5}{2}x + 17\right) dx - \int_2^6 \left(-\frac{3}{4}x + \frac{9}{2}\right) dx$$

$$Area = \left[\frac{x^2}{2} + 3x\right]_2^4 + \left[-\frac{5}{2}\frac{x^2}{2} + 17x\right]_4^6 - \left[-\frac{3}{4}\frac{x^2}{2} + \frac{9x}{2}\right]_2^6$$

$$Area = \frac{16}{2} + 12 - \frac{4}{2} - \frac{5}{2} \times \frac{36}{2} + 17 \times 6 + \frac{5}{2} \times \frac{16}{2} - 17 \times 4 + \frac{3}{4} \times \frac{36}{2} - \frac{9}{2} \times 6 - \frac{3}{4}\left(\frac{4}{2}\right) + \frac{9}{2}(2)$$

Area = 21 square units.

OR

Given Curves: $C_1: x^2 + y^2 = 8x$

$C_2: y^2 = 4x$

Let us find the intersection point of the curves.

Putting the value of y^2 from second curve in first curve,

$x^2 + 4x = 8x$

$x^2 - 4x = 0$

$x(x - 4) = 0$

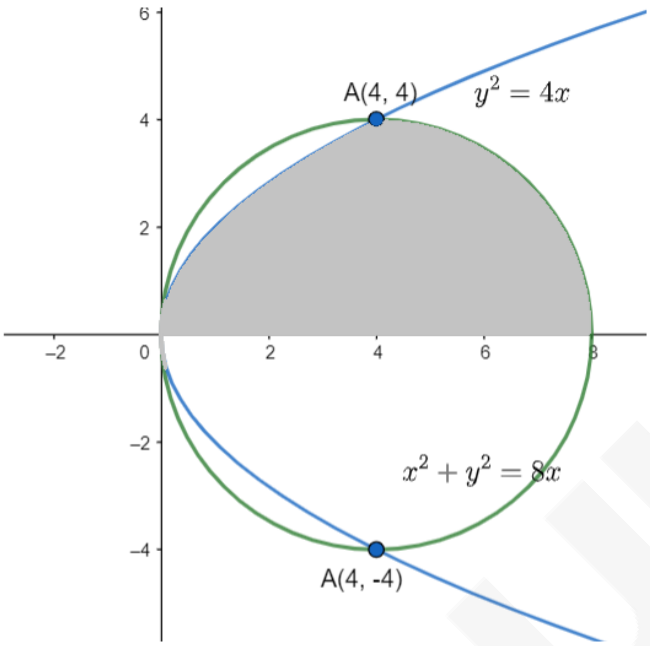
$x = 0$ or $x = 4$

And,

$y^2 = 4.0$, **$y = 0$**

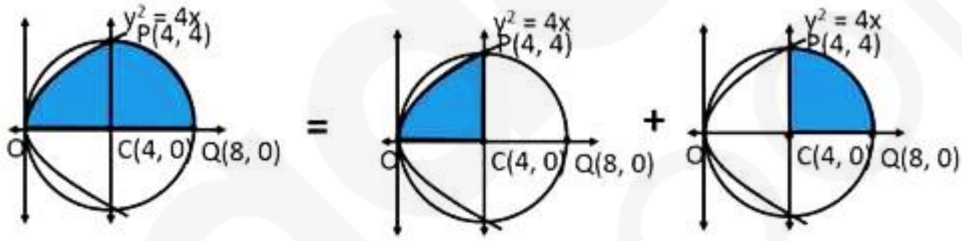
$y^2 = 4.4$, **$y = \pm 4$**

On plotting we will get,



Area required is shared in the given figure,

Area required will be found by:



Therefore,

Area required = Area OPC + Area PCQ

For parabola, y = ±√4x

y = ±2√x

Area OPC $= \int_0^4 2\sqrt{x}\,dx$

$$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$$

$$= 2 \times \frac{2}{3}\left[4^{\frac{3}{2}} - 0\right] \quad = \frac{32}{3}$$

Now, for circle,

$y = \pm\sqrt{8x - x^2}$ Area of PCQ $= \int_4^8 \sqrt{8x - x^2}\,dx$

$$= \int_4^8 \sqrt{-(x^2 - 8x)}\, dx \ = \int_4^8 \sqrt{16 - (x-4)^2}\, dx$$

$$= \int_4^8 \sqrt{4^2 - (x-4)^2}\, dx$$

Now, we know that,

$$\int (a^2 - x^2) dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$$

Therefore,

Area PCQ $= \left[\frac{x-4}{2}\sqrt{4^2 - (x-4)^2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8$

$$= \left[\frac{4}{2}\sqrt{4^2 - 4^2} + 8\sin^{-1}(1) - 0 - 8\sin^{-1} 0\right]$$

$$= \frac{8\pi}{2} - 8 \times 0 \ = \frac{8\pi}{2}$$

$= 4\pi$

$$\boldsymbol{Area\ required = \frac{32}{3} + 4\pi}$$

Solutions (Set - 1)

1. **Given:** AB = 2I, |A| = 2

   Formula used: |AB| = |A|.|B| and |kA| = k^n.|A|, where n is the order of matrix.

   As, AB = 2I

   Taking determinant, we get,

   |AB| = |2I|

   |A|.|B| = 2^3.|I|

   2.|B| = 8.1

   |B| = 4

3. **Given equation:** $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$

   The **order** is the highest numbered derivative in the **equation**, while the degree is the highest power to which a derivative is raised.

   The highest numbered derivative: $\frac{d^2y}{dx^2}$

   Therefore, order = 2

   Also,

   Highest power to which $\frac{d^2y}{dx^2}$ is raised = 1

   Degree = 1

   Hence, order is 2 and degree is 1 of the given differential equation.

7. Let $I = \int \frac{\sec^2 x}{\sqrt{\tan^2 x + 4}} dx$

Let tanx = t

Differentiating both sides, we get,

$\sec^2 x \, dx = dt$

Therefore, we have,

$$I = \int \frac{dt}{\sqrt{t^2 + 4}}$$

$$I = \int \frac{dt}{\sqrt{t^2 + 2^2}}$$

Now, we know the formula that, $\int \frac{dx}{\sqrt{x^2+a^2}} = \log\left|x + \sqrt{x^2 + a^2}\right| + C$

Therefore,

$$I = \log\left|t + \sqrt{t^2 + 4}\right| + C$$

Putting t, we get,

$$I = \log\left|tanx + \sqrt{\tan^2 x + 4}\right| + C$$

8. Let $I = \int \sqrt{1 - \sin 2x} \, dx$

Now, we know that, $\sin^2 x + \cos^2 x = 1$

Therefore,

$$I = \int \sqrt{\sin^2 x + \cos^2 x - \sin 2x} \, dx$$

Also, sin2x = 2.sinx.cosx

Therefore,

$$I = \int \sqrt{\sin^2 x + \cos^2 x - 2.sinx.cosx} dx$$

$$I = \int \sqrt{(sinx - cosx)^2} dx$$

$$I = \int (sinx - cosx) dx$$

Now, we know that, $\int sinx\, dx = -cosx + C$ and $\int cosx\, dx = sinx + C$

Hence, $I = -cosx + sinx + C$

OR

Let $I = \int \sin^{-1} 2x\, dx$

Let $\sin^{-1}2x = t$, $2x = \sin t$

Differentiating both sides, we get,

2dx = cost dt

Therefore,

$$I = \frac{1}{2}\int t.cost\, dt$$

Now, Integrating, by parts, taking t as 1st function and cost as the second function.

We know that,

$$\int u.v\, dx = \left[u.\int v.dx\right] - \int\left[\frac{du}{dx}.\int vdx\right]dx$$

Therefore,

$$I = \frac{1}{2}\left\{\left[t.\int cost.dt\right] - \int\left[\frac{d}{dt}t.\int cost.dt\right]dt\right\}$$

$$I = \frac{1}{2}[t.sint - sint] + C$$

Putting the value of t, we get,

$$I = \frac{1}{2}[2x.\sin^{-1}2x - 2x] + C$$

Hence,

$I = x.\sin^{-1}2x - x + C$

15. To Prove: $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

$$L.H.S = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

Applying, $R_1 \rightarrow R_1 - R_2$, we get,

$$= \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} (a+1)(a-1) & a-1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking (a – 1) common from the first row, we get,

$$= (a-1)\begin{vmatrix} a+1 & 1 & 0 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

Now, applying $R_2 \rightarrow R_2 - R_3$, we get,

$$= (a-1)\begin{vmatrix} a+1 & 1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Taking (a – 1) common from the second row, we get,

$$= (a-1)^2\begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

On expanding, we get,

$= (a-1)^2\,[(a+1)(1-0) - 1(2-0) + 0]$

$= (a-1)^2\,(a-1)$

$= (a-1)^3 = R.H.S$

Hence, Proved.

19. Let $I = \int \frac{3x+5}{x^2+3x-18}dx$

$$I = \int \frac{3x+5}{x^2+(6x-3x)-18}dx$$

$$I = \int \frac{3x+5}{x(x+6)-3(x+6)}dx$$

$$I = \int \frac{3x+5}{(x-3)(x+6)}dx$$

We will use integration by parts for solving this integral,

Therefore,

$$\frac{3x+5}{(x-3)(x+6)} = \frac{A}{x-3} + \frac{B}{x+6}$$

$$\frac{3x+5}{(x-3)(x+6)} = \frac{[A(x+6) + B(x-3)]}{(x-3)(x+6)}$$

Therefore,

3x + 5 = Ax + Bx + 6A – 3B

Comparing the coefficients, we get,

A + B = 3……(1)

6A – 3B = 5……(2)

Multiplying equation 1 by 6 and subtracting equation 2 from it, we get,

6A + 6B – 6A + 3B = 18 – 5

9B = 13

$$B = \frac{13}{9}$$

Putting the value of B in equation 1, we get,

$$A + \frac{13}{9} = 3$$

$$A = 3 - \frac{13}{9} = \frac{14}{9}$$

Now,

$$I = \int \frac{14}{9} \times \frac{1}{x-3} dx + \int \frac{13}{9} \times \frac{1}{x+6} dx$$

$$I = \frac{14}{9} \log|x-3| + \frac{13}{9} \log|x+6| + C$$

Hence,

$$\int \frac{3x+5}{x^2+3x-18} dx = \frac{14}{9} \log|x-3| + \frac{13}{9} \log|x+6| + C$$

21. Given equation: $xdy - ydx = \sqrt{x^2 + y^2}dx$

On - rearranging the term, we get,

$$x\frac{dy}{dx} - y = \sqrt{x^2 + y^2}$$

$$\frac{dy}{dx} - \frac{y}{x} = \sqrt{1 + \frac{y^2}{x^2}}$$

$$\frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}$$

Now, this is a homogenous differential equation of order 1.

Let y = vx and $\frac{dy}{dx} = v + x\frac{dv}{dx}$

Therefore,

$$v + x\frac{dv}{dx} = v + \sqrt{1 + v^2}$$

$$x\frac{dv}{dx} = \sqrt{1 + v^2}$$

$$\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$$

Integrating both sides, we get,

$$\int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$$

Now, we know that,

$\int \frac{dx}{\sqrt{1+x^2}} = \log\left|x + \sqrt{1 + x^2}\right| + C$ and $\int \frac{dx}{x} = \log x$

Therefore,

$$\log\left|v + \sqrt{1 + v^2}\right| = logx + logC$$

$$\left|v + \sqrt{1 + v^2}\right| = Cx$$

Putting the value of y, we get,

$$\left|\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right| = Cx$$

At x = 1, y = 0,

$$|0+1| = C$$

C = 1

Hence,

$$\left|\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}}\right| = x$$

$$\boldsymbol{y + \sqrt{x^2+y^2} = x^2}$$

OR

Given Equation: $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$

Dividing the whole equation by (1 + x^2), we get,

$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{4x^2}{1+x^2}$$

Now, this is a linear equation of the form,

$$\frac{dy}{dx} + Py = Q(x)$$

We know that the solution of this equation is given by,

$$y \times IF = \int (Q \times IF)dx + C$$

Where $IF = e^{\int pdx}$

Therefore, for a given equation,

$$IF = e^{\int \frac{2x}{1+x^2}dx}$$

Let 1 + x^2 = t

Differentiating both sides, we get,

2x dx = dt

Therefore,

$$IF = e^{\int \frac{dt}{t}} = e^{\log t}$$

$IF = e^{\log|1+x^2|} = 1 + x^2$

The solution of the equation:

$$y \times (1 + x^2) = \int \frac{4x^2}{1 + x^2} \times (1 + x^2) dx + C$$

$$y \times (1 + x^2) = \int 4x^2 dx + C$$

$$y \times (1 + x^2) = \frac{4x^3}{3} + C$$

At x = 0, y = 0

Therefore,

$$0 \times 1 = \frac{4}{3} + C$$

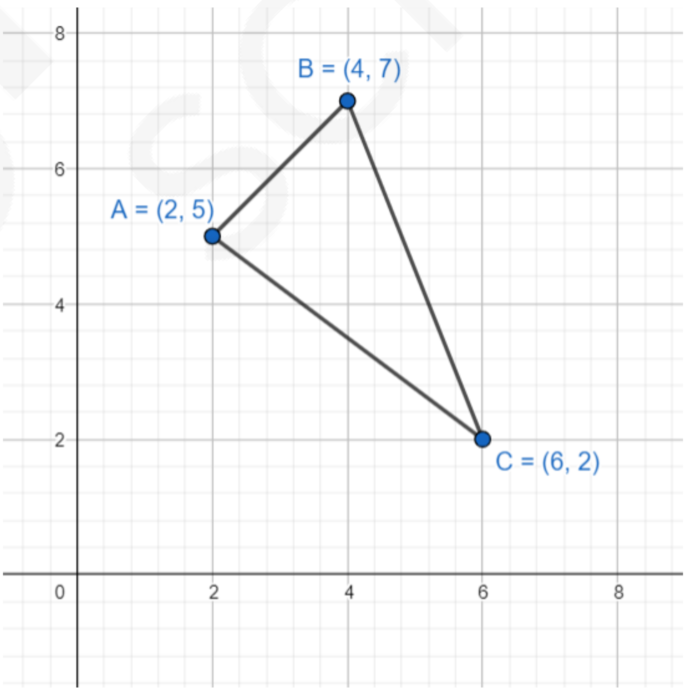
$$C = -\frac{4}{3}$$

Hence,

$$y \times (1 + x^2) = \frac{4}{3}x^3 - \frac{4}{3}$$

$$y = \frac{4}{3}\frac{x^3 - 1}{(1 + x^2)}$$

26. Plotting the points, we get,



Let us first find the equation of sides of the triangle,

Equation of line passing from points P(x_1, y_1) and Q(x_2, y_2) is given by:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Therefore,

Equation of AB: $y - 5 = \frac{7-5}{4-2}(x - 2)$

y – 5 = (x – 2)

y = x + 3

Equation of BC: $y - 7 = \frac{2-7}{6-4}(x - 4)$

$$y - 7 = -\frac{5}{2}(x - 4)$$

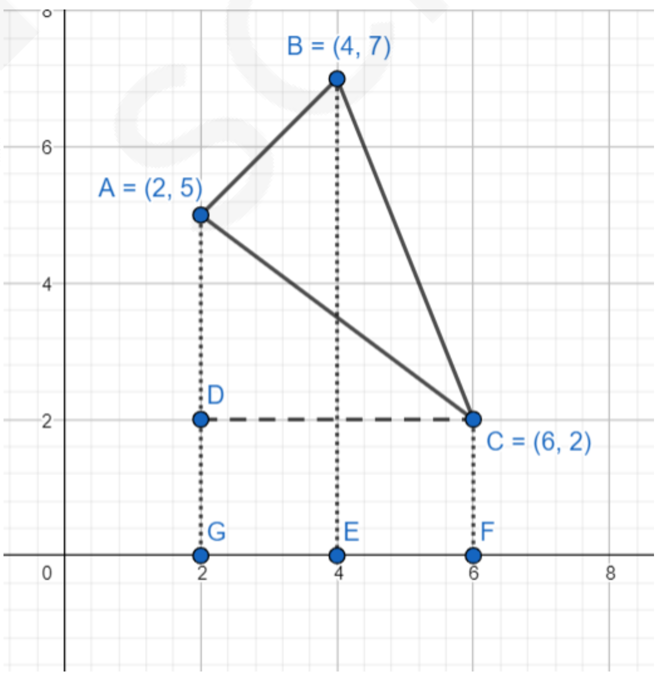
$$y = -\frac{5}{2}x + 17$$

Equation of AC: $y - 2 = \frac{5-2}{2-6}(x - 6)$

$$y - 2 = -\frac{3}{4}(x - 6)$$

$$y = -\frac{3}{4}x + \frac{9}{2}$$

Now, let us look at the limits that we have to take,



Therefore,

$$Area = \int_2^4 (x+3)dx + \int_4^6 \left(-\frac{5}{2}x + 17\right)dx - \int_2^6 \left(-\frac{3}{4}x + \frac{9}{2}\right)dx$$

$$Area = \left[\frac{x^2}{2} + 3x\right]_2^4 + \left[-\frac{5}{2}\frac{x^2}{2} + 17x\right]_4^6 - \left[-\frac{3}{4}\frac{x^2}{2} + \frac{9x}{2}\right]_2^6$$

$$Area = \frac{16}{2} + 12 - \frac{4}{2} - \frac{5}{2} \times \frac{36}{2} + 17 \times 6 + \frac{5}{2} \times \frac{16}{2} - 17 \times 4 + \frac{3}{4} \times \frac{36}{2} - \frac{9}{2} \times 6 - \frac{3}{4}\left(\frac{4}{2}\right) + \frac{9}{2}(2)$$

Area = 21 square units.

OR

Given Curves: C_1: $x^2 + y^2 = 8x$

C_2: $y^2 = 4x$

Let us find the intersection point of the curves.

Putting the value of y^2 from second curve in first curve,

$x^2 + 4x = 8x$

$x^2 – 4x = 0$

$x(x – 4) = 0$

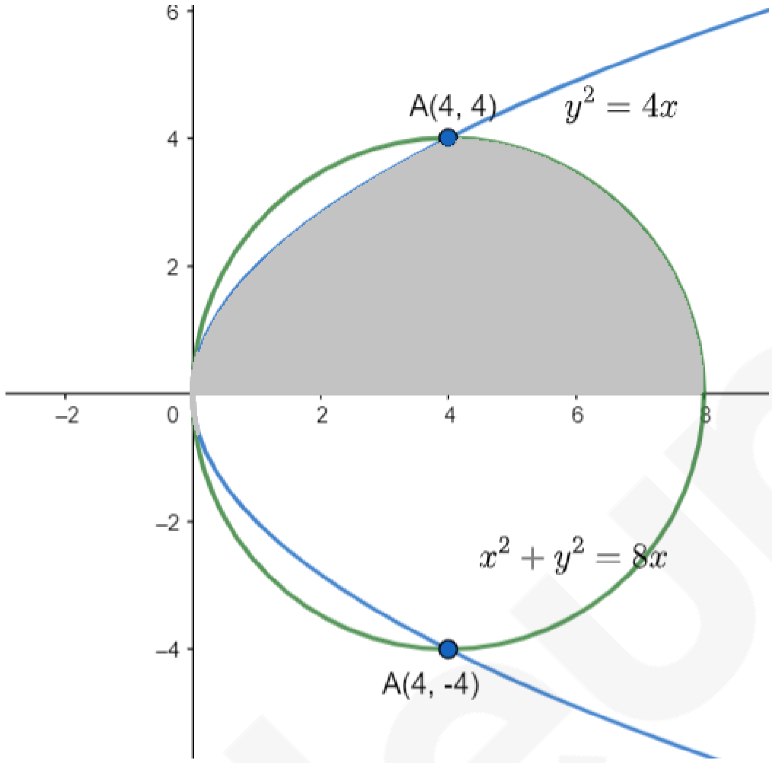
$x = 0$ or $x = 4$

And,

$y^2 = 4.0$, **$y = 0$**

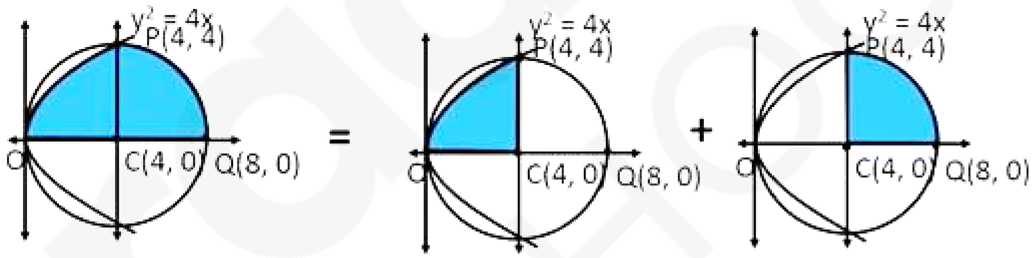
$y^2 = 4.4$, **$y = \pm 4$**

On plotting we will get,



Area required is shared in the given figure,

Area required will be found by:



Therefore,

Area required = Area OPC + Area PCQ

For parabola, y = ±√4x

y = ±2√x

Area OPC $= \int_0^4 2\sqrt{x}\,dx$

$= 2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4$

$= 2 \times \frac{2}{3}\left[4^{\frac{3}{2}} - 0\right]$

$= \frac{32}{3}$

Now, for circle,

$y = \pm\sqrt{8x - x^2}$ Area of PCQ $= \int_4^8 \sqrt{8x - x^2}\,dx$

$= \int_4^8 \sqrt{-(x^2 - 8x)}\,dx$

$= \int_4^8 \sqrt{16 - (x-4)^2}\,dx$

$= \int_4^8 \sqrt{4^2 - (x-4)^2}\,dx$

Now, we know that,

$\int (a^2 - x^2)dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + C$

Therefore,

Area PCQ $= \left[\frac{x-4}{2}\sqrt{4^2 - (x-4)^2} + \frac{4^2}{2}\sin^{-1}\left(\frac{x-4}{4}\right)\right]_4^8$

$= \left[\frac{4}{2}\sqrt{4^2 - 4^2} + 8\sin^{-1}(1) - 0 - 8\sin^{-1}0\right]$

$= \frac{8\pi}{2} - 8 \times 0$

$= \frac{8\pi}{2}$

$= 4\pi$

$\boldsymbol{Area\ required = \frac{32}{3} + 4\pi}$

28. Let E_1 be the event that machine A is defective. $P(E_1) = 1\%$

Let E_2 be the event that machine B is defective. $P(E_2) = 5\%$

Let E_3 be the event that machine C is defective. $P(E_3) = 7\%$

Let A be the event that the item chosen is defective.

P(the defective item is manufactured by A) = $P(A|E_1) = 50\%$

P(the defective item is manufactured by B) = $P(A|E_2) = 30\%$

P(the defective item is manufactured by C) = $P(A|E_3)$ = 20%

Now, According to the Bayes' theorem,

Given E_1, E_2, E_3.......E_n are mutually exclusive and exhaustive events, we can find the conditional probability $P(E_i|A)$ for any event A associated with E_i is given by:

$$P\left(\frac{E_i}{A}\right) = \frac{P\left(\frac{A}{E_i}\right)P(E_i)}{P\left(\frac{A}{E_1}\right)P(E_1) + P\left(\frac{A}{E_2}\right)P(E_2) + \cdots \ldots \ldots + P\left(\frac{A}{E_n}\right)P(E_n)}$$

Therefore,

The probability that defective item is produced by A,

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{100} \times \frac{50}{100}}{\frac{1}{100} \times \frac{50}{100} + \frac{5}{100} \times \frac{30}{100} + \frac{7}{100} \times \frac{20}{100}}$$

$$P\left(\frac{E_1}{100}\right) = \frac{\frac{1}{200}}{\frac{1}{200} + \frac{3}{200} + \frac{7}{500}} = 1000 \times \frac{1}{200} \times \frac{1}{34} = \frac{5}{34}$$

Hence, the probability that the defective item is produced by machine A is $\frac{5}{34}$.