

Mathematics | Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x -coordinate changes by an amount A , the output or y -coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y -intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Know that there are numbers that are not rational, and approximate them by rational numbers.
(8.NS.A)

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. **(8.NS.A.1) (DOK 1)**

- a. Example: [Solution](#) (DOK 3)

Decide whether each of the following numbers is rational or irrational.

If it is rational, explain how you know.

a. $0.\overline{333}$

b. $\sqrt{4}$

c. $\sqrt{2} = 1.414213\dots$

d. $1.\overline{414213}$

e. $\pi = 3.141592\dots$

f. 11

g. $\frac{1}{7} = 0.\overline{142857}$

h. $12.\overline{34565656}$

- b. Example: [Solution](#) (DOK 1)

Represent each of the following rational numbers in fraction form.

a. $0.\overline{333}$

b. $0.\overline{317}$

c. $2.\overline{16}$

- c. Example: [Solution](#) (DOK 3)

Leanne makes the following observation:

I know that

$$\frac{1}{11} = 0.0909\dots$$

where the pattern 09 repeats forever. I also know that

$$\frac{1}{9} = 0.1111\dots$$

where the pattern 11 repeats forever. I wonder if this is a coincidence?

- a. What is the decimal expansion of $\frac{1}{99}$? Use this to explain the patterns Leanne observes for the decimals of $\frac{1}{9}$ and $\frac{1}{11}$.
- b. What is the decimal expansion of $\frac{1}{999}$? Use this to help you calculate the decimal expansions of $\frac{1}{27}$ and $\frac{1}{37}$. How does this relate to Leanne's observations?

d. Example: [Solution](#) (DOK 3)

Tiffany said,

I know that 3 thirds equals 1 so $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$



I also know that $\frac{1}{3} = 0.333\dots$ where the 3's go on forever. But if I add them up as decimals, I get $0.999\dots$

$$\begin{array}{r} 0.333\dots \\ 0.333\dots \\ +0.333\dots \\ \hline 0.999\dots \end{array}$$

I just added up the tenths, then the hundredths, then the thousands, and so on. What went wrong?

- a. Write $0.999\dots$ in the form of a fraction $\frac{a}{b}$ where a and b are whole numbers. Are Tiffany's calculations consistent with what you find? Explain.
 - b. Use Tiffany's idea of adding decimals to write $\frac{1}{3} + \frac{1}{6}$ as a repeating decimal. Can this also be written as a terminating decimal?
- e. Example: For each number, indicate whether it is rational or irrational.

	Rational	Irrational
$\frac{4}{7}$	<input type="checkbox"/>	<input type="checkbox"/>
$\sqrt{30}$	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{21}{\sqrt{4}}$	<input type="checkbox"/>	<input type="checkbox"/>
π	<input type="checkbox"/>	<input type="checkbox"/>
-27	<input type="checkbox"/>	<input type="checkbox"/>

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key																		
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2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.* **(8.NS.A.2) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 3)

For each pair of numbers, decide which is greater without using a calculator. Explain your choices.

a. π^2 or 9

b. $\sqrt{50}$ or $\sqrt{51}$

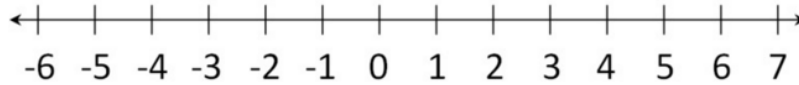
c. $\sqrt{50}$ or 8

d. -2π or -6

- b. Example: [Solution](#) (DOK 1)

Without using your calculator, label approximate locations for the following numbers on the number line.

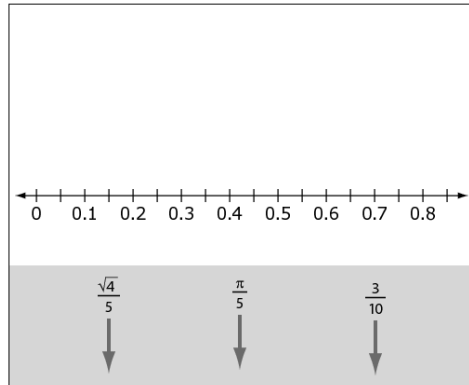
- a. π
- b. $-(\frac{1}{2} \times \pi)$
- c. $2\sqrt{2}$
- d. $\sqrt{17}$



c. Example: [Solution](#) (DOK 1)

Place $\sqrt{28}$ on a number line, accurate to one decimal point.

d. Example: Write each number on its correct position along the number line.



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
# 1	1	NS	A	2	8.NS.A.2	6	

e. Example: ([Former NAEP question](#)) (DOK 1)

$\sqrt{19}$ is between which of the following pairs of numbers?

- A. 4 and 5
- B. 9 and 10
- C. 18 and 20
- D. 360 and 362

Answer: A. 4 and 5

Work with radicals and integer exponents. (8.EE.A)

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.* **(8.EE.A.1) (DOK 1)**

- a. Example: [Solution](#) (DOK 2)

Marco and Seth are lab partners studying bacterial growth. They were surprised to find that the population of the bacteria doubled every hour.

a. The table shows that there were 2,000 bacteria at the beginning of the experiment. What was the size of population of bacteria after 1 hour? After 2, 3 and 4 hours? Enter this information into the table:

Hours into study	.	.	.	0	1	2	3	4
Population (thousands)				2				

b. If you know the size of the population at a certain time, how do you find the population one hour later?

c. Marco said he thought that they could use the equation $P = 2t + 2$ to find the population at time t . Seth said he thought that they could use the equation $P = 2 \cdot 2^t$. Decide whether either of these equations produces the correct populations for $t = 1, 2, 3, 4$.

d. Assuming the population doubled every hour before the study began, what was the population of the bacteria 1 hour *before* the students started their study? What about 3 hours before?

e. If you know the size of the population at a certain time, how do you find the population one hour *earlier*?

f. What number would you use to represent the time 1 hour before the study started? 2 hours before? 3 hours before? Finish filling in the table if you haven't already.

g. Now use Seth's equation to find the population of the bacteria 1 hour before the study started. Use the equation to find the population of the bacteria 3 hours before. Do these values produce results consistent with the arithmetic you did earlier?

h. Use the context to explain why it makes sense that $2^{-n} = (\frac{1}{2})^n = \frac{1}{2^n}$. That is, describe why, based on the population growth, it makes sense to define 2 raised to a negative integer exponent as repeated multiplication by $\frac{1}{2}$.

- b. Example: [Solution](#) (DOK 2)

The average mass of an adult human is about 65 kilograms while the average mass of an ant is approximately 4×10^{-3} grams. The total human population in the world is approximately 6.84 billion, and it is estimated there are currently about 10,000 trillion ants alive.¹

Based on these values, how does the total the total mass of all living ants compare to the total mass of all living humans?

1: Holldobler, B., E. Wilson, *Journey to the Ants: A Story of Scientific Exploration* (London, England: The Belknap Press of Harvard University, 1994).

c. Example: [Solution](#) (DOK 2)

In this problem a represents a positive number.

The quotient rule for exponents says that if m and n are positive integers with $m > n$, then

$$\frac{c^m}{c^n} = c^{m-n}.$$

After explaining to yourself why this is true, complete the following exploration of the quotient rule when $m \leq n$:

- What expression does the quotient rule provide for $\frac{c^m}{c^n}$ when $m = n$?
 - If $m = n$, simplify $\frac{c^m}{c^n}$ without using the quotient rule.
 - What do parts (a) and (b) above suggest is a good definition for c^0 ?
 - What expression does the quotient rule provide for $\frac{c^0}{c^n}$?
 - What expression do we get for $\frac{c^0}{c^n}$ if we use the value for c^0 found in part (c)?
 - Using parts (d) and (e), propose a definition for the expression c^{-n} .
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. **(8.EE.A.2) (DOK 1)**
- Example: A square with side length s has an area of 324 square centimeters. This equation shows the area of the square.

$$s^2 = 324$$

What is the side length of the square in centimeters?

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#3	1	EE	B	1	8.EE.A.2	N/A	18

- Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9 , and*

determine that the world population is more than 20 times larger. **(8.EE.A.3) (DOK 1,2)**

a. Example: [Solution](#) (DOK 2)

An ant has a mass of approximately 4×10^{-3} grams and an elephant has a mass of approximately 8 metric tons.

a. How many ants does it take to have the same mass as an elephant?

b. An ant is 10^{-1} cm long. If you put all these ants from your answer to part (a) in a line (front to back), how long would the line be? Find two cities in the United States that are a similar distance apart to illustrate this length.

Note: 1 kg = 1000 grams, 1 metric ton = 1000 kg, 1 m = 100 cm, 1 km = 1000 m

b. Example: [Solution](#) (DOK 2)

A penny is about $\frac{1}{16}$ of an inch thick.

a. In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

b. In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

c. The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?

c. Example: [Solution](#) (DOK 3)

It is said that the average person blinks about 1000 times an hour. This is an *order-of-magnitude* estimate, that is, it is an estimate given as a power of ten. Consider:

- 100 blinks per hour, which is about two blinks per minute.
- 10,000 blinks per hour, which is about three blinks per second.

Neither of these are reasonable estimates for the number of blinks a person makes in an hour. Make order-of-magnitude estimates for each of the following:

a. Your age in hours.

b. The number of breaths you take in a year.

c. The number of heart beats in a lifetime.

d. The number of basketballs that would fill your classroom.

Can you think of others questions like these?

d. Example: Kyle was given the following problem to solve.

A company sells baseball gloves and bats. The gloves regularly cost \$30 and the bats regularly cost \$90. The gloves are on sale for \$4 off, and the bats are on sale for 10% off. The goal is to sell

\$1200 worth of bats and gloves each week. Last week, the store sold 14 gloves and 9 bats.

Did the store meet its goal?

The steps Kyle used to solve the problem are shown. Select the first step that shows an error.

Step 1:

$$\begin{array}{r} \$30 \\ - \$4 \\ \hline \$26 \end{array}$$

Step 2:

$$\begin{array}{r} \$26 \\ \times 14 \\ \hline \$364 \end{array}$$

Step 3:

$$\begin{array}{r} \$90 \\ \div 0.9 \\ \hline \$100 \end{array}$$

Step 4:

$$\begin{array}{r} \$100 \\ \times 9 \\ \hline \$900 \end{array}$$

Step 5: Yes, the store met its goal.

$$\begin{array}{r} \$900 \\ + \$364 \\ \hline \$1264 \end{array}$$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#27	3	EE	E	2	7.EE.B.3	3, 6	Step 3

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. **(8.EE.A.4) (DOK 1,2)**
 - a. Example: [Solution](#) (DOK 3)

This headline appeared in a newspaper.

Every day 7% of Americans eat at Giantburger restaurants

Decide whether this headline is true using the following information.

- There are about 8×10^3 Giantburger restaurants in America.
- Each restaurant serves on average 2.5×10^3 people every day.
- There are about 3×10^8 Americans.

Explain your reasons and show clearly how you figured it out.

b. Example: [Solution](#) (DOK 2)

The average mass of an adult human is about 65 kilograms while the average mass of an ant is approximately 4×10^{-3} grams. The total human population in the world is approximately 6.84 billion, and it is estimated there are currently about 10,000 trillion ants alive.¹

Based on these values, how does the total the total mass of all living ants compare to the total mass of all living humans?

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c. Example: [Solution](#) (DOK 2)

A penny is about $\frac{1}{16}$ of an inch thick.

a. In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

b. In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?

c. The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?

d. Example: [Solution](#) (DOK 3)

- a. A computer has 128 gigabytes of memory. One gigabyte is 1×10^9 bytes. A floppy disk, used for storage by computers in the 1970's, holds about 80 kilobytes. There are 1000 bytes in a kilobyte. How many kilobytes of memory does a modern computer have? How many gigabytes of memory does a floppy disk have? Express your answers both as decimals and using scientific notation.
- b. George told his teacher that he spent over 21,000 seconds working on his homework. Express this amount using scientific notation. What would be a more appropriate unit of time for George to use? Explain and convert to your new units.
- c. A certain swimming pool contains about 3×10^7 teaspoons of water. Choose a more appropriate unit for reporting the volume of water in this swimming pool and convert from teaspoons to your chosen units.
- d. A helium atom has a diameter of about 62 picometers. There are one trillion picometers in a meter. The diameter of the sun is about 1,400,000 km. Express the diameter of a helium atom and of the sun in meters using scientific notation. About many times larger is the diameter of the sun than the diameter of a helium atom?
- e. Example: Approximately 7.5×10^5 gallons of water flow over a waterfall each second. There are 8.6×10^4 seconds in 1 day. Select the approximate number of gallons of water that flow over the waterfall in 1 day.

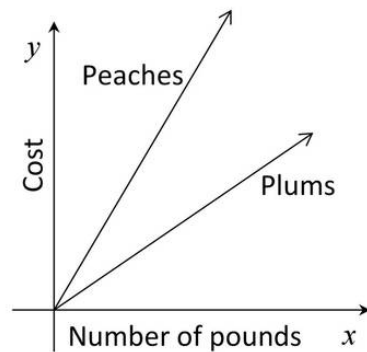
- (A) 6.45×10^{21}
- (B) 6.45×10^{20}
- (C) 6.45×10^{10}
- (D) 6.45×10^9

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#4	1	EE	B	1	8.EE.A.4	N/A	C

Understand the connections between proportional relationships, lines, and linear equations.
(8.EE.B)

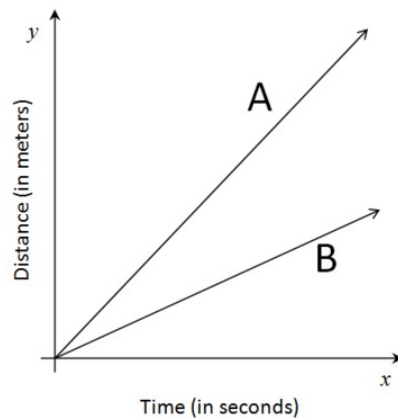
5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* (8.EE.B.5) (DOK 1,2,3)
- a. Example: [Solution](#) (DOK 3)

The graphs below show the cost y of buying x pounds of fruit. One graph shows the cost of buying x pounds of peaches, and the other shows the cost of buying x pounds of plums.



- a. Which kind of fruit costs more per pound? Explain.
 - b. Bananas cost less per pound than peaches or plums. Draw a line alongside the other graphs that might represent the cost y of buying x pounds of bananas.
- b. Example: [Solution](#) (DOK 3)

The graphs below show the distance two cars have traveled along the freeway over a period of several seconds. Car A is traveling 30 meters per second.



Which equation from those shown below is the best choice for describing the distance traveled by car B after x seconds? Explain.

- a. $y = 85x$
- b. $y = 60x$
- c. $y = 30x$
- d. $y = 15x$

c. Example: [Solution](#) (DOK 3)

Nia and Trey both had a sore throat so their mom told them to gargle with warm salt water.

. Nia mixed 1 teaspoon salt with 3 cups water.

. Trey mixed $\frac{1}{2}$ teaspoon salt with $1\frac{1}{2}$ cups of water.

Nia tasted Trey's salt water. She said,

"I added more salt so I expected that mine would be more salty, but they taste the same."

- Explain why the salt water mixtures taste the same.
- Find an equation that relates s , the number of teaspoons of salt, with w , the number of cups of water, for both of these mixtures.
- Draw the graph of your equation from part b.
- Your graph in part c should be a line. Interpret the slope as a unit rate.

d. Example: [Solution](#) (DOK 2)

Lena paid \$18.96 for 3 pounds of coffee.

- What is the cost per pound for this coffee?
- How many pounds of coffee could she buy for \$1.00?
- Draw a graph in the coordinate plane of the relationship between the number of pounds of coffee and the total cost.
- In this situation, what is the meaning of the slope of the line you drew in part (c)?

e. Example: [Solution](#) (DOK 3)

Kell works at an after-school program at an elementary school. The table below shows how much money he earned every day last week.

Time worked	1.5 hours	2.5 hours	4 hours
Money earned	\$12.60	\$21.00	\$33.60

Mariko has a job mowing lawns that pays \$7 per hour.

- Who would make more money for working 10 hours? Explain or show work.
- Draw a graph that represents y , the amount of money Kell would make for working x hours, assuming he made the same hourly rate he was making last week.
- Using the same coordinate axes, draw a graph that represents y , the amount of money Mariko would make for working x hours.
- How can you see who makes more per hour just by looking at the graphs? Explain.

f. Example: [Solution](#) (DOK 3)

Anna and Jason have summer jobs stuffing envelopes for two different companies. Anna earns \$14 for every 400 envelopes she finishes. Jason earns \$9 for every 300 envelopes he finishes.

- Draw graphs and write equations that show the earnings, y , as functions of the number of envelopes stuffed, n , for Anna and Jason.
 - Who makes more from stuffing the same number of envelopes? How can you tell this from the graph?
 - Suppose Anna has savings of \$100 at the beginning of the summer and she saves all her earnings from her job. Graph her savings as a function of the number of envelopes she stuffed, n . How does this graph compare to her previous earnings graph? What is the meaning of the slope in each case?
- g. Example: This graph shows a proportional relationship between the amount of money in Jack's savings account and the number of weeks Jack has been saving money.

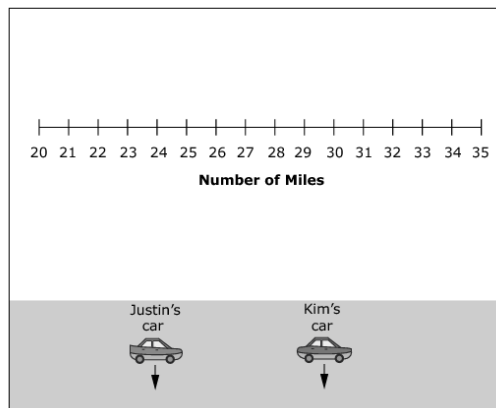


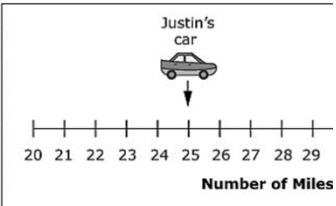
Select the statement that correctly reflects what is shown in the graph.

- (A) The slope of the line is $\frac{6}{1}$, so Jack's savings rate is \$6 every week.
- (B) The slope of the line is $\frac{6}{1}$, so Jack's savings rate is \$1 every 6 weeks.
- (C) The slope of the line is $\frac{1}{6}$, so Jack's savings rate is \$6 every week.
- (D) The slope of the line is $\frac{1}{6}$, so Jack's savings rate is \$1 every 6 weeks.

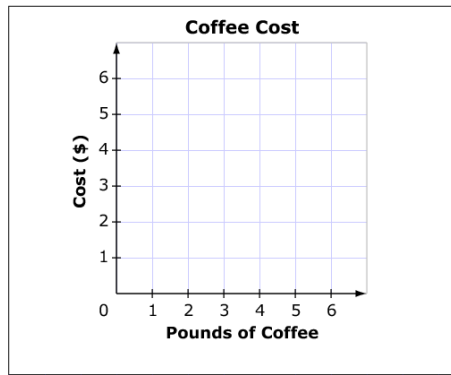
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#29	1	EE	C	1	8.EE.B.5	N/A	A

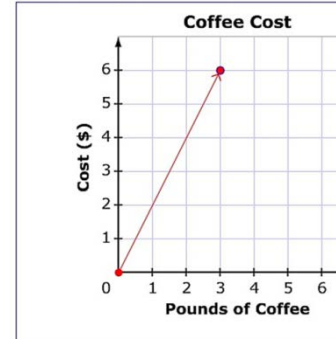
- h. Example: Justin's car can travel $77\frac{1}{2}$ miles with $3\frac{1}{10}$ gallons of gas.
 Kim's car can travel $99\frac{1}{5}$ miles with $3\frac{1}{5}$ gallons of gas.
 Draw the cars on the number line to show the number of miles each car can travel with 1 gallon of gas.



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#26	2	EE	A	2	8.EE.B.5	6	

- i. Example: Coffee costs \$2.00 per pound at a coffee shop.
 Draw a ray that shows the proportional relationship between the number of pounds of coffee purchased and the total cost.



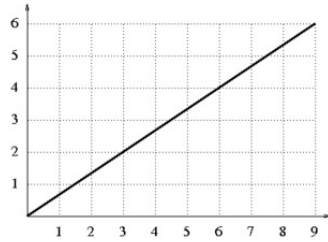
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#8	1	EE	C	1	8.EE.B.5	N/A	

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b . **(8.EE.B.6) (DOK 1,2,3)**
- a. Example: [Solution](#) (DOK 3)

The slope between two points is calculated by finding the change in y -values and dividing by the change in x -values. For example, the slope between the points $(7, -15)$ and $(-8, 22)$ can be computed as follows:

- The difference in the y -values is $-15 - 22 = -37$.
- The difference in the x -values is $7 - (-8) = 15$.
- Dividing these two differences, we find that the slope is $-\frac{37}{15}$.

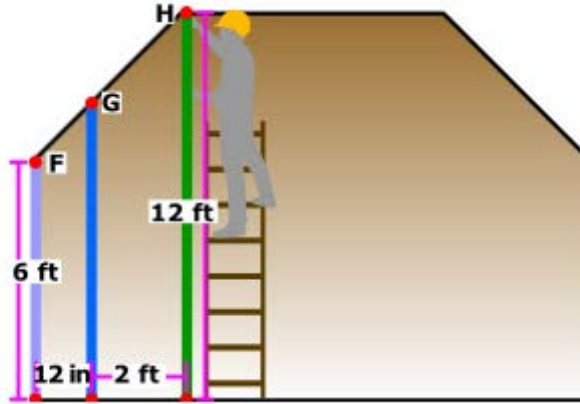
Eva, Carl, and Maria are computing the slope between pairs of points on the line shown below.



Eva finds the slope between the points $(0,0)$ and $(3,2)$. Carl finds the slope between the points $(3,2)$ and $(6,4)$. Maria finds the slope between the points $(3,2)$ and $(9,6)$. They have each drawn a triangle to help with their calculations (shown below).



- Which student has drawn which triangle? Finish the slope calculation for each student. How can the differences in the x - and y -values be interpreted geometrically in the pictures they have drawn?
 - Consider any two points (x_1, y_1) and (x_2, y_2) on the line shown above. Draw a triangle like the triangles drawn by Eva, Carl, and Maria. What is the slope between these two points? Why should this slope be the same as the slopes calculated by the three students?
- b. Example: A construction worker is using wooden beams to reinforce the back wall of a room.



Determine the height, in feet, of the beam that ends at point G. Explain how you found your answer.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#3	3	NS	3A, 3B	2	8.EE.6		See Below

Sample Top-Score Response:

Two right triangles can be formed by extending a line from point F that is perpendicular to the beam that ends at point H.

Now you have a right triangle with a height of 6 and a base of 3 and a smaller right triangle with a height of x and a base of 1.

The larger triangle and the smaller triangle are similar since they are both right triangles and share an angle. The proportion $3:1 = 6:x$ can be used to find the smaller portion of the beam ending at point G. Solving this proportion gives $x=2$. The remaining portion of the beam is 6 feet so the length of the beam that ends at point G is 8 feet.

For full credit (2 points):

The response demonstrates a full and complete understanding of communicating reasoning. The response contains the following evidence:

- The student determines the height of the beam that ends at point G is 8 feet.

AND

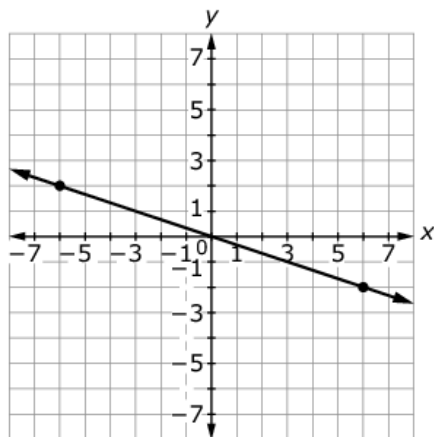
- The student provides sufficient reasoning to support this conclusion.

For partial credit (1 point):

The response demonstrates a partial understanding of communicating reasoning. The response contains the following evidence:

- The student determines the height of the beam that ends at point G is 8 feet but, the student does not provide sufficient reasoning to support this conclusion. OR

- The student determines an incorrect length of the beam that ends at point G but provides reasoning to support this answer that contains a minor conceptual or computation error.
- c. Example: Consider this graph of a line.



Write an equation for the line.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#7	1	EE	C	1	8.EE.B.6	N/A	$Y = \frac{1}{3}x$

Analyze and solve linear equations and pairs of simultaneous linear equations. (8.EE.C)

7. Solve linear equations in one variable.
 - a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).
 - b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

(8.EE.C.7) (DOK 1,2)

1. Example: [Solution](#) (DOK 2)

In elementary school, students often draw pictures of the arithmetic they do. For instance, they might draw the following picture for the problem $2 + 3$:

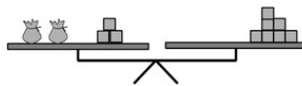


In this picture, each square represents a tile.

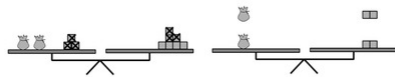
We can do the same thing for algebraic expressions, but we need to be careful about how we represent the unknown. If we assume that an unknown number of tiles x are contained in a bag, we could draw the following picture for $2x + 3$:



When we have an equation to solve, we assume that the two sides of the equation are equal. We can represent this by showing them level on a balance. For example, we the equation $2x + 3 = 7$ could be shown as:



When we solve equations, we can add, subtract, multiply or divide both sides of the equation by the same thing in order to maintain the equality. This can be shown in pictures by keeping the balance level. For example, we could solve the equation $2x + 3 = 7$ using pictures by first removing (subtracting) 3 from each side, and then splitting (dividing) the remaining blocks into two equal groups:



From this picture, we can see that, in order to keep the balance level, each bag must contain 2 tiles, which means that $x = 2$.

- Solve $5x + 1 = 2x + 7$ in two ways: symbolically, the way you usually do with equations, and also with pictures of a balance. Show how each step you take symbolically is shown in the pictures.
- Solve the equation $4x = x + 1$ using pictures and symbols. Discuss any issues that arise.
- What issues arise when you try to solve the equation $2 = 2x - 4$ using pictures? Do the same issues arise when you solve this equation symbolically?
- Make up a linear equation that has no solutions. What would happen if you solved this equation with pictures? How is this different than an equation that has infinitely many solutions?

Use pictures to show why the following solution to the equation $2x + 4 = 10$ is incorrect:

$$\begin{array}{r} \underline{2x + 4 = 10} \\ 2 \quad \quad 2 \\ x + 4 = 5 \\ \underline{-4 \quad -4} \\ x = 1 \end{array}$$

2. Example: [Solution](#) (DOK 1)

Without solving them, say whether these equations have a positive solution, a negative solution, a zero solution, or no solution.

- a. $3x = 5$
- b. $5z + 7 = 3$
- c. $7 - 5w = 3$
- d. $4a = 9a$
- e. $y = y + 1$

3. Example: [Solution](#) (DOK 1)

You have a coupon worth \$18 off the purchase of a scientific calculator. At the same time the calculator is offered with a discount of 15%, but no further discounts may be applied. For what tag price on the calculator do you pay the same amount for each discount?

4. Example: [Solution](#) (DOK 1)

For a science project, Sammy observed a chipmunk and a squirrel stashing acorns in holes. The chipmunk hid 3 acorns in each of the holes it dug. The squirrel hid 4 acorns in each of the holes it dug. They each hid the same number of acorns, although the squirrel needed 4 fewer holes. How many acorns did the chipmunk hide?

5. Example: For each linear equation in the table, select whether the equation has no solution, one solution, or infinitely many solutions.

Equation	No Solution	One Solution	Infinitely Many Solutions
$36x + 24 = 12(x + 2 + 2x)$			
$x = x + 1$			
$-12(x + 2) = -14x + 2$			

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#30	1		1D	2	8.EE.7a		a. Infinitely many solutions b. No solution c. One solution

6. Example: Consider this equation.

$$c = ax - bx$$

Joseph claims that if a , b , and c are non-negative integers, then the equation has exactly one solution for x .

Select **all** cases that show Joseph's claim is **incorrect**.

- a. $a - b = 1, c = 0$
- b. $a = b, c \neq 0$
- c. $a = b, c = 0$
- d. $a = b, c = 0$
- e. $a - b = 1, c \neq 1$
- f. $a \neq b, c = 0$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#18	3	EE	D	3	8.EE.C.7a	1, 3	<input type="checkbox"/> $a - b = 1, c =$ <input checked="" type="checkbox"/> $a = b, c \neq 0$ <input checked="" type="checkbox"/> $a = b, c = 0$ <input type="checkbox"/> $a - b = 1, c \neq$ <input type="checkbox"/> $a \neq b, c = 0$

7. Example: Write a number into each box to create an equation that has no solution.

$$8x - 3x + 2 - x = \boxed{}x + \boxed{}$$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#14	1	EE	D	2	8.EE.C.7a	N/A	$8x - 3x + 2 - x = \boxed{4}x + \boxed{8}$

8. Analyze and solve pairs of simultaneous linear equations.

- a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
- b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution*

because $3x + 2y$ cannot simultaneously be 5 and 6.

- c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. **(8.EE.C.8) (DOK 1,2,3)**

1. Example: [Solution](#) (DOK 2)

Kimi and Jordan are each working during the summer to earn money in addition to their weekly allowance, and they are saving all their money. Kimi earns \$9 an hour at her job, and her allowance is \$8 per week. Jordan earns \$7.50 an hour, and his allowance is \$16 per week.

- a. Complete the two tables shown below.

Number of hours worked in a week, h	0	1	2	3	4	5	6	7
Kimi's weekly total savings, K								

Number of hours worked in a week, h	0	1	2	3	4	5	6	7
Jordan's weekly total savings, J								

- b. Write an equation that can be used to calculate the total of Kimi's allowance and job earnings at the end of one week given the number of hours she works.
- c. Write an equation that can be used to calculate the total of Jordan's allowance and job earnings at the end of one week given the number of hours worked.
- d. Sketch the graphs of your two equations on one pair of axes.
- e. Jordan wonders who will save more money in a week if they both work the same number of hours. Write an answer for him.

2. Example: [Solution](#) (DOK 2)

You are a representative for a cell phone company and it is your job to promote different cell phone plans.

a. Your boss asks you to visually display three plans and compare them so you can point out the advantages of each plan to your customers.

- Plan A costs a basic fee of \$29.95 per month and 10 cents per text message
- Plan B costs a basic fee of \$90.20 per month and has unlimited text messages
- Plan C costs a basic fee of \$49.95 per month and 5 cents per text message
- All plans offer unlimited calling
- Calling on nights and weekends are free
- Long distance calls are included

b. A customer wants to know how to decide which plan will save her the most money. Determine which plan has the lowest cost given the number of text messages a customer is likely to send.

3. Example: [Solution](#) (DOK 3)

Ivan's furnace has quit working during the coldest part of the year, and he is eager to get it fixed. He decides to call some mechanics and furnace specialists to see what it might cost him to have the furnace fixed. Since he is unsure of the parts he needs, he decides to compare the costs based only on service fees and labor costs. Shown below are the price estimates for labor that were given to him by three different companies. Each company has given the same time estimate for fixing the furnace.

- Company A charges \$35 per hour to its customers.
- Company B charges a \$20 service fee for coming out to the house and then \$25 per hour for each additional hour.
- Company C charges a \$45 service fee for coming out to the house and then \$20 per hour for each additional hour.

For which time intervals should Ivan choose Company A, Company B, Company C? Support your decision with sound reasoning and representations. Consider including equations, tables, and graphs.

4. Example: [Solution](#) (DOK 2)

Consider the equation $5x - 2y = 3$. If possible, find a second linear equation to create a system of equations that has:

- Exactly 1 solution.
- Exactly 2 solutions.
- No solutions.
- Infinitely many solutions.

Bonus Question: In each case, how many such equations can you find?

5. Example: [Solution](#) (DOK 1)

A type of pasta is made of a blend of quinoa and corn. The pasta company is not disclosing the percentage of each ingredient in the blend but we know that the quinoa in the blend contains 16.2% protein, and the corn in the blend contains 3.5% protein. Overall, each 57 gram serving of pasta contains 4 grams of protein. How much quinoa and how much corn is in one serving of the pasta?

6. Example: [Solution](#) (DOK 3)

The local swim center is making a special offer. They usually charge \$7 per day to swim at the pool. This month swimmers can pay an enrollment fee of \$30 and then the daily pass will only be \$4 per day.

- Suppose you do not take the special offer. Write an equation that represents the amount of money you would spend based on how many days you go to the pool if the passes were bought at full price.
- Write a second equation that represents the amount of money you would spend if you decided to take the special offer.
- Graph your two equations from part (a) and (b).
- After how many days of visiting the pool will the special offer be a better deal? How can you tell algebraically? How can you see this graphically?
- You only have \$60 to spend for the summer on visiting this pool. Which offer would you take? Explain.

7. Example: [Solution](#) (DOK 3)

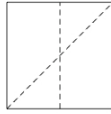
- Draw the two lines that intersect only at the point $(1, 4)$. One of the lines should pass through the point $(0, -1)$.
- Write the equation of each of the lines you created in part (a).
- If we consider two or more equations together we have a *system of equations*. A *solution to a system of equations in x and y* is a pair of values a and b for x and y that make all of the equations true.

Do you think such a solution exists for the system of equations in part (b)? Explain.

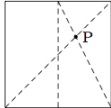
- Can you determine whether a system of equations has a solution by looking at the graph of the equations? Explain.

8. Example: [Solution](#) (DOK 3)

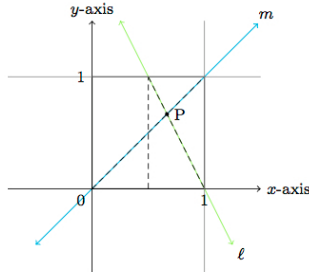
Suppose we take a square piece of paper and fold it in half vertically and diagonally, leaving the creases shown below:



Next we make a fold that joins the top of the vertical crease to the bottom right corner, leaving the crease shown below. The point P is the intersection of this new crease with the first diagonal fold.



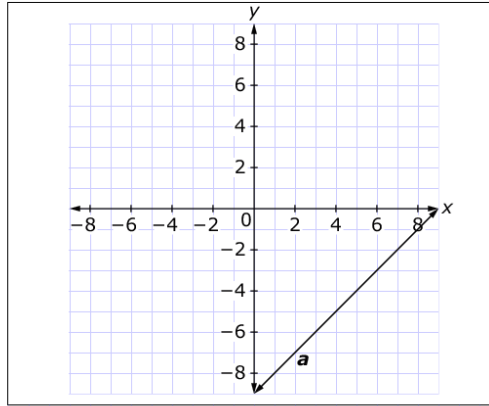
a. Place the lower left corner of the square at $(0,0)$ on a coordinate grid with the upper right corner at $(1,1)$ as pictured below:



The lines l and m labelled in the picture contain the two diagonal folds. Find equations defining l and m and use these to calculate the coordinates of the point P .

b. Explain how to use part (a) in order to fold the square into thirds.

9. Example: Line a is shown on the graph. Construct line b on the graph so that:
- Line a and b represent a system of linear equations with a solution of $(7, -2)$.
 - The slope of line b is greater than -1 and less than 0 .
 - The y -intercept of line b is positive.



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#22	2	EE	D, B	2	8.EE.C.8	1	

10. Example: Joe solved this linear system correctly.

$$6x + 3y = 6$$

$$y = -2x + 2$$

These are the last two steps of his work.

$$6x - 6x + 6 = 6$$

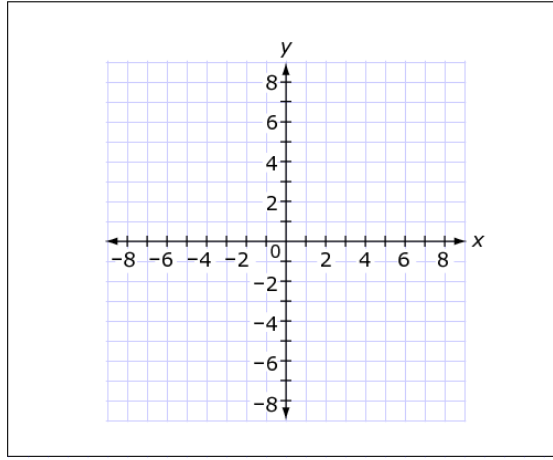
$$6 = 6$$

Which statement about this linear system must be true?

- X must equal 6
- Y must equal 6
- There is no solution to this system
- There are infinitely many solutions to this system

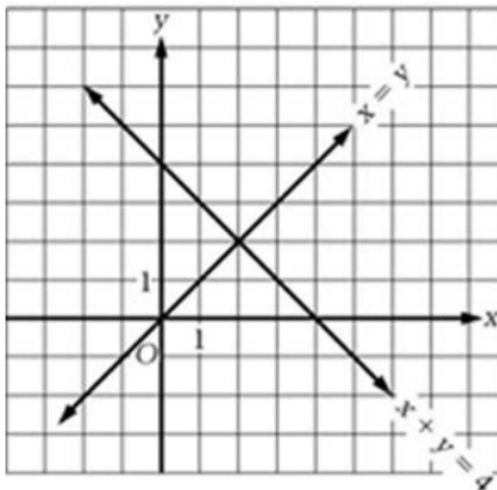
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#13	1	EE	D	2	8.EE.C.8b	N/A	D

11. Example: Graph a system of two equations that has a single solution of $(-2, -3)$.



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#12	1	EE	D	1	8.EE.C.8a	4	

12. Example: ([Former NAEP question](#)) (DOK 1)



Which point is the solution to both equations shown on the graph above?

- F. (0, 0)
- G. (0, 4)
- H. (1, 1)
- I. (2, 2)
- J. (4, 0)

Answer: I. (2, 2)

13. Example: ([Former NAEP question](#)) (DOK 1)

At Jorge's local video store, "New Release" video rentals cost \$2.50 each and "Movie Classic" video rentals cost \$1.00 each (including tax). On Saturday evening, Jorge rented 5 videos and spent a total of \$8.00.

How many of the 5 rentals were New Releases and how many were Movie Classics?

New Releases _____

Movie Classics _____

Answer: New Releases: 2, Movie Classics: 3

14. Example: ([Former NAEP question](#)) (DOK 2)

What is the solution to the system of equations $\begin{cases} 3x - 2y = -7 \\ x + y = 11 \end{cases}$?

Answer: x = _____ y = _____

Answer: x=3 , y=8

Define, evaluate, and compare functions. (8.F.A)

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.¹ (8.F.A.1) (DOK 1,2)
 - a. Example: [Solution](#) (DOK 3)

A certain business keeps a database of information about its customers.

a. Let C be the rule which assigns to each customer shown in the table his or her home phone number. Is C a function? Explain your reasoning.

Customer Name	Home Phone Number
Heather Baker	3105100091
Mike London	3105200256
Sue Green	3234132598
Bruce Swift	3234132598
Michelle Metz	2138061124

b. Let P be the rule which assigns to each phone number in the table above, the customer name(s) associated with it. Is P a function? Explain your reasoning.

c. Explain why a business would want to use a person's social security number as a way to identify a particular customer instead of their phone number.

- b. Example: [Solution](#) (DOK 3)

Given below is a table that gives the populations of foxes and rabbits in a national park over a 12 month period. Note that each value of t corresponds to the beginning of the month.

t , month	1	2	3	4	5	6	7	8	9	10	11	12
R , number of rabbits	1000	750	567	500	567	750	1000	1250	1433	1500	1433	1250
F , number of foxes	150	143	125	100	75	57	50	57	75	100	125	143

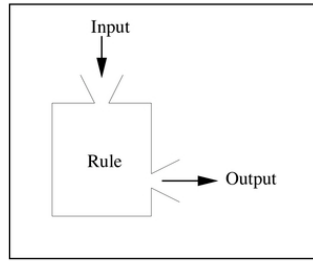
a. According to the data in the table, is F a function of R ? Is R a function of F ? Explain.

b. Are either R or F functions of t ? Explain.

- c. Example: [Solution](#) (DOK 2)

¹ Function notation is not required in Grade 8.

A function machine takes an input, and based on some rule produces an output.



The tables below show some input-output pairs for different functions. For each table, describe a function rule in words that would produce the given outputs from the corresponding inputs. Then fill in the rest of the table values as inputs and outputs which are consistent with that rule.

a. Input values can be any English word. Output values are letters from the English alphabet.

input	cat	house	you	stem	.	.	.
output	t	e	u		z		

b. Input values can be any real number. Output values can be any real number.

input	2	5	-1.53	0	-4	.	.
output	7	10	3.47	5		8	

c. Input values can be any whole number. Output values can be any whole number.

input	1	2	3	4	5	6	7
output	2	1	4	.			

d. Input values can be any whole number between 1 and 365. Output values can be any month of the year.

input	25	365	35	95	330	66	
output	January	December	February	April	November	.	October

For at least one of the tables, describe a second rule which fits the given pairs but ultimately produces different pairs than the first rule for the rest of the table.

d. Example: [Solution](#) (DOK 2)

The following table shows the amount of garbage that was produced in the US each year between 2002 and 2010 (as reported by the EPA).

t (years)	2002	2003	2004	2005	2006	2007	2008	2009	2010
G (million tons)	239	242	249	254	251	255	251	244	250

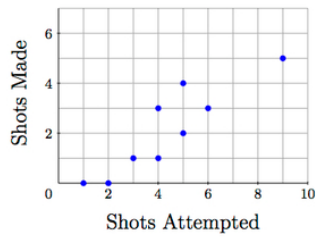
Let's define a function which assigns to an input t (a year between 2002 and 2010) the total amount of garbage, G , produced in that year (in million tons). To find these values, you can look them up in the table.

- How much garbage was produced in 2004?
 - In which year did the US produce 251 million tons of garbage?
 - Does the table describe a linear function?
 - Draw a graph that shows this data.
- e. Example: [Solution](#) (DOK 2)
- A penny is about $\frac{1}{16}$ of an inch thick.
- In 2011 there were approximately 5 billion pennies minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?
 - In the past 100 years, nearly 500 billion pennies have been minted. If all of these pennies were placed in a single stack, how many miles high would that stack be?
 - The distance from the moon to the earth is about 239,000 miles. How many pennies would need to be in a stack in order to reach the moon?
- f. Example: [Solution](#) (DOK 3)
- For each situation below, fill in the missing information in the tables.
 - A parking meter takes only dimes and each dime is worth 6 minutes on the meter.

Number of Dimes	Minutes of Parking
2	
3	
7	

Minutes of Parking	Number of Dimes
0	
12	
54	

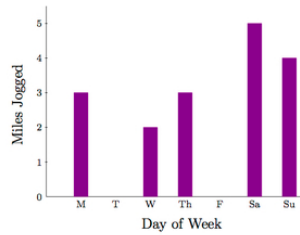
- Each point on the graph below shows how many shots a player on a basketball team took and made in the first half of a game.



Shots Attempted	Shots Made
2	
4	
7	

Shots Made	Shots Attempted
1	
4	
6	

iii. Below is a graph showing how far Marika jogged each day in a week:



Day of Week	Miles Jogged
Monday	
Friday	
Sunday	

Miles Jogged	Day of Week
1	
3	
5	

b. What, if any, difficulties did you encounter filling out these tables? Explain.

- Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a

linear function represented by an algebraic expression, determine which function has the greater rate of change.

(8.F.A.2) (DOK 1,2)

- a. Example: [Solution](#) (DOK 2)

Sam wants to take his MP3 player and his video game player on a car trip. An hour before they plan to leave, he realized that he forgot to charge the batteries last night. At that point, he plugged in both devices so they can charge as long as possible before they leave.

Sam knows that his MP3 player has 40% of its battery life left and that the battery charges by an additional 12 percentage points every 15 minutes.

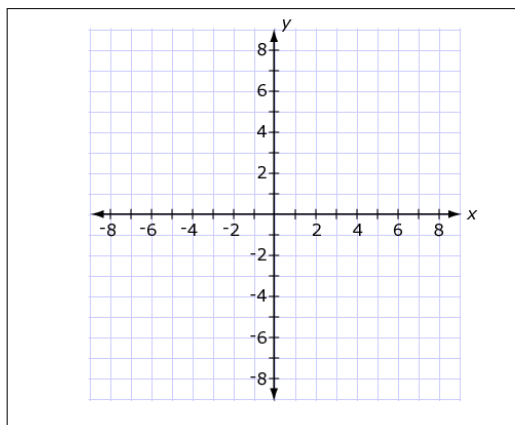
His video game player is new, so Sam doesn't know how fast it is charging but he recorded the battery charge for the first 30 minutes after he plugged it in.

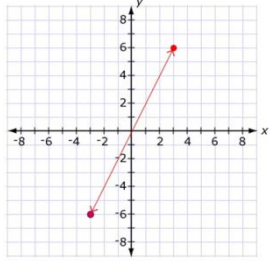
time charging (minutes)	0	10	20	30
video game player battery charge (%)	20	32	44	56

- a. If Sam's family leaves as planned, what percent of the battery will be charged for each of the two devices when they leave?
- b. How much time would Sam need to charge the battery 100% on both devices?
- b. Example: John and Kim wrote down two different functions that have the same rate of change. John's function is represented by the table shown.

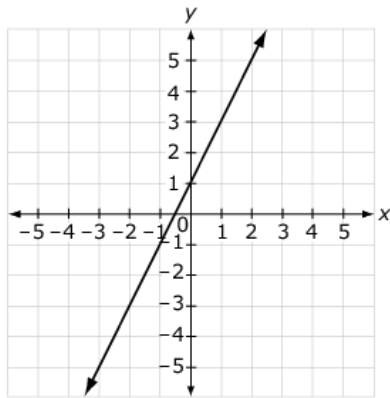
x	y
-1	-5
1	-1
3	3

Graph a function that could be Kim's function.



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#19	1	F	E	2	8.F.A.2	N/A	

c. Example: Consider this graph of a line.



Which equation has a rate of change **greater than** the rate of change for the line shown?

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#17	1	F	E	1	8.F.A.2	N/A	A

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points $(1,1)$, $(2,4)$ and $(3,9)$, which are not on a straight line.*

(8.F.A.3) (DOK 1,2)

a. Example: [Solution](#) (DOK 3)

a. Decide which of the following points are on the graph of the function $y = 2x + 1$:

- $(0, 1), (2, 5), (\frac{1}{2}, 2), (2, -1), (-1, -1), (0.5, 1)$
- Find 3 more points on the graph of the function.

b. Find several points that are on the graph of the function $y = 2x^2 + 1$.

- Plot the points in the coordinate plane. Is this a linear function?
- Support your conclusion.

c. Graph both functions and list as many differences between the two functions as you can.

b. Example: [Former NAEP question](#) (DOK 1)

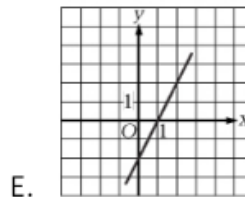
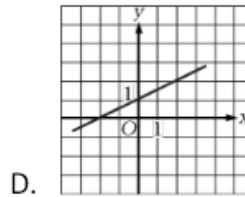
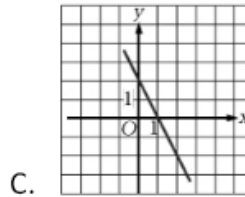
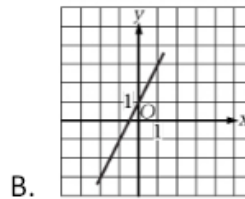
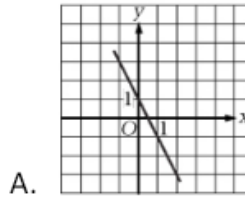
Which of the following is an equation of a line that passes through the point (0, 5) and has a negative slope?

- A. $y = 5x$
- B. $y = 5x - 5$
- C. $y = 5x + 5$
- D. $y = -5x - 5$
- E. $y = -5x + 5$

Answer: E. $y = -5x + 5$

- a. Example: ([Former NAEP question](#)) (DOK 1)

Which of the following is the graph of the line with equation $y = -2x + 1$?



Answer: A.

Use functions to model relationships between quantities. (8.F.B)

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values. **(8.F.B.4) (DOK 1,2,3)**

- a. Example: [Solution](#) (DOK 3)

A car is traveling down a long, steep hill. The elevation, E , above sea level (in feet) of the car when it is d miles from the top of the hill is given by $E = 7500 - 250d$, where d can be any number from 0 to 6. Find the slope and intercepts of the graph of this function and explain what they mean in the context of the moving car.

- b. Example: [Solution](#) (DOK 3)

You work for a video streaming company that has two monthly plans to choose from:

- Plan 1: A flat rate of \$7 per month plus \$2.50 per video viewed
- Plan 2: \$4 per video viewed

a. What type of functions model this situation? Explain how you know.

b. Define variables that make sense in the context, and then write an equation with cost as a function of videos viewed, representing each monthly plan.

c. How much would 3 videos in a month cost for each plan? 5 videos?

d. Compare the two plans and explain what advice you would give to a customer trying to decide which plan is best for them, based on their viewing habits.

c. Example: [Solution](#) (DOK 3)

The SLV High School graduation started at 1:00PM. After some speeches, the principal started reading off the names of the students, alphabetically by last name. When he finishes, the graduation will end.

a. Use the bulletin shown below to estimate when the graduation will end.

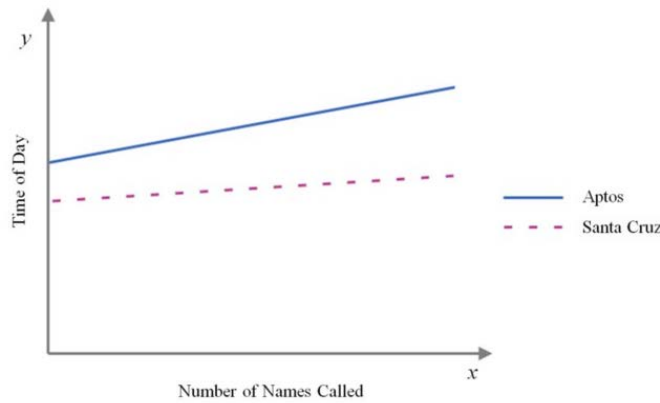
**San Lorenzo Valley High School
Class of 2011**

Micah Stephen Adams	Amber Anne Chase	Nicholas Gregory Hargraves	Matthew Alexander Lippert
Xavier Josue Alvin	Faye Elise Chavez	Alexander Samuel Harnett	Madeline Elizabeth Lopez
Connor James Antisdal	<u>David Thomas Chaves</u>	David Michael Hart	Emily Marie Madison
David Zachary Baker	<u>Zoe Darlene Chinn</u>	Melissa Lorraine Harvey	Rebecca Lynn Makita
Bradley Garrett Barnard	Emily Elizabeth Clements	Olivia Louise Herrera	Savanna San Mangelsen
Joseph Barnes	Claire Elizabeth Cloud	Kasia Adair Hill	Zane Michael Markham
James Alexander Barnes	Steven Dennis Connolly	Ludmila Eliana Hipsley	Alfredo Martinez
Michael Barton	Conor Thomas D'Amato	Jazelle Gina Hooper	Tyler Cameron Mattson
Sarah Raisha Beasley	Cassandra Suzanne Davis	Mariah Donna May Hopkins	Wayne Thomas McCarthy
Allison Rose Beasley	Lauren Marie Dempewolf	Lindsay Marie Hoppin	Theos Christopher McClish
Emily Anne Bechtie	Blake Bradley Dennis	Bryn Kathryn Horton	Annelise Marie McFarland
Brandon Scott Beavers	Jake Madison Debreuil	Thomas John Housek	Alexander Michael McIntosh
Patrick Fereva Belardi	Shannon Elaine Eisner	Ellen Joanne Innis	Dominique Sharee McIntosh
Kaitlyn Elia Benson	Emily Anne Engel	Jordan Isaacson	Victoria McKenzie
Jordan Starr Thomas Bereman	Samantha Marie Ferguson	Joseph Clifford Jansen	Violet Joseette McNally
Taylor Renee Foster Berritto	Dominique Angelina Jones	Allison Kirstine Janus	Kendra Dee Messimer
Ashley Michelle Ann Billington	Cherina Marie Freitas	Nettie Pearl Mitchell Johnson	Marisa Grace Brudnick
Morgen King Bawa	Anju Friend	Lacey Marie Johnson	Sage Nicole Monack
Jaclyn Kate Black	Nathaniel Justin Fruzza	Felipe Johnson	Thomas Ocean Moreno
Xitali Borreson	Mariah Nichole Galmez	Shelby Danielle Johnson	Rainbow Roxanna Muchamuel
Chare Nicole Btandon	Robin Lavendar Garcia	Zachary David Johnson	Nicholas Jeffrey Newberry
Sydney Lauren Garcia Brell	Ronja Andrea Franklin	Dakota Shea Jones	George Austin Norfleet
Curtis Martin Brewer	Mason Garrett	Tanner William Jones	Travis James Nugent
Katherine Evelyn Brown	Kayle Anne Genis	Clara Joy Kamau	Joseph Carl Olson
Annika Bruce	Melanie Lauren Gleim	Mia Anne Kellogg	Lacie Marie Orlando
Amanda Lynn Bruce	Andrea Serena Godbout	Maxine Elaine Kelly	Mary Elaine Ivy Orr
David Michael Burgler	Angela Gonzales	Ryan Scott Kennedy	Austin Leonard Overton
Jourdan Donovan Burk	<u>Chase Charles Gaudreau</u>	Jenny Rae Kersten	Haley Lauren Pace
Nicholas Ryan Burks	<u>Dakota Makua Gorman</u>	Andrew Benjamin King	James Evan Paolini
Joseph Thomas Burton	Dennis Jesse Greer	Russell Theodore Klair	Timothy William Parker
McLean Avery Camacho	Michael Marie Grindy	Zachariah Waya Tagli Klus	Carson Taylor Paynter
Alexander Austin Campbell	Erin Cassidy Groszird	Christina Rose Knoll	Zachary Alan Peabody
Jason William McGregor	Krista Marie Grunberger	Jessica Danielle Kraft	Katrina May Pearce
Taylor James Casey	Tyler Alan Hagen	Jessica Ann Lucy	Samuel Jackson Peiphrey
Kory Daniel Chadwick	Hazel Gladys Jane Haikkila	Joseph Allan Landry	Chandler Elizabeth Perazzo
Audrey Elizabeth Chapin	Marcus Taylor Halversen	Abbie Mae Leveque	Sebastian Thomas Peterson

**San Lorenzo Valley High School
Class of 2011**

Hannah Mae Petras Rachael Danielle Pfister Jesse Dean Phillips Sarae Iris Putter Kirstin Jane Prather Halley Brooke Prunella Brittany Lynette Ramirez Sondra Marie Raymond Nathan Dawes Reader Kristy Marie Rector Sergio Aldo Ringelman Christopher Ross Rinkert Joshua David Roberts Lara Elaine Rodriguez Brandon Bouchard Ruiz Kayla Irene Salangsiang Erik Mark Sanchez Fiona Darlene Sans James Trevor Sawyer Jared Wainor Schell Lial Melissa Sherrell Jessica Marie Simonson Megan Danielle Skrabak David Tayamen Staschewick Logan James Smith Elizabeth Smith Dillon Sean Sochoza Nicolas Aaron Sovulewski Miles Randon Staggs Bruce Louis Steinberg Matthew McCrea Stewart Kelsey Anne Stillier Daniel Todd Sutton Deanna Marie Tindell Arianna Jordan Townsend Joseph Ryan Tumbale	Rachel Ana Uvill Natalie Vernazza Rochelle Whitney Viole Carson Alexander Walker Daniel Herrera Wetterhorn Quillen Arthur White Jessica Lynne Whitehill Stephanie Anne Wildman Jessica Kayla Wood Alajha Kathleen Wyllie Peter Francis Zavoroni	Graduation coordinators: Principal Assistant Principal Counselors Board of Trustees Superintendent Director Student Activities Program cover artwork	Class President - Hazel Haikkila Vice President - Lindsey Hopkin Secretary - Morgan Blawa Treasurer - Bradley Barnard Michael Billings Keri Arredondo Leslie Ebbage Noreen Rorden Annabel Nolan Jeff Burns Kip Dakota, President Lisa Teltz, Clerk Laura Ritchie, Trustee Kathy Wyllie, Trustee George Dolson, Trustee Julie Belardi Leslie Ebbage Shannon Hall '11
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- b. Estimate how long the speeches took. How do you know?
- c. Write an equation that the parents could use to find the approximate time the principal will call their child's name given the child's position in the list in the graduation program.
- d. Aptos High School and Santa Cruz High School started their graduations at the same time. The graphs shown below show the time of day as a function of the number of names the principal has read at each school. Write down as many differences between the two graduations as you can based on differences in the two graphs. Give your reasons for each.



d. Example: [Solution](#) (DOK 2)

You have \$100 to spend on a barbeque where you want to serve chicken and steak. Chicken costs \$1.29 per pound and steak costs \$3.49 per pound.

- a. Find a function that relates the amount of chicken and the amount of steak you can buy.
- b. Graph the function. What is the meaning of each intercept in this context? What is the meaning of the slope in this context? Use this (and any other information represented by the equation or graph) to discuss what your options are for the amounts of chicken and amount of steak you can buy for the barbeque.

- e. Example: [Solution](#) (DOK 3)

A student has had a collection of baseball cards for several years. Suppose that B , the number of cards in the collection, can be described as a function of t , which is time in years since the collection was started. Explain what each of the following equations would tell us about the number of cards in the collection over time.

a. $B = 200 + 100t$

b. $B = 100 + 200t$

c. $B = 2000 - 100t$

d. $B = 100 - 200t$

- f. Example: [Solution](#) (DOK 3)

Consider the relationship between the number of pounds of chicken and the number of pounds of steak that can be purchased for a barbecue on a fixed budget.

a. Write an equation for this relationship given that chicken costs \$1.29 per pound, steak costs \$3.49 per pound, and the total allotment for both is \$100.

b. Sketch a graph for the equation. Describe how the amount of steak that can be purchased depends on the amount of chicken purchased. What is the significance of the graph's x - and y -intercepts?

c. Re-write the equation to show the amount of steak purchased in terms of the amount of chicken purchased. Use this equation to describe how the amount of steak purchased changes with each pound of chicken purchased.

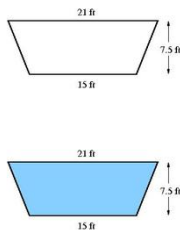
d. Both the equation and the graph can be used to find amounts of chicken and steak that can be purchased for the barbecue on a fixed budget of \$100. Explain how important features of the graph can be seen or found with the equation and describe what they say about the relationship between chicken and steak purchases.

- g. Example: [Solution](#) (DOK 3)

In science class, Mrs. Winkler's students are studying the water flow in a channel close to their school (see the photo below found [here](#)).



A cross section of the channel approximates the shape of an isosceles trapezoid with dimensions noted on the diagrams below: one showing an empty channel and the other the channel with water to the top of its banks.



The students found that there is a linear relationship between the *water depth* and the *distance across the channel at water level*. This means that we can write y , the distance across the channel at water level (in feet), as a function of the water depth, d (in feet).

- Use the information in the diagram to find two depth-distance pairs (d, y) that correspond with each other. In other words, find two input-output pairs for the function.
- Find an equation that describes y as a function of d . Draw the graph of the equation.
- Explain the meaning of the slope and the vertical intercept of the line in the context of the situation. In other words, what does each tell you about the depth of the water and the distance across the channel at water level?
- What is the distance across the channel at water level when the water is 2 feet deep?
- Formulate and answer another question about this situation.

h. Example: [Solution](#) (DOK 2)

Joshua's mail truck travels 14 miles every day he works, and is not used at all on days he does not work. At the end of his 100th day of work the mail truck shows a mileage of 76,762.

a. Fill in the blanks to express the mileage y as a linear function of the number of days x that Joshua has worked:

$$y = [\text{blank 1}]x + [\text{blank 2}].$$

b. What are the units of the number [the number the student typed into in blank 1] that appears in your equation?

c. What are the units of the number [the number the student typed into in blank 2] that appears in your equation?

d. Which of the following is a correct interpretation of the number [the number the student typed into in blank 1] that appears in your equation? (Select all that apply.)

- i. The mileage at the end of Joshua's first day of work.
- ii. The number of miles Joshua drives the truck each day he works.
- iii. The mileage at the beginning of Joshua's first day of work.
- iv. The number of days Joshua works for each mile he drives.
- v. The number of miles Joshua drives at work over 100 days.

e. In this context, which of the following is a correct interpretation of the number [the number the student typed into in blank 2] that appears in your equation? (Select all that apply.)

- i. The mileage at the end of Joshua's first day of work.
- ii. The number of miles Joshua drives the truck each day he works.
- iii. The mileage at the beginning of Joshua's first day of work.
- iv. The number of days Joshua works for each mile he drives.
- v. The number of miles Joshua drives at work over 100 days.

i. Example: This table shows the linear relationship of the water level in a tank and time.

Time (hr)	Water Level (ft)
0	50
2	40
4	30
6	20

Write the rate of change of the water level, in feet per hour.

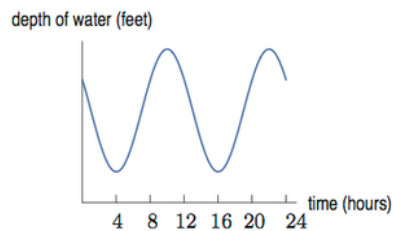
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#24	1	F	F	1	8.F.B.4	N/A	-5

- j. Example: ([Former NAEP question](#)) (DOK 2)

The number of gallons of water, y , in a tank after x hours may be modeled by the linear equation $y = 800 - 50x$. Which of the following statements about the tank is true?

- A. It is filling at a rate of 800 gallons per hour.
 - B. It is filling at a rate of 50 gallons per hour.
 - C. It is emptying at a rate of 16 gallons per hour.
 - D. It is emptying at a rate of 50 gallons per hour.
 - E. It is emptying at a rate of 800 gallons per hour.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally. **(8.F.B.5) (DOK 1,2,3)**
- a. Example: [Solution](#) (DOK 2)

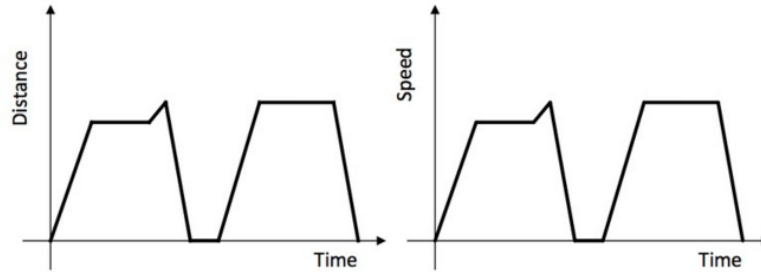
The figure below gives the depth of the water at Montauk Point, New York, for a day in November.



- a. How many high tides took place on this day?
- b. How many low tides took place on this day?
- c. How much time elapsed in between high tides?

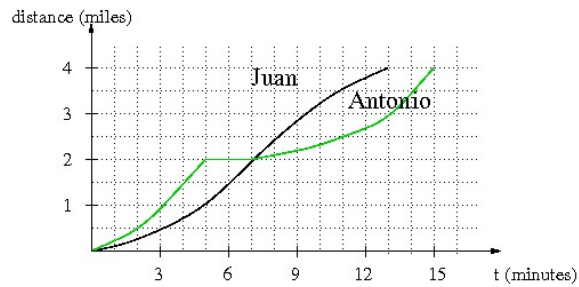
- b. Example: [Solution](#) (DOK 2)

Below are two graphs that look the same. Note that the first graph shows the speed of a car as a function of time and the second graph shows the distance of a different car from home as a function of time. Describe what someone who observes the car's movement would see in each case.



- c. Example: [Solution](#) (DOK 3)

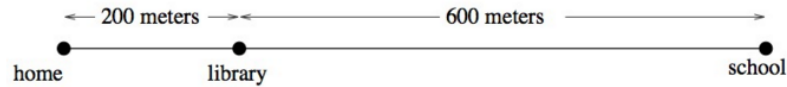
Antonio and Juan are in a 4-mile bike race. The graph below shows the distance of each racer (in miles) as a function of time (in minutes).



- Who wins the race? How do you know?
- Imagine you were watching the race and had to announce it over the radio, write a little story describing the race.

- d. Example: [Solution](#) (DOK 2)

Nina rides her bike from her home to school passing by the library on the way, and traveling at a constant speed for the entire trip. (See map below.)



- Sketch a graph of Nina's distance from school as a function of time.
- Sketch a graph of Nina's distance from the library as a function of time.

- e. Example: SPEEDING TICKETS

New York State wants to change its system for assigning speeding fines to drivers. The current system allows a judge to assign a fine that is within the ranges shown in Table 1.

Table 1. New York Speeding Fines

Miles per Hour over Speed Limit	Minimum Fine	Maximum Fine
1 - 10	\$45	\$150
11 - 30	\$90	\$300
31 or more	\$180	\$600

Some people have complained that the New York speeding fine system is not fair. The New Drivers Association (NDA) is recommending a new speeding fine system. The NDA is studying the Massachusetts system because of claims that it is fairer than the New York system.

Table 2. Massachusetts Speeding Fines

Miles per Hour over Speed Limit	Fine
1 - 10	\$100 flat charge
11 or more	\$100 flat charge plus \$10 for each additional mph above the first 10 mph

In this task, you will:

- Analyze the speeding fine systems for both New York and Massachusetts.
- Use data to propose a fairer speeding fine system for New York state.

Table 2. Massachusetts Speeding Fines

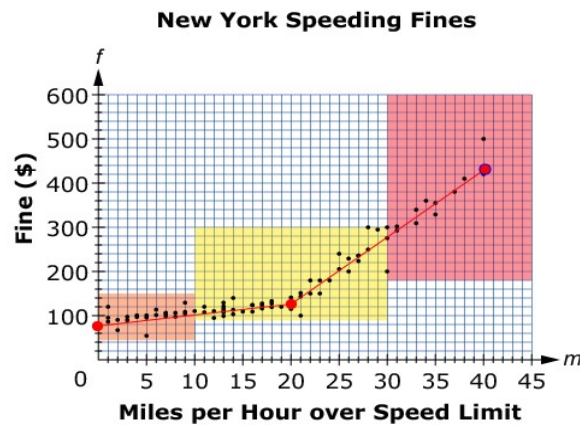
Miles per Hour over Speed Limit	Fine
1 - 10	\$100 flat charge
11 or more	\$100 flat charge plus \$10 for each additional mph above the first 10 mph

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#4	4	NS	4A, 4B, 4D, 4F	2	8.F.B.5,		See exemplar

For this item, a full-credit response includes (1 point) includes

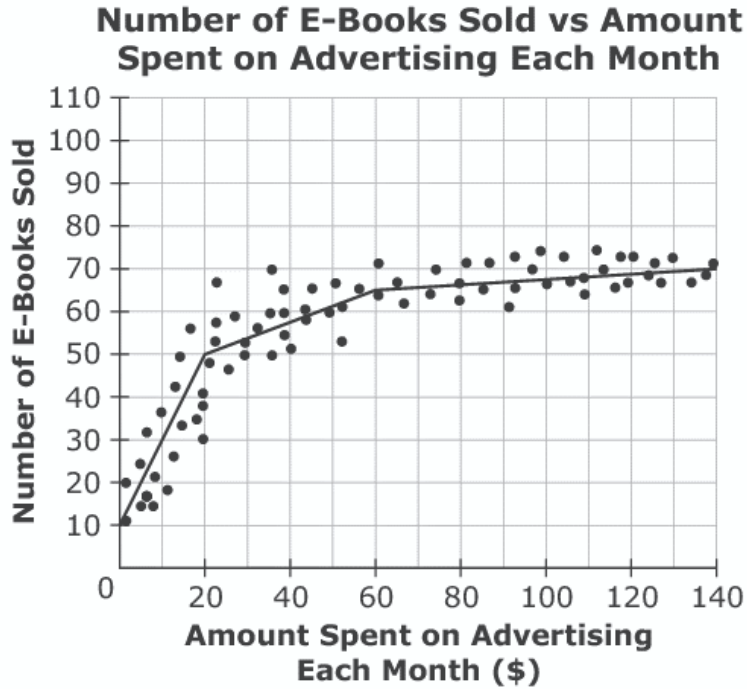
- the graph of a piecewise linear function that approximates the data points on the graph. (Note: There is a range of acceptable answers, near $f = 2m + 90$ for $1 \leq m \leq 20$; $f = 15m - 170$ for $20 \leq m \leq 40$.)

For example:



For this item, a no-credit response (0 points) includes none of the features of a full-credit response.

- f. Example: Tyler earns \$3.00 for every e-book he sells on his website. (E-books are books that are available electronically.) He investigated the relationship between the amount spent on advertising each month and the number of e-books sold. He used this information to determine the lines of best fit shown in this graph.



What is the greatest amount Tyler should spend on advertising each month? *Show your work or explain how you found your answer.*

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#5	4	NS	4A, 4B, 4D, 4F	2	8.SP.1, 8.sp.3, 8.F.5		See Below

Sample Top-Score Response:

Sample Top-Score Response:

Tyler should spend \$60 on monthly advertising. The slope of the line from \$0 to \$20 is $\frac{40}{20} = 2$. He earns \$3 for every book sold, he makes $2(3) = \$6$ for every dollar spent on advertising. The slope of the line from \$20 to \$60 is $\frac{15}{40} = \frac{3}{8}$. So, he earns $\frac{3(3)}{8} = \frac{9}{8} \approx \1.13 for every dollar spent on advertising. The slope of the line from \$60 to \$140 is $\frac{5}{80} = \frac{1}{16}$. So, he earns $\frac{5(3)}{16} = \frac{15}{16} \approx \0.94 for every dollar spent on advertising. So, he will earn the most profits by spending \$60 per month. He will earn \$135 in profits, which is \$5 more than spending \$20 on advertising. If he spends more than \$60, then he will be spending more on advertising than he is making on sales.

For Full credit (2 points):

The response demonstrates a full and complete understanding of modeling scenarios of this type. The response contains the following evidence:

- The student indicates either \$20 or \$60 (or indicates that any value in this range is acceptable) should be

spent on advertising.

AND

- The student provides sufficient support for this conclusion.

For partial credit (1 point):

The response demonstrates a partial understanding of modeling scenarios of this type. The response contains the following evidence:

- The student provides work or explanations demonstrating a clear understanding of how to analyze the profits in the scenario, but does not resolve the analysis into a final recommendation.

OR

- The student indicates an incorrect amount of money should be spent on advertising, but provides reasoning to support this conclusion that contains a minor conceptual or calculation error.

- g. Example: The table shows the relationship between the average number of hours students study for a mathematics test and their average grade.

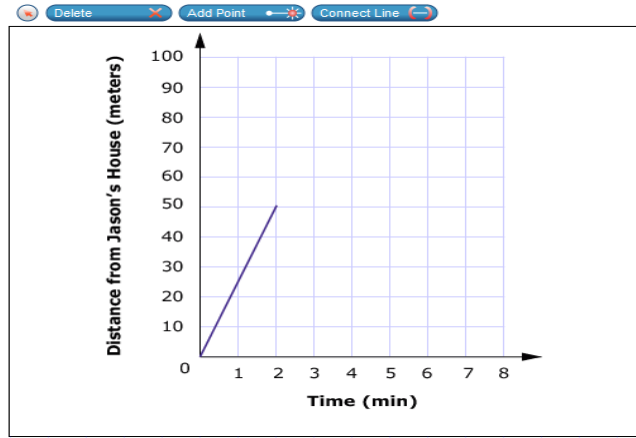
Hours Studying	Average Grade
0	62
1	78
2	85
5	74

Which type of function is most likely to model these data?

- a. Linear function with positive slope
- b. Linear function with negative slope
- c. Non-linear function that decreases then increases
- d. Non-linear function that increases then decreases

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#23	4	F	D	2	8.F.B.5	4	D

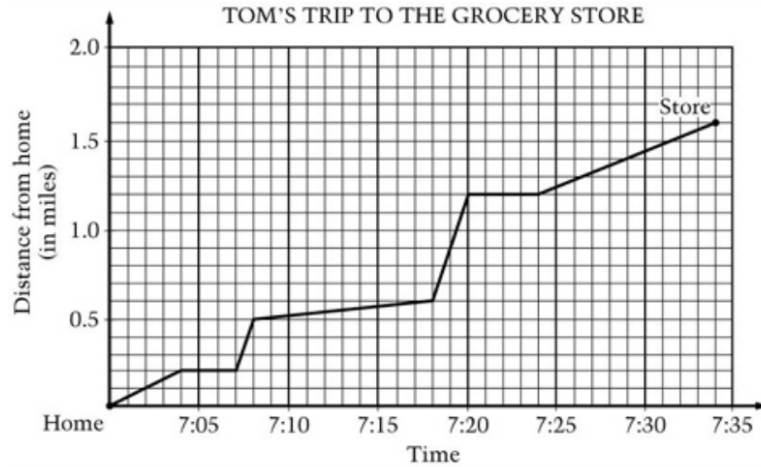
- h. Example: The school is 100 meters from Jason’s house. The following describes his most recent trip:
- He walked 50 meters toward school in 2 minutes. He realized that he left a book at home.
 - He turned around and walked home at the same speed.
 - He spent 1 minute looking for his book.
 - He walked all the way to school at twice his original speed.
- Draw a line that accurately represents Jason’s trip.



Item	Claim	Domain	Target	DOK	CONTENT	M P	Key
#5	1	F	F	2	8.F.B.5	N/ A	<p>A line graph with a grid. The vertical axis is labeled 'Distance from Jason's House (meters)' and ranges from 0 to 100 in increments of 10. The horizontal axis is labeled 'Time (min)' and ranges from 0 to 8 in increments of 1. The graph consists of five connected line segments: from (0,0) to (2,50), from (2,50) to (4,0), from (4,0) to (5,0), and from (5,0) to (7,100). Red dots are placed at each of these points.</p>

h. Example: ([Former NAEP question](#)) (DOK 2)

Tom went to the grocery store. The graph below shows Tom's distance from home during his trip.



Tom stopped twice to rest on his trip to the store. What is the total amount of time that he spent resting?

- A. 5 minutes
- B. 7 minutes
- C. 8 minutes
- D. 10 minutes
- E. 25 minutes

Answer: B. 7 minutes

- i. Example: ([Former NAEP question](#)) (DOK 3)

The linear graph below describes Josh's car trip from his grandmother's home directly to his home.



(a) Based on this graph, what is the distance from Josh's grandmother's home to his home?

(b) Based on this graph, how long did it take Josh to make the trip?

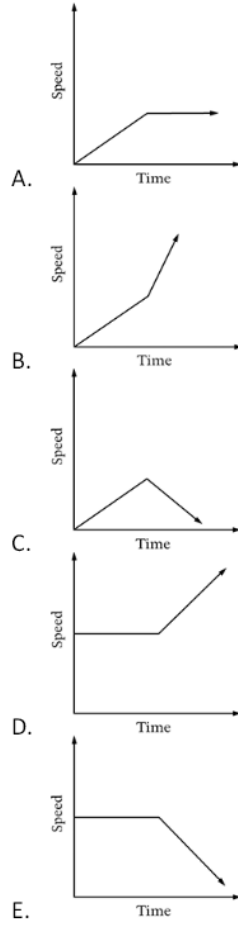
(c) What was Josh's average speed for the trip? Explain how you found your answer.

(d) Explain why the graph ends at the x-axis.

Answer: a. 160 miles, b. 4 hours, c. 40 mph, d. The graph ends at the x-axis because Josh is home and has zero more miles to go until he makes it back home.

j. Example: ([Former NAEP question](#)) (DOK 2)

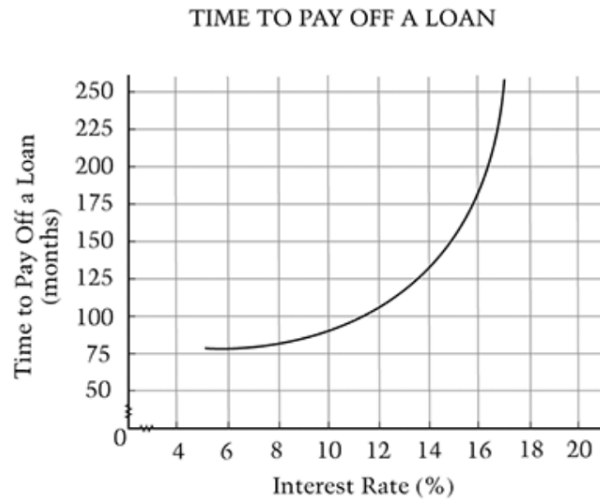
For 2 minutes, Casey runs at a constant speed. Then she gradually increases her speed. Which of the following graphs could show how her speed changed over time?



Answer: C.

k. Example: ([Former NAEP question](#)) (DOK 1)

This question refers to the following graph.



According to the graph, between which of the following pairs of interest rates will the increase in the number of months to pay off a loan be greatest?

- A. 7% and 9%
- B. 9% and 11%
- C. 11% and 13%
- D. 13% and 15%
- E. 15% and 17%

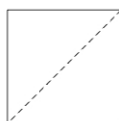
Understand congruence and similarity using physical models, transparencies, or geometry software.
(8.G.A)

1. Verify experimentally the properties of rotations, reflections, and translations:
 - a. Lines are taken to lines, and line segments to line segments of the same length.
 - b. Angles are taken to angles of the same measure.
 - c. Parallel lines are taken to parallel lines. **(8.G.A.1) (DOK 2)**

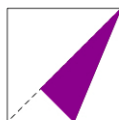
1. Example: [Solution](#) (DOK 2)

This task examines the mathematics behind an origami construction of a rectangle whose sides have the ratio $(\sqrt{2} : 1)$. Such a rectangle is called a *silver rectangle*.

Beginning with a square piece of paper, first fold and unfold it leaving the diagonal crease as shown here:



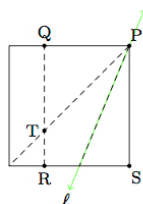
Next fold the bottom right corner up to the diagonal:



After unfolding then fold the left hand side of the rectangle over to the crease from the previous fold:



Here is a picture, after the last step has been unfolded, with all folds shown and some important points marked. In the picture T is the reflection of S about ℓ .



- a. Suppose s is the side length of our square. Show that $|PT| = s$.
- b. Show that $\triangle PQT$ is a 45-45-90 isosceles triangle.
- c. Calculate $|PQ|$ and conclude that $PQRS$ is a silver rectangle.

2. Example: [Solution](#) (DOK 3)

In this task, using computer software, you will apply reflections, rotations, and translations to a triangle. You will then study what happens to the side lengths and angle measures of the triangle after these transformations have been applied. In each part of the question, a sample picture of the triangle is supplied along with a line of reflection, angle of rotation, and segment of translation: the attached GeoGebra software will allow you to experiment with changing the location of the line of reflection, changing the measure of the angle of rotation, and changing the location and length of the segment of translation.

a. Below is a triangle ABC and a line \overleftrightarrow{DE} :



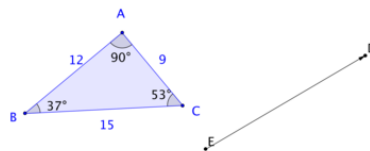
Use the supplied GeoGebra application to reflect $\triangle ABC$ over \overleftrightarrow{DE} . Label the reflected triangle $A'B'C'$. What are the side lengths and angle measures of triangle $A'B'C'$? What happens when you change the location of one of the vertices of $\triangle ABC$? What happens when you change the location of line \overleftrightarrow{DE} ?

b. Below is a triangle ABC and a point E . Draw the rotation of $\triangle ABC$ about E through an angle of 85 degrees in the counterclockwise direction.



Label the image of $\triangle ABC$ as $\triangle A'B'C'$. What happens to the side lengths and angle measures of $\triangle A'B'C'$ when you change the measure of the angle of rotation? What happens when you move the center of rotation E ?

c. Below is a triangle ABC and a directed line segment \overrightarrow{ED} .



Draw the translation of $\triangle ABC$ by \overline{ED} and label it $\triangle A'B'C'$. What happens to the side lengths and angle measures of triangle $A'B'C'$ when you change one of the vertices, A , B , or C ? What if you change the position, length, or direction of the directed line segment \overline{ED} ?

3. Example: Segment FG begins at point $F(-2, 4)$ and ends at point $G(-2, -3)$. The segment is translated by $\langle x - 3, y + 2 \rangle$ and then reflected across the y -axis to form segment $F'G'$. How many units long is segment $F'G'$?
- 0
 - 2
 - 3
 - 7

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#15	1	G	G	2	8.G.A.1a	N/A	D

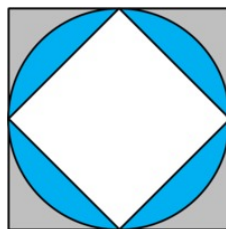
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them. **(8.G.A.2) (DOK 1,2)**
- Example: [Solution](#) (DOK 2)



Line segments AB and CD have the same length. Describe a sequence of reflections that exhibits a congruence between them.

- Example: [Solution](#) (DOK 2)

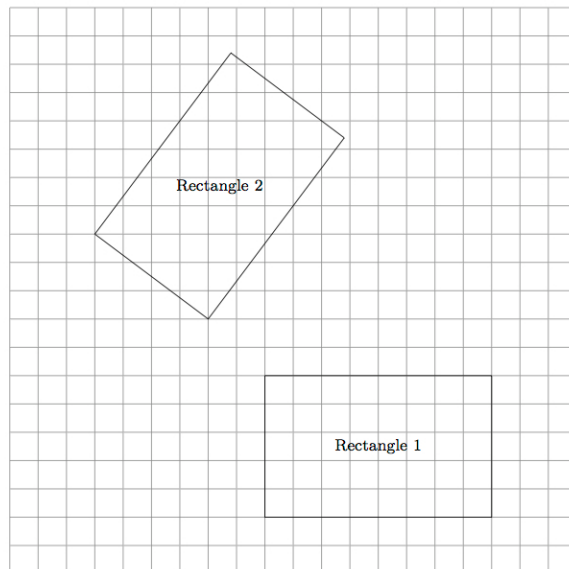
A square is inscribed in a circle which is inscribed in a square as shown below. Note that the vertices of the inner square meet the midpoints of the outer square's sides.



Consider the area of the region left by removing the interior of the small square from the interior of the big square. Is the area of the blue region more or less than $\frac{1}{2}$ of that?

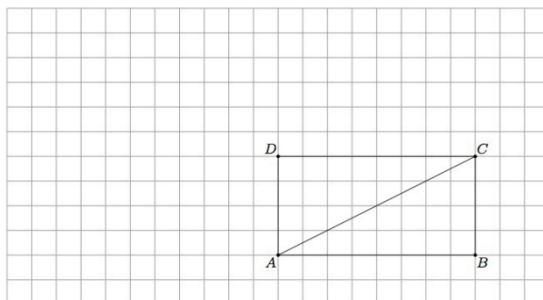
- Example: [Solution](#) (DOK 3)

Below is a picture of two rectangles with the same length and width:



- Show that the rectangles are congruent by finding a translation followed by a rotation which maps one of the rectangles to the other.
 - Explain why the congruence of the two rectangles can not be shown by translating Rectangle 1 to Rectangle 2.
 - Can the congruence of the two rectangles be shown with a single reflection? Explain.
- d. Example: [Solution](#) (DOK 3)

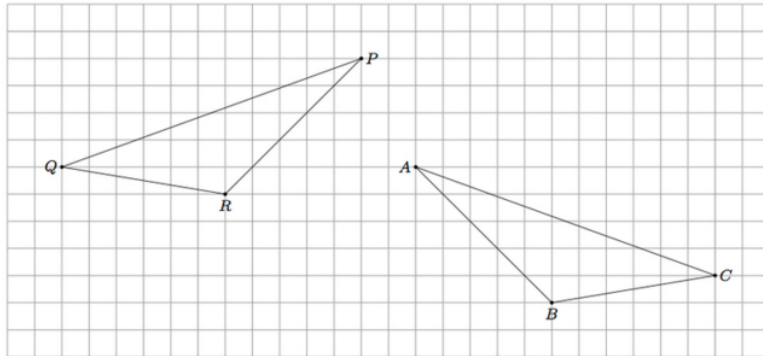
Below is a picture of rectangle $ABCD$ with diagonal AC .



- Draw the image of triangle ACD when it is rotated 180° about vertex D . Call A' the image of point A under the rotation and C' the image of point C .
- Explain why $\overleftrightarrow{DA'} \cong \overleftrightarrow{DA}$ and why $\overleftrightarrow{DC'}$ is parallel to \overleftrightarrow{AB} .
- Show that $\triangle A'C'D$ can be translated to $\triangle CAB$. Conclude that $\triangle ACD$ is congruent to $\triangle CAB$.
- Show that $\triangle ACD$ is congruent to $\triangle CAB$ with a sequence of translations, rotations, and/or reflections different from those chosen in parts (a) and (c).

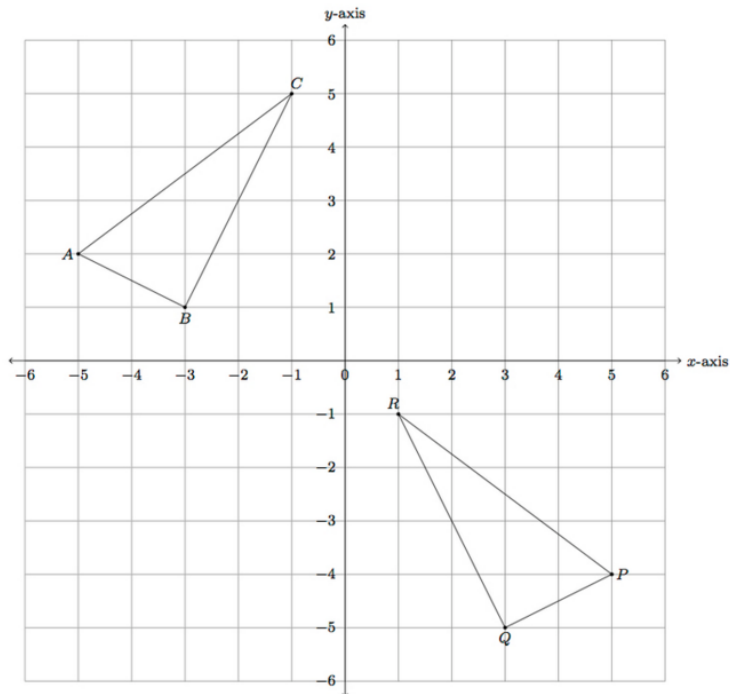
- e. Example: [Solution](#) (DOK 3)

The two triangles in the picture below are congruent:



- Give a sequence of rotations, translations, and/or reflections which take $\triangle PQR$ to $\triangle ABC$.
 - Is it possible to show the congruence in part (a) using only translations and rotations? Explain.
- f. Example: [Solution](#) (DOK 2)

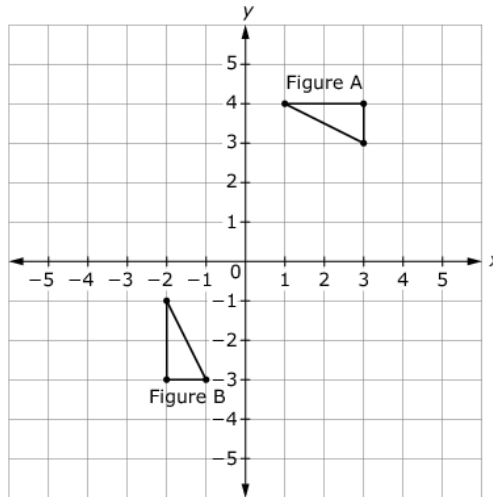
Triangles ABC and PQR are shown below in the coordinate plane:



- Show that $\triangle ABC$ is congruent to $\triangle PQR$ with a reflection followed by a translation.

- b. If you reverse the order of your reflection and translation in part (a) does it still map $\triangle ABC$ to $\triangle PQR$?
- c. Find a second way, different from your work in part (a), to map $\triangle ABC$ to $\triangle PQR$ using translations, rotations, and/or reflections.

g. Example: Two figures are shown on the coordinate grid.



Show that Figure A and Figure B are congruent by describing a sequence of basic transformations that maps Figure A onto Figure B. In your response, be sure to identify the transformations in the order they are performed.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#20	3	G	B	3	8.G.A.2	1, 2, 3	See exemplar

Exemplar 1: 1st transformation is to reflect over the y -axis. 2nd transformation is to rotate 90° counter-clockwise about the origin. 3rd transformation is to translate right by 2 units.

Exemplar 2: 1st transformation is to reflect over the x -axis. 2nd transformation is to rotate 90° clockwise about the origin. 3rd transformation is to translate right by 2 units.

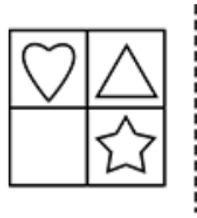
Other correct series of transformations are possible.

Rubric:

(2 points) Student describes three transformations with sufficient detail to prove that Figure A and Figure B are congruent.

(1 point) Student either describes all three transformations in general terms, without the degree of precision necessary to prove congruency (e.g., rotation, reflection, and translation) or correctly describes two out of three transformations and incorrectly describes the third (e.g., states the rotation is 180° instead of 90° or translates in the wrong direction or an incorrect number of units).

h. Example: ([Former NAEP question](#)) (DOK 1)



Tony flips the figure over the dotted line. Which picture shows the result of the flip?

- A.

△	♥
☆	□
- B.

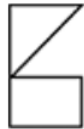
△	♥
□	☆
- C.

♥	▽
♥	□
△	☆
- D.

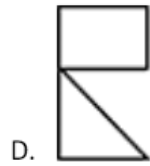
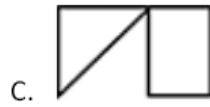
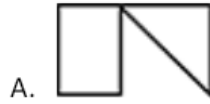
♥	□
△	☆

Answer: A.

i. Example: ([Former NAEP question](#)) (DOK 1)



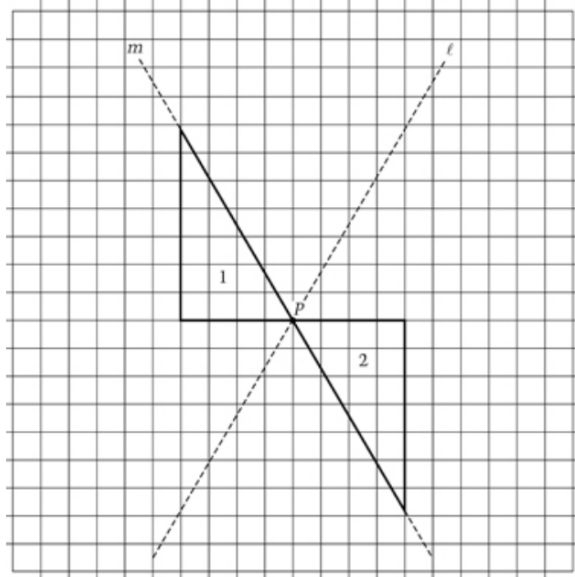
2. When the figure above is rotated 90 degrees clockwise, which of the following is the resulting figure?



Answer: A.

j. Example: ([Former NAEP question](#)) (DOK 2)

The figure below shows two triangles, labeled 1 and 2.

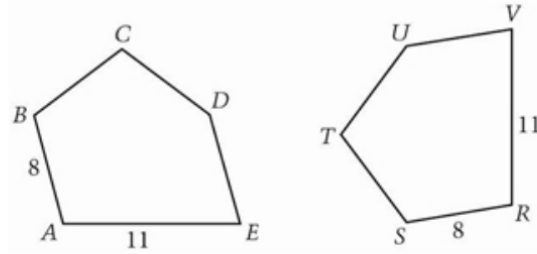


Which one of the following describes a way to move triangle 1 so that it completely covers triangle 2?

- F. Turn (rotate) 180 degrees about point P .
- G. Flip (reflect) over line ℓ .
- H. Slide (translate) 5 units to the right followed by 8 units down.
- I. Flip (reflect) over line m .
- J. Slide (translate) 10 units to the right followed by 16 units down.

Answer: F. Turn (rotate) 180 degrees about point P .

- k. Example: ([Former NAEP question](#)) (DOK 2)



In the figure above, polygons $ABCDE$ and $RSTUV$ are congruent. Which side must have the same length as side BC ?

- A. CD
- B. DE
- C. ST
- D. TU
- E. UV

Answer: C. ST

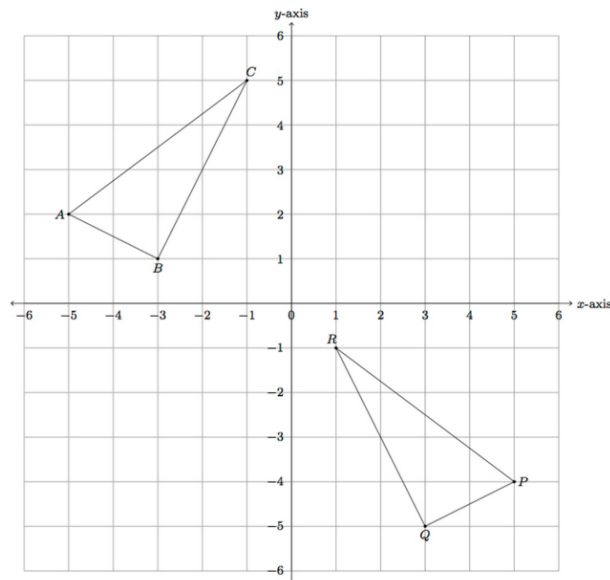
4. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates. **(8.G.A.3) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 2)

The point in the x - y plane with coordinates $(1000, 2012)$ is reflected across the line $y = 2000$. What are the coordinates of the reflected point?

b. Example: [Solution](#) (DOK 3)

Triangles ABC and PQR are shown below in the coordinate plane:



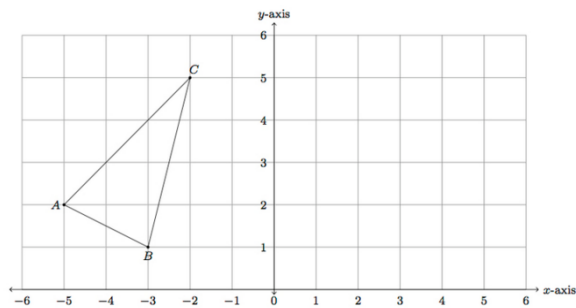
a. Show that $\triangle ABC$ is congruent to $\triangle PQR$ with a reflection followed by a translation.

b. If you reverse the order of your reflection and translation in part (a) does it still map $\triangle ABC$ to $\triangle PQR$?

c. Find a second way, different from your work in part (a), to map $\triangle ABC$ to $\triangle PQR$ using translations, rotations, and/or reflections.

c. Example: [Solution](#) (DOK 3)

Below is a picture of a triangle on a coordinate grid:



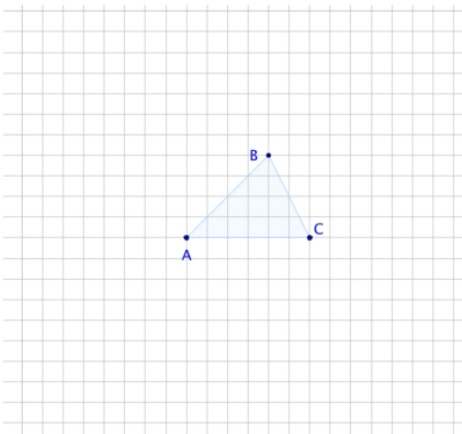
a. Draw the reflection of $\triangle ABC$ over the line $x = -2$. Label the image of A as A' , the image of B as B' and the image of C as C' .

b. Draw the reflection of $\triangle A'B'C'$ over the line $x = 2$. Label the image of A' as A'' , the image of B' as B'' and the image of C' as C'' .

c. What single rigid transformation of the plane will map $\triangle ABC$ to $\triangle A''B''C''$? Explain.

- d. Example: [Solution](#) (DOK 3)

Consider triangle ABC .



- a. Draw a dilation of ABC with:
- Center A and scale factor 2.
 - Center B and scale factor 3.
 - Center C and scale factor $\frac{1}{2}$.

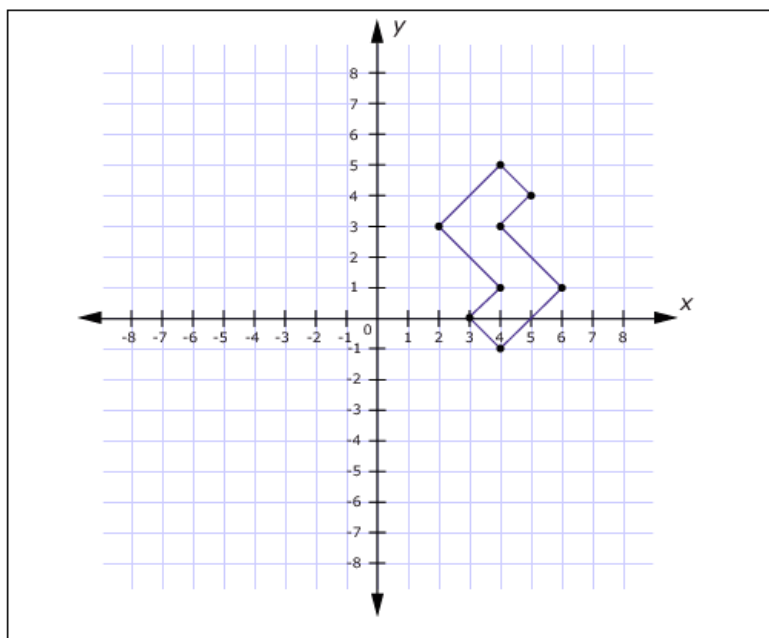
b. For each dilation, answer the following questions:

- By what factor do the base and height of the triangle change? Explain.
 - By what factor does the area of the triangle change? Explain.
 - How do the angles of the scaled triangle compare to the original? Explain.
- e. Example: A sequence of transformations is applied to a polygon. Select **all** statements which indicate a sequence of transformations where the resulting polygon has an area greater than the original polygon.
- Reflect over the x -axis, dilate about the origin by a scale factor of $\frac{1}{2}$, translate up 5 units
 - Rotate 90° counterclockwise around the origin, dilate about the origin by a scale factor of $\frac{3}{2}$
 - Dilate about the origin by a scale factor of $\frac{2}{3}$, rotate 180° clockwise around the origin, translate down 2 units
 - Dilate about the origin by a scale factor of 2, reflect over the y -axis, dilate about the origin by a scale factor of $\frac{2}{3}$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#16	3	G	G	3	8.G.A.3	2	<input type="checkbox"/> Reflect over the x -axis, dilate about the origin by a scale factor of $\frac{1}{2}$, translate up 5 units. <input checked="" type="checkbox"/> Rotate 90° counterclockwise around the origin, dilate about the origin by a scale factor of $\frac{3}{2}$. <input type="checkbox"/> Dilate about the origin by a scale factor of $\frac{2}{3}$, rotate 180° clockwise around the origin, translate down 2 units. <input checked="" type="checkbox"/> Dilate about the origin by a scale factor of 2, reflect over the y -axis, dilate about the origin by a scale factor of $\frac{2}{3}$.

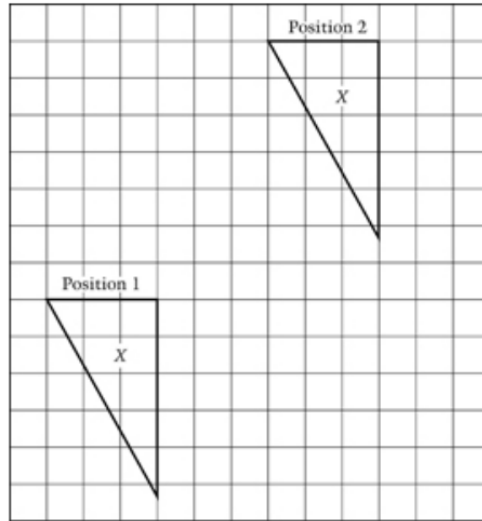
f. Example: Draw the image of the figure after the following transformations:

- a reflection over the x -axis
- a horizontal translation 7 units to the left



Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#6	1	G	G	2	8.G.A.3	N/A	

g. Example: ([Former NAEP question](#)) (DOK 1)



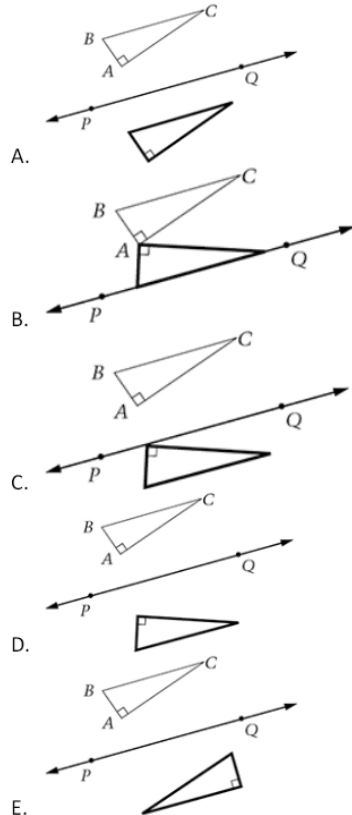
Which word best describes how to move the piece labeled X from position 1 to position 2 ?

- A. Flip
- B. Fold
- C. Slide
- D. Turn

Answer: C. Slide

h. Example: ([Former NAEP question](#)) (DOK 2)

Which of the following figures shows the reflection of triangle ABC over line PQ?



Answer: D.

- i. Example: ([Former NAEP question](#)) (DOK 2)

The point (3, 7) is a vertex of a triangle. When the triangle is reflected over the y-axis, what are the coordinates of the image of the vertex?

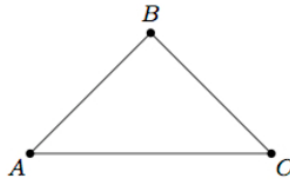
- A. (-3, -7)
- B. (-3, 7)
- C. (3, -7)
- D. (3, 7)
- E. (7, 3)

Answer: B.

5. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **(8.G.A.4) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 3)

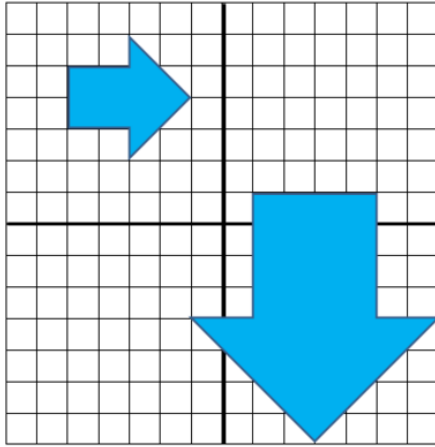
In triangle ABC below, $\angle B$ is a right angle and $|AB| = |BC|$:



Draw a line segment joining one of the vertices of $\triangle ABC$ to the opposite side so that it divides $\triangle ABC$ into two triangles which are both similar to $\triangle ABC$. Explain, using rigid motions and dilations, why the triangles are similar.

- b. Example: [Solution](#) (DOK 2)

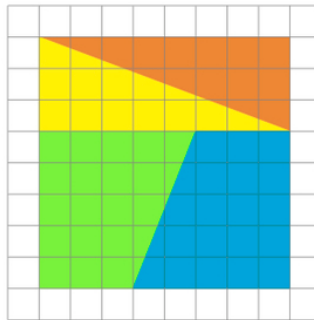
Determine, using rotations, translations, reflections, and/or dilations, whether the two polygons below are similar.



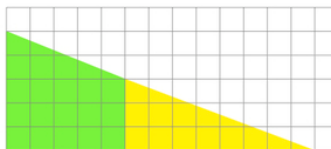
The intersection of the dark lines on the coordinate plane represents the origin (0,0) in the coordinate plane.

c. Example: [Solution](#) (DOK 3)

Below is a picture of an 8 by 8 square divided into four polygons:



The green and yellow polygons can be combined along their 3 unit edges as shown below:



a. What is the area of the green and yellow shape above? What about the area of the blue quadrilateral and orange triangle?

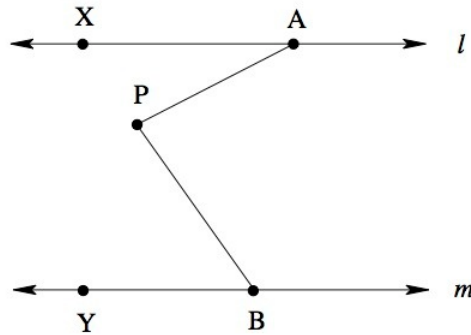
b. What is the area of the original square?

c. Are the answers to (a) and (b) consistent? Explain.

6. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
(8.G.A.5) (DOK 1,2,3)

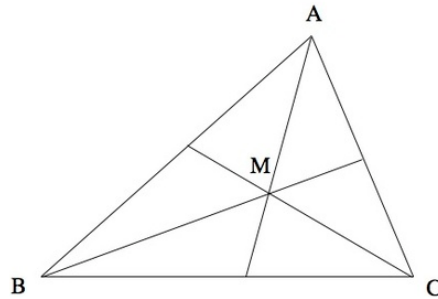
- a. Example: [Solution](#) (DOK 1)

In the picture below, lines l and m are parallel. The measure of angle $\angle PAX$ is 31° , and the measure of angle $\angle PBY$ is 54° . What is the measure of angle $\angle APB$?



- b. Example: [Solution](#) (DOK 1)

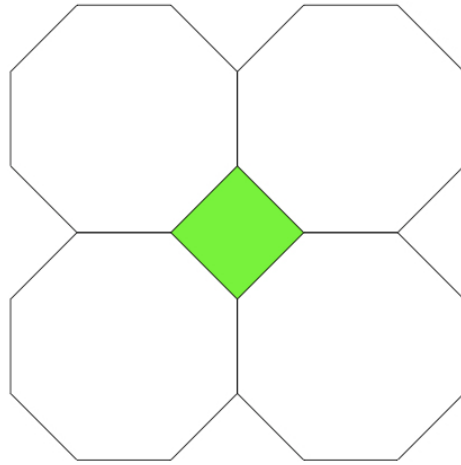
In triangle $\triangle ABC$, point M is the point of intersection of the bisectors of angles $\angle BAC$, $\angle ABC$, and $\angle ACB$. The measure of $\angle ABC$ is 42° , and the measure of $\angle BAC$ is 64° . What is the measure of $\angle BMC$?



This task adapted from a problem published by the Russian Ministry of Education.

- c. Example: [Solution](#) (DOK 2)

One interesting tile pattern (see for example [this Tile Pattern](#)) has four octagons surrounding what appears to be a square as in the picture below:

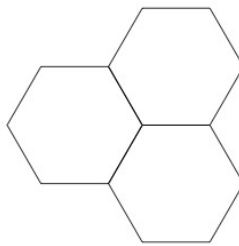


The pattern can then be extended up, down, to the left, and to the right.

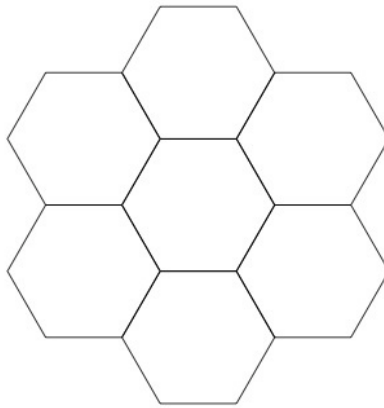
- a. Suppose each octagon is a regular octagon, that is an octagon where all 8 sides are congruent and all 8 angles are congruent. Find the measure of the 8 interior angles.
- b. Show that the space between the four octagons, shaded in the picture, is in fact a square.

d. Example: [Solution](#) (DOK 2)

A common tiling pattern with hexagons is pictured below:

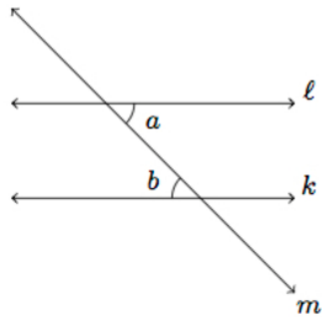


- a. A regular hexagon is a hexagon with 6 congruent sides and 6 congruent interior angles. Find the measure of the interior angles in a regular hexagon.
- b. Show that three equally sized regular hexagons sharing a common vertex can be arranged in the configuration of the above picture.
- c. Show that a single regular hexagon can be surrounded by six regular hexagons with no spaces and no overlap, as in the picture below:



e. Example: [Solution](#) (DOK 3)

In the picture below, ℓ and k are parallel lines:

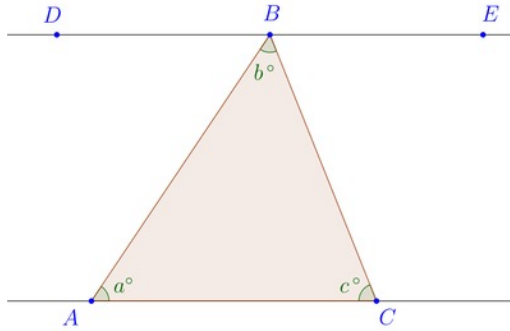


a. Show that angle a is congruent to angle b using rigid motions.

b. Which other angles, made by the intersection of ℓ and m or by the intersection of k and m , are congruent to a ? Explain using rigid motions.

f. Example: [Solution](#) (DOK 3)

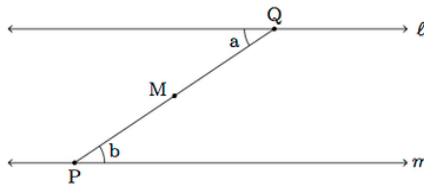
Given that $\overleftrightarrow{DE} \parallel \overleftrightarrow{AC}$ in the diagram below, prove that $a + b + c = 180$.



Explain why this result holds for any triangle, not just the one displayed above.

g. Example: [Solution](#) (DOK 3)

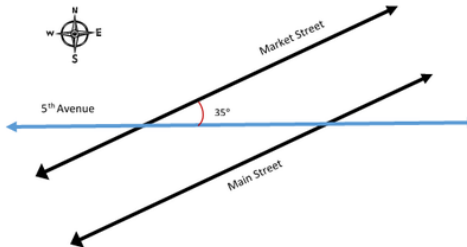
Suppose ℓ and m are parallel lines with Q a point on ℓ and P a point on m as pictured below:



Also labelled in the picture is the midpoint M of \overline{PQ} and a pair of angles. Explain why rotating the picture about M by 180 degrees demonstrates that angles a and b are congruent.

h. Example: [Solution](#) (DOK 3)

Market Street runs parallel to Main Street and both are intersected by 5th Avenue as shown below:

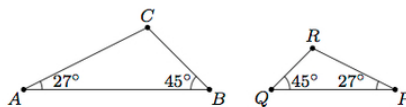


If a car traveling northeast on Market Street turns right to go east on 5th Avenue, it turns (clockwise) through a 35 degree angle as indicated in the picture.

- Suppose a car is traveling southwest on Market Street and turns left to go east on 5th Avenue. Draw the angle of turn in the picture. What is the measure of this angle? Explain how you know.
- Suppose a car is traveling southwest on Main Street and turns right onto 5th Avenue. Draw the angle of turn in the picture. What is the measure of this angle? Explain using rigid motions.
- A car makes a 35° angle of turn going through the intersection of Main Street and 5th Avenue. Assuming that the car is following these two roads, what can you conclude about the car's route through this intersection? Explain.

i. Example: [Solution](#) (DOK 3)

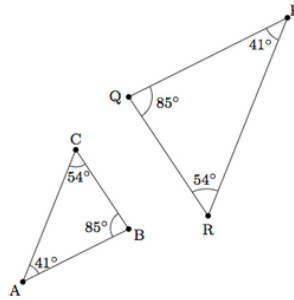
Triangles ABC and PQR below share two pairs of congruent angles as marked:



- Explain, using dilations, translations, reflections, and/or rotations, why $\triangle PQR$ is similar to $\triangle ABC$.
- Are angles C and R congruent?
- Can you show the similarity in part **a** *without* using a reflection? What about *without* using a dilation? Explain.
- Suppose DEF and KLM are two triangles with $m(\angle D) = m(\angle K)$ and $m(\angle E) = m(\angle L)$. Are triangles DEF and KLM similar?

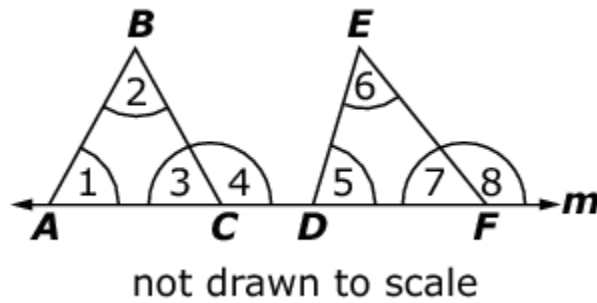
j. Example: [Solution](#) (DOK 2)

The two triangles pictured below have three pairs of congruent angles:



Are these two triangles similar?

- k. Example: The base of triangle ABC and the base of triangle DEF lie on line m , as shown in the diagram.



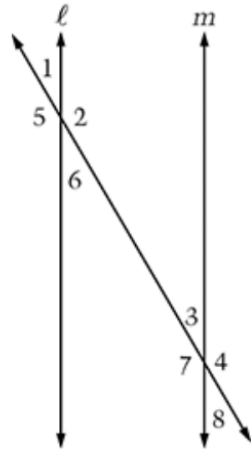
The measure of $\angle 4$ is less than the measure of $\angle 8$.

For each comparison, select the symbol ($<$, $>$, $=$) that makes the relationship between the first quantity and the second quantity true.

First Quantity	Comparison	Second Quantity
$m\angle 3$	<div style="border: 1px solid black; padding: 5px; width: 30px; margin: 0 auto;"> $<$ $=$ $>$ </div>	$m\angle 7$
$m\angle 1 + m\angle 2$	<div style="border: 1px solid black; padding: 5px; width: 30px; margin: 0 auto;"> $<$ $=$ $>$ </div>	$m\angle 5 + m\angle 6$

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key									
#21	3	G	F	2	7.G.A.5	1, 2	<table border="1"> <thead> <tr> <th>First Quantity</th> <th>Comparison</th> <th>Second Quantity</th> </tr> </thead> <tbody> <tr> <td>$m\angle 3$</td> <td></td> <td>$m\angle 7$</td> </tr> <tr> <td>$m\angle 1 + m\angle 2$</td> <td></td> <td>$m\angle 5 + m\angle 6$</td> </tr> </tbody> </table>	First Quantity	Comparison	Second Quantity	$m\angle 3$		$m\angle 7$	$m\angle 1 + m\angle 2$		$m\angle 5 + m\angle 6$
First Quantity	Comparison	Second Quantity														
$m\angle 3$		$m\angle 7$														
$m\angle 1 + m\angle 2$		$m\angle 5 + m\angle 6$														

l. Example: ([Former NAEP question](#))(DOK 1)

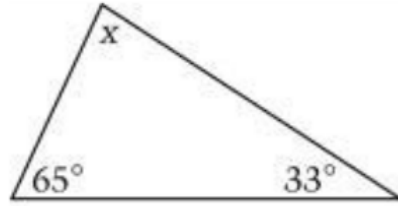


In the figure above, line ℓ is parallel to line m . Which of the following pairs of angles must have the same measure?

- A. Angles 1 and 2
- B. Angles 1 and 5
- C. Angles 2 and 3
- D. Angles 4 and 5
- E. Angles 4 and 8

Answer: D. 4 and 5

m. Example: ([Former NAEP question](#)) (DOK 1)



What is the value of x in the triangle above?

- A. 65°
- B. 82°
- C. 90°
- D. 92°
- E. 98°

Answer: B. 82

Understand and apply the Pythagorean Theorem. (8.G.B)

- 7. Explain a proof of the Pythagorean Theorem and its converse. (8.G.B.6) (DOK 2,3)
 - a. Example: [Solution](#) (DOK 3)

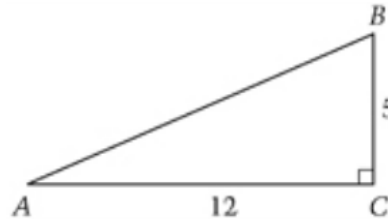
A Pythagorean triple (a, b, c) is a set of three positive whole numbers which satisfy the equation

$$a^2 + b^2 = c^2.$$

Many ancient cultures used simple Pythagorean triples such as (3,4,5) in order to accurately construct right angles: if a triangle has sides of lengths 3, 4, and 5 units, respectively, then the angle opposite the side of length 5 units is a right angle.

- a. State the Pythagorean Theorem and its converse.
- b. Explain why this practice of constructing a triangle with side-lengths 3, 4, and 5 to produce a right angle uses the converse of the Pythagorean Theorem.
- c. Explain, in this particular case, why the converse of the Pythagorean Theorem is true.

- b. Example: ([Former NAEP question](#)) (DOK 2)



In the right triangle above, what is the length of AB ?

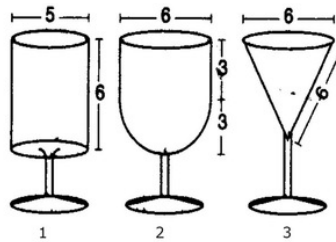
- A. 8.5
- B. 12
- C. 13
- D. 17
- E. 30

Answer: C. 13

8. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. **(8.G.B.7) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 2)

The diagram shows three glasses (not drawn to scale). The measurements are all in centimeters.



The bowl of glass 1 is cylindrical. The inside diameter is 5 cm and the inside height is 6 cm.

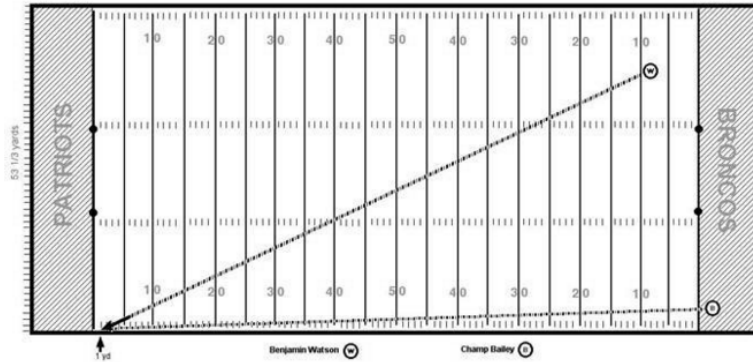
The bowl of glass 2 is composed of a hemisphere attached to cylinder. The inside diameter of both the hemisphere and the cylinder is 6 cm. The height of the cylinder is 3 cm.

The bowl of glass 3 is an inverted cone. The inside diameter is 6 cm and the inside slant height is 6 cm.

- a. Find the vertical height of the bowl of glass 3.
 - b. Calculate the volume of the bowl of each of these glasses.
 - c. Glass 2 is filled with water and then half the water is poured out. Find the height of the water.
- b. Example: [Solution](#) (DOK 2)

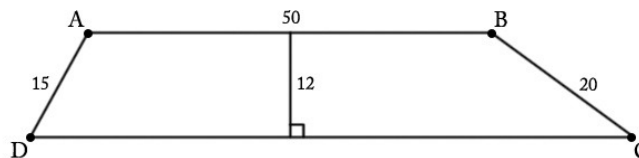
During the 2005 Divisional Playoff game between The Denver Broncos and The New England Patriots, Bronco player Champ Bailey intercepted Tom Brady around the goal line (see the circled B). He ran the ball nearly all the way to other goal line. Ben Watson of the New England Patriots (see the circled W) chased after Champ and tracked him down just before the other goal line.

In the image below, each hash mark is equal to one yard: note too the the field is $53\frac{1}{3}$ yards wide.



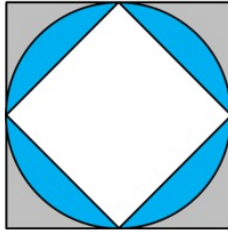
- How can you use the diagram and the Pythagorean Theorem to find approximately how many yards Ben Watson ran to track down Champ Bailey?
 - Use the Pythagorean Theorem to find approximately how many yards Watson ran in this play.
 - Which player ran further during this play? By approximately how many more yards?
- c. Example: [Solution](#) (DOK 2)
 Let A be the area of a triangle with sides of length 25, 25, and 30. Let B be the area of a triangle with sides of length 25, 25, and 40. Find A/B .
- d. Example: [Solution](#) (DOK 2)
 Quadrilateral $ABCD$ is a trapezoid, $AD = 15$, $AB = 50$, $BC = 20$, and the altitude is

- What is the area of the trapezoid?



- Example: [Solution](#) (DOK 2)

A square is inscribed in a circle which is inscribed in a square as shown below. Note that the vertices of the inner square meet the midpoints of the outer square's sides.



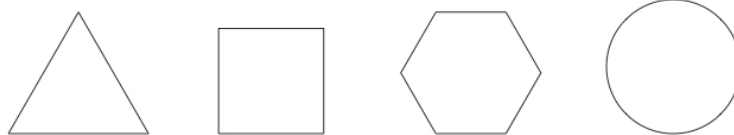
Consider the area of the region left by removing the interior of the small square from the interior of the big square. Is the area of the blue region more or less than $\frac{1}{2}$ of that?

- f. Example: [Solution](#) (DOK 2)

Point B is due east of point A . Point C is due north of point B . The distance between points A and C is $10\sqrt{2}$ meters, and $\angle BAC = 45^\circ$. Point D is 20 meters due north of point C . The distance $|AD|$ is between which two integers?

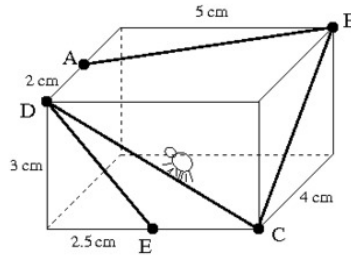
- g. Example: [Solution](#) (DOK 3)

Below are pictures of an equilateral triangle, a square, a regular hexagon, and a circle each having the same perimeter:



- Find the area of the equilateral triangle whose perimeter is 1 unit.
 - Find the area of the square whose perimeter is 1 unit.
 - Find the area of the regular hexagon whose perimeter is one unit.
 - Find the area of the circle whose perimeter is 1 unit.
 - List the answers to (a), (b), (c) and (d) in increasing size. How do you think the area of a regular octagon with perimeter 1 unit would compare to the triangle, square, hexagon, and circle?
- h. Example: [Solution](#) (DOK 2)

A spider walks on the outside of a box from point A to B to C to D and finally to point E as shown in the picture below.



- Draw a net of the box and map out the path of the spider on the net.
 - Compare your net with those of some other people.
 - Cut out the net and put the box together to see if you are right.
 - How long is the path of the spider?
- i. Example: Samantha invented a new outdoor game. The game requires attaching a rope between the tops of two poles of different heights. Read the instructions Samantha created. Use all the given information to determine the maximum allowable distance between the base of pole A and the base of pole B.

Game Instructions

Materials needed: Pole A, Pole B, 10 feet of rope

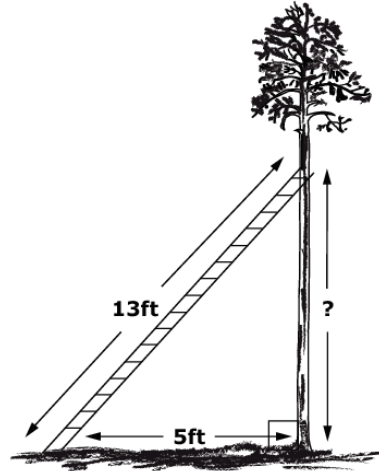
Setup:

- Place pole A perpendicular to the ground so that its height is 3 feet.
- Place pole B perpendicular to the ground so that its height is 7 feet.
- The length of the rope must extend at least 6 inches past the top of each pole for proper assembly.
- Attach the rope to the top of the two poles.

Write the **maximum** distance between the base of pole A and the base of pole B to the nearest whole foot.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#25	4	G	E	2	8.G.B.7	2, 4, 7	8

- j. Example: A 13-foot ladder is leaning on a tree. The bottom of the ladder is on the ground at a distance of 5 feet from the base of the tree. The base of the tree and the ground form a right angle as shown.



Write the distance, in feet, between the ground and the top of the ladder.

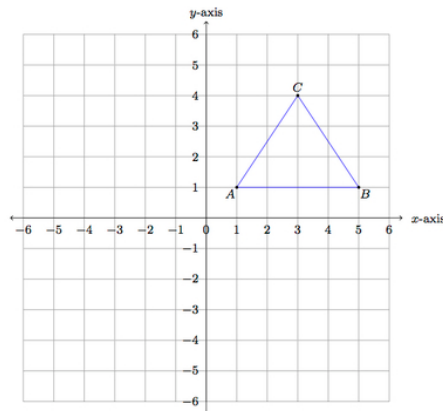
Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#9	1	G	H	2	8.G.B.7	N/A	12

9. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
(8.G.B.8) (DOK 1,2)

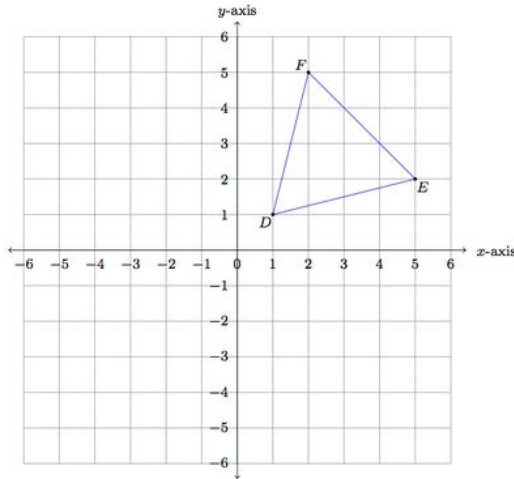
- a. Example: [Solution](#) (DOK 2)

Mrs. Lu has asked students in her class to find isosceles triangles whose vertices lie on a coordinate grid. For each student example below, explain why the triangle is isosceles.

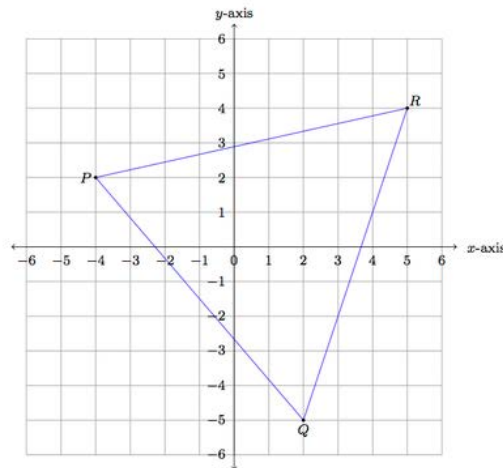
- a. Jessica draws the following triangle:



- b. Bruce's picture is here:



c. Yan draws this picture:



b. Example: [Solution](#) (DOK 3)

a. Plot the points $(5, 3)$, $(-1, 1)$, and $(2, -3)$ in the coordinate plane and find the lengths of the three segments connecting the points.

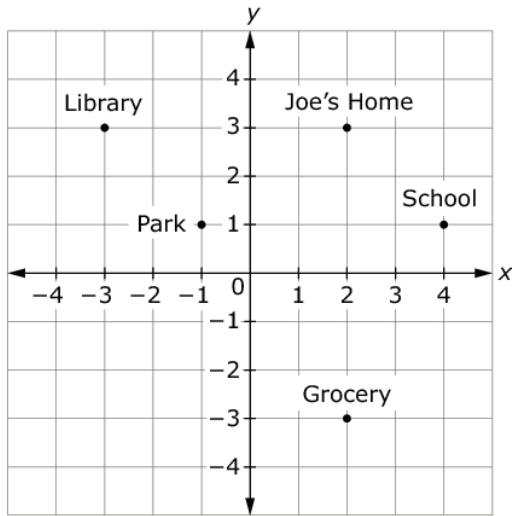
b. Find the distance between $(5, 9)$ and $(-4, 2)$ without plotting the points.

c. If (u, v) and (s, t) are two distinct points in the plane, what is the distance between them? Explain how you know.

d. Does your answer to (c) agree with your calculations in parts (a) and (b)? Explain.

c. Example: The points show different locations in Joe's town. Each unit represents 1 mile.

Places in Joe's Town



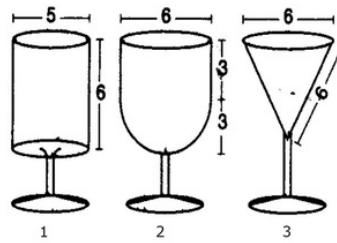
Write the shortest distance, in miles, between Joe's home and the park. Round your answer to the nearest tenth.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#10	1	G	H	2	8.G.B.8	N/A	3.6

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres. (8.G.C)

10. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. **(8.G.C.9) (DOK 1,2)**
 - a. Example: [Solution](#) (DOK 2)

The diagram shows three glasses (not drawn to scale). The measurements are all in centimeters.



The bowl of glass 1 is cylindrical. The inside diameter is 5 cm and the inside height is 6 cm.

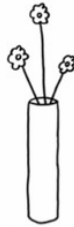
The bowl of glass 2 is composed of a hemisphere attached to cylinder. The inside diameter of both the hemisphere and the cylinder is 6 cm. The height of the cylinder is 3 cm.

The bowl of glass 3 is an inverted cone. The inside diameter is 6 cm and the inside slant height is 6 cm.

- a. Find the vertical height of the bowl of glass 3.
- b. Calculate the volume of the bowl of each of these glasses.
- c. Glass 2 is filled with water and then half the water is poured out. Find the height of the water.

b. Example: [Solution](#) (DOK 3)

My sister's birthday is in a few weeks and I would like to buy her a new vase to keep fresh flowers in her house. She often forgets to water her flowers and needs a vase that holds a lot of water. In a catalog there are three vases available and I want to purchase the one that holds the most water. The first vase is a cylinder with diameter 10 cm and height 40 cm. The second vase is a cone with base diameter 16 cm and height 45 cm. The third vase is a sphere with diameter 18 cm.



Cylinder Vase
Show off your flowers in this beautiful vase.
10cm X 40cm
\$9.95
4KE09



Cone Vase
This vase holds your flowers in place!
16cm X 45cm
\$9.95
4KE08



Sphere Vase
Doesn't get any more symmetric than this!
18cm X 18cm
\$9.95
4KE07

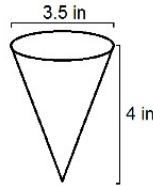
- Which vase should I purchase?
- How much more water does the largest vase hold than the smallest vase?
- Suppose the diameter of each vase decreases by 2 cm. Which vase would hold the most water?
- The vase company designs a new vase that is shaped like a cylinder on bottom and a cone on top. The catalog states that the width is 12 cm and the total height is 42 cm. What would the height of the cylinder part have to be in order for the total volume to be $1224\pi \text{ cm}^3$?



Pencil Vase
The perfect gift for your math teacher!
12cm X 42cm
\$9.95
4KE06

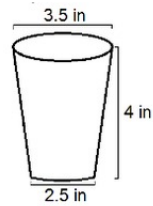
- Design your own vase with composite shapes, determine the volume, and write an ad for the catalog.
- c. Example: [Solution](#) (DOK 2)

Pablo's Icy Treat Stand sells home-made frozen juice treats as well as snow-cones. Originally, Pablo used paper cone cups with a diameter of 3.5 inches and a height of 4 inches.



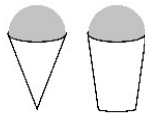
Conical Cup A

His supply store stopped carrying these paper cones, so he had to start using more standard paper cups. These are truncated cones (cones with the "pointy end" sliced off) with a top diameter of 3.5 inches, a bottom diameter of 2.5 inches, and a height of 4 inches.



Cup B

Because some customers said they missed the old cones, Pablo put a sign up saying "The new cups hold 50% more!" His daughter Letitia wonders if her father's sign is correct. Help her find out.



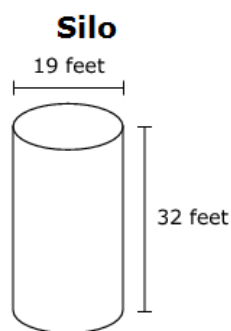
- How much juice can cup A hold? (While cups for juice are not usually filled to the top, we can assume frozen juice treats would be filled to the top of the cup.)
- How much juice can cup B hold?
- By what percentage is cup B larger in volume than cup A?
- Snow cones have ice filling the cup as well as a hemisphere of ice sticking out of the top of each cup. How much ice is in a snow cone for each cup?
- By what percentage is the snow cone in cup B larger than the snow cone in conical cup A?
- Is Pablo's sign accurate?

d. Example: [Solution](#) (DOK 2)

Rolled oats (dry oatmeal) come in cylindrical containers with a diameter of 5 inches and a height of $9\frac{1}{2}$ inches. These containers are shipped to grocery stores in boxes. Each shipping box contains six rolled oats containers. The shipping company is trying to figure out the dimensions of the box for shipping the rolled oats containers that will use the least amount of cardboard. They are only considering boxes that are rectangular prisms so that they are easy to stack.

- What is the surface area of the box needed to ship these containers to the grocery store that uses the least amount of cardboard?
- What is the volume of this box?

e. Example: An empty corn silo in the shape of a cylinder is being filled with corn.



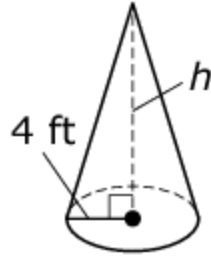
The silo is filled at a constant rate for a total of 10 hours. The table shows the amount of corn, in cubic feet, in the silo at the given number of hours after filling started.

Number of Hours	Amount of Corn (cu ft)
0	0
3	2475
5	4125
8	6600

Write the **percent** of the silo that is filled with corn at 10 hours.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#25	4	G, RP	A	2	8.G.C.9, 7.RP.A.3	1, 4	90.9 – 91

f. Example: A cone with radius 4 feet is shown. Its approximate volume is 165 cubic feet.



Write the height of the cone, in feet. Round your answer to the nearest hundredth.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#10	1	G	H	2	8.G.B.8	N/A	3.6

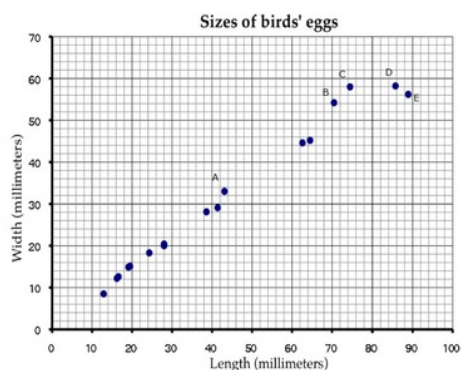
g. Example: What is the value of $\frac{3}{4} + \frac{7}{12} - (-4)$?

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#1	2		2A, 2B	1	8.G.9, A-REI.4	N/A	5

Investigate patterns of association in bivariate data. (8.SP.A)

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. **(8.SP.A.1) (DOK 1,2,3)**
 - a. Example: [Solution](#) (DOK 3)

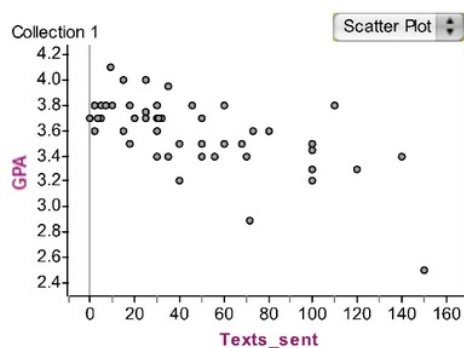
This scatter diagram shows the lengths and widths of the eggs of some American birds.



- a. A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width of 41.6 millimeters. Use an X to mark a point that represents this on the scatter diagram.
- b. What does the graph show about the relationship between the lengths of birds' eggs and their widths?
- c. Another sample of eggs from similar birds has an average length of 35 millimeters. If these bird eggs follow the trend in the scatter plot, about what width would you expect these eggs to have, on average?
- d. Describe the differences in shape of the two eggs corresponding to the data points marked C and D in the plot.
- e. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

- b. Example: [Solution](#) (DOK 2)

Medhavi suspects that there is a relationship between the number of text messages high school students send and their academic achievement. To explore this, she asks each student in a random sample of 52 students from her school how many text messages he or she sent yesterday and what his or her grade point average (GPA) was during the most recent marking period. The data are summarized in the scatter plot of number of text messages sent versus GPA shown below.



Describe the relationship between number of text messages sent and GPA. Discuss both the overall pattern and any deviations from the pattern.

c. Example: [Solution](#) (DOK 3)

Do taller people tend to have bigger hands? To investigate this question, each student in your class should measure his or her hand span (in cm) and height (in inches). Record these values in the table below.

Student	Hand Span (cm)	Height (inches)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		

- Create a clearly labeled graph that displays the relationship between height and hand span.
- Based on the graph, how would you answer the question about whether taller people tend to have bigger hands?
- Based on your graph, would you describe the relationship between hand span and height as linear or nonlinear? Explain your choice.

d. Example: [Solution](#) (DOK 3)

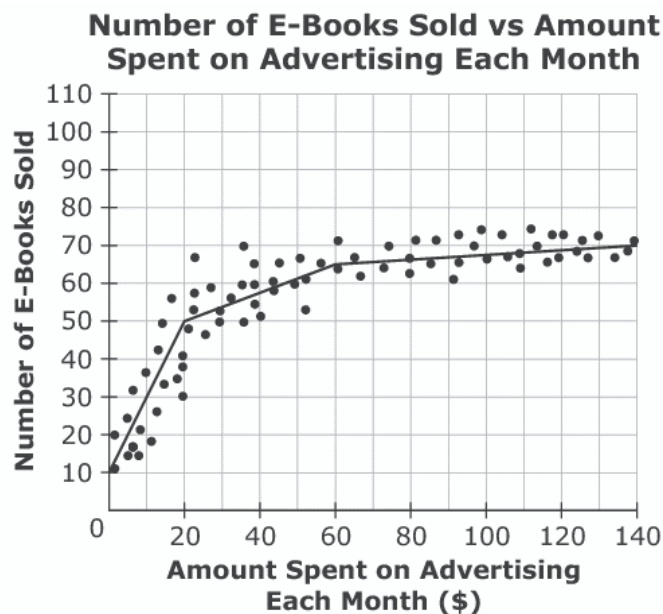
Is there an association between the weight of an animal's body and the weight of the animal's brain? 1. Make a scatterplot using the following data.

Body and brain weight by animal. Source:
<http://mste.illinois.edu/malcz/DATA/BIOLOGY/Animals.html>

Animal	Body Weight (kg)	Brain Weight (g)
Mountain beaver	1.35	8.1
Cow	465	423
Grey wolf	36.33	119.5
Goat	27.66	115
Guinea pig	1.04	5.5
Asian elephant	2547	4603
Donkey	187.1	419
Horse	521	655
Potar monkey	10	115
Cat	3.3	25.6
Giraffe	529	680
Gorilla	207	406
Human	62	1320
African elephant	6654	5712
Rhesus monkey	6.8	179

Kangaroo	35	56
Golden hamster	0.12	1
Mouse	0.023	0.4
Rabbit	2.5	12.1
Sheep	55.5	175
Jaguar	100	157
Chimpanzee	52.16	440
Mole	0.122	3
Pig	192	180

- Do there appear to be outliers in this data? Which animals appear to be outliers? Explain how you identified these outliers.
 - Removing the outliers from the data set, make a new scatterplot of the remaining animal body and brain weights.
 - Does there appear to be a relationship between body weight and brain weight? If yes, write a brief description of the relationship.
 - Take a piece of uncooked spaghetti and use that spaghetti to informally fit a line to the data. Attempt to place your line so that the vertical distances from the points to the line are as small as possible.
 - How well does the spaghetti line appear to fit the data? Explain.
- e. Example: Tyler earns \$3.00 for every e-book he sells on his website. (E-books are books that are available electronically.) He investigated the relationship between the amount spent on advertising each month and the number of e-books sold. He used this information to determine the lines of best fit shown in this graph.



What is the greatest amount Tyler should spend on advertising each month? *Show your work or explain how you found your answer.*

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#5	4	NS	4A, 4B, 4D, 4F	2	8.SP.1, 8.sp.3, 8.F.5		See Below

Sample Top-Score Response:

Tyler should spend \$60 on monthly advertising. The slope of the line from \$0 to \$20 is $\frac{40}{20} = 2$. Since he earns \$3 for every book sold, he makes $2(3) = \$6$ for every dollar spent on advertising. The slope of the line from \$20 to \$60 is $\frac{15}{40} = \frac{3}{8}$. So, he earns $\frac{3(3)}{8} = \frac{9}{8} \approx \1.13 for every dollar spent on advertising. The slope of the line from \$60 to \$140 is $\frac{5}{80} = \frac{1}{16}$. So, he earns $\frac{5(3)}{16} = \frac{15}{16} \approx \0.94 for every dollar spent on advertising. So, he will earn the most profits by spending \$60 per month. He will earn \$135 in profits, which is \$5 more than spending \$20 on advertising. If he spends more than \$60, then he will be spending more on advertising than he is making on sales.

For Full credit (2 points):

The response demonstrates a full and complete understanding of modeling scenarios of this type. The response contains the following evidence:

- The student indicates either \$20 or \$60 (or indicates that any value in this range is acceptable) should be spent on advertising.

AND

- The student provides sufficient support for this conclusion.

For partial credit (1 point):

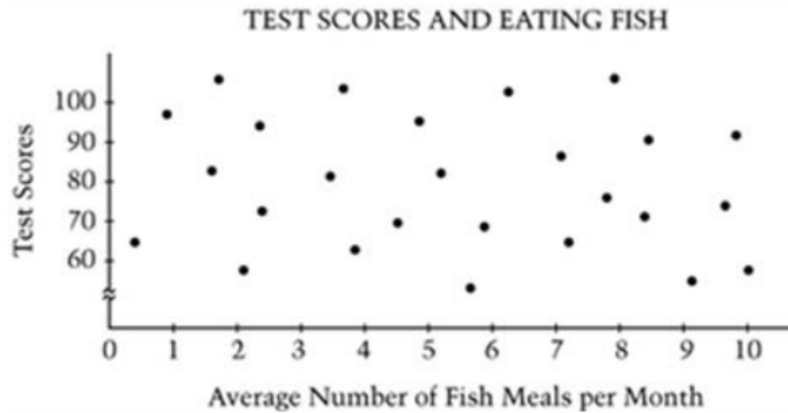
The response demonstrates a partial understanding of modeling scenarios of this type. The response contains the following evidence:

- The student provides work or explanations demonstrating a clear understanding of how to analyze the profits in the scenario, but does not resolve the analysis into a final recommendation.

OR

- The student indicates an incorrect amount of money should be spent on advertising, but provides reasoning to support this conclusion that contains a minor conceptual or calculation error.

- f. Example: ([Former NAEP question](#)) (DOK 1)

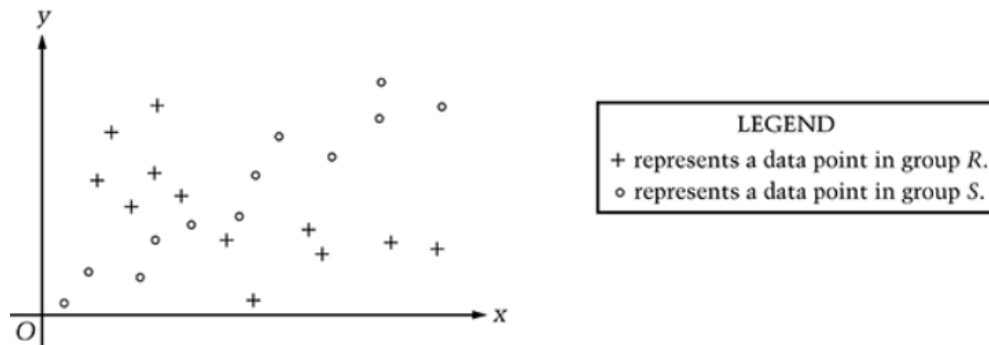


For a science project, Marsha made the scatterplot above that gives the test scores for the students in her math class and the corresponding average number of fish meals per month. According to the scatterplot, what is the relationship between test scores and the average number of fish meals per month?

- A. There appears to be no relationship.
- B. Students who eat fish more often score higher on tests.
- C. Students who eat fish more often score lower on tests.
- D. Students who eat fish 4-6 times per month score higher on tests than those who do not eat fish that often.
- E. Students who eat fish 7 times per month score lower on tests than those who do not eat fish that often.

Answer: A. There appears to be no relationship.

g. Example: ([Former NAEP question](#)) (DOK 1)



The scatterplot above shows data for groups R and S. Which of the following statements is true about the correlation between the x and y values of group R and the correlation between the x and y values of group S?

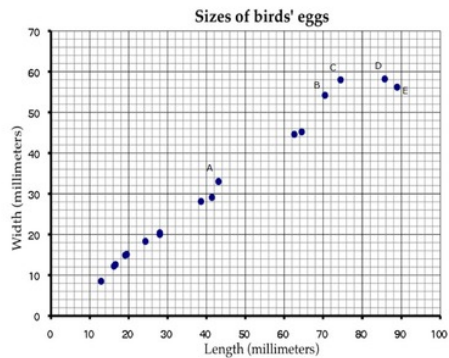
- A. The x and y values appear to be negatively correlated in both R and S.
- B. The x and y values appear to be positively correlated in both R and S.
- C. The x and y values appear to be negatively correlated in R, but positively correlated in S.
- D. The x and y values appear to be positively correlated in R, but negatively correlated in S.
- E. The x and y values appear to be more highly correlated in R than in S.

Answer: C. The x and y values appear to be negatively correlated in R, but positively correlated in S.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. **(8.SP.A.2) (DOK 1,2)**

- a. Example: [Solution](#) (DOK 3)

This scatter diagram shows the lengths and widths of the eggs of some American birds.



a. A biologist measured a sample of one hundred Mallard duck eggs and found they had an average length of 57.8 millimeters and average width of 41.6 millimeters. Use an X to mark a point that represents this on the scatter diagram.

b. What does the graph show about the relationship between the lengths of birds' eggs and their widths?

c. Another sample of eggs from similar birds has an average length of 35 millimeters. If these bird eggs follow the trend in the scatter plot, about what width would you expect these eggs to have, on average?

d. Describe the differences in shape of the two eggs corresponding to the data points marked C and D in the plot.

e. Which of the eggs A, B, C, D, and E has the greatest ratio of length to width? Explain how you decided.

- b. Example: [Solution](#) (DOK 3)

Is there an association between the weight of an animal's body and the weight of the animal's brain? 1. Make a scatterplot using the following data.

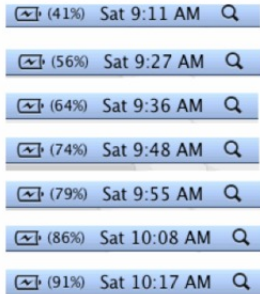
Body and brain weight by animal. Source:
<http://mste.illinois.edu/malcz/DATA/BIOLOGY/Animals.html>

Animal	Body Weight (kg)	Brain Weight (g)
Mountain beaver	1.35	8.1
Cow	465	423
Grey wolf	36.33	119.5
Goat	27.66	115
Guinea pig	1.04	5.5
Asian elephant	2547	4603
Donkey	187.1	419
Horse	521	655
Potar monkey	10	115
Cat	3.3	25.6
Giraffe	529	680
Gorilla	207	406
Human	62	1320
African elephant	6654	5712
Rhesus monkey	6.8	179
Kangaroo	35	56
Golden hamster	0.12	1
Mouse	0.023	0.4
Rabbit	2.5	12.1
Sheep	55.5	175
Jaguar	100	157
Chimpanzee	52.16	440
Mole	0.122	3
Pig	192	180

- Do there appear to be outliers in this data? Which animals appear to be outliers? Explain how you identified these outliers.
- Removing the outliers from the data set, make a new scatterplot of the remaining animal body and brain weights.
- Does there appear to be a relationship between body weight and brain weight? If yes, write a brief description of the relationship.
- Take a piece of uncooked spaghetti and use that spaghetti to informally fit a line to the data. Attempt to place your line so that the vertical distances from the points to the line are as small as possible.
- How well does the spaghetti line appear to fit the data? Explain.

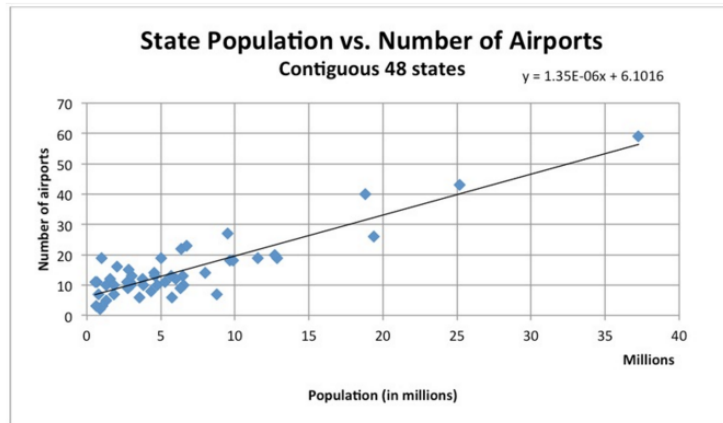
- c. Example: [Solution](#) (DOK 2)

Jerry forgot to plug in his laptop before he went to bed. He wants to take the laptop to his friend's house with a full battery. The pictures below show screenshots of the battery charge indicator after he plugs in the computer at 9:11 a.m.



- a. The screenshots suggest an association between two variables. What are the two variables in this situation?
- b. Make a scatter plot of the data.
- c. Draw a line that fits the data and find the equation of the line.
- d. When can Jerry expect to have a fully charged battery?
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.* **(8.SP.A.3) (DOK 1,2)**
- a. Example: [Solution](#) (DOK 2)

The scatter plot below shows the relationship between the number of airports in a state and the population of that state according to the 2010 Census. Each dot represents a single state. The number of airports in each state comes from data on <http://www.nationalatlas.gov/atlasftp.html?openChapters=chptrans#chptrans>. The data for population comes from the 2010 census: <http://www.census.gov/2010census/data/>



- a. How would you characterize the relationship between the number of airports in a state and the state's population? (Select one):
- i. The variables are positively associated; states with higher populations tend to have fewer airports.
 - ii. The variables are negatively associated; states with higher populations tend to have fewer airports.

- iii. The variables are positively associated; states with higher populations tend to have more airports.
- iv. The variables are negatively associated; states with higher populations tend to have more airports.
- v. The variables are not associated.

LaToya uses the function $y = (1.35 \times 10^{-6})x + 6.1$ to model the relationship between the number of airports, y and the population in a state, x .

b. How many airports does LaToya's model predict for a state with a population of 30 million people? [_____].

c. What does the number 6.1 that appears in LaToya's function mean in the context of airports vs. populations? (Select one.)

- i. The average number of airports in a state is 6.1.
- ii. The median number of airports in a state is 6.1.
- iii. The model predicts a population of 6.1 people in a state with no airports.
- iv. The model predicts 6.1 airports in a state with no people.
- v. The model predicts that 6.1 states have no airports.
- vi. The model predicts 6.1 more airports, on average, for each additional person in a state.
- vii. The model predicts 6.1 fewer airports, on average, for each additional person in a state.

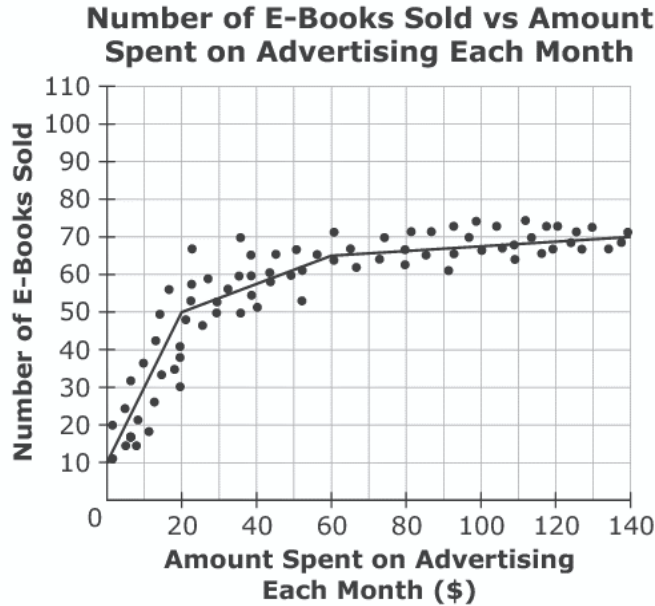
d. What does the number 1.35×10^{-6} that appears in LaToya's function mean in the context of airports vs. populations? (Select one.)

- i. The average number of airports in a state is 1.35×10^{-6} .
- ii. The median number of airports in a state is 1.35×10^{-6} .
- iii. The model predicts 1.35×10^{-6} airports in a state with no people.
- iv. The model predicts 1.35×10^{-6} people in a state with no airports.
- v. The model predicts that 1.35×10^{-6} states have no airports.
- vi. The model predicts 1.35×10^{-6} more airports, on average, for each additional person in a state.
- vii. The model predicts 1.35×10^{-6} fewer airports, on average, for each additional person in a state.
- viii. The number 1.35×10^{-6} cannot be interpreted in this context.

e. Fill in the following newspaper headline based on this relationship:

On average, a state in the contiguous 48 US states has 1 additional airport for every _____ additional people.

- b. Example: Tyler earns \$3.00 for every e-book he sells on his website. (E-books are books that are available electronically.) He investigated the relationship between the amount spent on advertising each month and the number of e-books sold. He used this information to determine the lines of best fit shown in this graph.



What is the greatest amount Tyler should spend on advertising each month? *Show your work or explain how you found your answer.*

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#5	4	NS	4A, 4B, 4D, 4F	2	8.SP.1, 8.sp.3, 8.F.5		See Below

Sample Top-Score Response:

Tyler should spend \$60 on monthly advertising. The slope of the line from \$0 to \$20 is $\frac{40}{20} = 2$. Since he earns \$3 for every book sold, he makes $2(3) = \$6$ for every dollar spent on advertising. The slope of the line from \$20 to \$60 is $\frac{15}{40} = \frac{3}{8}$. So, he earns $\frac{3(3)}{8} = \frac{9}{8} \approx \1.13 for every dollar spent on advertising. The slope of the line from \$60 to \$140 is $\frac{5}{80} = \frac{1}{16}$. So, he earns $\frac{5(3)}{16} = \frac{15}{16} \approx \0.94 for every dollar spent on advertising. So, he will earn the most profits by spending \$60 per month. He will earn \$135 in profits, which is \$5 more than spending \$20 on advertising. If he spends more than \$60, then he will be spending more on advertising than he is making on sales.

For Full credit (2 points):

The response demonstrates a full and complete understanding of modeling scenarios of this type. The response contains the following evidence:

- The student indicates either \$20 or \$60 (or indicates that any value in this range is acceptable) should be spent on advertising.

AND

- The student provides sufficient support for this conclusion.

For partial credit (1 point):

The response demonstrates a partial understanding of modeling scenarios of this type. The response contains the following evidence:

- The student provides work or explanations demonstrating a clear understanding of how to analyze the profits in the scenario, but does not resolve the analysis into a final recommendation.

OR

- The student indicates an incorrect amount of money should be spent on advertising, but provides reasoning to support this conclusion that contains a minor conceptual or calculation error.
- c. Example: Claire is filling bags with sand. All the bags are the same size. Each bag must weigh less than 50 pounds. One sand bag weighs 58 pounds, another sand bag weighs 41 pounds, and another sand bag weighs 53 pounds. Explain whether Claire can pour sand between sand bags so that the weight of each bag is less than 50 pounds.

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key
#31	2		2A	3	6.SP.3		See below

Key and Rubric:

1. Sample top-score response: since the mean is more than 50, $(58+41+53)/3=50\ 2/3$ pounds, it is not possible to move sand between bags so that each sandbag weighs no more than 50 pounds.
2. For full credit: The response demonstrates a full and complete understanding of solving problems of this type. The response demonstrates a full and complete understanding of solving problems of this type. The response contains the following evidence: Student provides sufficient support for the conclusion that it is not possible to have less than 50 pounds or less in each sandbag (e.g. applying the mean, explaining how much weight cannot be transferred, or other valid supporting explanation).
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?* **(8.SP.A.4) (DOK 1,2,3)**
 - a. Example: [Solution](#) (DOK 3)

All the students at a middle school were asked to identify their favorite academic subject and whether they were in 7th grade or 8th grade. Here are the results:

Favorite Subject by Grade

Grade	English	History	Math/Science	Other	Totals
7th Grade	38	36	28	14	116
8th Grade	47	45	72	18	182
Totals	85	81	100	32	298

Is there an association between favorite academic subject and grade for students at this school? Support your answer by calculating appropriate relative frequencies using the given data.

b. Example: [Solution](#) (DOK 2)

Is there an association between whether a student plays a sport and whether he or she plays a musical instrument? To investigate these questions, each student in your class should answer the following two questions:

- a. Do you play a sport? (yes or no)
- b. Do you play a musical instrument? (yes or no)

Record the answers in the table below.

Student	Sport?	Musical Instrument?
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		

12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
23		
24		

- Summarize the data into a clearly labeled table.
 - Of those students who play a sport, what proportion play a musical instrument?
 - Of those students who do not play a sport, what proportion play a musical instrument?
 - Based on the class data, do you think there is an association between playing a sport and playing an instrument?
 - Create a graph that would help visualize the association, if any, between playing a sport and playing a musical instrument.
- c. Example: All 8th-grade students at a school answered Yes or No to the two survey questions shown.
- Do you have a cell phone? Yes or No
 - Do you have an MP3 player? Yes or No

The same students responded to both questions. Complete the two-way frequency table to show the correct totals for the given data. You must complete **all** five cells of the table for a full credit response.

	MP3 Player	No MP3 Player	Total
Cell Phone	57	122	<input type="text"/>
No Cell Phone	30	65	<input type="text"/>
Total	<input type="text"/>	<input type="text"/>	<input type="text"/>

Item	Claim	Domain	Target	DOK	CONTENT	MP	Key		
							MP3 Player	No MP3 Player	Total
#28	1	SP	J	1	8.SP.A.4	N/A	<input type="text"/>	<input type="text"/>	<input type="text"/>
							57	122	179
							30	65	95
							87	187	274

Performance Task Example:

HEARTBEATS

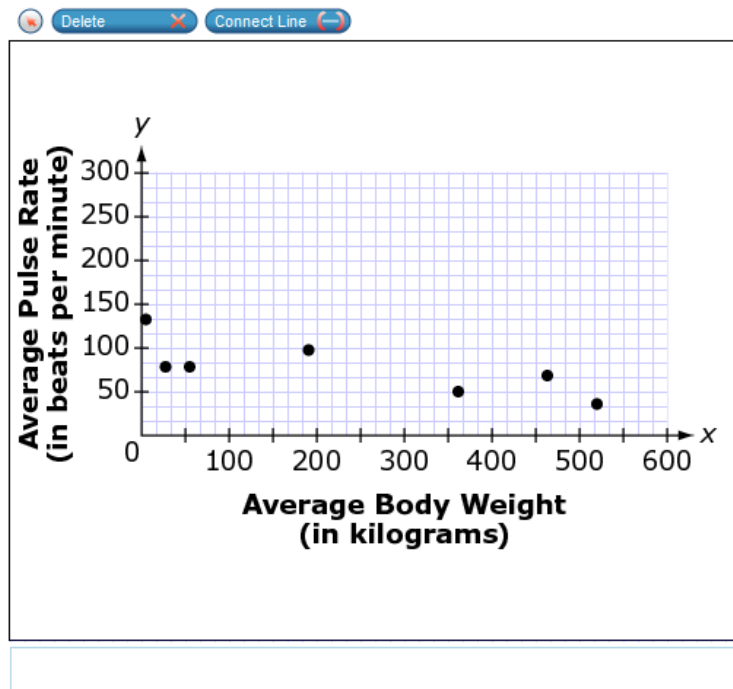
In this task, you will use data to create a model that shows the relationship between animal body weight and pulse rate measures. Then you will examine additional data to evaluate your model.

A study states that the relationship between an animal's pulse rate and body weight is approximately linear. The study data are below.

Table 1. Average Body Weight and Average Pulse Rate of Seven Animals

Animal	Average Body Weight (in kilograms)	Average Pulse Rate (in beats per minute)
Cat	3	130
Goat	28	75
Sheep	56	75
Pig	192	95
Ox	362	48
Cow	465	66
Horse	521	34

1. The data from Table 1 are plotted below. Draw a line to create a linear model of these data.



2. What is the equation of the line you drew in question 1?
3. Interpret the slope of the line from Item 1 in the context of the situation.
4. **Part A:** Based on the equation from Item 2, predict the average rate in beats per minute of an animal that weighs 6000 kilograms.

5. **Part B:** Explain whether the predicted average pulse rate in Part A is reasonable in the context of the situation.

6. The body weight and pulse rate of a guinea pig and rabbit are given in the table below.

Animal	Average Body Weight (in kg)	Average Pulse Rate (in beats per minute)
Guinea Pig	1	250
Rabbit	2.5	265

If the study had included these data, would this change the model relating average body weight and average pulse rate? How do you know?