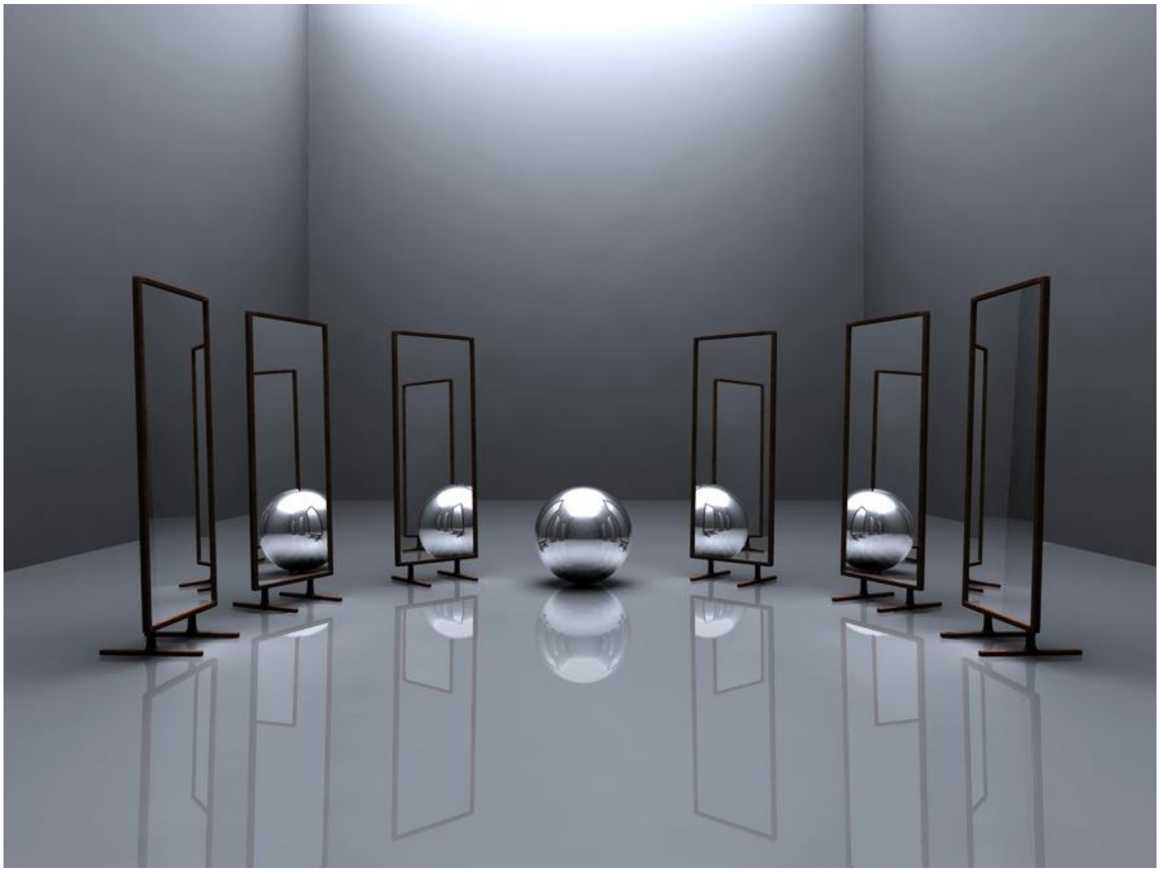
Grade 11 Revision



Getting a different perspective
on Mathematics exams

Prepared by Sarel van Greunen

Table of Contents

Equations and inequalities

Quadratic or polynomial equations

Polynomial equations are equations of the form $a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 = 0$ where $a_n \neq 0$ and $n \in \mathbb{N}$, or an equation that can be written in this form. I know this looks horribly complicated, but here's a few examples:

- $x^2 - 8 = 0$
- $3x^2 - 2x = 5$
- $\frac{2}{x} + x - 3 = 0$
- $x - 3 - \frac{1}{x-3} = 0$
- $x^3 - 1 = 0$
- $x(x-2)(x+3) = 0$
- $2x^3 - 3x^2 + x - 1 = 0$

How to solve polynomial equation

Factorization:

1. Write the equation in standard form, i.e. manipulate the equation and get the equation equal to 0.
2. Factorize the equation.
3. Set each of the different factorized terms equal to 0.
4. Solve each of the resultant equations.

Quadratic formula

In the case of a quadratic equation that can't be factorized or when it's difficult to determine the correct factors or when you are too lazy to factorize, you can use the quadratic formula for step 2-4 in the factorization method:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where you get the values of a, b and c from the standard form $ax^2 + bx + c = 0$.

Completing the square

This method is used when asked. It is the long way of solving a quadratic equation. The best way to show you the method is by doing an example. Solve $2x^2 + 3x - 8 = 0$ by completing the square.

1. Get the variables alone $\quad 2x^2 + 3x = 8$
2. Divide by the coefficient of x^2 $\quad x^2 + \frac{3}{2}x = 4$
3. Take the coefficient of x divide by 2 and square it: $\left(+\frac{3}{2} \div 2\right)^2 = \left(+\frac{3}{2} \times \frac{1}{2}\right)^2 = \left(+\frac{3}{4}\right)^2$
4. Add that answer to both sides: $\quad x^2 + \frac{3}{2}x + \left(+\frac{3}{4}\right)^2 = 4 + \left(\frac{3}{4}\right)^2$
5. The left side must be factorized: $\quad \left(x + \frac{3}{4}\right)^2 = \frac{73}{16}$

6. Take a square root on both sides: $\sqrt{\left(x+\frac{3}{4}\right)^2} = \pm\sqrt{\frac{73}{16}}$

NB: Don't forget the $\pm$ on the right side

7. Solve the resulting equation:

$x+\frac{3}{4} = \pm\frac{\sqrt{73}}{4}$

$x = \frac{-3\pm\sqrt{73}}{4}$

$x \approx 1{,}39\ or\ 2{,}89$

Systems of equations

- Choose one of the equations and get an x or y alone;
- Substitute the equation you got above into the second equation;
- Solve this equation;
- Substitute the value(s) into one of the equations given;
- Solve the other variable.

K-method

We use the k-method to make solving certain equations easier. We replace whatever repeats itself with "k" and then solve the equation.

Examples where k-method is useful:

- $x^2-3x=\frac{1}{x^2-3x}$ ... Let $x^2-3x=k$ then the equation becomes:
  $k=\frac{1}{k}$
- $2x^2-8x-\frac{3}{2x(x-4)}=2$ ... On the left we have $2x(x-4)=2x^2-8x$...
  Let $2x^2-8x=k$ then the equation becomes:
  $k-\frac{3}{k}=2$
- Here are a few interesting ones:
  - $x^{\frac{2}{3}}-x^{\frac{1}{3}}=6$ ... since $x^{\frac{2}{3}}=\left(x^{\frac{1}{3}}\right)^2$ ...Let $x^{\frac{1}{3}}=k$... Then the equation becomes
    $k^2-k=6$;
  - $2x+x^{\frac{1}{2}}-3=0$... since $x=\left(x^{\frac{1}{2}}\right)^2$ ...Let $x^{\frac{1}{2}}=k$... then the equation becomes
    $2k^2-k-3=0$;
  - $3x^{\frac{1}{5}}+x^{-\frac{1}{5}}=2$... since $x^{-\frac{1}{5}}=\frac{1}{x^{\frac{1}{5}}}$...Let $x^{\frac{1}{5}}=k$... then the equation becomes
    $3k+\frac{1}{k}=2$

Nature of the roots

The nature of the roots of an equation is basically a quick peek into how the roots of an equation will look WITHOUT having to solve the actual equation. To determine the nature of the roots, or if asked to solve variables based on the nature of the roots we follow a few basic steps:

1. Get the equation in standard form: $ax^2 + bx + c = 0$
2. Determine the discriminant $\Delta = b^2 - 4ac$
3. Interpret the discriminant:

$\Delta < 0$	$\Delta = 0$	$\Delta > 0$	
No real roots	Real roots	Real roots	
	2 Equal roots	2 Unequal roots	
	Rational roots	If Δ is perfect square, rational roots	If Δ is not a perfect square, irrational roots

Polynomial inequalities

Solving polynomial inequalities are somewhat challenging since the quickest way to solve this is by drawing a polynomial graph, i.e. a parabola or cubic graph.

Tips to remember:

- When you divide or times by a negative, the inequality sign swops around.
- When you have 0 on one side and you have a polynomial divided by a polynomial, then you treat them as if it is a polynomial times by polynomial, e.g. $\frac{x-1}{x+3} \leq 0$ can be treated as: $(x-1)(x+3) \leq 0$
- When you have 0 on one side and you have a polynomial divided or times by a polynomial AND one of the polynomials are ALWAYS positive, then you can "ignore" the polynomial that's always positive, e.g.
    - $\frac{x-1}{(x+3)^2} < 0$: since $(x+3)^2 \geq 0$ the inequality can be treated as $x - 1 < 0$;
    - $\frac{3x-1}{2x^2+1} \geq 0$: since $2x^2 + 1 \geq 0$ the inequality can be treated as $3x - 1 \geq 0$;
    - $4x(x+2)^2 > 0$: since$(x+2)^2 \geq 0$ the inequality can be treated as $4x > 0$;
    - $(3^x + 5)(x^2 - x - 2) \leq 0$: since $(3^x + 5) > 0$ the inequality can be treated as $x^2 - x - 2 \leq 0$;

Functions, relations and inverses

There are certain questions, which you must be able to answer for all 9 graphs:

1. **x-intercept:**
    - Let $y = 0$.
2. **y-intercept:**
    - Let $x = 0$.
3. **Definition/Domain:**
    - All x-values for which the function is defined.
4. **Range:**
    - All y-values for which the function is defined.
5. **Intersection between graphs:**
    - Set the y-values equal to each other or
    - Use substitution to solve simultaneously.

Specific graphs and their unique questions

- **Straight lines** will be handled during analitical geometry

- **Parabola**

  Equation:

  $y = a(x-p)^2 + q$ with (p;q) as stationary point;

  OR

  $$y = ax^2 + bx + c$$

    - The stationary point can be calculated by calcuting the **axis of symmetry** $\boldsymbol{x = -\frac{b}{2a}}$ and substituting the x-value into the original equation.
        - **Finding equation:** To find the equation depends on what has been given to you. It will always be given either:
            - **Turning point(p;q):** $y = a(x-p)^2 + q$
            - **2 $\boldsymbol{x}$-intercepts($\boldsymbol{x_1 and\ x_2}$)**: $y = a(x-x_1)(x-x_2)$

- **Hyperbola**
    - **Equation:**

  $$y = \frac{a}{x-p} + q$$

    - **Equations of assymptotes:**
        - Horizontal $y = q$; and
        - Vertical $x = p$

- A very important thing to remember is that hyperbolas are symmetrical.

- Hyperbolas have 2 axes of symmetry:
  - One with positive gradient: $y = x - p + q$
  - One with negative gradient: $y = -x + p + q$

- **Exponential**
  - **Equation:**

$$y = b.a^{x-p} + q$$

  - **Equation of asymptote:**
    - Horizontal $y = q$

  - The p-value is the number of units the graph has been moved left or right
  - The q-value is the number of units the graph has been moved up or down

- **Trigonometrical graphs**

  - The standard **sine and cosine** graphs are very similar:
    - Period= 360˚
    - Amplitude=1

  - The standard **tangent** is a strange graph
    - Period= 180˚
    - Amplitude=∞
    - Asymptotes of **standard tangent graph** at $x = 90^\circ + k.180°; k \in \mathbb{Z}$

  - There are several alterations you are going to be asked

1. **Period:** $y = \sin ax; y = \cos ax$ or $y = \tan ax$ then the new period is:

$$\textbf{Original period} \div \boldsymbol{a}$$

2. **Amplitude:** $y = b \sin x$ and $y = b \cos x$ then the new amplitude is b, if b is positive. If b is negative then we make b positive and that will be the new amplitude.

3. **Move of graph:**
   a. Left $y = \cos(x + 30^\circ)$ moved graph left by 30°
   b. Right $y = \sin(s - 40^\circ)$ moved graph right by 40°
   c. Up $y = \tan x + 2$ moved graph up by 2 units
   d. Down $y = \sin x - 1$ moved graph down by 1 unit

4. Very important in sketching the graph is finding the Critical points by dividing the period by 4. This gives you the interval between “special” happenings on the graph.

Sequences and Series

Three types of sequences

- Quadratic
- Arithmetic
- Geometric

Quadratic patterns:

Definition: Second differences are equal where the first differences form an arithmetic sequence.

General Term: $T_n = an^2 + bn + c$

To calculate the values of a,b and c:

$$\mathbf{2}\boldsymbol{a} = \textbf{Second difference}$$
$$\mathbf{3}\boldsymbol{a} + \boldsymbol{b} = \textbf{First first difference, ie Term1} - \textbf{Term2}$$
$$\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \textbf{Term1}$$

Arithmetic patterns:

Definition: All first differences are equal, i.e. you always add a constant difference.

NB: $T_2 - T_1 = T_3 - T_2$

General Term: $T_n = a + (n-1)d$

$d = T_2 - T_1$

Geometric patterns:

Definition: There exists a constant ratio, i.e. you multiply by a ratio to get from one term to the next.

NB: $\boldsymbol{T_3}/\boldsymbol{T_2} = \boldsymbol{T_2}/\boldsymbol{T_1}$

General Term: $T_n = a.r^{n-1}$

$r = T_2/T_1$

Exponents

The reason for exponents is to shorten the way of writing any number.

Exponents laws

1. $3^2 \times 3^4 = 3^{4+2} = 3^6$
2. $x^6 \div x^3 = x^{6-3} = x^3$
3. $(xy^2)^2 = x^{1\times 2} y^{2\times 2} = x^2 y^4$
4. $\sqrt[3]{x^2 y} = x^{2/3} y^{1/3}$

Basic definitions

1. $x^0 = 1$ except for $x = 0$
2. $a^{-2} = \dfrac{1}{a^2}$ or $\dfrac{1}{a^{-1}} = a^1 = a$

   A shortcut for this is, if you have $a^{-2}bc^4 = \dfrac{bc^4}{a^2}$
3. $\left(\dfrac{2}{3}\right)^{-1} = \left(\dfrac{3}{2}\right)$

How to handle exponential sums:

Simplification:

- Only multiplication and division of exponential terms:
    i. Change the bases into prime numbers
    ii. Apply law 3 or 4
    iii. Apply laws 1 and 2
    iv. Find an answer

- Add or subtract of terms of exponential terms
    i. Split the terms with exponents up
    ii. Factorise
    iii. Cancel out
    iv. Find an answer

Equations:

- You handle each of the different types of equations the same as in simplification, but your fourth steps change, instead of finding an answer, you :
    - Get the bases the same
    - Leave away the bases
    - Solve the sum

- When you have an x as base, you simply:
    - Get the term with the x, or the variable, and its exponent alone on a side;
    - Put both sides to the power of the reciprocal, inverse, of the exponent.

Financial Mathematics

Financial Mathematics is simply about the loan and investment of money in banks or financial institutions. In your grade 11 syllabus we are only doing once-off financial transactions.

Once-off financial transactions include all transactions, which occur once and then interest is added on the Principal investment or loan amount. The following will be seen as once-off transactions:

- Normal investments
- Loans at a bank(Not home loans)
- Inflation
- Depreciation

Normal investments

Simple Interest $A = P(1 + i.n)$

Compound interest $A = P(1 + i)^n$

Depreciation

Cost price/ Straight line method $A = P(1 - i.n)$

Reducing balance method $A = P(1 - i)^n$

Inflation $A = P(1 + i)^n$

Converting between nominal interest rate and effective interest:

A beloved question to be asked is to switch between nominal and effective interest. The formula is as follows:

$$1 + i_e = \left(1 + \frac{i^{(m)}}{m}\right)^m$$

In the formula: i_e = Effective interest rate

m= How many times a year the interest is compounded

$i^{(m)}$= Nominal interest rate

Sinking funds

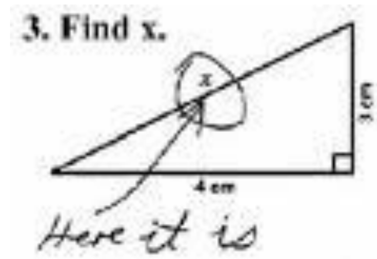
A sinking fund is a fund set up to replace an asset (Vehicle or equipment) after a period of time. Sinking funds questions consists of four parts:

1. Replacing value — How much the will cost to replace after the period (Inflation)
2. Bookvalue — The value of the car(Depreciation)
3. Sinking fund value — Replacing value – Bookvalue
4. Monthly installment — Do a timeline sum with all the payment and their interest to equal the value of the sinking fund.

Trigonometry

CAST diagram

Sin $180^\circ - \theta$ $90^\circ + \theta$	**All** $\mathbf{90^\circ - \theta}$
Tan $180^\circ + \theta$	**Cos** $180^\circ + \theta$ $\theta - 90^\circ$



Negative angles

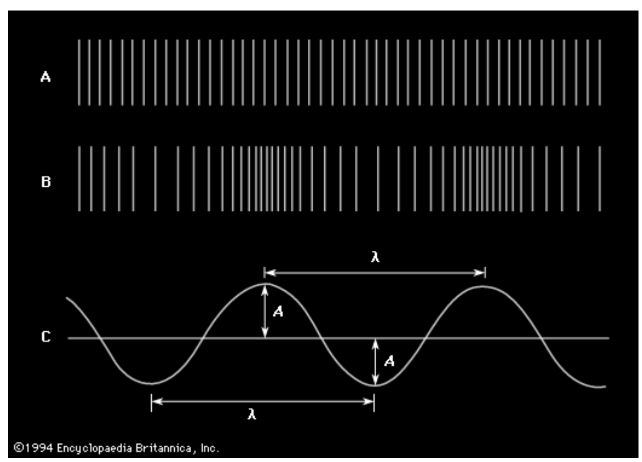
The best strategy you can follow is just adding 360˚ to the angle, since adding a revolution doesn't change the angle.

Angles greater than 360°

You simply subtract 360° until your angle is between 0° and 360°.

Co-functions

$$\cos(90^\circ - x) = \sin x$$
$$\sin(90^\circ - x) = \cos x$$



Identities

Quotient identity

$$\tan x = \frac{\sin x}{\cos x}$$

Square identities

$$\sin^2 x + \cos^2 x = 1$$
$$\sin^2 x = 1 - \cos^2 x$$
$$\cos^2 x = 1 - \sin^2 x$$

The big problem is manipulation, the secret is changing your sum so that you can get to a stage where you can solve your question. A very important part of your syllabus is being able to factorise trigonometrical expressions. There are 2 methods you can use:

1. Substitute the $\cos x$ and/or $\sin x$ with another variable, like a or b; or
2. Straight factorising

Trigonomentic equations

When it comes to solving trigonometrical equations, the first step would be to get an identity alone or factorise to solve the equation. To be able to factorise you will be expected to manipulate your sum to get it in factorisable form.

After you have the identity alone, you must find the reference angle. You use shift/2nd function to find the reference angle. After finding the reference angle you check in which quadrant you want to work.

Now you must see what the exact question is, if asked for:

General solutions

- sine and cosine, you add $k.360^\circ$
- tangent is special, since you only have to work in 1 quad and then add $k.180^\circ$
- Remember: $k \in \mathbb{Z}$

Specific solutions

- Find the general solutions
- Choose values for k such that your answer falls in the desired interval

Non-right angled triangles

Sine Rule

Formula	**When is the formula used**
$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	Working with 2 Angles

Cosine Rule

Formula	**When is the formula used**
$a^2 = b^2 + c^2 - 2bc.\cos A$	Working with 3 Sides

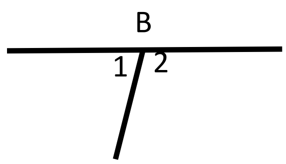
Area Rule

Formula	**When is the formula used**
Area of ΔABC= $\frac{1}{2}ab.\sin C$	When asked

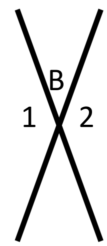
Euclidean Geometry

Euclidean Geometry is that special section in Mathematics where we study the different shapes and their properties, specifically the properties relating to the sizes of the angles and the lengths of the sides. Definitions, theorems, axioms and proofs form the basics of Euclidean geometry. In this section we will look at the basic theorems that you need to know to answer the questions asked in the exams.

Straight lines



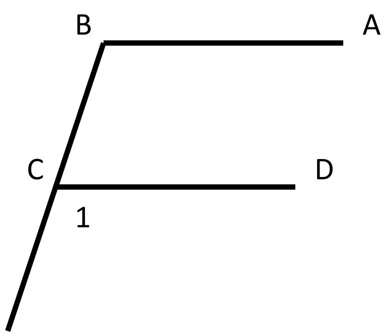
$\hat{B}_1 + \hat{B}_2 = 180^\circ$
Reason: Angles on a straight line



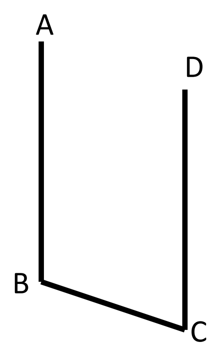
$\hat{B}_1 = \hat{B}_2$
Reason: Vertically opposite angles

Parallel lines

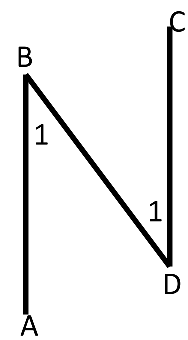
Assume in the theorems that AB ∥ CD:



$\hat{C}_1 = \hat{B}$
Reason: Corresponding angles

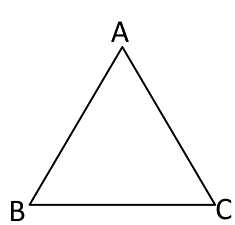


$\hat{C} + \hat{B} = 180^\circ$
Reason: Co-interior angles



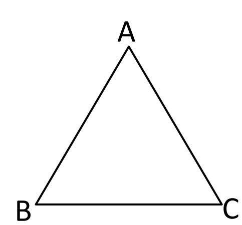
$\hat{D}_1 = \hat{B}_1$
Reason: Alternate angles

Triangles



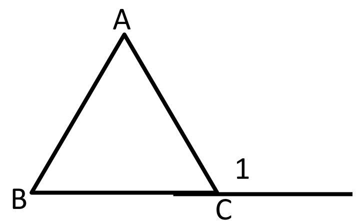
$\hat{A} + \hat{B} + \hat{C} = 180^\circ$
Reason: Interior ∠s of Δ

Isosceles Δ



If $\hat{B} = \hat{C}$ then $AB = AC$
Reason: Sides opp = ∠s

If $AB = AC$ then $\hat{B} = \hat{C}$
Reason: ∠s opp =sides



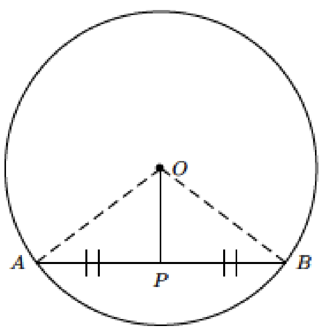
$\hat{B} + \hat{A} = \hat{C}_1$
Reason: Exterior ∠ of Δ

Circle theorems

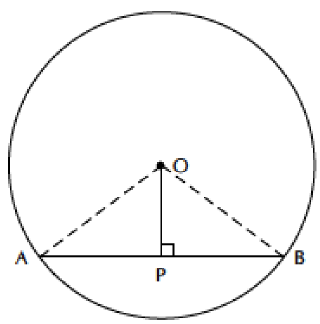
There are 9 theorems in total with quite a few of them also having converses. When answering a question in an exam, it is quite handy to think of the theorems in 3 sections:

Center of circle theorems

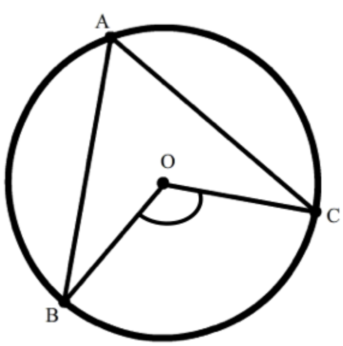
Theorem 1: If AP=PB then $OP \perp AB$.
Reason: Line from center of circle ⊥ to chord



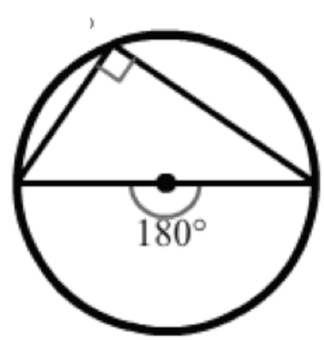
Theorem 1 converse: If $OP \perp AB$ then AP=PB
Reason: Line from center of circle to center of chord



Theorem 2: $B\hat{O}C = 2 \times \hat{A}$.
Reason: ∠ at center= 2 × ∠ at circ

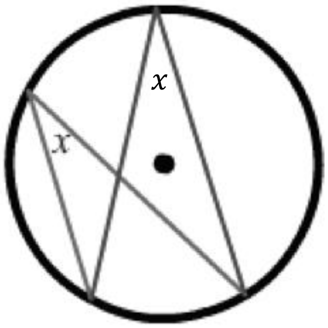


Theorem 3:
Reason: ∠ in semi-circle

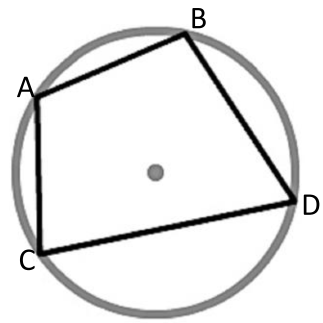


Cyclic Quadrilateral theorems

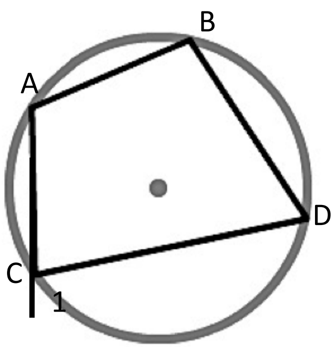
Theorem 4:
Reason: $\angle$s on circ are =



Theorem 5: $\hat{B} + \hat{C} = 180^\circ$ and $\hat{A} + \hat{D} = 180^\circ$
Reason: Opp $\angle s$ of cyclic quad

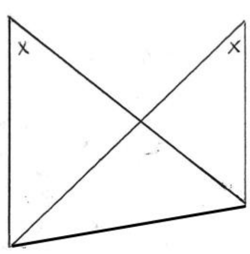


Theorem 6: $\angle C_1 = \angle B$
Reason: Ext $\angle$ of cyclic quad

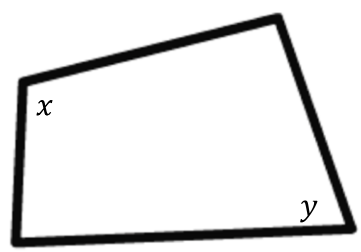


Cyclic Quadrilateral Converse Theorems

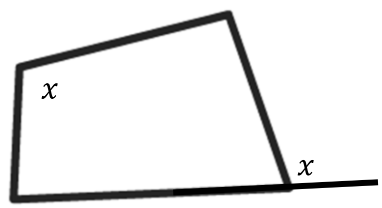
Theorem 4 Converse: If the same chord subtends equal angles in the quadrilateral, then the quadrilateral is cyclic.
Reason: Equal $\angle$s subtended by chord.



Theorem 5 converse: If $x + y = 180^\circ$ then the quadrilateral is cyclic
Reason: Opp $\angle s$ are supplementary

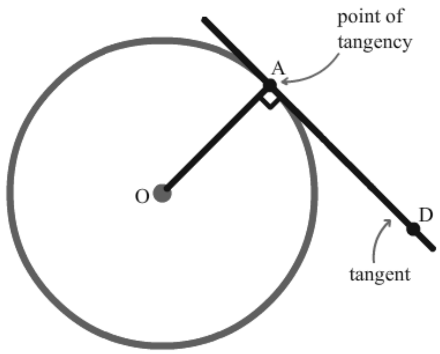


Theorem 6 Converse: If the exterior angle of a quadrilateral equals the interior opposite angle, then the quadrilateral is cyclic.
Reason: Ext ∠=Interior opp ∠

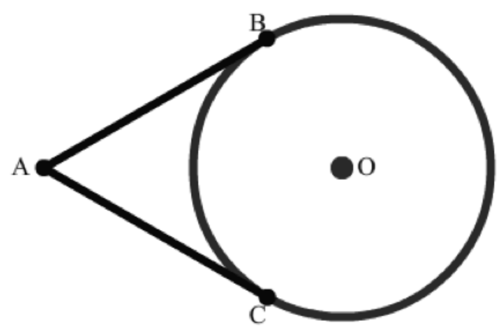


Tangent to circle theorems

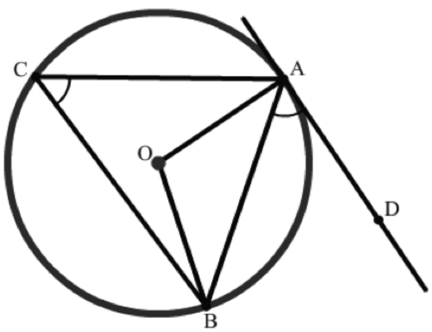
Theorem 7:
Reason: Radius ⊥ tangent



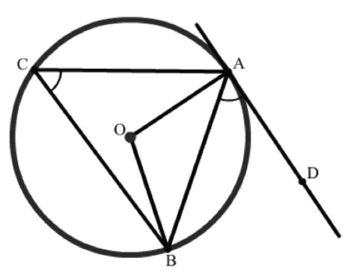
Theorem 8: $AB = AC$
Reason: Tangents from same point



Theorem 9: $D\hat{A}B = \hat{C}$
Reason: tan-chord theorem



Theorem 9 Converse: If $D\hat{A}B = \hat{C}$, then AD is a tangent
Reason: ∠ between line and chord=∠ in opp circle segment.



Analytical Geometry

Distance:

Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint:

Formula: $M = \left(\frac{x_2+x_1}{2}; \frac{y_2+y_1}{2}\right)$

OR

$x_M = \frac{x_2+x_1}{2}$ and $y_M = \frac{y_2+y_1}{2}$

Gradient:

To find gradient there are 4 different methods:

1. Two coordinates: $m = \frac{y_2-y_1}{x_2-x_1}$
2. Parallel lines: $m_1 = m_2$
3. Perpendicular line: $m_1 \times m_2 = -1$
4. Inclination angle: $m = \tan\theta$

Equation of straight line

Formula: $y - y_1 = m(x - x_1)$where $(x_1; y_1)$ is a coordinate on the straight line

Inclination angle

Formula: $m = \tan\theta$ if m is negative then $\theta = 180^\circ - \text{reference angle}$

Very important, is to find ways to calculate θ by use of Grade 8-10 geometry theorems

Special straight lines

There are special lines which you must be able to find the equation for:

1. **Altitudes** – a line from a point of a triangle, perpendicular to the opposite side
   a. Here we use the formula for perpendicular lines to calculate the second gradient
   b. We then use the formula and the coordinate on the line to find the equation of the straight line

2. **Median** – a line from a point of triangle, which bisects the opposite side
   a. Calculate the midpoint of the bisecting side
   b. Find the gradient between the midpoint and the opposite coordinate
   c. Use the formula and the coordinate on the line to find the equation of the straight line

3. **Perpendicular bisector** – ANY line which bisects a side and is perpendicular to that side
   a. Calculate the midpoint of the bisecting side
   b. Calculate the gradient of the side which is bisected
   c. Now use the formula for perpendicular lines to calculate the second gradient

Statistics and linear regression

Individual stats

Average/Mean

Formula: $\bar{x} = \frac{\sum x}{n}$ or with frequencies(f) $\bar{x} = \frac{\sum fx}{n}$

Median

The number in the middle

Formula: $\frac{n+1}{2}$-th number

Mode:

The number that occurs most

Range:

The width of the population

Formula: Maximum – minimum

Lower Quartile(Q1)

The number on a quarter of the population

Formula: $\frac{n+1}{4}$-th number

Upper Quartile(Q3)

The number on 3 quarters of the population

Formula: $\frac{3(n+1)}{4}$-th number

Inter-Quartile Range(IQR)

The width between the upper and lower quartile

Formula: $Q_3 - Q_1$

Variance:

The square of the average number by which the numbers differ from average

Formula: $\sigma^2 = \frac{\sum(x-\bar{x})^2}{n}$ or if you work with frequencies: $\sigma^2 = \frac{\sum f(x-\bar{x})^2}{n}$

Standard deviation:

The actual number by which a number varies from the average

Formula: $\sigma = \sqrt{variance} = \sqrt{\sigma^2}$

Interval Stats:

When all the numbers are given in intervals instead of individually

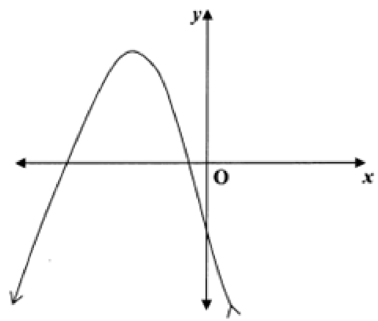
All the above stays the same except the class midpoint is now used as x, instead of the actual number.

Class Midpoint

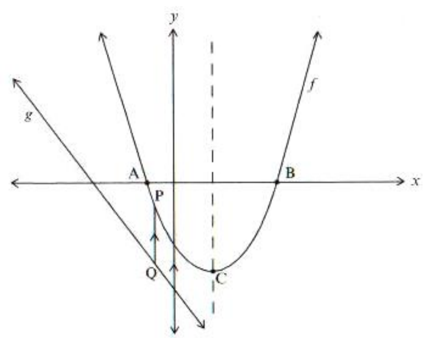
Formula: $\text{Class middle value (CMV)} = \frac{\text{Start value+End value}}{2}$

Functions, relations and inverses - Questions

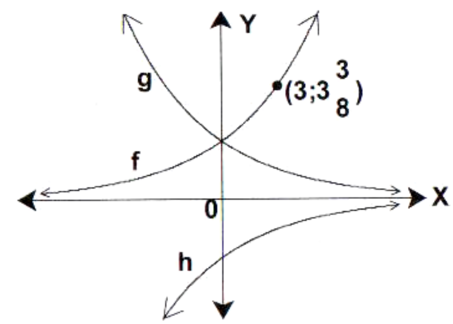
1. Sketch the following graphs:
   a. $y = 3x^2 - 2x - 5$
   b. $y = 2x^2 - 2$
   c. $y = -3x^2 - 2x$
   d. $y = \dfrac{2}{x-1}$
   e. $y = 3.2^{x-1} + 1$
2. In the following sketch you've been given the graph of $f(x) = -x^2 - 6x - 4$



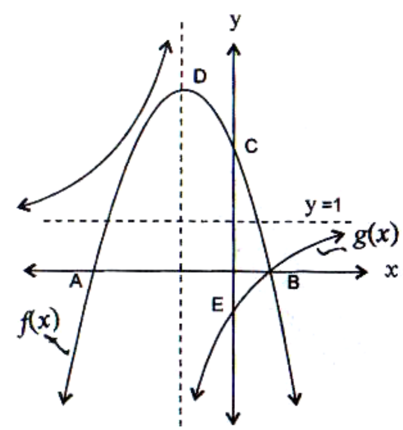
   a. Change the graph into the form $y = a(x-p)^2 + q$
   b. Prove that $f(x) \leq 5$ for all values of x
3. The sketch gives the graphs of f and g where $f(x) = x^2 - 2x - 3$ and $g(x) = -4x - 6$ and $PQ \parallel y-axis$.



   Determine:
   a. The length of AB
   b. The coordinate of C
   c. For which value(s) of x is f(x)>0?
4. Given $f(x) = 4.2^{x+1} - 2$. Determine the following:
   a. x-intercept
   b. y-intercept
   c. Equation of the asymptote
   d. $f(-1)$
   e. Sketch the graph of f
   f. Give the equation of $g(x)$ if g is f moved 2 units right.
5. The following sketch represents the graph of f with $f(x) = a^x$; g is the reflection of f in the y-axis and h, the reflection of g in the x-axis.

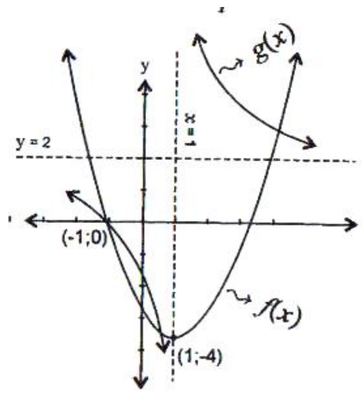


   a. Calculate the value of a
   b. Write down the equation of g and h
6. Given the graphs of f(x) and g(x) with $f(x) = -x^2 - 2x + 3$ and $g(x) = \dfrac{-2}{x-p} + q$



   a. Find the value of p and q
   b. Hence, find the coordinate of E

7. Determine the equation of $f(x)$ and $g(x)$



8. Given that $h(x) = \cos(x + 30^\circ)$ and $g(x) = -2\sin x$
   a. Determine without the use of a calculator the general solution of $h(x) = g(x)$
   b. Sketch the graphs of h and g for all $x \in [-120^\circ; 180^\circ]$

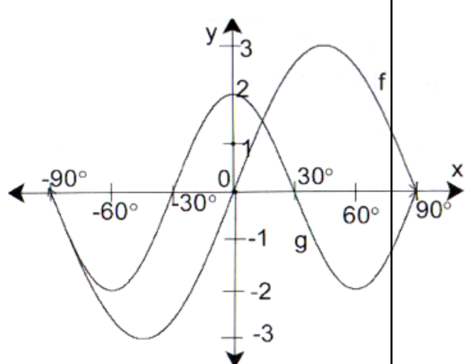
9. Given that $f(x) = \tan x$ and $g(x) = \sin 2x$
   a. Sketch f and g for $x \in [-180^\circ; 90^\circ]$
   b. For which value(s) of x is both f and g increasing for $x \in [-90^\circ; 90^\circ]$

10. The graphs represented are:

$f(x) = a\sin bx$

and

$g(x) = d\cos cx$



   a. Write down the values of a, b, c and d
   b. Write down 2 values of x where $\sin 2x - \frac{2}{3}\cos 3x = 0$

   c. What is the period of g?
   d. For which negative values of x will g(x) decrease in value if x increases?

Sequences and series - Questions

1. A ball falls from a height of 10 meters; it bounces 6 meters and then continues to fall $\frac{3}{5}$ of its previous height.

   Determine after how many bounces the ball's height will be less than 1 cm.

2. A fitness test requires that athletes repeatedly run a distance of 20m. They finish the distance 5 times in the first min, 6 times in the second min and 7 times in the third min. They carry on in this manner. Determine after how minutes have the athletes ran 2200m.

3. Write down the next two terms and the general term (Tn) in the following sequences:
   a. 3; 12; 35; 52; …
   b. 100; 80; 58; 34; …

4. Given: 2;3;2;5;2;7;…. Answer the following questions:
   a. Write down the next three terms
   b. Determine the 43rd term
   c. Calculate the sum of the first 40 terms

Exponents and logarithms - Questions

1. Simply the following:
    a. $\frac{\sqrt{27}+\sqrt{12}}{\sqrt{75}}$
    b. $\left[\frac{81x^3}{16x^{-1}}\right]^{\frac{3}{4}}$
    c. $\frac{36^{x-1}.49^x.8^x}{81^{\frac{1}{2}x}.16^{x-1}.98^{x-1}}$
    d. $\frac{10.2^{p-1}-24.2^{p-3}}{2^{p+1}.3-2^p}$

2. Solve for x in each of the following equations:
    a. $3^{x-1}+3^{x+1}=\sqrt{300}$
    b. $2^{2x+1}-3.2^x+1=0$
    c. $2^x.3^{x+1}=10$
    d. $\sqrt{3}x^{\frac{3}{4}}-\sqrt{24}=0$
    e. $5^{x+1}+4=5^{-x}$

Financial Mathematics - Questions

1. You invest R 5000 and it doubles in a certain amount of years @ 12% p.a. compounded monthly. Answer the following questions:
    a. How much is the effective interest rate?
    b. For how many years was the investment?

2. At which monthly compounded interest rate should I invest to double my principal amount in 4 years time?

3. You invest R 60 000 for 5 years @ an effective interest rate of 10,8% pa. Calculate the following:
    a. The monthly compounded interest rate
    b. Amount necessary to accumulate to the same value in 5 years, but at an interest rate of 9% pa compounded quarterly?

4. Which is a better investment option, 9% pa compounded monthly or 9,3% pa compounded quarterly?

5. A basic car is valued at R 80 000 at present. In 3 years time the same car will cost you R95 000. Market norm for depreciation is 15% pa on reducing balance method. Answer the following questions:
    a. What will the book value be after 3 years?
    b. Calculate the inflation rate
    c. You want to replace the car in 3 years time, so you set up a sinking fund. The bank offers you 6,6% pa compounded daily. How much will your annual installments be?

6. A Chev Aveo is available for you. In 4 years time the car will be valued at R 125 000. Market norm for depreciation is 10% pa on reducing balance method and inflation is at 5% pa. Answer the following questions:
    a. Calculate the cost price?
    b. Calculate the replacing value
    c. You want to replace the car in 4 years time, so you set up a sinking fund. The bank offers you 8,4% pa compounded monthly. How much will your annual installments be?

Trigonometry - Questions

1. Simplify each of the following expressions:

   a. $\frac{\cos(\alpha+720^\circ)}{\sin^2(180^\circ+\theta).\cos(\alpha+90^\circ)}$ if $\alpha+\theta=90^\circ$

   b. $\cos(\theta-90^\circ).\sin(\theta-180^\circ)+\dfrac{\cos(720^\circ+\theta)}{\sin(90^\circ-\theta)}$

   c. $\sqrt{\dfrac{\tan(-207^\circ)}{\tan 333^\circ}-\dfrac{\sin^2(x-360^\circ)}{\cos x.\sin(x-90^\circ)}}$

2. Given that $\cos 61^\circ = p$, express the following in terms of p:

   a. $\sin 209^\circ$

   b. $\dfrac{1}{\sin(-421^\circ)}$

   c. $\tan 299^\circ$

3. Prove the following identities and state where the identity is undefined:

   a) $\dfrac{1+\sin\theta}{\cos\theta}=\dfrac{\cos\theta}{1-\sin\theta}$

   b) $\sin^2\alpha+(\cos\alpha-\tan\alpha)(\cos\alpha+\tan\alpha)=1-\tan^2\alpha$

   c) $\dfrac{1}{\cos\theta}-\dfrac{\cos\theta\tan^2\theta}{1}=\cos\theta$

   d) $\dfrac{2\sin\theta\cos\theta}{\sin\theta+\cos\theta}=\sin\theta+\cos\theta-\dfrac{1}{\sin\theta+\cos\theta}$

   e) $\left(\dfrac{\cos\beta}{\sin\beta}+\tan\beta\right)\cos\beta=\dfrac{1}{\sin\beta}$

4. Solve the following equations, find the general solutions:

   a. $2\sin x+\dfrac{1}{\sin x}-3=0$

   b. $4\sin x.\cos x-2\sin x+3\cos x=6\cos^2 x$

   c. $\cos(54^\circ-x)=\sin 2x$

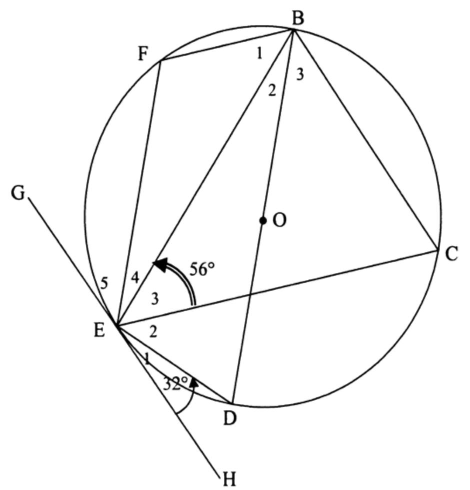
Euclidean Geometry – Questions

All the Euclidean Geometry questions come from previous year's Grade 12 Examination papers set up by the Department of Education.

QUESTION 10

In the diagram below, O is the centre of the circle. BD is a diameter of the circle. GEH is a tangent to the circle at E. F and C are two points on the circle and FB, FE, BC, CE and BE are drawn.

$\hat{E}_1 = 32°$ and $\hat{E}_3 = 56°$.



Calculate, with reasons, the values of:

10.1 $\hat{E}_2$ (2)

10.2 $E\hat{B}C$ (3)

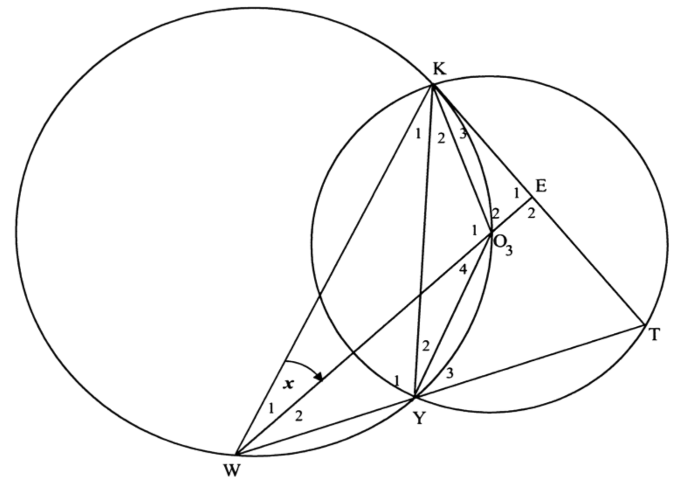
10.3 $\hat{F}$ (4)

[9]

QUESTION 12

In the diagram below, two circles intersect at K and Y. The larger circle passes through O, the centre of the smaller circle. T is a point on the smaller circle such that KT is a tangent to the larger circle. TY produced meets the larger circle at W.
WO produced meets KT at E.

Let $\hat{W}_1 = x$



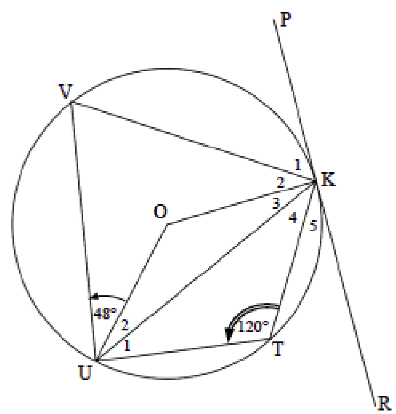
12.1 Determine FOUR other angles, each equal to x. (8)

12.2 Prove that $\hat{T} = 90° - x$. (3)

12.3 Prove that KE = ET. (3)

QUESTION 8

In the diagram below, O is the centre of the circle KTUV. PKR is a tangent to the circle at K.
$O\hat{U}V = 48°$ and $K\hat{T}U = 120°$.

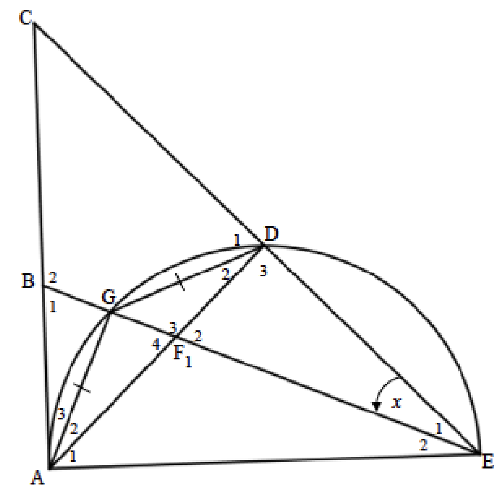


Calculate, with reasons, the sizes of the following angles:

8.1	$\hat{V}$	(2)
8.2	$K\hat{O}U$	(2)
8.3	$\hat{U}_2$	(2)
8.4	$\hat{K}_1$	(2)
8.5	$\hat{K}_2$	(2)
		[10]

QUESTION 10

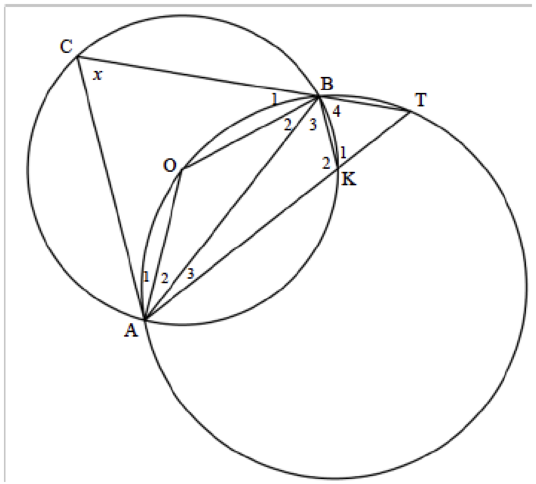
In the figure AGDE is a semicircle. AC is the tangent to the semicircle at A and EG produced intersects AC at B. AD intersects BE in F.
$AG = GD$. $\hat{E}_1 = x$.



10.1 Write down, with reasons, FOUR other angles each equal to x. (8)

QUESTION 9

O is the centre of the circle CAKB.
AK produced intersects circle AOBT at T.
$A\hat{C}B = x$

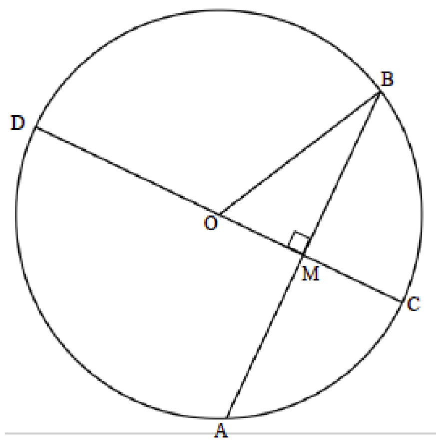


9.1 Prove that $\hat{T} = 180° - 2x$. (3)

9.2 Prove AC ∥ KB. (5)

QUESTION 10

In the diagram below, O is the centre of the circle. Chord AB is perpendicular to diameter DC. CM : MD = 4 : 9 and AB = 24 units.



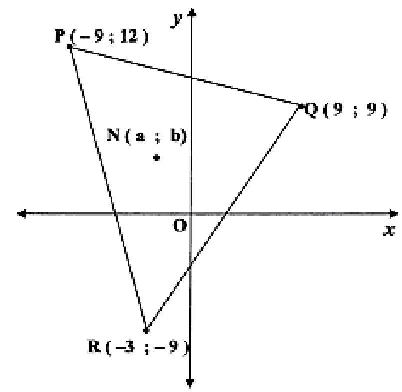
10.1 Determine an expression for DC in terms of x if CM = $4x$ units. (1)

10.2 Determine an expression for OM in terms of x. (2)

10.3 Hence, or otherwise, calculate the length of the radius. (4)
[7]

Analytical Geometry - Questions

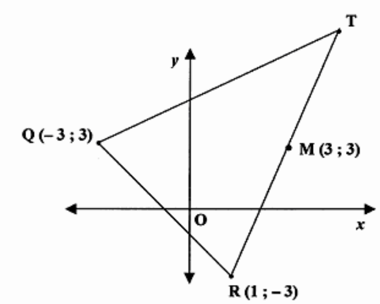
1. In the diagram we have P (-9 ; 12), Q(9 ; 9) and R(-3; -9) the corner of ΔPQR. N(a ; b) is a coordinate in the second quadrant.



Determine the following:

a. Gradient of PQ
b. Magnitude of $\hat{Q}$
c. The coordinate of M, the midpoint of QR
d. The equation of the median PM
e. The coordinate of N if P, N and M are co-linear and QN=$5\sqrt{5}$ units

2. The diagram shows ΔTQR, where Q(-3 ; 3) and R(1 ; -3). M(3 ; 3) is the midpoint of RT.

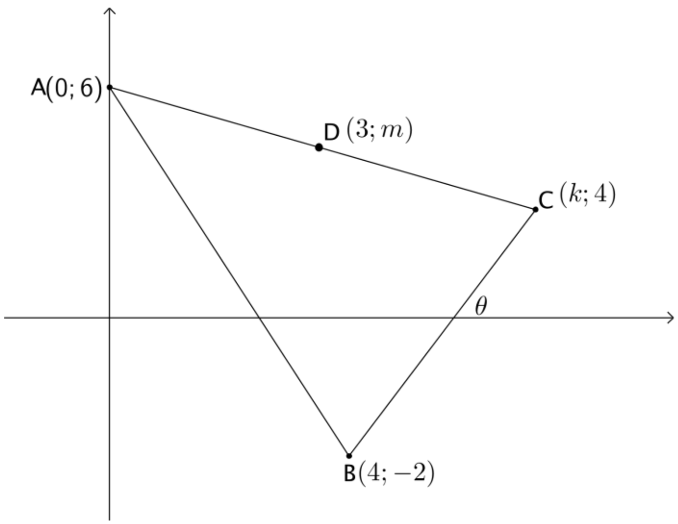


a. Calculate:
   i. Length of TR
   ii. Size of $\hat{R}$
   iii. Size of
b. Find the following:
   i. Equation of the median from T to RQ
   ii. Hence, or otherwise, determine the intersection point of the medians of the ΔTQR

c. Find the equation of the perpendicular bisector of RQ

Question 3

In the sketch below you are given the coordinates of A $(0; 6)$, B$(4; -2)$ and C $(3k; k)$. Furthermore it is given that AB⊥BC.



a) Determine the gradient of AB

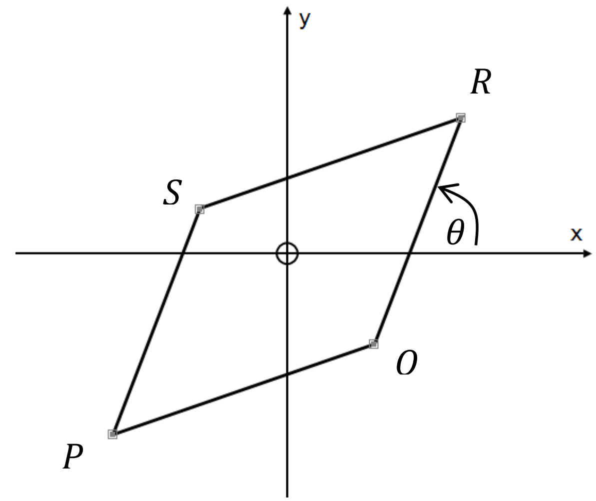
b) Determine the equation of the line parallel to AB passing through (0;-4).

c) If D$(3; m)$ is the midpoint of AC, determine thevalue of k and m.

d) If M is the midpoint of AB, prove by using analytical methods that $AB = 2MD$.

Question 4

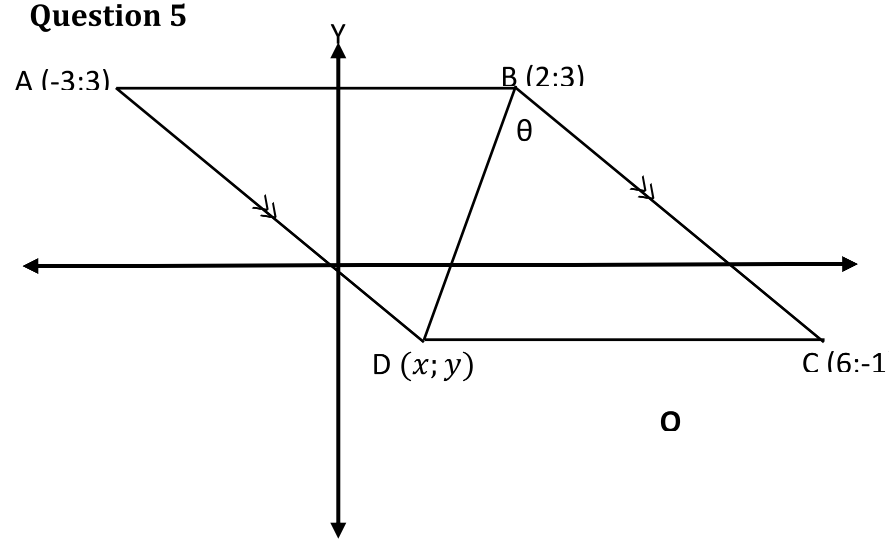
In the diagram $PQRS$ is a parallelogram. With vertices $P(-2; -4)$, $Q(1; -2)$, $R(2; 3)$ and $S(x; y)$.



Determine:

(a) The distance PQ, in simplest surd form.
(b) The midpoint of PR.
(c) The equation of the line QR.
(d) The angle of inclination (θ) of line QR.
(e) The point S.

Question 5



A (-3;3); B(2;3); C (6;-1) and D$(x; y)$ are vertices of quadrilateral ABCD in a Cartesian plane.

(a) Determine the equation of AD

(b) Determine the coordinates of D if D is equidistant from B and C.

(c) If it is given that the coordinates of D are $\left(\frac{3}{2};\ -\frac{3}{2}\right)$, determine the size of θ, the angle between BD and BC, rounded off to one decimal place.

Statistics - Questions

1. Age of 40 people:

20	17	53	65	16	18	33	69	50	45
66	25	43	48	45	53	26	38	19	41
52	60	40	38	48	53	48	27	35	38
50	69	27	29	35	41	36	39	42	53

a. Complete the table:

Interval	**Tally**	**Frequency**	**Cumulative Frequency**
10-19			
20-29			
30-39			
40-49			
50-59			
60-69			

b. Draw an ogive for the data

c. Use the letters A and B and indicate the first quartile and median is read off

d. Determine the mean and the deviation of the grouped data

2. The following is the number of marbles by certain kids on the playground:

4 86 27 21 29 37 29 44 31 42
35 38 41 29 40

Determine the following:

a. Mean
b. Mode
c. Median
d. Q1 and Q3
e. Range
f. IQR
g. Standard deviation
h. Outliers
i. Draw a box and whisker diagram for the data
j. Comment on the shape of the set of data
k. Which of the central values is most appropriate to use? Motivate your answer

3. Here is 2 learners' marks:

a.	43	61	31	79
b.	32	22	34	28

i. Which one has the higher average?
ii. Which learner is more consistent?
iii. If you're looking for a stable performer, which child would you choose?

