2.6 Properties of Equality and Congruence

Goal
Use properties of equality and congruence.

Key Words
- Reflexive Property
- Symmetric Property
- Transitive Property

Reflexive Property

Jean is the same height as Jean.

Symmetric Property

If Jean is the same height as Pedro, then Pedro is the same height as Jean.

Transitive Property

If Jean is the same height as Pedro and Pedro is the same height as Chris, then Jean is the same height as Chris.

The photos above illustrate the *Reflexive*, *Symmetric*, and *Transitive Properties* of Equality. You can use these properties in geometry with statements about equality and congruence.

Student Help

LOOK BACK
To review the difference between equality and congruence, see p. 30.

PROPERTIES OF EQUALITY AND CONGRUENCE

Reflexive Property

Equality $AB = AB$ **Congruence** $\overline{AB} \cong \overline{AB}$

$m\angle A = m\angle A$ $\angle A \cong \angle A$

Symmetric Property

Equality

If $AB = CD$, then $CD = AB$.

If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.

Congruence

If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$.

If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.

Transitive Property

Equality

If $AB = CD$ and $CD = EF$, then $AB = EF$.

If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Congruence

If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

EXAMPLE 1 **Name Properties of Equality and Congruence**

Name the property that the statement illustrates.

  a. If $\overline{GH} \cong \overline{JK}$, then $\overline{JK} \cong \overline{GH}$.

  b. $DE = DE$

  c. If $\angle P \cong \angle Q$ and $\angle Q \cong \angle R$, then $\angle P \cong \angle R$.

Solution

  a. Symmetric Property of Congruence

  b. Reflexive Property of Equality

  c. Transitive Property of Congruence

Checkpoint ✓ *Name Properties of Equality and Congruence*

Name the property that the statement illustrates.

  1. If $DF = FG$ and $FG = GH$, then $DF = GH$.

  2. $\angle P \cong \angle P$

  3. If $m\angle S = m\angle T$, then $m\angle T = m\angle S$.

Logical Reasoning In geometry, you are often asked to explain why statements are true. Reasons can include definitions, theorems, postulates, or properties.

EXAMPLE 2 **Use Properties of Equality**

In the diagram, N is the midpoint of $\overline{MP}$, and P is the midpoint of $\overline{NQ}$. Show that $MN = PQ$.
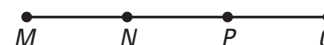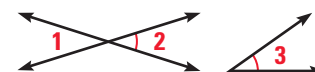
M   N   P   Q

Solution

$MN = NP$	**Definition of midpoint**
$NP = PQ$	**Definition of midpoint**
$MN = PQ$	**Transitive Property of Equality**

Checkpoint ✓ *Use Properties of Equality and Congruence*

  4. $\angle 1$ and $\angle 2$ are vertical angles, and $\angle 2 \cong \angle 3$. Show that $\angle 1 \cong \angle 3$.

$\angle 1 \cong \angle 2$	__?__ **Theorem**
$\angle 2 \cong \angle 3$	**Given**
$\angle 1 \cong \angle 3$	__?__ **Property of Congruence**

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PROPERTIES OF EQUALITY

Addition Property

Adding the same number to each side of an equation produces an equivalent equation.

Example

$x - 3 = 7$
$x - 3 + 3 = 7 + 3$

Subtraction Property

Subtracting the same number from each side of an equation produces an equivalent equation.

Example

$y + 5 = 11$
$y + 5 - 5 = 11 - 5$

Multiplication Property

Multiplying each side of an equation by the same nonzero number produces an equivalent equation.

Example

$\frac{1}{4}z = 6$
$\frac{1}{4}z \cdot 4 = 6 \cdot 4$

Division Property

Dividing each side of an equation by the same nonzero number produces an equivalent equation.

Example

$8x = 16$
$\frac{8x}{8} = \frac{16}{8}$

Substitution Property

Substituting a number for a variable in an equation produces an equivalent equation.

Example

$x = 7$
$2x + 4 = 2(7) + 4$

EXAMPLE 3 Justify the Congruent Supplements Theorem

$\angle 1$ and $\angle 2$ are both supplementary to $\angle 3$. Show that $\angle 1 \cong \angle 2$.
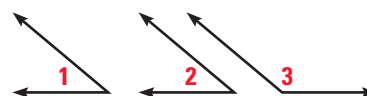
Solution

$m\angle 1 + m\angle 3 = 180°$	**Definition of supplementary angles**
$m\angle 2 + m\angle 3 = 180°$	**Definition of supplementary angles**
$m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$	**Substitution Property of Equality**
$m\angle 1 = m\angle 2$	**Subtraction Property of Equality**
$\angle 1 \cong \angle 2$	**Definition of congruent angles**

Checkpoint ✓ Use Properties of Equality and Congruence

5. In the diagram, M is the midpoint of $\overline{AB}$. Show that $AB = 2 \cdot AM$.

$MB = AM$	**Definition of _?_**
$AB = AM + MB$	**_?_ Postulate**
$AB = AM + AM$	**_?_ Property of Equality**
$AB = 2 \cdot AM$	**Distributive property**

2.6 **Exercises**

Guided Practice

Vocabulary Check   **Match the statement with the property it illustrates.**

1. $m\angle DEF = m\angle DEF$ **A.** Symmetric Property of Equality

2. If $\overline{PQ} \cong \overline{ST}$, then $\overline{ST} \cong \overline{PQ}$.   **B.** Reflexive Property of Equality

3. $\overline{XY} \cong \overline{XY}$ **C.** Transitive Property of Equality

4. If $\angle J \cong \angle K$ and $\angle K \cong \angle L$,   **D.** Reflexive Property of
then $\angle J \cong \angle L$.   Congruence

5. If $PQ = QR$ and $QR = RS$,   **E.** Symmetric Property of
then $PQ = RS$.   Congruence

6. If $m\angle X = m\angle Y$, then   **F.** Transitive Property of
$m\angle Y = m\angle X$.   Congruence

Skill Check   **Name the property that the statement illustrates.**

7. $\angle ABC \cong \angle ABC$

8. If $m\angle B = m\angle D$ and $m\angle D = m\angle F$, then $m\angle B = m\angle F$.

9. If $\overline{GH} \cong \overline{JK}$, then $\overline{JK} \cong \overline{GH}$.

Practice and Applications

Extra Practice
See p. 676.

Completing Statements Use the property to complete the statement.

10. *Reflexive Property of Equality: JK =* __?__

11. *Symmetric Property of Equality: If $m\angle P = m\angle Q$, then* __?__ *=* __?__.

12. *Transitive Property of Equality: If $AB = BC$ and $BC = CD$, then* __?__ *=* __?__.

13. *Reflexive Property of Congruence:* __?__ $\cong \angle GHJ$

14. *Symmetric Property of Congruence: If* __?__ $\cong$ __?__, *then $\angle XYZ \cong \angle ABC$.*

15. *Transitive Property of Congruence: If $\overline{GH} \cong \overline{IJ}$ and* __?__ $\cong$ __?__, *then $\overline{GH} \cong \overline{PQ}$.*

Naming Properties Name the property that the statement illustrates.

16. If $AB = CD$, then $AB + EF = CD + EF$.

Homework Help

Example 1: Exs. 10–18
Example 2: Exs. 19–24
Example 3: Exs. 19–24

17. If $m\angle C = 90°$, then $2(m\angle C) + 15° = 2(90°) + 15°$.

18. If $XY = YZ$, then $3 \cdot XY = 3 \cdot YZ$.

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**19. Using Properties** In the diagram, $m\angle 1 + m\angle 2 = 132°$, and $m\angle 2 = 105°$. Complete the argument to show that $m\angle 1 = 27°$.

$m\angle 1 + m\angle 2 = 132°$	**Given**
$m\angle 2 = 105°$	**Given**
$m\angle 1 + 105° = 132°$	**_?_ Property of Equality**
$m\angle 1 = 27°$	**_?_ Property of Equality**
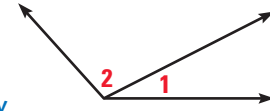
20. Using Properties of Congruence In the diagram, $\overline{AB} \cong \overline{FG}$, and $\overleftrightarrow{BF}$ bisects $\overline{AC}$ and $\overline{DG}$. Complete the argument to show that $\overline{BC} \cong \overline{DF}$.

$\overline{BC} \cong \overline{AB}$	**Definition of _?_**
$\overline{AB} \cong \overline{FG}$	**_?_**
$\overline{FG} \cong \overline{DF}$	**Definition of _?_**
$\overline{BC} \cong \overline{DF}$	**_?_ Property of Congruence**
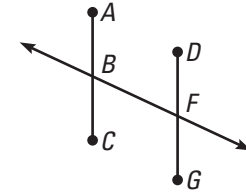
21. Unscramble the Steps In the diagram below, $PQ = RS$. Copy the diagram and arrange the statements and reasons in order to make a logical argument to show that $PR = QS$.

$P \quad Q \quad R \quad S$

PR = QS

Given

PQ + QR = RS + QR

PQ + QR = PR

Addition Property of Equality

Segment Addition Postulate
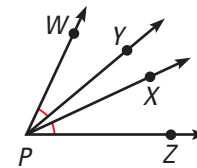
RS + QR = QS

PQ = RS

Substitution Property of Equality

Segment Addition Postulate

22. Using Properties of Equality In the diagram at the right, $m\angle WPY = m\angle XPZ$. Complete the argument to show that $m\angle WPX = m\angle YPZ$.

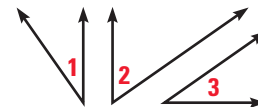|	
$m\angle WPY = m\angle XPZ$	**Given**
$m\angle WPX = m\angle WPY + m\angle YPX$	_?_
$m\angle YPZ = m\angle YPX + m\angle XPZ$	_?_
$m\angle WPY + m\angle YPX = m\angle YPX + m\angle XPZ$	_?_
$m\angle WPX = m\angle YPZ$	_?_

23. Congruent Complements Theorem Show that the Congruent
Complements Theorem is true. Use Example 3 on page 90
as a model. Provide a reason for each step.

In the diagram, $\angle 1$ is complementary to
$\angle 2$, and $\angle 3$ is complementary to $\angle 2$.
Show that $\angle 1 \cong \angle 3$.

24. Error Analysis In the diagram, $\overline{SR} \cong \overline{CB}$ and $\overline{AC} \cong \overline{QR}$. Explain
what is wrong with the student's argument.

Because $\overline{SR} \cong \overline{CB}$ and $\overline{AC} \cong \overline{QR}$,
then $\overline{CB} \cong \overline{AC}$ by the Transitive
Property of Congruence.
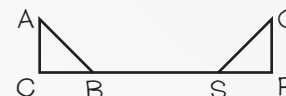
Using Algebra **Find the value of the variable using the given
information. Provide a reason for each step.**
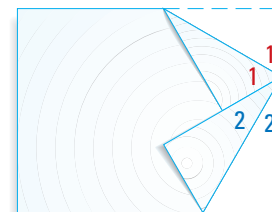
25. $AB = BC, BC = CD$

A **3t + 1** B   C **7** D

26. $QR = RS, ST = RS$

Q **23** R   S **5n − 2** T

27. Challenge Fold two corners of a piece
of paper so their edges match as shown
at the right.

What do you notice about the angle
formed by the fold lines?

Show that the angle measure is always
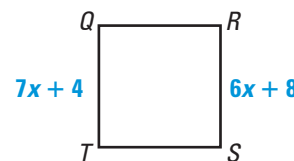the same. Provide a reason for each step.

**Standardized Test
Practice**

28. Multiple Choice Which statement illustrates the Symmetric
Property of Congruence?

Ⓐ If $\overline{AD} \cong \overline{BC}$, then $\overline{DA} \cong \overline{CB}$.

Ⓑ If $\overline{WX} \cong \overline{XY}$ and $\overline{XY} \cong \overline{YZ}$, then $\overline{WX} \cong \overline{YZ}$.

Ⓒ If $\overline{AB} \cong \overline{GH}$, then $\overline{GH} \cong \overline{AB}$.

Ⓓ $\overline{AB} \cong \overline{BA}$

29. Multiple Choice In the figure below, $\overline{QT} \cong \overline{TS}$ and $\overline{RS} \cong \overline{TS}$.
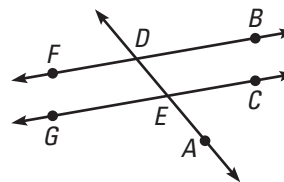What is the value of x?

Ⓕ 4   Ⓖ 12

Ⓗ 16   Ⓙ 32

7x + 4   **6x + 8**

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Mixed Review

Naming Collinear Points Use the diagram to name a point that is collinear with the given points. *(Lesson 1.3)*

30. G and E **31.** F and B

32. A and D **33.** B and D

Sketching Intersections Sketch the figure described. *(Lesson 1.4)*

34. Three lines that do not intersect but lie in the same plane.

35. Two lines that intersect at one point, and another line that intersects both of those lines at different points.
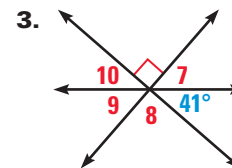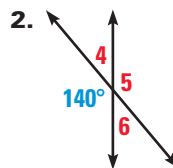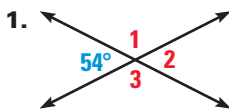
Algebra Skills

Plotting Points Plot the point in a coordinate plane. Then determine which quadrant, if any, the point lies in. *(Skills Review, p. 664)*

36. $(5, 2)$ **37.** $(0, -7)$ **38.** $(1, -4)$ **39.** $(-8, -3)$

40. $(-6, 7)$ **41.** $(10, 2)$ **42.** $(-1, 1)$ **43.** $(9, -4)$

Quiz 2

Find the measures of the numbered angles. *(Lesson 2.4)*

1.   **2.**   **3.**

In Exercises 4 and 5, rewrite the statement as an if-then statement. *(Lesson 2.5)*

4. A square is a four-sided figure.   **5.** The value of x^2 is 25 if $x = 5$.

6. Use the Law of Syllogism to write the statement that follows from the pair of true statements. *(Lesson 2.5)*

   If we charter a boat, then we will go deep sea fishing.

   If we go deep sea fishing, then we will be gone all day.

7. In the diagram, $\overrightarrow{KM}$ bisects $\angle JKN$, and $\overrightarrow{KN}$ bisects $\angle MKL$. Complete the argument to show that $m\angle JKM = m\angle NKL$. *(Lesson 2.6)*

   $m\angle JKM = m\angle MKN$ Definition of __?__

   $m\angle MKN = m\angle NKL$ Definition of __?__

   $m\angle JKM = m\angle NKL$ __?__ Property of Equality