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Greg Byrd Lynn Byrd and Chris Pearce
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1 Integers, powers and roots

The first primes are 2 3 5 7 11 13 17 19 23 29 …

Prime numbers have just two factors: 1 and the number itself.

Every whole number that is not prime can be written as a product of prime numbers in exactly one way (apart from the order of the primes).

$8 = 2 \times 2 \times 2 \quad 65 = 5 \times 13 \quad 132 = 2 \times 2 \times 3 \times 11$
$2527 = 7 \times 19 \times 19$

It is easy to multiply two prime numbers. For example, $13 \times 113 = 1469$.

It is much harder to do the inverse operation. For example, 2021 is the product of two prime numbers. Can you find them?

This fact is the basis of a system that is used to encode messages sent across the internet.

The **RSA cryptosystem** was invented by Ronald Rivest, Adi Shamir and Leonard Adleman in 1977. It uses two large prime numbers with about 150 digits each. These are kept secret. Their product, N, with about 300 digits, is made public so that anyone can use it.

If you send a credit card number to a website, your computer performs a calculation with N and your credit card number to encode it. The computer receiving the coded number will do another calculation to decode it. Anyone else, who does not know the factors, will not be able to do this.

Key words

Make sure you learn and understand these key words:

integer
inverse
multiple
common multiple
lowest common multiple (LCM)
factor
common factor
highest common factor (HCF)
prime number
prime
factor tree
power
index (indices)
square
cube
square root
cube root

Prime numbers more than 200 are 211 223 227 229 233 239 241 251 257 263 269 271 … …

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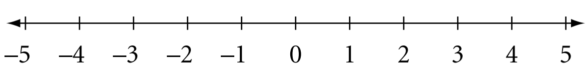
1.1 Arithmetic with integers

Integers are whole numbers. They may be positive or negative. Zero is also an integer.

You can show integers on a number line.



Look at the additions in the box to the right. The number added to 2 decreases, or goes down, by 1 each time. The answer also decreases, or goes down, by 1 each time.

$2 + 3 = 5$
$2 + 2 = 4$
$2 + 1 = 3$
$2 + 0 = 2$
$2 + -1 = 1$
$2 + -2 = 0$
$2 + -3 = -1$
$2 + -4 = -2$

Now see what happens if you subtract. Look at the first column.

The number subtracted from 5 goes down by 1 each time. The answer goes up by 1 each time. Now look at the two columns together.

You can change a subtraction into an addition by adding the **inverse** number. The inverse of 3 is –3. The inverse of –3 is 3.
For example, 5 – –3 = 5 + 3 = 8.

$5 - 3 = 2$	$5 + -3 = 2$
$5 - 2 = 3$	$5 + -2 = 3$
$5 - 1 = 4$	$5 + -1 = 4$
$5 - 0 = 5$	$5 + 0 = 5$
$5 - -1 = 6$	$5 + 1 = 6$
$5 - -2 = 7$	$5 + 2 = 7$
$5 - -3 = 8$	$5 + 3 = 8$

Worked example 1.1a

Work these out.	**a** $3 + -7$	**b** $-5 - 8$	**c** $-3 - -9$
a $3 + -7 = -4$	Subtract 7 from 3.		$3 - 7 = -4$
b $-5 - 8 = -13$	The inverse of 8 is –8.		$-5 - 8 = -5 + -8 = -13$
c $-3 - -9 = 6$	The inverse of –9 is 9.		$-3 - -9 = -3 + 9 = 6$

Look at these multiplications.

$3 \times 5 = 15$
$2 \times 5 = 10$
$1 \times 5 = 5$
$0 \times 5 = 0$

The pattern continues like this.

$-1 \times 5 = -5$
$-2 \times 5 = -10$
$-3 \times 5 = -15$
$-4 \times 5 = -20$

You can see that negative integer × positive integer = negative answer.

Now look at this pattern.

$-3 \times 4 = -12$
$-3 \times 3 = -9$
$-3 \times 2 = -6$
$-3 \times 1 = -3$
$-3 \times 0 = 0$

The pattern continues like this.

$-3 \times -1 = 3$
$-3 \times -2 = 6$
$-3 \times -3 = 9$
$-3 \times -4 = 12$
$-3 \times -5 = 15$

You can see that negative integer × negative integer = positive answer.

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Here is a simple rule, which also works for division.

> When you multiply two integers:
> if they have same signs → positive answer
> if they have different signs → negative answer

Worked example 1.1b

Work these out. **a** 12 × −3 **b** −8 × −5 **c** −20 ÷ 4 **d** −24 ÷ −6

a	12 × −3 = −36	12 × 3 = 36	The signs are different so the answer is negative.
b	−8 × −5 = 40	8 × 5 = 40	The signs are the same so the answer is positive.
c	−20 ÷ 4 = −5	20 ÷ 4 = 5	The signs are different so the answer is negative.
d	−24 ÷ −6 = 4	24 ÷ 6 = 4	The signs are the same so the answer is positive.

Warning: This rule works for multiplication and division. It does not work for addition or subtraction.

Exercise 1.1

1 Work out these additions.
a 3 + −6 **b** −3 + −8 **c** −10 + 4 **d** −10 + −7 **e** 12 + −4

2 Work out these additions.
a 30 + −20 **b** −100 + −80 **c** −20 + 5 **d** −30 + −70 **e** 45 + −40

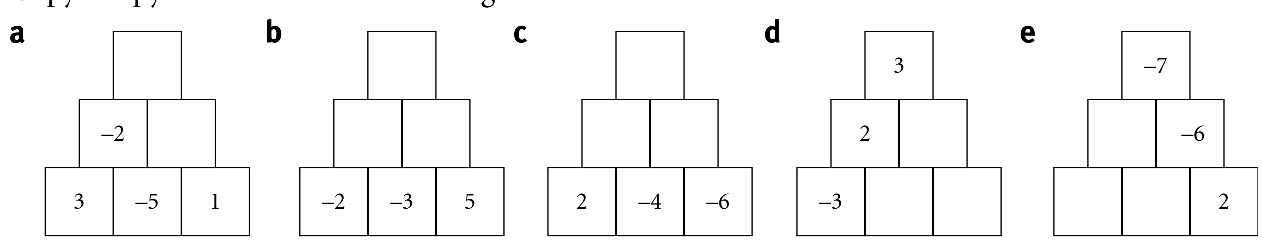
3 Work out these subtractions.
a 4 − 6 **b** −4 − 6 **c** 6 − 4 **d** −6 − 6 **e** −2 − 10

4 Write down additions that have the same answers as these subtractions. Then work out the answer to each one.
a 4 − −6 **b** −4 − −6 **c** 8 − −2 **d** −4 − −6 **e** 12 − −10

5 Work out these subtractions.
a 7 − −2 **b** −5 − −3 **c** 12 − −4 **d** −6 − −6 **e** −2 − −10

6 Here are some addition pyramids. Each number is the sum of the two in the row below it.
Copy the pyramids. Fill in the missing numbers.

> In part **a**, 3 + −5 = −2



7 Here is a subtraction table. Two answers have already been filled in: 4 − −4 = 8 and −4 − 2 = −6.
Copy the table and complete it.

		second number				
	−	**−4**	**−2**	**0**	**2**	**4**
first number	**4**	8				
	2					
	0					
	−2					
	−4				−6	

8 Work out these multiplications.
a 5 × −4 **b** −8 × 6 **c** −4 × −5 **d** −6 × −10 **e** −2 × 20

9 Work out these divisions.
a 20 ÷ −10 **b** −30 ÷ 6 **c** −12 ÷ −4 **d** −50 ÷ −5 **e** 16 ÷ −4

10 Write down two correct division expressions.
a 4 × −10 **b** −20 ÷ 5 **c** −20 × 5 **d** −40 ÷ −8 **e** −12 × −4

11 Here are some multiplications. In each case, use the same numbers to write down two correct division expressions.
a 5 × −3 = −15 **b** −8 × −4 = 32 **c** −6 × 7 = −42

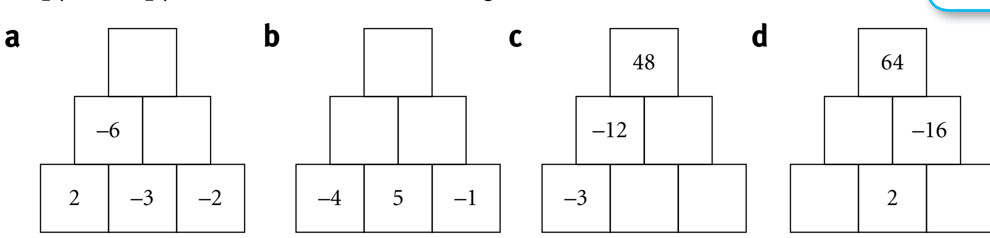
12 Here is a multiplication table. Three answers have already been filled in.

×	−3	−2	−1	0	1	2	3
3						6	
2							
1		−2					
0							
−1	3						
−2							
−3							

a Copy the table and complete it.
b Colour all the 0 answers in one colour, for example, green.
c Colour all the positive answers in a second colour, for example, blue.
d Colour all the negative answers in a third colour, for example, red.

13 These are multiplication pyramids. Each number is the product of the two in the row below it.
Copy each pyramid. Fill in the missing numbers.

The product is the result of multiplying two numbers
In part **a**, 2 × −3 = −6



14 a What integers will replace the symbols to make this multiplication correct? ○ × Δ = −12
b How many different pairs of numbers can you find that give this answer?

15 Work these out.
a 5 × −3 **b** 5 + −3 **c** −4 − −5 **d** −60 ÷ −10 **e** −2 + 18 **f** −10 − 4

16 Write down the missing numbers.
a 4 × □ = −20 **b** □ ÷ −2 = −6 **c** □ − −5 = −2 **d** □ × −3 = 12
e −2 + □ = 2 **f** □ − 4 = −3

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1.2 Multiples, factors and primes

$6 \times 1 = 6$ $6 \times 2 = 12$ $6 \times 3 = 18$ … …
$9 \times 1 = 9$ $9 \times 2 = 18$ $9 \times 3 = 27$ … …

The **multiples** of 6 are 6, 12, 18, 24, 30, 36, …, …

The multiples of 9 are 9, 18, 27, 36, 45, 54, …, …

The **common multiples** of 6 and 9 are 18, 36, 54, 72, …, …

18 36 54 … … are in both lists of multiples.

The **lowest common multiple (LCM)** of 6 and 9 is 18.

The **factors** of a number divide into it without a remainder.

$3 \times 6 = 18$ so 3 and 6 are factors of 18

The factors of 18 are 1, 2, 3, 6, 9 and 18.

The factors of 27 are 1, 3, 9 and 27.

The **common factors** of 18 and 27 are 1, 3 and 9.

The **highest common factor (HCF)** of 18 and 27 is 9.

Some numbers have just two factors. Examples are 7 (1 and 7 are factors), 13 (1 and 13 are factors) and 43. Numbers with just two factors are called **prime numbers** or just **primes**. The first ten primes are 2, 3, 5, 7, 11, 13, 17, 19, 23 and 29.

Worked example 1.2a

a Find the factors of 45. **b** Find the prime factors of 48.

a The factors of 45 are 1, 3, 5, 9, 15 and 45.

$45 = 1 \times 45$ so 1 and 45 are factors. (1 is always a factor.)
Check 2, 3, 4, … in turn to see if it is a factor.
2 is not a factor. (45 is an odd number.)
$45 = 3 \times 15$ 3 and 15 are factors.
4 is not a factor.
$45 = 5 \times 9$ 5 and 9 are factors.
6, 7 and 8 are not factors. The next number to try is 9 but we already have 9 in the list of factors. You can stop when you reach a number that is already in the list.

b The prime factors of 48 are 2 and 3.

You only need to check prime numbers.
$48 = 2 \times 24$ 2 is a prime factor. 24 is not.
$48 = 3 \times 16$ 3 is a prime factor. 16 is not.
5 and 7 are not factors.
Because 7×7 is bigger than 48, you can stop there.

Worked example 1.2b

Find the LCM and HCF of 12 and 15.

The LCM is 60.

The multiples of 12 are 12, 24, 36, 48, $\boxed{60}$, …, ….
The multiples of 15 are 15, 30, 45, $\boxed{60}$, 75, …, …
60 is the first number that is in both lists.

The HCF is 3.

The factors of 12 are 1, 2, $\boxed{3}$, 4, 6 and 12.
The factors of 15 are 1, $\boxed{3}$, 5 and 15.
3 is the largest number that is in both lists.

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Exercise 1.2

1 Find the factors of each number.
a 20 **b** 27 **c** 75 **d** 23 **e** 100 **f** 98

2 Find the first four multiples of each number.
a 8 **b** 15 **c** 7 **d** 20 **e** 33 **f** 100

3 Find the lowest common multiple of each pair of numbers.
a 6 and 8 **b** 9 and 12 **c** 4 and 14
d 20 and 30 **e** 8 and 32 **f** 7 and 11

4 The LCM of two numbers is 40. One of the numbers is 5. What is the other number?

5 Find:
a the factors of 24 **b** the factors of 32 **c** the common factors of 24 and 32
d the highest common factor of 24 and 32.

6 List the common factors of each pair of numbers.
a 20 and 25 **b** 12 and 18 **c** 28 and 35
d 8 and 24 **e** 21 and 32 **f** 19 and 31

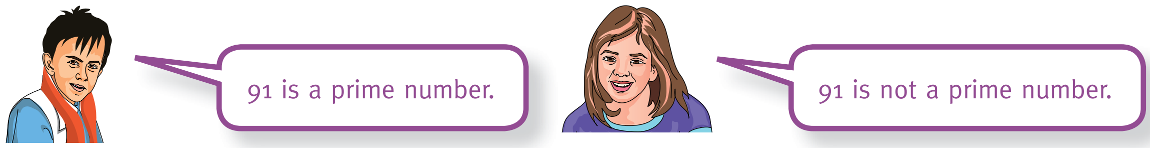
7 Find the HCF of the numbers in each pair.
a 8 and 10 **b** 18 and 24 **c** 40 and 50
d 80 and 100 **e** 17 and 33 **f** 15 and 30

8 The HCF of two numbers is 8. One of the numbers is between 20 and 30. The other number is between 40 and 60. What are the two numbers?

9 31 is a prime number. What is the next prime after 31?

10 List the prime numbers between 60 and 70.

11 Read what Xavier and Alicia say about the number 91.



Who is correct? Give a reason for your answer.

12 73 and 89 are prime numbers. What is their highest common factor?

13 7 is a prime number. No multiple of 7, except 7 itself, can be a prime number. Explain why not.

14 List the prime factors of each number.
a 12 **b** 15 **c** 21 **d** 49 **e** 30 **f** 77

15 a Write down three numbers whose <u>only</u> prime factor is 2.
b Write down three numbers whose <u>only</u> prime factor is 3.
c Write down three numbers whose <u>only</u> prime factor is 5.

16 Find a number bigger than 10 that has an <u>odd</u> number of factors.

17 Find a number that has three prime factors.

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1.3 More about prime numbers

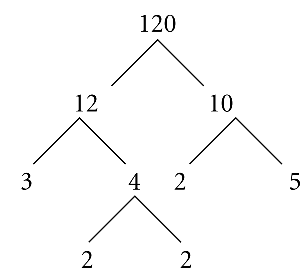
Any integer bigger than 1, that is not prime, can be written as a product of prime numbers.

Here are some examples.

$84 = 2 \times 2 \times 3 \times 7$ $\quad 45 = 3 \times 3 \times 5$ $\quad 196 = 2 \times 2 \times 7 \times 7$

You can use a **factor tree** to find and show factors.

This is how to draw a factor tree for 120.

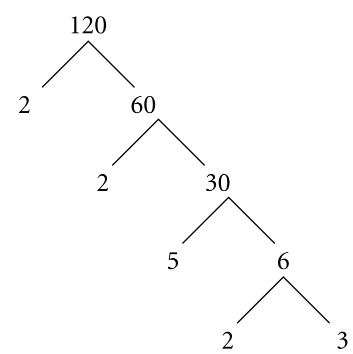


1 Draw branches to two numbers that multiply to make 120. Here 12 and 10 are chosen.
2 Do the same with 12 and 10. $12 = 3 \times 4$ and $10 = 2 \times 5$
3 3, 2 and 5 are prime, so stop.
4 $4 = 2 \times 2$ so draw two branches.
5 Stop, because all the end numbers are prime.
6 Multiply all the numbers at the ends of the branches.

$120 = 2 \times 2 \times 2 \times 3 \times 5$

You can draw the tree in different ways.

Here is a different tree for 120.



The numbers at the ends of the branches are the same.

You can write the result like this. $\quad 120 = 2^3 \times 3 \times 5$.

The small number ^{3} next to the 2 is called an **index**. 2^3 means $2 \times 2 \times 2$.

Check that these are correct.

$60 = 2^2 \times 3 \times 5$ $\quad 75 = 3 \times 5^2$

You can use these expressions to find the LCM and HCF of 60 and 75.

For the LCM, take the larger frequency of each prime factor and multiply them all together.

$60 = \boxed{2^2} \times \boxed{3} \times 5$
$75 = 3 \times \boxed{5^2}$
Two 2s, one 3, two 5s

$\text{LCM} = 2^2 \times 3 \times 5^2 = 4 \times 3 \times 25$
$= 300$

For the HCF, take the smaller frequency of each prime factor that occurs in both numbers and multiply them all together.

$60 = 2^2 \times \boxed{3} \times \boxed{5}$
$75 = \boxed{3} \times 5^2$
No 2s, one 3, one 5

$\text{HCF} = 3 \times 5$
$= 15$

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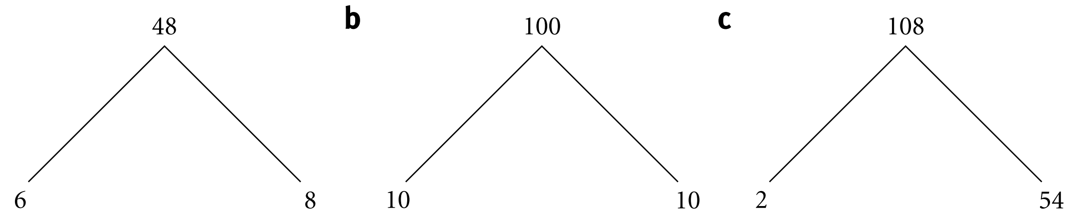
Exercise 1.3

1 Copy and complete each of these factor trees.

a **b** **c**



2 **a** Draw a <u>different</u> factor tree for each of the numbers in question **1**.
b Write each of these numbers as a product of primes.
i 48 **ii** 100 **iii** 108

3 Match each number to a product of primes.
One has been done for you.

20 • ———	• $2^2 \times 5$
24 •	• $2 \times 3 \times 7$
42 •	• $2^2 \times 3^2 \times 5$
50 •	• 2×5^2
180 •	• $2^3 \times 3$

4 Write down the number that is being represented.
a $2^2 \times 3 \times 5$ **b** 2×3^3 **c** 3×11^2 **d** $2^3 \times 7^2$ **e** $2^4 \times 3^2$ **f** $5^2 \times 13$

5 Write each number as a product of primes.
a 24 **b** 50 **c** 72
d 200 **e** 165 **f** 136

You can use a factor tree to help you.

6 **a** Write each number as a product of primes. **i** 45 **ii** 75
b Find the LCM of 45 and 75.
c Find the HCF of 45 and 75.

7 **a** Write each number as a product of primes. **i** 90 **ii** 140
b Find the LCM of 90 and 140.
c Find the HCF of 90 and 140.

8 37 and 47 are prime numbers.
a What is the HCF of 37 and 47? **b** What is the LCM of 37 and 47?

1.4 Powers and roots

A number multiplied by itself several times is called a **power** of that number. You use **indices** to show powers.

The plural of index is indices: one index, two indices.

Here are some powers of 5.

$5^2 = 5 \times 5 = 25$ This is five **squared** or the square of five.

$5^3 = 5 \times 5 \times 5 = 125$ This is five cubed or the **cube** of five.

$5^4 = 5 \times 5 \times 5 \times 5 = 625$ This is five to the power four.

$5^5 = 5 \times 5 \times 5 \times 5 \times 5 = 3125$ This is five to the power five.

The square of 5 is $5^2 = 25$.

Therefore the **square root** of 25 is 5 and you write this as $\sqrt{25} = 5$.

The cube of 5 is $5^3 = 125$.

Therefore the **cube root** of 125 is 5 and you write this as $\sqrt[3]{125} = 5$.

$\sqrt{}$ means square root.
$\sqrt[3]{}$ means cube root.

5 is not the only square root of 25.
$(-5)^2 = -5 \times -5 = 25$ so 25 has <u>two</u> square roots, 5 and -5.
$\sqrt{25}$ means the positive square root.

125 only has one integer cube root. -5 is <u>not</u> a cube root because $-5 \times -5 \times -5 = -125$.

Square numbers have square roots that are integers.
Examples: $13^2 = 169$ so $\sqrt{169} = 13$ $19^2 = 361$ so $\sqrt{361} = 19$

Try to memorise these five cubes and their corresponding cube roots:

$1^3 = 1$ so $\sqrt[3]{1} = 1$ $2^3 = 8$ so $\sqrt[3]{8} = 2$ $3^3 = 27$ so $\sqrt[3]{27} = 3$
$4^3 = 64$ so $\sqrt[3]{64} = 4$ $5^3 = 125$ so $\sqrt[3]{125} = 5$

Exercise 1.4

1 Find the value of each power.
a 3^2 **b** 3^3 **c** 3^4 **d** 3^5

2 Find the value of each power.
a 10^2 **b** 10^3 **c** 10^4

3 10^6 is one million and 10^9 is one billion.
Write down these two numbers in full.

Over one billion people live in India.

4 In each pair, which of the two numbers is larger?
a 3^5 or 5^3 **b** 2^6 or 6^2 **c** 5^4 or 4^5

5 **a** N^3 is 27. What number is N?
b 6^M is 1296. What number is M?

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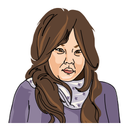
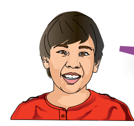
6 Can you find two different integers, A and B, so that $A^B = B^A$?

7 Write down <u>two</u> square roots for each of these numbers.
a 9 **b** 36 **c** 81 **d** 196 **e** 225 **f** 400

8 Read what Maha says about her number. What could her number be?



9 Read what Hassan says about his number. What is the largest possible value of his number?



10 Find the value of each of these.
a $\sqrt{100}$ **b** $\sqrt{400}$ **c** $\sqrt[3]{27}$ **d** $\sqrt[3]{125}$ **e** $\sqrt[3]{1000}$

11 Read what Oditi says about her number. Find a possible value for her number.

12 $2^{10} = 1024$. Use this fact to find:
a 2^{11} **b** 2^{12} **c** 2^9

13 **a** Find the value of each expression. **i** $1^3 + 2^3$ **ii** $\sqrt{1^3+2^3}$
b Find the value of $\sqrt{1^3+2^3+3^3}$. **c** Find the value of $\sqrt{1^3+2^3+3^3+4^3}$.
d Can you see an easy way to work out the value of $\sqrt{1^3+2^3+3^3+4^3+5^3}$? If so, describe it.

Summary

You should now know that:

- You can multiply or divide two integers. If they have the same sign the answer is positive ($-5 \times -2 = 10$). If they have different signs the answer is negative ($-5 \times 2 = -10$).
- You can subtract a negative number by adding the corresponding positive number.
- You can find multiples of a number by multiplying by 1, 2, 3, etc.
- Prime numbers have just two factors.
- You can write every positive integer as a product of prime factors.
- You can use the products of prime factors to find the lowest common factor and highest common multiple.
- 5^4 means $5 \times 5 \times 5 \times 5$.
- Positive integers have two square roots.
- $\sqrt{49} = 7$ and $\sqrt[3]{64} = 4$.

You should be able to:

- Add, subtract, multiply and divide integers.
- Identify and use multiples and factors.
- Identify and use primes.
- Find common factors and highest common factors (HCF).
- Find lowest common multiples (LCM).
- Write a number in terms of its prime factors, for example, $500 = 2^2 \times 5^3$.
- Calculate squares, positive and negative square roots, cubes and cube roots.
- Use index notation for positive integer powers.
- Calculate accurately, choosing operations and mental or written methods appropriate to the numbers and context.
- Manipulate numbers and apply routine algorithms, for example, to find the HCF and LCM of two numbers.

End-of-unit review

1 Work these out.
a $5 + -3$ **b** $-3 - 5$ **c** $-8 + -7$ **d** $3 - 13$ **e** $-7 - 7$

2 Work these out.
a $2 - -5$ **b** $-3 - -4$ **c** $12 - -5$ **d** $-5 - -12$ **e** $-9 - -9$

3 Work these out.
a -3×-9 **b** $8 \div -4$ **c** -20×4 **d** $-30 \div -5$ **e** $-16 \div 8$

4 Copy and complete this multiplication table.

×	−2	3	5
−4			
−3			
6			30

5 Here is a number chain. Each number is the product of the previous two numbers.
$-1 \longrightarrow -2 \longrightarrow 2 \longrightarrow -4 \longrightarrow \square \longrightarrow \square$
Write down the next two numbers in the chain.

6 Find all the factors of each number.
a 42 **b** 52 **c** 55 **d** 29 **e** 64 **f** 69

7 **a** Find two prime numbers that add up to 40.
b Find another two prime numbers that add up to 40.
c Are there any more pairs of prime numbers that add up to 40? If so, what are they?

8 Write each of these numbers as a product of its prime factors.
a 18 **b** 96 **c** 200 **d** 240 **e** 135 **f** 175

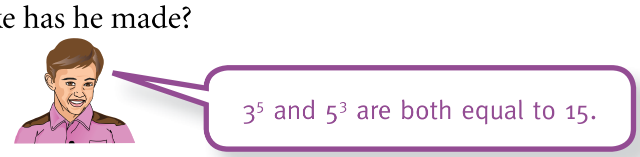
9 Use your answers to question **8** to find:
a the highest common factor of 200 and 240
b the highest common factor of 135 and 175
c the lowest common multiple of 18 and 96
d the lowest common multiple of 200 and 240.

10 Find the square roots of each number.
a 25 **b** 81 **c** 169 **d** 256

11 Find the value of each number.
a $\sqrt{64}$ **b** $\sqrt[3]{64}$

12 In computing, 2^{10} is called 1K. Write down as a number:
a 1K **b** 2K **c** 4K.

13 **a** Read Shen's comment. What mistake has he made?
b Correct the statement.



14 The HCF of two numbers is 6. The LCM is 72. One of the numbers is 24.
Find a possible value of the other number.