Name: _____ Date: _____

CHAPTER

Algebraic Linear Equations

Lesson 3.1 Solving Linear Equations with One Variable

Explain why each pair of equations is equivalent or not equivalent.

1. $x - 4 = 2$ and $x + 3 = 9$

2. $\frac{1}{2}x + 1 = 5$ and $2x = 5$

3. $3x = 6$ and $x + \frac{1}{2} = \frac{5}{2}$

4. $\frac{1}{3}x = 9$ and $2(x + 3) = 40$

Write a linear equation for each situation. State the independent and dependent variables for each equation.

5. A square crate has sides of x centimeters. Write the perimeter, P, of the crate in centimeters in terms of x.

6. A boy is k years old. His father is presently four times as old as he is. Find L, the father's age in three years' time, in terms of k.

7. The cost of 1 kilogram of salmon is r dollars. The cost of 1 kilogram of clams is \$5 less than the cost of 1 kilogram of salmon. Find the cost, S, of 8 kilograms of clams in terms of r.

Name: _____ Date: _____

Solve each equation.

8. $5x = 2x - 8$

9. $\frac{2}{3}s - 3 = 6 + \frac{1}{3}s$

10. $k - \frac{1}{4}(2k + 15) = \frac{3}{4}$

11. $4\left(\frac{1}{3}y - 9\right) = \frac{1}{2}(y - 7)$

Write the decimal for each fraction. Use bar notation.

12. $\frac{2}{3}$

13. $\frac{14}{9}$

14. $\frac{23}{11}$

15. $\frac{50}{18}$

Name: _____ Date: _____

Solve linear equations involving the distributive property.

Example

Solve the equation $\frac{2x-1}{3}+\frac{x+4}{4}=\frac{3}{2}$.

$$\frac{2x-1}{3}+\frac{x+4}{4}=\frac{3}{2}$$

$\frac{4(2x-1)}{12}+\frac{3(x+4)}{12}=\frac{3}{2}$ Write equivalent fractions with a common denominator.

$\frac{4(2x-1)+3(x+4)}{12}=\frac{3}{2}$ Rewrite the left side as a single fraction.

$\frac{8x-4+3x+12}{12}=\frac{3}{2}$ Use the distributive property.

$\frac{11x+8}{12}=\frac{3}{2}$ Simplify the numerator.

$\frac{11x+8}{12}\cdot 12=\frac{3}{2}\cdot 12$ Multiply both sides by 12.

$11x+8=18$ Simplify.

$11x+8-8=18-8$ Subtract 8 from both sides.

$\frac{11x}{11}=\frac{10}{11}$ Divide both sides by 11.

$x=\frac{10}{11}$ Simplify.

Name: _____ Date: _____

Complete.

16. Solve the equation $\frac{7x}{2} - \frac{1-x}{2} = 4$.

$$\frac{7x}{2} - \frac{1-x}{2} = 4$$

$$\frac{7x - \square + \square}{\square} = \_\_\_\_\_$$ Rewrite the left side as a single fraction.

$$\frac{\square - \square}{\square} = \_\_\_\_\_$$ Simplify the numerator.

$$\frac{\square - \square}{\square} \cdot \_\_\_\_\_ = \_\_\_\_\_ \cdot \_\_\_\_\_$$ Multiply both sides by _____.

_____ − _____ = _____ Simplify.

_____ − _____ + _____ = _____ + _____ Add _____ on both sides.

$$\frac{\square}{\square} = \frac{\square}{\square}$$ Divide both sides by _____.

x = _____ Simplify. Express your answer as a mixed number

Name: _____ Date: _____

Solve each linear equation involving the distributive property.

17. $2x + \frac{x}{2} = \frac{5}{2}$

18. $\frac{5x+1}{3} - \frac{3+4x}{6} = \frac{3}{4}$

19. $\frac{4}{5} + \frac{x}{4} = x - 1$

Name: _____ Date: _____

Write repeating decimals as fractions using linear equations.

Example

Write the decimal $0.\overline{8}$ as a fraction.

STEP 1 Assign a variable to the repeating decimal.

Let $x =$ $0.\overline{8}$.

$x =$ 0.888888...; $10x =$ 8.888888...

Notice that if you multiply both sides of this equation by 10, the infinite number of repeating digits does not change. So you can subtract one equation from the other to eliminate the infinite string of digits.

STEP 2 Subtract x from $10x$ to get a terminating decimal.

$10x - x = 8.\overline{8} - 0.\overline{8}$

$9x = 8$

STEP 3 Solve for x.

$\frac{9x}{9} = \frac{8}{9}$ Divide both sides by 9.

$x = \frac{8}{9}$ Simplify.

Therefore, $0.\overline{8} = \frac{8}{9}$.

Name: _____ Date: _____

Complete.

20. Write the decimal $1.\overline{3}$ as a fraction.

Let x = _____

x = _____

$10x$ = _____

_____ − _____ = _____ − _____ Subtract _____ from _____ to get aterminating decimal.

_____ = _____ Simplify.

_____ = _____ Divide both sides by _____.

x = _____ Simplify.

Therefore, _____ = _____.

Write repeating decimals as fractions using linear equations.

21. $0.\overline{2}$

22. $2.\overline{7}$

23. $0.\overline{46}$

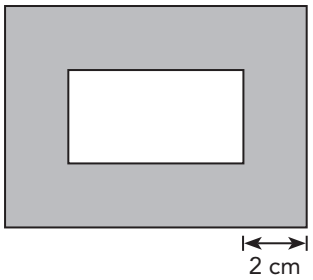
24. $4.\overline{81}$

Name: _____ Date: _____

Solve real-world problems involving linear equations with one variable.

Example

A picture is framed in a rectangular wooden frame. The border of the frame is 2 centimeters. The picture has a length 5 centimeters longer than its width. The perimeter of the picture is 42 centimeters. Find the length of the wooden frame.



Let the length of the picture be x centimeters.

So, the width of the picture is $x - 5$ centimeters.

$x + x + (x - 5) + (x - 5) = 42$	Write an equation.
$x + x + x - 5 + x - 5 = 42$	Use the distributive property.
$4x - 10 = 42$	Simplify.
$4x - 10 + 10 = 42 + 10$	Add 10 to both sides.
$4x = 52$	Simplify.
$\frac{4x}{4} = \frac{52}{4}$	Divide both sides by 4.
$x = 13$	Simplify.

Length of frame = Length of picture + 2 + 2 cm

= 13 + 2 + 2 cm

= 17 cm

The length of the wooden frame is 17 centimeters.

Complete.

25. Howard's home is 12 miles from the shopping mall. His school is located on the same road as his home and as the shopping mall. His school is two times closer to his home than to the shopping mall. How far is his school to the shopping mall?

Let the distance from Howard's school to the shopping mall be x miles.

So, the distance from Howard's home to his school is $\frac{1}{2}x$ miles.

_____ = 12	Write an equation.
_____ = 12	Simplify.
_____ · _____ = _____ · _____	Multiply both sides by _____.
_____ = _____	Simplify.
_____ = _____	Divide both sides by _____.
x = _____	Simplify.

The distance from Howard's school to the shopping mall is _____ miles.

Name: _____ Date: _____

Solve. Show your work.

26. A bucket of sand has a mass 3 times the mass of a bucket of water. To make the mass of both buckets the same, Meghan removed 18 kilograms of sand from the sand bucket. What was the mass of the bucket of sand before the sand was removed?

27. Fiona bought five carrots and seven potatoes for $4.10. Each carrot cost 10 cents more than a potato. How much does a carrot cost?

28. A mother is presently four times as old as her daughter. The sum of their ages six years ago was 48. Find their present ages.

7. Difference in thickness of a page from the storybook and a page from the dictionary

= Thickness of a page from the storybook − Thickness of a page from the dictionary

$= 2.5 \cdot 10^{-2} - 5 \cdot 10^{-3}$

$= 25 \cdot 10^{-3} - 5 \cdot 10^{-3}$

$= (25 - 5) \cdot 10^{-3}$

$= 20 \cdot 10^{-3}$

$= 2 \cdot 10^{1} \cdot 10^{-3}$

$= 2 \cdot 10^{1-3}$

$= 2 \cdot 10^{-2}$ cm

The storybook has a thicker page and it is $2 \cdot 10^{-2}$ centimeter thicker.

8. **a)** $3.3 \cdot 10^{-3}$ g

b) $1.7 \cdot 10^{-3}$ g

9. Width of a nylon fiber:

1.3 μm $= 1.3 \cdot 10^{-6}$ m

Approximate difference in width

= Width of synthetic fiber − Width of nylon fiber

$= 1.6 \cdot 10^{-5} - 1.3 \cdot 10^{-6}$

$= 16 \cdot 10^{-6} - 1.3 \cdot 10^{-6}$

$= (16 - 1.3) \cdot 10^{-6}$

$= 14.7 \cdot 10^{-6}$

$= 1.47 \cdot 10^{1} \cdot 10^{-6}$

$= 1.47 \cdot 10^{1-6}$

$= 1.47 \cdot 10^{-5}$ m

So, the synthetic fiber has a greater width than the nylon fiber by approximately $1.47 \cdot 10^{-5}$ meter.

10. $1.3 \cdot 10^{4}$ L

11. **a)** $3.47 \cdot 10^{5}$ kg

b) $7.4 \cdot 10^{4}$ kg

c) $1.5 \cdot 10^{4}$ kg

Lesson 2.3

1. Approximate surface area of *E. coli* bacterium

= Length · Width

$= 4 \cdot 10^{-3} \cdot 2.3 \cdot 10^{-3}$

$= 4 \cdot 2.3 \cdot 10^{-3} \cdot 10^{-3}$

$= 9.2 \cdot 10^{-3} \cdot 10^{-3}$

$= 9.2 \cdot 10^{-3+(-3)}$

$= 9.2 \cdot 10^{-6}$

The area of the surface of the bacterium is approximately $9.2 \cdot 10^{-6}$ square millimeter.

2. $8.7 \cdot 10^{7}$

3. $6.435 \cdot 10^{3}$

4. $2.475 \cdot 10^{5}$ cm^2

5. $5.29 \cdot 10^{4}$ yd^2

6. $7.7 \cdot 10^{3}$ cm^2

7. $$\frac{\text{Thickness of dictionary}}{\text{Thickness of storybook}} = \frac{\boxed{2.9} \cdot \boxed{10^3}}{\boxed{1.1} \cdot \boxed{10^3}} = \frac{\boxed{2.9}}{\boxed{1.1}} \cdot \frac{\boxed{10^3}}{\boxed{10^3}} \approx 2.6 \cdot 10^{3-3} = 2.6 \cdot 10^{0} = 2.6$$

The thickness of the dictionary is about 2.6 times greater than the thickness of the storybook.

8. $2 \cdot 10^{1}$

9. $3.1 \cdot 10^{-4}$

10. 650 times as long

11. 1.6 times as great

12. 4.8 times as large

Chapter 3

Lesson 3.1

1. Yes. The solution to both equations is 6.

2. No. The solution to $\frac{1}{2}x + 1 = 5$ is 8 and the solution to $2x = 5$ is $2\frac{1}{2}$.

3. Yes. The solution to both equations is 2.

4. No. The solution to $\frac{1}{3}x = 9$ is 27 and the solution to $2(x + 3) = 40$ is 17.

5. $P = 4x$; Independent variable: x; Dependent variable: P

6. $L = 4k + 3$; Independent variable: k; Dependent variable: L

7. $S = 8(r - 5)$; Independent variable: r; Dependent variable: S

8. $x = -2\frac{2}{3}$

9. $s = 9$

10. $k = 11\frac{1}{2}$

11. $y = 39$

12. $y = \frac{31}{39}$

13. $1.\overline{5}$

14. $2.\overline{09}$

15. $2.\overline{7}$

16. $\frac{7x}{2} - \frac{1-x}{2} = 4$

$\frac{7x - (1) + (x)}{(2)} = \underline{4}$

$\frac{(8x) - (1)}{(2)} = 4$

$\frac{(8x) - (1)}{(2)} \cdot \underline{2} = \underline{4} \cdot \underline{2}$

$\underline{8x} - \underline{1} = \underline{8}$

$\underline{8x} - \underline{1} + \underline{1} = \underline{8} + \underline{1}$

$\frac{(8x)}{(8)} = \frac{(9)}{(8)}$

$x = 1\frac{1}{\underline{8}}$

17. $x = 1$

18. $x = \frac{11}{12}$

19. $x = 2\frac{6}{15}$

20. Let $x = 1.\overline{3}$

$x = \underline{1.333333}$

$\underline{10x} = \underline{13.\ 33333}$

$\underline{10x} - \underline{x} = \underline{13.\overline{3}} - \underline{1.\overline{3}}$

$\underline{9x} = \underline{12}$

$\frac{\underline{9x}}{\underline{9}} = \frac{\underline{12}}{\underline{9}}$

$x = 1\frac{1}{\underline{3}}.$

Therefore, $\underline{1.\overline{3}} = 1\frac{1}{\underline{3}}$.

21. $\frac{2}{9}$

22. $\frac{25}{9}$

23. $\frac{7}{15}$

24. $\frac{53}{11}$

25. $\underline{x + \frac{x}{2}} = 12$

$\underline{\frac{3x}{2}} = 12$

$\underline{\frac{3x}{2}} \cdot \underline{2} = \underline{12} \cdot \underline{2}$

$\underline{3x} = \underline{24}$

$\underline{\frac{3x}{3}} = \underline{\frac{24}{3}}$

$x = \underline{8}$

The distance from Howard's school to the shopping mall is $\underline{8}$ miles.

26. 27 kilograms

27. 40 cents or $0.40

28. Mother: 48 years old; Daughter: 12 years old

Lesson 3.2

1. $\underline{10} - \underline{8x} + \underline{26} = \underline{8x} + \underline{88}$

$\underline{36} - \underline{8x} = \underline{8x} + \underline{88}$

$\underline{36} - \underline{8x} + \underline{8x} = \underline{88} + \underline{8x} + \underline{8x}$

$\underline{36} = \underline{16x} + \underline{88}$

$\underline{36} - \underline{88} = \underline{16x} + \underline{88} - \underline{88}$

$\underline{-52} = \underline{16x}$

$\frac{(-52)}{(16)} = \frac{(16x)}{(16)}$

$\underline{x = -3\frac{1}{4}}$

Because $\underline{x = -3\frac{1}{4}}$, the equation has $\underline{\text{one}}$ solution.

So, the equation is $\underline{\text{consistent}}$.

2. $0 \neq 4$, the equation has no solution. The equation is inconsistent.

3. $x = \frac{1}{20}$, the equation has one solution. The equation is consistent.

4. $6 \neq 1$, the equation has no solution. The equation is inconsistent.

5. $x = 3$, the equation has one solution. The equation is consistent.

6. $\frac{1}{5}x + 3 = \frac{1}{10}(2x - 1) + \frac{31}{10}$

$\underline{\frac{1}{5}}x + 3 = \underline{\frac{2}{10}}x - \underline{\frac{1}{10}} + \underline{\frac{31}{10}}$

$\underline{\frac{1}{5}}x + \underline{3} = \underline{\frac{2}{10}}x + \underline{\frac{30}{10}}$

$\underline{\frac{1}{5}}x + \underline{3} - \underline{3} = \underline{\frac{2}{10}}x + \underline{\frac{30}{10}} - \underline{3}$

$\underline{\frac{1}{5}}x = \underline{\frac{2}{10}}x$

$\underline{\frac{1}{5}}x = \underline{\frac{2}{10}}x$, the equation has $\underline{\text{infinitely many}}$ solution(s). So, the equation is an $\underline{\text{identity}}$.

7. $-8 \neq -2$, the equation has no solution. So, the equation is inconsistent.

8. $-1 = -1$, the equation has infinitely many solutions. So, the equation is an identity.

9. $x = 4\frac{2}{3}$, the equation has one solution. So, the equation is consistent.

10. No. $4p + 16 = 4p + 16$ which implies that $16 = 16$. So, the equation has infinitely many solutions. Since the equation is an identity you are not able to find the width of the hamster's cage.