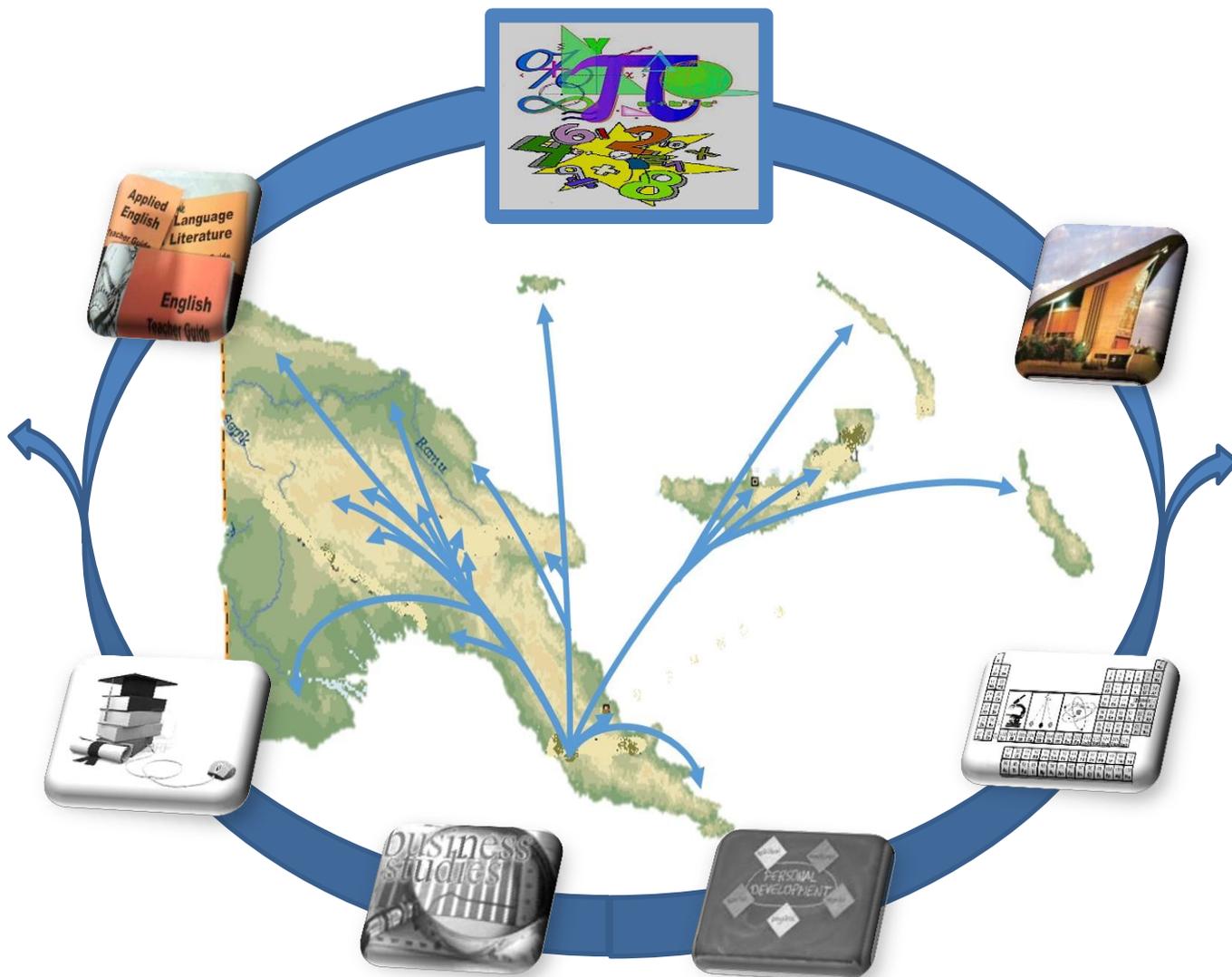




DEPARTMENT OF EDUCATION

GRADE 11 GENERAL MATHEMATICS

11.5: TRIGONOMETRY



FODE DISTANCE LEARNING



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PAPUA NEW GUINEA
2017



GRADE 11

MATHEMATICS B

UNIT MODULE 5

Trigonometry

TOPIC 1: TRIGONOMETRIC IDEAS

TOPIC 2: SOLUTION OF RIGHT TRIANGLES

TOPIC 3: SOLUTIONS OF OBLIQUE TRIANGLES

TOPIC 4: MAPS, CONTOURS AND VECTORS



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Mathematics Department- CDAD

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MR. DEMAS TONGOGO
Principal-FODE



Flexible Open and Distance Education
Papua New Guinea

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SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the National OBE Curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as an alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many path ways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.

Secretary for Education



UNIT INTRODUCTION

The study of trigonometry enables us to compare similar triangles so that lengths that are difficult or impossible to measure directly can be calculated. We can use trigonometry to find the heights of tall objects, such as trees, flagpoles and buildings. Trigonometry is an important tool for evaluating measurements of height and distance.

Topic 1 TRIGONOMETRIC IDEAS

The topic defines what a right triangle is, discusses application of Pythagoras Theorem, the use of three basic Trigonometric ratios, and Reciprocal (Inverse) ratios in a right triangle. It goes on further to identify exact values of specific trigonometric functions and insight into trigonometric functions of complimentary angles.

Topic 2 SOLUTION OF RIGHT TRIANGLES

This topic provides skills in solving unknown sides of right triangles and unknown angles. It also provide skills in problems related to angles of elevation and depression; and bearings and back bearings. Skills learned in solving triangles can be applied in solving other geometric planes and to navigate by air, sea or land.

Topic 3 SOLUTIONS OF OBLIQUE TRIANGLES

This topic provides you the skills required to find sides or angles when Pythagoras Theorem and Trigonometric ratios are not applicable at first sight in oblique triangles. These can be solved by either Sine Rule or Cosine Rule. It goes further to discuss areas of Triangles by Sine Rule for area.

Topic 4 MAPS, CONTOURS AND VECTORS

The topic provides skills in reading maps and being able to identify distances and sizes of a land mass, and be able to sketch planes using scales and scale factors; and it also describes and attempts to visualize gradient and altitudes in contours. It then introduces scalar and vector quantities, and goes on to provide examples of addition and subtraction of column vectors, and multiplying vectors by a scalar quantity which are prerequisite to solving 3 x 3 square matrix problems.

Having completed this topic will provide you the foundation mathematics skills required in your further study in geometry in relation to surveying, navigation, engineering, astronomy and business applications.



Student Learning Outcomes

On successful completion of this module, students will be able to:

- measure and calculate angles of elevation and depression
- solve supplication problems on right angled triangles
- derive sine and cosine rule
- apply sine and cosine rule to solve practical problems
- discuss and measure conventional and compass bearing
- discuss, illustrate and interpret contour maps
- calculate average slopes and distance of contour
- use vector notation and position vector
- use scalar multiplication to explain and apply parallel vectors



Time Frame

This unit should be completed within 10 weeks.

If you set an average of 3 hours per day, you should be able to complete the unit comfortably by the end of the assigned week.

Try to do all the learning activities and compare your answers with the ones provided at the end of the unit. If you do not get a particular exercise right in the first attempt, you should not get discouraged but instead, go back and attempt it again. If you still do not get it right after several attempts then you should seek help from your tutor or even your friend. Do not pass any question without solving it first.



11.5. 1: TRIGONOMETRIC IDEAS

Trigonometry is the study of triangles. The word comes from Greek *trigonon* "triangle" + *metron* "measure". It means triangle measurement. Trigonometry is a branch of mathematics that combines arithmetic, algebra and geometry.

Greek, Persian and Hindu astronomers first developed trigonometry around 200 BC. **Hipparchus** is credited with being the originator of the science at that time. Today, it plays an important role in surveying, navigation, engineering, astronomy and many other branches of physical science. Wave theory also uses trigonometry.

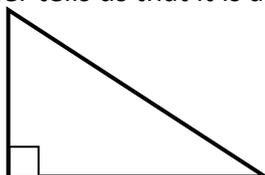
Basic Trigonometry involves the ratios of the sides of right triangles. In these units we will revise the definition of right angled triangles and its properties. The three basic trigonometric ratios are called tangent, sine and cosine. It can then be extended to other ratios such as the reciprocal trigonometric ratios namely cosecant, secant and cotangent. Also some properties of trigonometric ratios, complementary ratios and trigonometric ratios of special angles such as 30° , 45° and 60° are introduced.

Triangles are made up of three line segments. They meet to form three angles. The sizes of the angles and the lengths of the sides are related to one another. If you know the measure of three out of the six parts of the triangle (at least one side must be included), you can find the sizes of the remaining sides and angles. If the triangle is a right triangle, you can use simple trigonometric ratios to find the missing parts. In a general triangle (acute or obtuse), you need to use other techniques, including the cosine rule and the sine rule. You can also find the area of triangles by using trigonometric ratios.

11.5.1.1 The Right Triangles

A **right triangle** is a triangle that has a right angle (90°) in it.

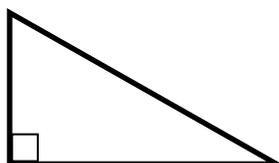
The **little square**  in the corner tells us that it is a right angled triangle.



There are two types of right angled triangle.

1. Scalene right triangle

One right angle
Two other **unequal** angles
No equal sides



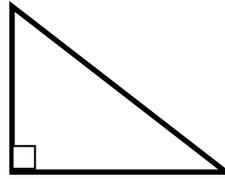


2. Isosceles right triangle

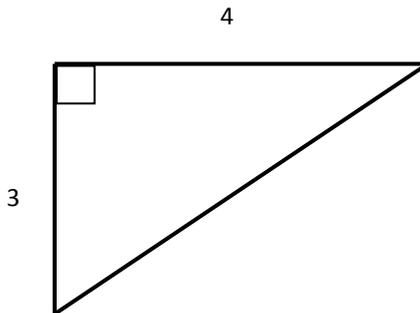
One right angle

Two other **equal** angles always of **45°**

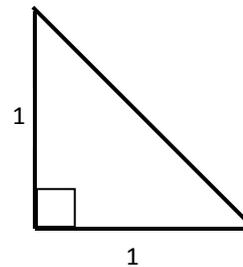
Two equal sides



Examples



scalene right triangle



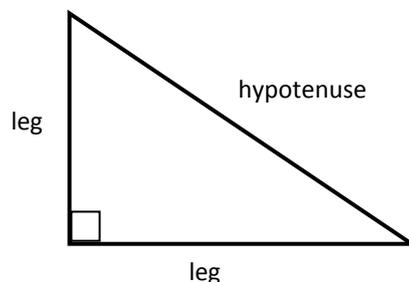
isosceles right triangle

The right angled triangle is one of the most useful shapes in all of geometry. It is therefore important to know how to label a triangle (name the sides and angles) likewise the properties of right triangles.

Naming sides of a right triangle

'Right Triangle' is a triangle with one internal angle equal to 90 degrees (right angle). The vertices are named using capital letters P, Q and R etc..., and these letters are written on the vertices of the triangle. The sides are named with reference to one of the acute angles and the angles can be labelled with three letters with the letter of the angle at the centre or with a single letter as the vertex.

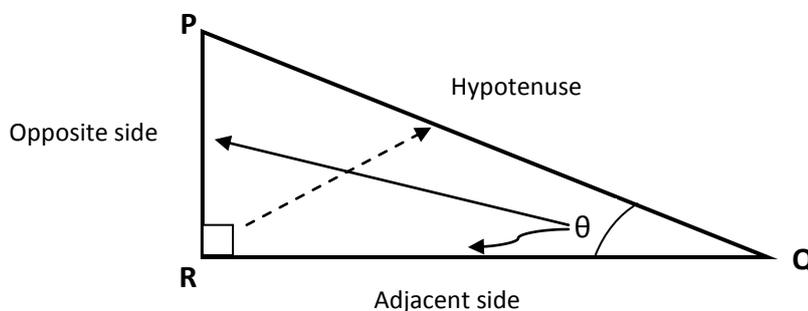
The side opposite to the right angle is called **hypotenuse** and the hypotenuse is the longest side of the right angled triangle. The other two sides adjacent to the right angle are called **legs**.



In the following right triangle PQR , θ is our reference angle. The Greek letter theta, θ , is just a symbol used as a variable name, much like x and y . For some reason it is most commonly used to denote an angle. But it does not have a constant value. Theta is most often used to represent unknown angles, especially in the study of trigonometry. Theta represents an angle in degrees.



When the angle θ is given or defined, the leg opposite it is identified as **opposite side**, and the leg adjacent to θ is identified as **adjacent side**.

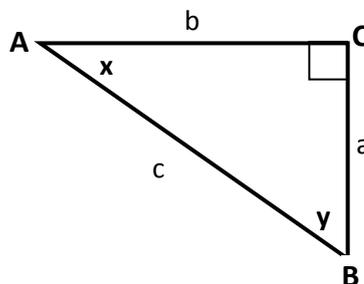


- The side PQ or r , which is opposite to the right angle PQR is called the **hypotenuse**. (The hypotenuse is the longest side of the right triangle.)
- The side RQ or p is called the **adjacent** side of angle θ or the angle **next** to angle θ
- The side PR or q is called the **opposite** side to angle θ .

Note: The adjacent and the opposite sides vary according to where the reference angle is marked.

Example 1 Identify the hypotenuse, adjacent side and opposite side in the following triangle:

- a) for angle x
- b) for angle y



Solution

- a) For angle x : AB or c is the hypotenuse, AC or b is the adjacent side, and BC or a is the opposite side.
- b) For angle y : AB is the hypotenuse, BC is the adjacent side, and AC is the opposite side

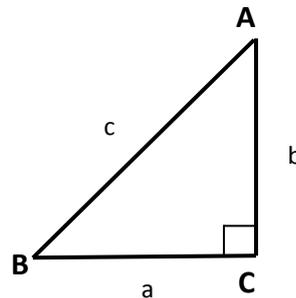
Properties of Right Triangle

1. If any two side lengths are given then we can find the third side length by using "**Pythagoras theorem**" discussed in Grade 10.

Example: If we let ' c ' be the length of the hypotenuse and ' a ' and ' b ' be the lengths of the two legs, then the theorem can be expressed as the given equation.



$$c^2 = a^2 + b^2$$



2. Acute angles of the right triangle are complimentary; the sum of the two acute angles is 90°.

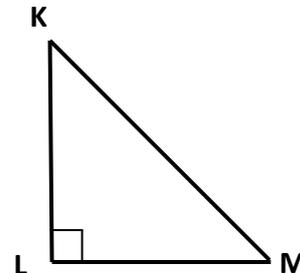
In Grade 10, you learned that complimentary angles are angles with a sum of 90°. ∠K and ∠M are acute angles.

Example: $\angle K + \angle M + \angle L = 180^\circ$ (angle sum in a triangle)

$$\angle K + \angle M + 90^\circ = 180^\circ$$

But $\angle L = 90^\circ$ so $\angle K + \angle M = 180^\circ - 90^\circ$

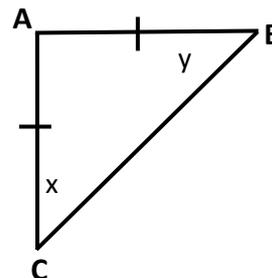
$$\angle K + \angle M = 90^\circ$$



3. If both acute angles are the same then both legs are of equal length and vice versa. Hence, both acute angles are 45 degrees.

$$\angle x + \angle y = 45^\circ$$

$$AB = AC$$



Example 2

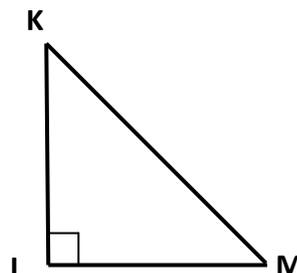
a) Which side is the hypotenuse in ΔKLM ?

b) Which of the following statements is correct based on the given triangle?

$$KL^2 = KM^2 + LM^2$$

$$KM^2 = KL^2 + LM^2$$

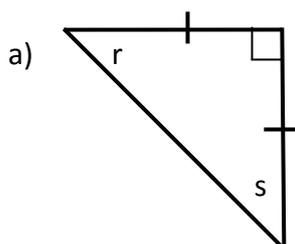
$$LM^2 = KL^2 + KM^2$$



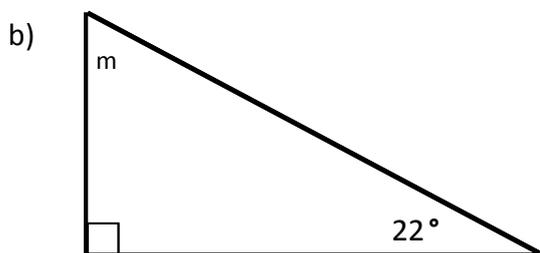
**Solution**

- a) KM is the hypotenuse.
- b) $KM^2 = KL^2 + LM^2$ (using the Pythagorean Theorem $c^2 = a^2 + b^2$; KM is **c** and KL and LM are **a** and **b** respectively. (Defining the sides as k, l and m, the theorem can be re-expressed as $l^2 = k^2 + m^2$).

Example 3 Find the values of all pronumerals .

**Solution:**

If both legs are of equal length, then both acute angles are the same and vice-versa. Hence both acute angles are 45 degrees. ($r = 45^\circ$; $s = 45^\circ$)

**Solution:**

Acute angles of the right angled triangle are complimentary.

Therefore : $m + 22^\circ = 90^\circ$

$$m = 90^\circ - 22^\circ$$

$$m = 68^\circ$$



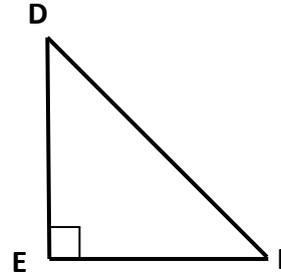
Learning Activity 11.5.1.1



20 minutes

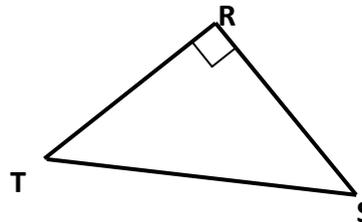
1. In $\triangle DEF$,

- a. name the hypotenuse
- b. which side is opposite angle D?
- c. which side is adjacent to angle D?



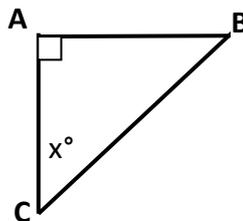
2. In $\triangle RST$;

- a. which angle is opposite to side RS?
- b. which side is adjacent to angle T?



3. For the given angle x° as reference, name the sides of this triangle.

- a. hypotenuse
- b. opposite
- c. Adjacent

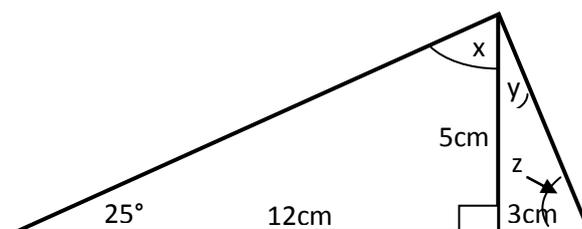


4. Draw a right angle triangle XYZ with sides $XY= 6$, $YZ= 8$ and $XZ = 10$ cm.

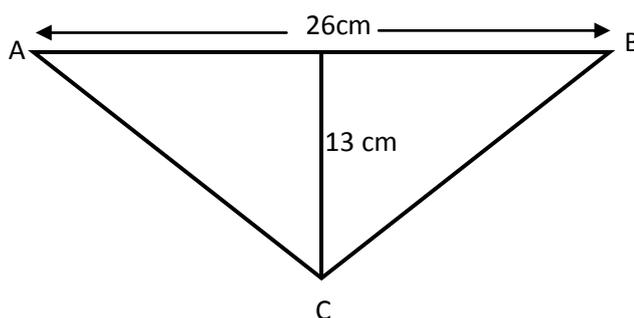


5. Using the diagram in number 4,
- name the hypotenuse
 - name the side opposite angle Z
 - name the adjacent side of angle X.

6. Find the values of the angles x , y , and z in the diagram below.



7. Study the diagram below and solve the angles and sides given that Segment CD bisects AB at E, AB is 26 cm and CE is 13 cm.

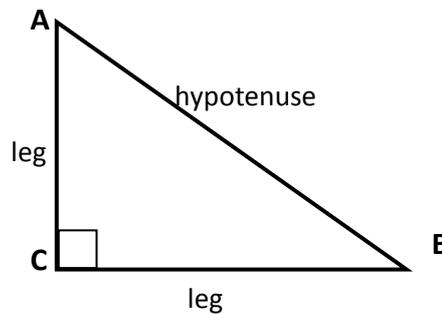


- Find
- Length AE
 - Angle CEA
 - Angle CBE
 - Angle ECB



11.5.1.2 The Pythagorean Theorem

Pythagoras was a Greek mathematician who lived about 2500 years ago, and who developed the most famous formula in Geometry and possibly in all of mathematics! He proved that, for a right triangle,



the sum of the squares of the two sides(legs) that join at a right angle equals the square of the longest side, hypotenuse.

$$c^2 = a^2 + b^2$$

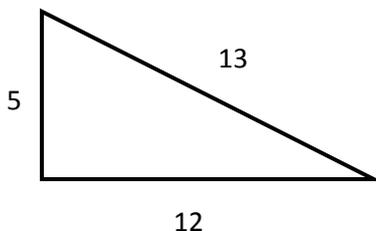
This formula is called the **Pythagorean Theorem** in honour of Pythagoras.

The Pythagorean Theorem has many uses. You can use it to **verify whether or not a triangle is a right triangle**. Or you can use it to **find the missing measures of sides**.

Example 1 Show that the triangle with sides of 5 and 12 cm, and a hypotenuse of 13 cm is a right triangle.

Solution

We can verify that the Pythagorean Theorem is true by substituting with the values. (for the pronumerals in the Pythagorean formula). The square root of 169 is 13, which is the measure of the hypotenuse in the triangle.



$$c^2 = a^2 + b^2$$

$$13^2 = 5^2 + 12^2$$

$$13^2 = 169$$

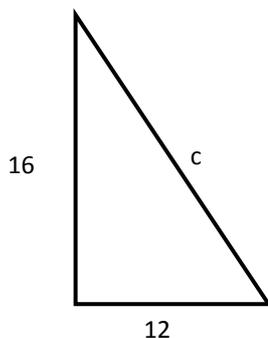
$$169 = 169$$



Example 2 Find the missing measure of the hypotenuse of the right triangle below:

Solution

Substitute the values into the formula and perform the calculations.



$$c^2 = a^2 + b^2$$

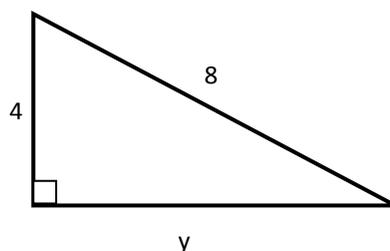
$$c^2 = 16^2 + 12^2$$

$$c^2 = 256 + 144$$

$$c^2 = 400$$

$$c = \sqrt{400} = 20$$

Example 3 Find the exact value of y .



Solution:

$$c^2 = a^2 + b^2$$

$$8^2 = y^2 + 4^2$$

$$64 = y^2 + 16$$

$$48 = y^2$$

Taking square roots on both sides

$$y = \sqrt{48}$$

Factoring 48 where one factor is a **perfect** square number

$$y = \sqrt{16} \times \sqrt{3}$$

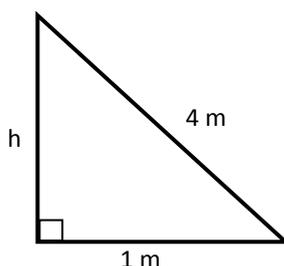
$$y = 4\sqrt{3}$$

(Leave the answer in simplified surd form for the exact answer.)

Solving problems using Pythagoras' Theorem

Many practical problems involve finding the side lengths of right angled triangles. A clear diagram including all information given is the starting point for solving such problems.

Example 1 A 4m ladder is leaning against a wall. The foot of the ladder is 1 m out from the wall, as the diagram shows. How far up the wall does the ladder reach?



**Solution**

A right angled triangle is drawn with h being the height of the ladder reaches up the wall.

Using Pythagorean Theorem:

$$c^2 = a^2 + b^2$$

$$4^2 = h^2 + 1^2$$

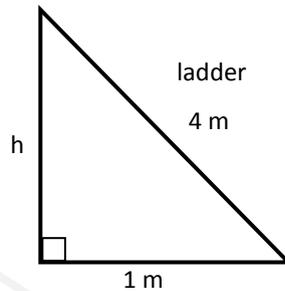
$$16 - 1 = h^2$$

$$15 = h^2$$

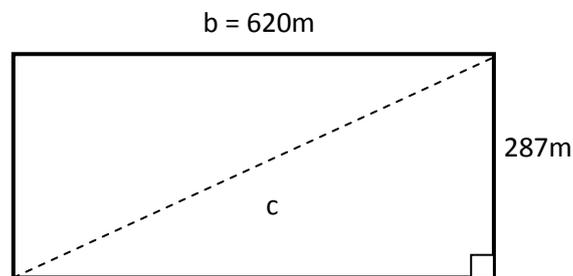
$$h = \sqrt{15}$$

$$h = 3.87$$

(2 decimal places)

**Example 2**

A rectangular park has a length of 630m and a width of 287m. If I walk diagonally across the park, how far do I walk?

**Solution**

A rectangular shape is drawn with c as the diagonal and hypotenuse because this diagonal line is opposite the right angle.

Using $c^2 = a^2 + b^2$

$$c^2 = 620^2 + 287^2$$

$$c^2 = 466\,769$$

$$c = 683.20 \text{ (2 decimal places)}$$

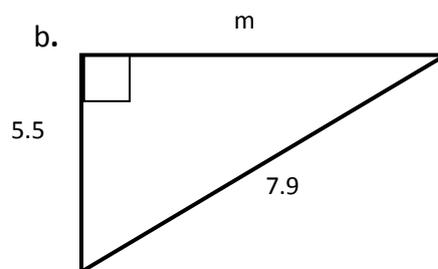
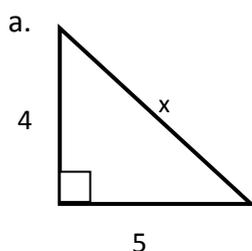


Student Learning Activity 11.5.1.2



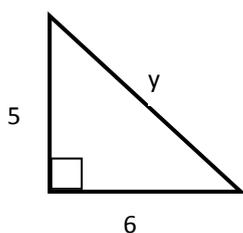
20 minutes

1. Find the value of all pronumerals correct to 2 decimal places.

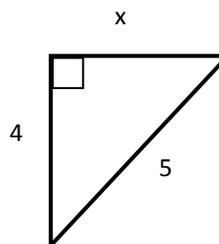


2. Find the exact value of all pronumerals.

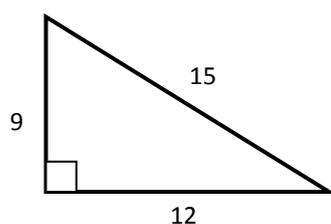
a.



b.



3. Prove that $\triangle ABC$ is a right angled triangle.



4. A ladder leans against a wall. Its foot is 2 metres from the base of the wall and it reaches 6 metres up the wall approximately. About what length is the ladder?

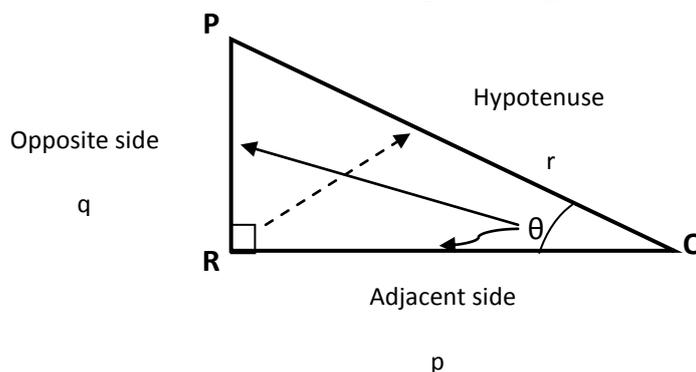
5. The sides of a rectangle are 10 cm and 4 cm. How long is the diagonal?



11.5.1.3 The Trigonometric Ratios of Acute Angle

Trigonometric ratios provide a relationship between angle size and side lengths in right-angled triangles.

In the triangle below $\angle Q$ and $\angle P$ are the acute angles. Using θ as the reference acute angle,



The trigonometric ratios derived from the triangle above are:

Definition	Ratio	Formula
The ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ is the SINE of the angle θ	$\text{Sin } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\text{Sin } \theta = \frac{PR}{PQ}$ $\text{sin } \theta = \frac{q}{r}$
The ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ is the COSINE of the angle θ	$\text{Cos } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\text{Cos } \theta = \frac{RQ}{PQ}$ $\text{cos } \theta = \frac{p}{r}$
The ratio $\frac{\text{opposite}}{\text{adjacent}}$ is the TANGENT of the angle θ	$\text{Tan } \theta = \frac{\text{opposite}}{\text{adjacent}}$	$\text{Tan } \theta = \frac{PR}{RQ}$ $\text{tan } \theta = \frac{q}{p}$

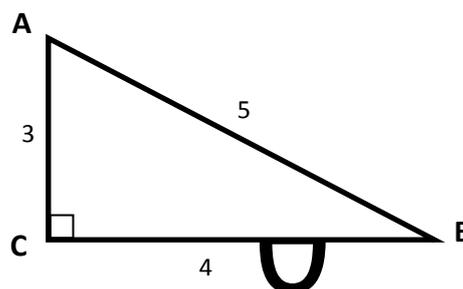
Example 1 Find $\sin \theta$, $\cos \theta$ and $\tan \theta$

Solution:

Identify the sides and their measures

$$AB = c = \text{hypotenuse} = 5$$

$$BC = a = \text{opposite side} = 3$$





$AC = b = \text{adjacent side} = 4$

Using θ as the reference acute angle, then

$$\sin \theta = \frac{a}{c} = \frac{3}{5} \quad \cos \theta = \frac{b}{c} = \frac{4}{5} \quad \tan \theta = \frac{a}{b} = \frac{3}{4}$$

Example 2 Refer to the triangle on the right and find the following:

- $\sin A$, $\cos A$ and $\tan A$
- $\sin B$, $\cos B$ and $\tan B$

Solution

- Using A as the reference acute angle,

Hypotenuse = 6

Adjacent side = 3

Opposite side = $3\sqrt{3}$

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{6} = \frac{1}{2}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

- Using B as the reference acute angle,

Hypotenuse = 6

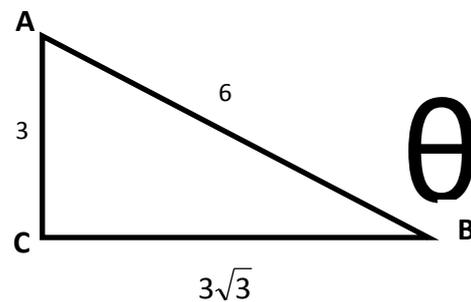
Adjacent side = $3\sqrt{3}$

Opposite side = 3

$$\sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{6} = \frac{1}{2}$$

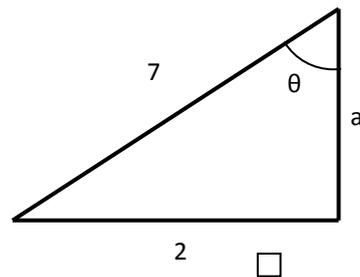
$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3\sqrt{3}}{6} = \frac{\sqrt{3}}{2}$$

$$\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}$$



Example 3

If $\sin \theta = \frac{2}{7}$, find the exact ratio of $\cos \theta$ and $\tan \theta$.

**Solution**

To find the other ratios, you need to find the missing side of the triangle and that is the adjacent side of angle θ .

Using the two given sides, the ratio is $\frac{2}{7} = \frac{\text{opposite}}{\text{hypotenuse}} = \sin \theta$

By Pythagoras Theorem:

$$c^2 = a^2 + b^2$$

$$7^2 = a^2 + b^2$$

$$49 = a^2 + 4$$

$$49 - 4 = a^2$$

$$45 = a^2$$

$$a = \sqrt{45}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{45}}{7}$$

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{2}{\sqrt{45}}$$



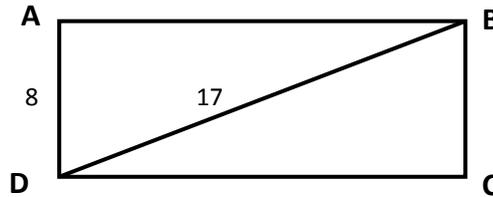
Student Learning Activity 11.5.1.3



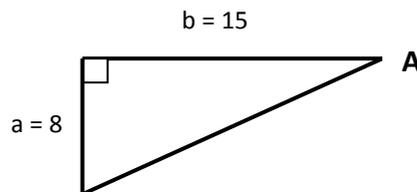
20 minutes

1. In the rectangle ABCD, find:

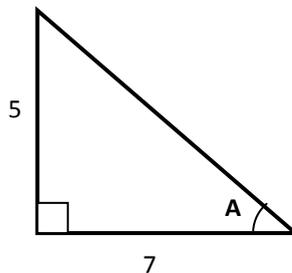
- a. $\tan \angle ABD$
- b. $\cos \angle BDC$
- c. $\sin \angle ADB$



2. If $a = 8$ and $b = 15$, find the six trigonometric ratios of angle A.



3. Find the exact ratios of $\sin A$, $\cos A$ and $\tan A$.



4. If $\tan \theta = \frac{4}{3}$ find $\sin \theta$ and $\cos \theta$.

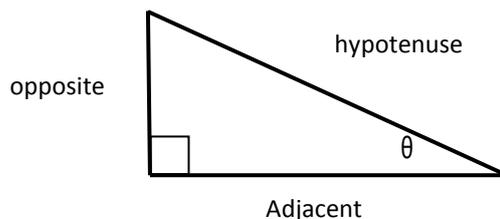
5. If $\cos \theta = \frac{2}{3}$ find exact values for $\sin \theta$ and $\tan \theta$.

(add more practice exercises)



11.5.1.4 The Reciprocal Trigonometric Ratios

In a right triangle, there are actually six possible trigonometric ratios, or functions. A Greek letter (such as theta θ) will again be used to represent the angle.



We have covered sine, cosine, and tangent ratios of a right triangle. Sine, cosine, and tangent each have a **reciprocal** function. Reciprocal means inverse. In fractions, inverse means interchanging the position of the numerator and denominator.

For example, the reciprocal or inverse of

a. $\frac{2}{3}$ is $\frac{3}{2}$

b. $\frac{1}{2}$ is $\frac{2}{1}$ or 2.

c. 6 is $\frac{1}{6}$

As well as the trigonometric ratios, there are three inverse ratios

$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \frac{\text{hypotenuse}}{\text{opposite}} = \text{cosecant}$$

$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \frac{\text{hypotenuse}}{\text{adjacent}} = \text{secant}$$

$$\tan = \frac{\text{opposite}}{\text{adjacent}} \qquad \frac{\text{adjacent}}{\text{opposite}} = \text{cotangent}$$

Cosecant is the reciprocal of sine.

Its abbreviation is **csc**.

To determine csc, just inverse sin over.

$$\boxed{\text{csc } A = \frac{1}{\sin A}}$$



Secant is the reciprocal of cosine.

Its abbreviation is **sec**.

To determine sec, just inverse cos over.

$$\sec A = \frac{1}{\cos A}$$

Cotangent is the reciprocal of tangent.

Its abbreviation is **cot**.

To determine cot, just inverse tan over.

$$\cot A = \frac{1}{\tan A}$$

Notice that the three new ratios at the right are reciprocals of the ratios on the left.

Applying a little algebra shows the connection between these functions.

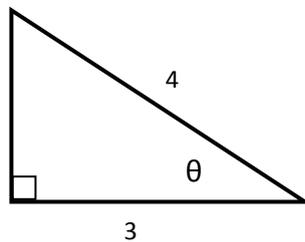
$$\csc A = \frac{1}{\sin A} = \frac{1}{\frac{\text{opposite}}{\text{hypotenuse}}} = \frac{\text{hypotenuse}}{\text{opposite}}$$

In summary, the reciprocal functions are :

$\sin A = \frac{1}{\csc A}$	$\csc A = \frac{1}{\sin A}$
$\cos A = \frac{1}{\sec A}$	$\sec A = \frac{1}{\cos A}$
$\tan A = \frac{1}{\cot A}$	$\cot A = \frac{1}{\tan A}$

Example 1

Given the triangle at the right, express the exact value of the six trigonometric functions in relation to angle θ .



Solution:

Step 1: Find the missing side of the right triangle using the Pythagorean Theorem.



Pythagorean Theorem

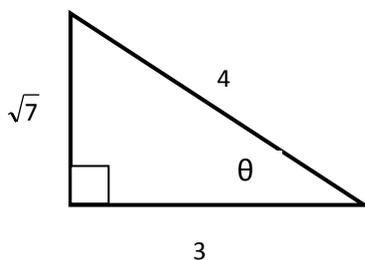
$$a^2 + b^2 = c^2$$

$$3^2 + b^2 = 4^2$$

$$b^2 = 16 - 9 = 7$$

$$b = \sqrt{7} \text{ (leave the answer as a surd which is the exact value)}$$

Step 2: Using the value found and the other given sides of the triangle, express each function as a ratio of the lengths of the sides. Do not "estimate" or convert to a decimal value the answers.



$$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{7}}{4} \quad \csc\theta = \frac{\text{hyp}}{\text{opp}} = \frac{4}{\sqrt{7}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{3}{4} \quad \sec\theta = \frac{\text{hyp}}{\text{adj}} = \frac{4}{3}$$

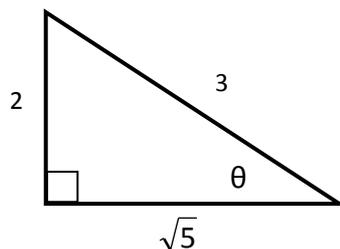
$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{7}}{3} \quad \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{3}{\sqrt{7}}$$

Example 2

Find $\sec\theta$ and $\cot\theta$, given $\sin\theta = \frac{2}{3}$ and $\cos\theta = \frac{\sqrt{5}}{3}$.

Solution

Step 1: Draw a diagram to get a better understanding of the given information.





Using the given ratio $\sin \theta = \frac{2}{3}$, which is $\frac{\text{opposite}}{\text{hypotenuse}}$ and

$$\cos \theta = \frac{\sqrt{5}}{3} \text{ which is } \frac{\text{adjacent}}{\text{hypotenuse}},$$

label the sides of the triangle in relation to angle θ .

Step 2: Now, using your diagram, read off the values for the secant and the cotangent.

$$\sec \theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{3}{\sqrt{5}} \qquad \cot \theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{\sqrt{5}}{2}$$

When flipping over your fractions to find the reciprocals, use the original fraction (with the surd form.)

Example 3

Given $\cos \theta = \frac{12}{13}$, find $\sec \theta$.

Solution: This is an easy problem since cosine and secant are reciprocal functions.

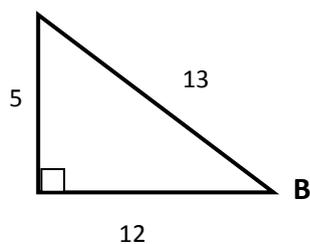
$$\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{12}{13}} \text{ (Apply division of fractions)}$$

$$\sec \theta = \frac{13}{12}$$

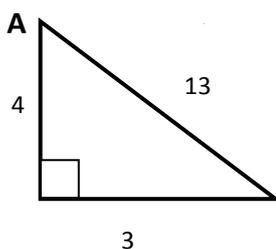
**Learning Activity 11.5.1.4**

20 minutes

1. Write down the six trigonometric ratios of angle B:



2. If side $a = 3$ and $b = 4$, find the hypotenuse and the six trigonometric ratios of angle A.

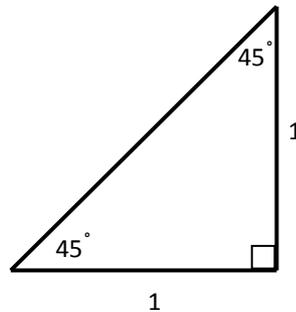


3. If $\tan \theta = \frac{3}{5}$, find the other six trigonometric ratios.
4. If $\sin \theta = \frac{1}{6}$, find the other exact trigonometric ratios.
5. If $\sin A = 0.7$, find the other exact trigonometric ratios of angle A



11.5.1.5 The Trigonometric Functions for 30°, 45°, 60°

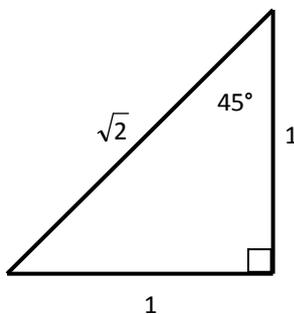
A right triangle has one angle equal to 90 degrees. A right triangle can also be an isosceles triangle which means that it has two sides that are equal. A right isosceles triangle has a 90 - degree angle and two 45-degree angles.



45°- 45°- 90° Triangle

If we are handling a 45°-45°-90° angle in a triangle, we have to be able to draw this reference triangle.

The trick? First, draw an isosceles right triangle. Isosceles triangles have two legs that are the same length. Label each leg 1.



Then, thanks to the Pythagorean Formula, we can find the hypotenuse by

$$c^2 = a^2 + b^2$$

$$c^2 = 1^2 + 1^2$$

$$c^2 = 1 + 1$$

$$c^2 = 2$$

$$c = \sqrt{2}$$

In summary,

The trigonometric ratios for 45°

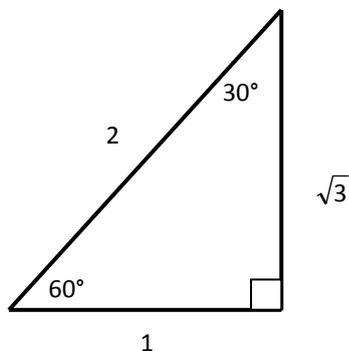
$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}}$$

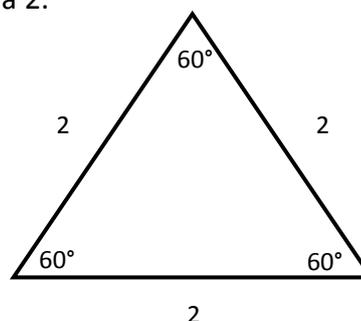
$$\tan 45^\circ = \frac{1}{1} = 1$$

**30°- 60°- 90° Triangle**

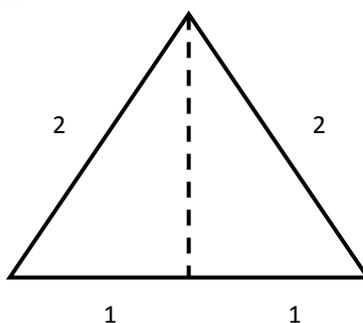
If we are working with a 30° or 60° angle, you must be able to draw this reference triangle.



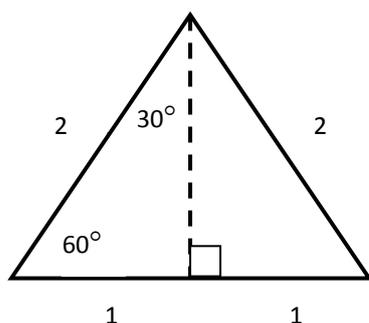
To sketch this reference triangle, first draw an equilateral triangle and label each side with a 2.



Now split the triangle in half by bisecting the vertex angle. Label each half of the base with a 1.



Now, find the altitude h by using the Pythagorean Formula



$$c^2 = a^2 + b^2$$

$$2^2 = h^2 + 1^2$$

$$4 - 1 = h^2$$

$$3 = h^2$$

$$\sqrt{3} = h$$

In summary,

The trigonometric ratios for 30°- 60°- 90°

$$\sin 30^\circ = \frac{1}{2}$$

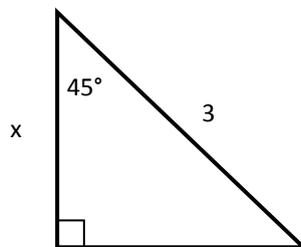
$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$



Below is the summary of the basic trigonometric functions and their reciprocals of the special angles.

	sin	cos	tan	csc	sec	cot
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\frac{\sqrt{3}}{1} = \sqrt{3}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{1} = \sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{\sqrt{3}}{1} = \sqrt{3}$

Example 1 Find the exact value of the pronumeral x.



Solution: $\cos 45^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{x}{3}$

But $\cos 45^\circ = \frac{1}{\sqrt{2}}$; substitute this value

$$\cos 45^\circ = \frac{x}{3}$$

$$\frac{1}{\sqrt{2}} = \frac{x}{3} \quad \text{by cross multiplication}$$

$$\sqrt{2}x = 3 \quad \text{dividing both sides by } \sqrt{2}$$



$$x = \frac{3}{\sqrt{2}}$$

Example 2 Simplify and find the exact value.

a. $\sin 60^\circ + \cos 60^\circ$

b. $\tan 45^\circ + \tan 60^\circ$

Solution

$$\sin 60^\circ + \cos 60^\circ$$

$$\frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3} + 1}{2}$$

Solution:

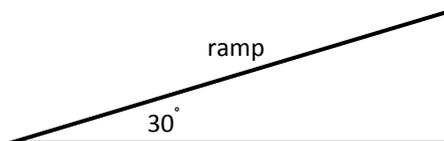
$$\tan 45^\circ + \tan 60^\circ$$

$$1 + \sqrt{3}$$

Example 3 A boat ramp is to be made with an angle of 30° and base length 5 m.
What is the exact length of the surface of the ramp?

Solution:

Draw the diagram and label all given parts so you can easily identify what ratio to use.



$$\cos 30^\circ = \frac{5}{x} \quad (\text{by cross multiplication})$$

$$x \cos 30^\circ = 5 \quad (\text{dividing both sides with } \cos 30^\circ)$$

$$x = \frac{5}{\cos 30^\circ} \quad (\text{substitute the value } \cos 30^\circ = \frac{\sqrt{3}}{2})$$

$$x = \frac{5}{\frac{\sqrt{3}}{2}} \quad (\text{division of fractions } 5 \div \frac{\sqrt{3}}{2})$$

$$= 5 \times \frac{2}{\sqrt{3}} = \frac{10}{\sqrt{3}}$$

So the exact length of the ramp is $\frac{10}{\sqrt{3}}$ m.

**Learning Activity 11.5.1.5**

20 minutes

1. Simplify and evaluate.
 - a. $\cos 45^\circ + \sin 45^\circ$
 - b. $\sin 30^\circ + \cos 60^\circ$
 - c. $\cot 30^\circ + \cot 60^\circ$
2. Evaluate,
 - a. $2 \cos 30^\circ$
 - b. $\cos^2 45^\circ$
 - c. $\frac{\tan 60^\circ}{\cos 60^\circ}$
 - d. $2 \sec 60^\circ$
3. A 1.8 m ladder is placed at 60° angle out from a wall. How far out from the wall is it?
4. A two- person tent is pitched at an angle of 45° . Each side of the tent is 2 m long. A pole of what height is needed for the centre of the tent?
5. One leg of an isosceles right triangle is 5 cm. How long is the other leg? How long is the hypotenuse?



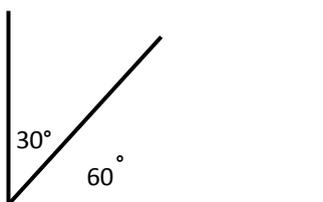
11.5.1.6 The Trigonometric Functions of Complementary Angles

Co-functions

"Co" is short for complementary. Co + function = complementary functions.

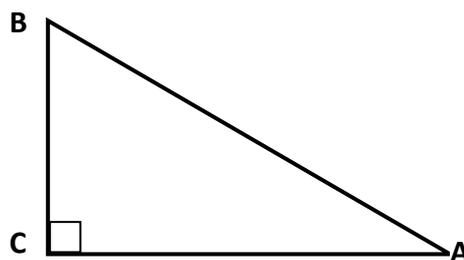
Does the word complementary ring a bell?

In geometry, we learned that complementary angles combine to form 90° angles.



From the diagram above $\angle 30^\circ + \angle 60^\circ = 90^\circ$ so $\angle 30^\circ$ and $\angle 60^\circ$ are complementary angles.

The sum of the angles in a triangle must be 180° . ($A + B + C = 180^\circ$)



Since angle C is a right angle, the other two angles must add up to the other 90° .

Since $A + B = 90$ angles A and B are complementary angles. And by rearranging things a little, we can see that

$$A + B = 90^\circ. \quad \text{Thus, } B = 90^\circ - A \text{ and } A = 90^\circ - B$$

Now let us apply our "co" to the trigonometric functions.

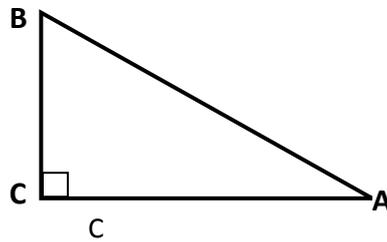
Cosine is the complementary function of sine.

Cosecant is the complementary function of secant.

Cotangent is the complementary function of tangent.



What is this all about? It means that for this triangle:



$$\cos A = \sin B$$

substituting in $B = 90^\circ - A$ gives us $\cos A = \sin(90^\circ - A)$

Similarly $\csc A = \sec B$ $\csc A = \sec(90^\circ - A)$

and $\cot A = \tan B$ $\cot A = \tan(90^\circ - A)$ and similarly

$$\sin A = \cos B = \cos(90^\circ - A)$$

$$\sec A = \csc B = \csc(90^\circ - A)$$

$$\tan A = \cot B = \cot(90^\circ - A)$$

Example 1 Write $\sin(30^\circ)$ in terms of its co-function.

Solution Using $\sin A = \cos(90^\circ - A)$

$$\sin(30^\circ) = \cos(90^\circ - 30^\circ)$$

$$\sin(30^\circ) = \cos(60^\circ)$$

Example 2 Write $\csc(70^\circ)$ in terms of its co-function.

Solution Using $\csc A = \sec(90^\circ - A)$

$$\csc(70^\circ) = \sec(90^\circ - 70^\circ)$$

$$\csc(70^\circ) = \sec(20^\circ)$$

Example 3 Write $\tan(10^\circ)$ in terms of its co-function.

Solution

Using $\tan A = \cot(90^\circ - A)$

$$\tan(10^\circ) = \cot(90^\circ - 10^\circ)$$

$$\tan(10^\circ) = \cot(80^\circ)$$



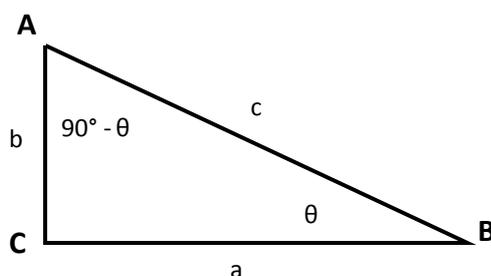
In the summary below, notice that $\sin 30^\circ = \cos 60^\circ$ and $\sin 60^\circ = \cos 30^\circ$. Why?

$$\begin{aligned}\sin 30^\circ &= \frac{1}{2} \\ \cos 30^\circ &= \frac{\sqrt{3}}{2} \\ \tan 30^\circ &= \frac{1}{\sqrt{3}}\end{aligned}$$

$$\begin{aligned}\sin 60^\circ &= \frac{\sqrt{3}}{2} \\ \cos 60^\circ &= \frac{1}{2} \\ \tan 60^\circ &= \frac{\sqrt{3}}{1}\end{aligned}$$

Can you find the relationship between 30° and 60° ? Yes, you are right. Their sum adds up to 90° and they are called complementary angles.

In $\triangle ABC$, if $\angle B = \theta$ then $\angle A = 90^\circ - \theta$ (Angle sum of a triangle)



Below are the six trigonometric ratios of θ and $(90-\theta)$

$\sin \theta = \frac{b}{c}$	$\sin (90-\theta) = \frac{a}{c}$
$\cos \theta = \frac{a}{c}$	$\cos (90-\theta) = \frac{b}{c}$
$\tan \theta = \frac{b}{a}$	$\tan (90-\theta) = \frac{a}{b}$
$\sec \theta = \frac{c}{a}$	$\sec (90-\theta) = \frac{c}{b}$
$\csc \theta = \frac{c}{b}$	$\csc (90-\theta) = \frac{c}{a}$
$\cot \theta = \frac{a}{b}$	$\cot (90-\theta) = \frac{b}{a}$



From these ratios, come the results of complementary ratios. Can you name them?

$$\begin{array}{ll} \sin \theta = \cos (90-\theta) = \frac{b}{c} & \sec \theta = \csc (90-\theta) = \frac{c}{a} \\ \cos \theta = \sin (90-\theta) = \frac{a}{c} & \csc \theta = \sec (90-\theta) = \frac{c}{b} \\ \tan \theta = \cot (90-\theta) = \frac{b}{a} & \cot \theta = \tan (90-\theta) = \frac{a}{b} \end{array}$$

Example 1

Name the complement of each angle.

- a) 70° b) 15° c) 45° d) θ

Solution

- a) The complement of 70° is 20° .
b) The complement of 15° is 75° ,
c) The complement of 45° is 45° .
d) The complement of θ is $(90^\circ - \theta)$

Example 2

Answer in terms of cofunctions.

- a) $\cos 5^\circ$ b) $\tan 60^\circ$ c) $\csc 12^\circ$ d) $\sin (90^\circ - \theta)$ e) $\cot \theta$

Solution :

- a) The cofunction of $\cos 5^\circ$ is $\sin 85^\circ$.
b) The cofunction of $\tan 60^\circ$ is $\cot 30^\circ$
c) The cofunction of $\csc 12^\circ$ is $\sec 78^\circ$
d) The cofunction of $\sin (90^\circ - \theta)^\circ$ is $\cos \theta$
e) The cofunction of $\cot \theta$ is $\tan (90^\circ - \theta)^\circ$

**Learning Activity 11.5.1.6**

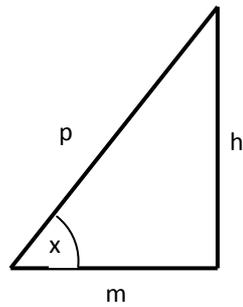
20 minutes

1. Answer in terms of co-functions.
 - a) $\cos 15^\circ$
 - b) $\tan 32^\circ$
 - c) $\csc 48^\circ$
 - d) $\tan (90^\circ - \theta)$
 - e) $\sec \theta$
2. Find the value of x if $\sin 80^\circ = \cos (90 - x)^\circ$
3. Find the value of y if $\tan 22^\circ = \cot (90 - y)^\circ$
4. Find the value of b if $\sin 35^\circ = \cos (b + 30)^\circ$
5. Find the value of t if $\cot(2t + 5)^\circ = \tan (3t-15)^\circ$

**SUMMATIVE TASK 1: Trigonometric Ideas**

I. MULTIPLE CHOICE Identify the choice that best completes the statement or answers the question.

In the figure below,



1. $\sin x$ is equal to

- a. $\frac{h}{p}$ b. $\frac{h}{m}$ c. $\frac{m}{p}$ d. $\frac{p}{h}$

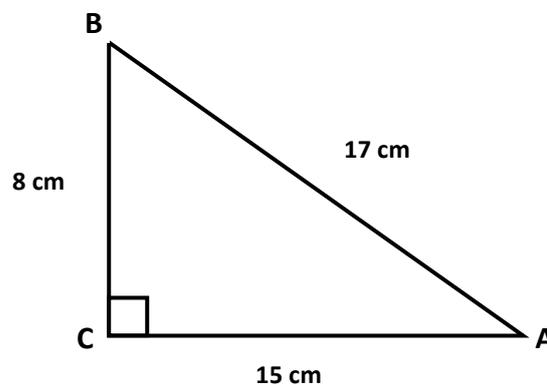
2. $\cos x$ is equal to

- a. $\frac{h}{p}$ b. $\frac{h}{m}$ c. $\frac{m}{p}$ d. $\frac{p}{h}$

3. $\tan x$ is equal to

- a. $\frac{h}{p}$ b. $\frac{h}{m}$ c. $\frac{m}{p}$ d. $\frac{p}{h}$

4. Which equation shows a correct trigonometric ratio for angle A in the right triangle below?



- a. $\sin A = \frac{15}{17}$ c. $\tan A = \frac{8}{17}$
b. $\cos A = \frac{15}{17}$ d. $\tan A = \frac{5}{8}$



5. The diagram below shows right triangle UPC.

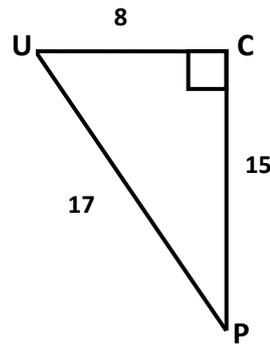
Which ratio represents the sine of angle PUC?

a. $\frac{8}{17}$

c. $\frac{8}{15}$

b. $\frac{15}{17}$

d. $\frac{8}{15}$



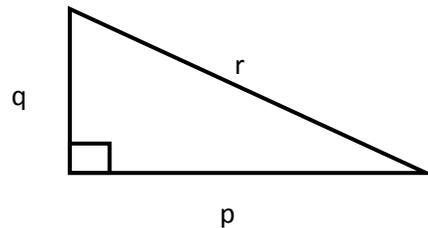
6. In the right triangle shown, $\cos \theta$ is equal to

a. $\frac{p}{r}$

b. $\frac{r}{p}$

c. $\frac{q}{r}$

d. $\frac{r}{q}$



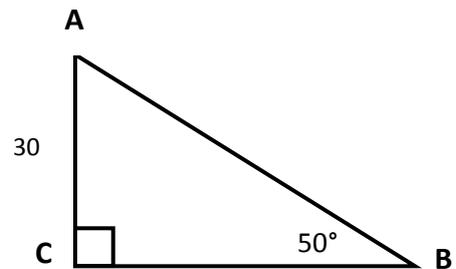
7. In the figure below, the expression for the length BC is

a. $30 \tan 50^\circ$

b. $30 \sin 50^\circ$

c. $\frac{30}{\tan 50^\circ}$

d. $\frac{30}{\sin 50^\circ}$



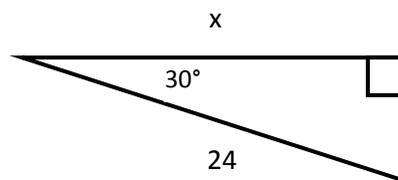
8. In the right triangle shown below, what is the value of x to the nearest degree?

a. 21

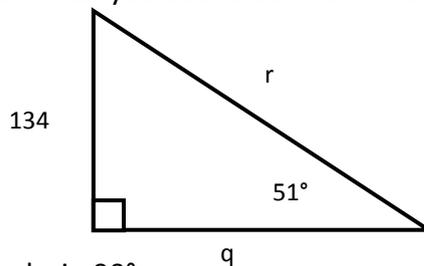
b. 14

c. 12

d. 28



9. Which primary trigonometric ratio can you use to calculate the length of side q?



a. $\tan 51^\circ$

b. $\sin 51^\circ$

c. $\cos 51^\circ$

d. $\sin 90^\circ$



11.5. 2: SOLUTION OF RIGHT TRIANGLES

All triangles are made up of three sides and three angles. If the three angles of the triangle are labelled $\angle A$, $\angle B$ and $\angle C$, then the three sides of the triangle should be labelled as a , b , and c . Figure 1 illustrates how lowercase letters are used to name the sides of the triangle that are opposite the angles named with corresponding uppercase letters. If any three of these six measurements are known (other than knowing the measures of the three angles), then you can calculate the values of the other three measurements.

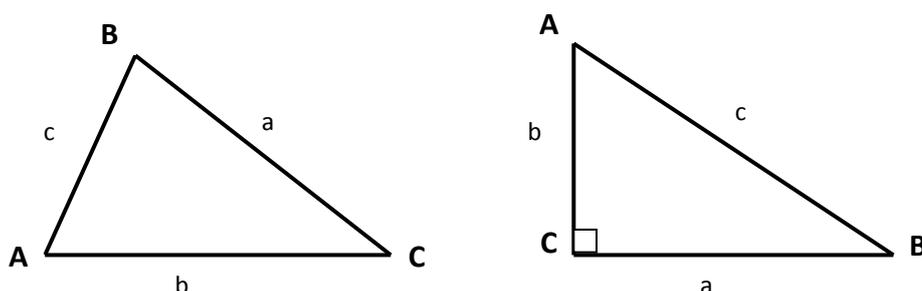


Figure 1

The process of finding the missing measurements is known as **solving the triangle**. If the triangle is right, then one of the angles is 90° . Therefore, you can solve the right triangle if you are given the measures of two of the three sides or if you are given the measure of one side and one of the other two angles.

One of the most important applications of trigonometric functions is to “solve” a right triangle. By now, you should know that every right triangle has five unknowns: the lengths of its three sides and the measures of its two acute angles. Solving the triangle means finding the values of these unknowns. You can use trigonometric functions to solve a right triangle if you are given either of the following sets of information:

1. The length of one side and the measure of one acute angle
2. The lengths of two sides

11.5.2.1 Finding the unknown sides of a Right Triangle

Angle Measurement

Angles are usually given in degrees and minutes. In this section you will practice rounding off angles and finding trigonometric ratios on the calculator.



Rounding off angles

Conversion:

$$60 \text{ minutes} = 1 \text{ degree } (60' = 1^\circ)$$

$$60 \text{ seconds} = 1 \text{ minute } (60'' = 1')$$

In normal rounding off, you round up to the next number if the number to the right is 5 or more. Angles are rounded off to the nearest degree by rounding up if there are 30 minutes or more. Similarly, angles are rounded off to the nearest minute by rounding up if there are 30 seconds or more.

Examples:

Solution:

Round off to the nearest minute.

1. $23^\circ 12' 22''$ ($22''$ is less than $30''$ so $12'$ remains as the rounded minute.) **$23^\circ 12'$**
2. $59^\circ 34' 41''$ ($44''$ is more than $30''$ so $34'$ is rounded up.) **$59^\circ 35'$**
3. $16^\circ 54' 30''$ ($30''$ is equal to $30''$ so $54'$ is rounded up.) **$16^\circ 55'$**

Round off to the nearest degree.

1. $23^\circ 12' 22''$ ($12'$ is less than $30'$ so 23° remains as the rounded degree) **23°**
2. $59^\circ 34' 41''$ ($34'$ is more than $30'$ so 60° is rounded up). **60°**
3. $16^\circ 54' 30''$ ($54'$ is more than $30'$ so 16° is rounded up). **17°**

Trigonometric Ratios and the Calculator

We could make use of a scientific calculator to obtain the trigonometric value of an angle.

Example 1

Find the value of $\cos 6.35^\circ$.

Solution:

Press

cos	6	.	3	5	=
-----	---	---	---	---	---

$$\cos 6.35^\circ = 0.9939 \text{ (correct to 4 decimal places)}$$

**Example 2**

Find the value of $\sin 40^\circ 32'$.

Solution

Press the following on your calculator .

sin	4	0	''	''	3	2	''	''	=
-----	---	---	----	----	---	---	----	----	---

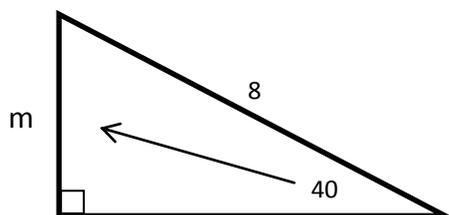
 $\sin 40^\circ 32' = 0.6499$ (correct to 4 decimal places)

FINDING THE UNKNOWN SIDES OF A RIGHT TRIANGLE

For any right angled triangle the length of an unknown side can be found if the length of another side and the size of another angle (other than the right angle) are known.

Given an acute angle and one side**Example 1**

Calculate the length of the unknown side in each of the following diagrams.

**Solution**

In the triangle the hypotenuse is 8 and relative to the given angle the opposite side is **m**. The ratio linking opposite side and hypotenuse is **sine**. (SOH)

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

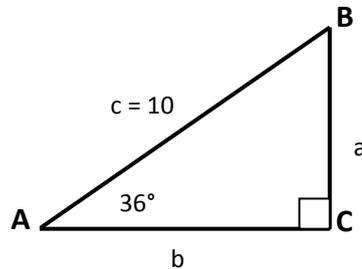
$$\sin 40 = \frac{m}{8} \quad (\text{multiplying by } 8)$$

$$8 \times \sin 40 = m$$

$$m = 5.14 \quad (\text{correct to 2 decimal places - using a calculator for the value of } \sin 40)$$

**Example 2**

Solve the right triangle ABC if angle A is 36° and side c is 10 cm.

**Solution**

Since angle A is 36° , then angle B is $90^\circ - 36^\circ = 54^\circ$.

To find an unknown side, say a , proceed as follows:

Make the **unknown** side the numerator of a fraction, and make the **known** side the denominator.

$$\frac{\text{Unknown}}{\text{Known}} = \frac{a}{10}$$

Name that function of the angle.

$$\frac{\text{Unknown}}{\text{Known}} = \frac{a}{10} = \sin 36^\circ$$

Use the trigonometric Table to evaluate that function.

$$\frac{\text{Unknown}}{\text{Known}} = \frac{a}{10} = \sin 36^\circ = .588$$

Solve for the unknown side.

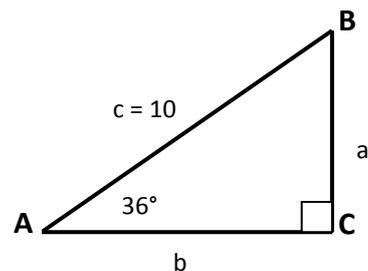
$$a = 10 \times .588 \text{ cm} = 5.88 \text{ cm}$$

Example 3 Solve the triangle for side b .

Solution

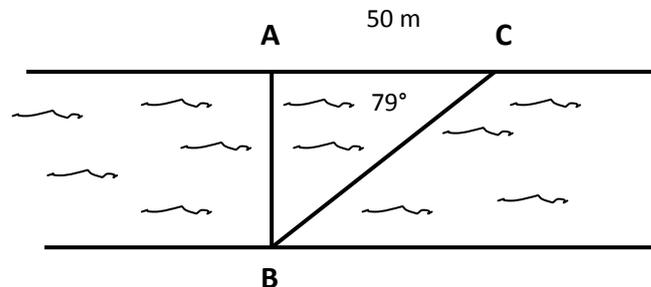
$$\frac{\text{Unknown}}{\text{Known}} = \frac{b}{10} = \cos 36^\circ = .809$$

$$b = 10 \times .809 = 8.09 \text{ cm}$$



**Example 4** To measure the width of a river

Two trees stand opposite one another, at points A and B, on opposite banks of a river.

**Solution**

Distance AC along one bank is perpendicular to BA, and is measured to be 50 m. $\angle ACB$ is measured to be 79° . How far apart are the trees, that is, what is the width w of the river?

$$\frac{\text{Unknown}}{\text{Known}} = \frac{w}{100} = \tan 79^\circ$$

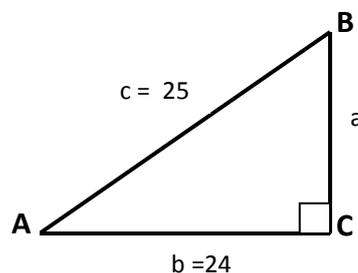
$$w = 100 \tan 79^\circ$$

$$w = 514 \quad (\text{nearest metre})$$

Sometimes we are asked to solve all the missing parts in a triangle. Start working out by using any of two given parts in a triangle.

Example 1

Solve $\triangle ABC$ given that side $c = 25$ cm and side $b = 24$ cm.

**Solution**

To find the remaining side a , use the Pythagorean Theorem

$$a^2 + 24^2 = 25^2$$

$$a^2 = 625 - 576 = 49$$

$$a = \sqrt{49} = 7$$



Next, to find angle A, we have

$$\begin{aligned}\cos A &= \frac{24}{25} = \frac{96}{100}, \text{ on multiplying each term by 4.} \\ &= .96\end{aligned}$$

We find $\cos 16^\circ = .961$

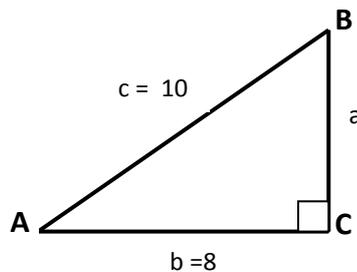
Therefore, Angle A $\approx 16^\circ$.

Finally, Angle B = $90^\circ - 16^\circ = 74^\circ$.

We have solved the triangle.

Example 2

Solve the right triangle ABC given that $c = 10$ cm and $b = 8$ cm.



To find the remaining side a , use the Pythagorean Theorem

$$\begin{aligned}a^2 + 8^2 &= 10^2 \\ a^2 &= 100 - 64 = 36 \\ a &= \sqrt{36} = 6 \text{ cm.}\end{aligned}$$

To find angle A, we have

$$\cos A = \frac{8}{10} = 0.8$$

$$\cos^{-1} A = 0.799$$

$$A = 37^\circ$$

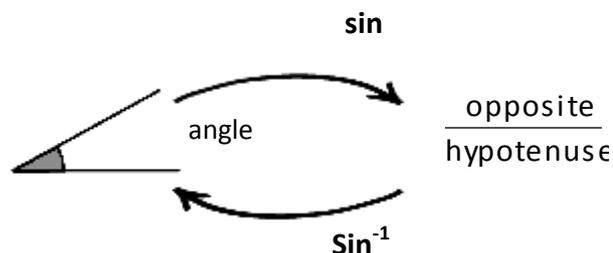
Therefore, angle A $\approx 37^\circ$.

Angle B = $90^\circ - 37^\circ = 53^\circ$.

And we have our answer! But what is the meaning of \sin^{-1} , \cos^{-1} and \tan^{-1} ?



Well, the Sine function "**sin**" takes an angle and gives us the **ratio** "opposite/hypotenuse",



But **sin⁻¹** (called "inverse sine") goes the other way. It takes the **ratio** "opposite/hypotenuse" and gives us an angle.

- Sine Function: $\sin(30^\circ) = 0.5$
- Inverse Sine Function: $\sin^{-1}(0.5) = 30^\circ$



On the calculator press one of the following (depending on your brand of calculator): either '2ndF sin' or 'shift sin'.

On your calculator, try using **sin** and **sin⁻¹** to see what results you get!

Also try **cos** and **cos⁻¹** and **tan** and **tan⁻¹**.

Example

Find θ in degrees and minutes.

1. $\sin \theta = 0.298$

2. $\tan \theta = 1.056$

3. $\cos \theta = 0.188$

Solution

$17^\circ 20'$

$46^\circ 34'$

$79^\circ 9'$



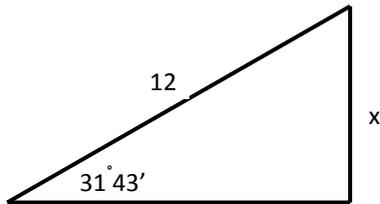
Learning Activity 11.5.2.1



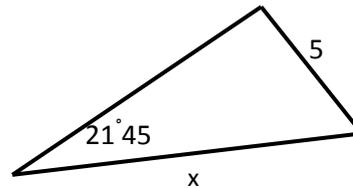
20 minutes

Find the values of all pronumerals, correct to one decimal place.

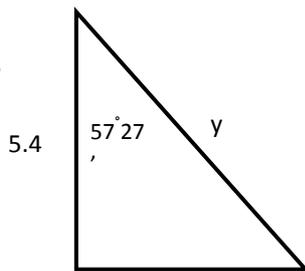
1.



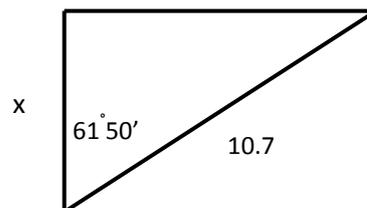
2.



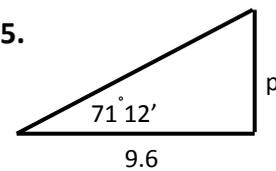
3.



4.



5.



11.5.2.2 Finding the Unknown Angles of a Right Triangle

If we know the lengths of the two sides of a right triangle, we can calculate the sizes of the other angles.

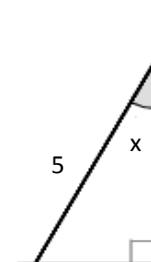
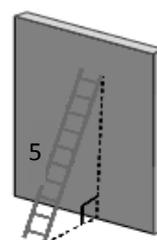
Given: Any Two Sides

We can find an **unknown angle** in a right-angled triangle, as long as we know the lengths of **two of its sides**.

Example 1

The ladder leans against a wall as shown.

What is the **angle** between the ladder and the wall?



The answer is to use Sine, Cosine or Tangent.

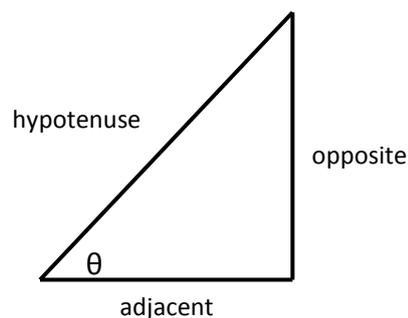


But which one to use? We have a special phrase "SOHCAHTOA" to help us, and we use it like this.

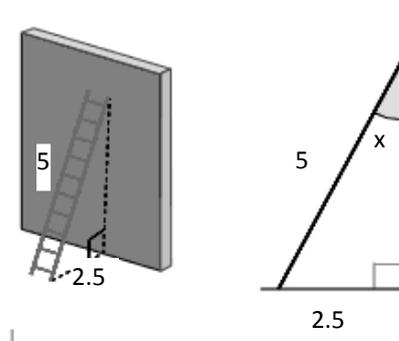
Step 1

Find the names of the two sides we know

- **Adjacent** is adjacent to the angle,
- **Opposite** is opposite the angle,
- And the longest side is the **Hypotenuse**.



In our ladder example we know the length of:



- the side **Opposite** the angle "x", which is **2.5**
- the longest side, called the **Hypotenuse**, which is **5**

Step 2:

Now use the first letters of those two sides (**O**pposite and **H**ypotenuse) and the phrase "SOHCAHTOA" to find which one of Sine, Cosine **or** Tangent to use:

SOH...

Sine: $\sin(\theta) = \text{Opposite} / \text{Hypotenuse}$

...CAH...

Cosine: $\cos(\theta) = \text{Adjacent} / \text{Hypotenuse}$

...TOA

Tangent: $\tan(\theta) = \text{Opposite} / \text{Adjacent}$

In our example that is **O**pposite and **H**ypotenuse, and that gives us "**SOH**cahtoa", which tells us we need to use **Sine**.



Step 3: Put our values into the Sine equation.

$$\sin(x) = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{2.5}{5} = 0.5$$

Step 4: Now solve that equation. $\sin(x) = 0.5$

Next we can re-arrange that into this. $x = \sin^{-1}(0.5)$

And then get our calculator, key in 0.5 and use the \sin^{-1} button to get the answer: $x = 30^\circ$

Step By Step

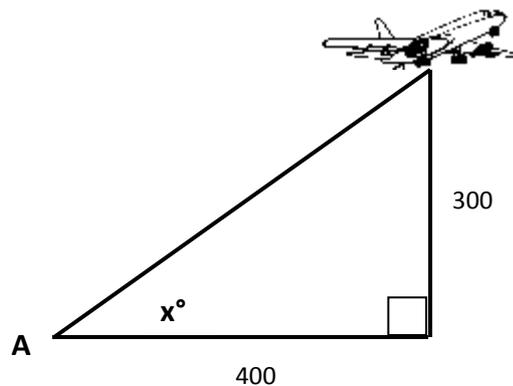
These are the four steps we need to follow.

- **Step 1** Find which two sides we know – out of Opposite, Adjacent and Hypotenuse.
- **Step 2** Use SOHCAHTOA to decide which one of Sine, Cosine **or** Tangent to use in this question.
- **Step 3** For Sine calculate Opposite/Hypotenuse, for Cosine calculate Adjacent/Hypotenuse **or** for Tangent calculate Opposite/Adjacent.
- **Step 4** Find the angle from your calculator, using one of \sin^{-1} , \cos^{-1} **or** \tan^{-1}

Let us look at two more examples.

Example 2

Find the angle x of the plane from point A on the ground.



Solution

Step 1 The two sides we know are **Opposite** (300) and **Adjacent** (400).



Step 2 SOHCAHTOA tells us we must use **Tangent**.

Step 3 Calculate $\frac{\text{opposite}}{\text{adjacent}} = \frac{300}{400} = 0.75$

Step 4 Find the angle from your calculator using \tan^{-1}

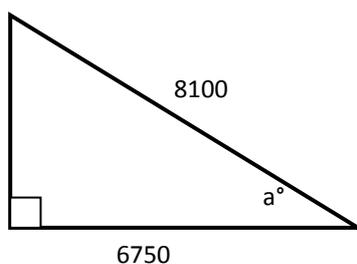
$$\tan x^\circ = \frac{\text{opposite}}{\text{adjacent}} = \frac{300}{400} = 0.75$$

\tan^{-1} of 0.75 = **36.9°** (correct to 1 decimal place)

Unless you are told otherwise, angles are usually rounded to one place of decimals.

Example 3

Find the size of angle a° .



Solution

Step 1 The two sides we know are **Adjacent** (6,750) and **Hypotenuse** (8,100).

Step 2 SOHCAHTOA tells us we must use **Cosine**.

Step 3 Calculate Adjacent / Hypotenuse = $6,750/8,100 = 0.8333$

Step 4 Find the angle from your calculator using \cos^{-1} of 0.8333.

$$\cos a^\circ = 6,750/8,100 = 0.8333$$

\cos^{-1} of 0.8333 = **33.6°** (to 1 decimal place)



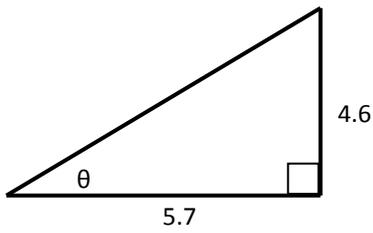
Learning Activity 11.5.2.2



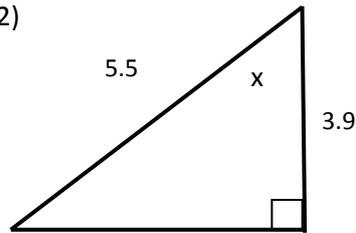
20 minutes

Find the value of each pronumeral, in degrees and minutes.

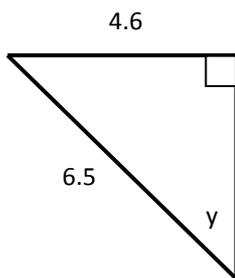
(1)



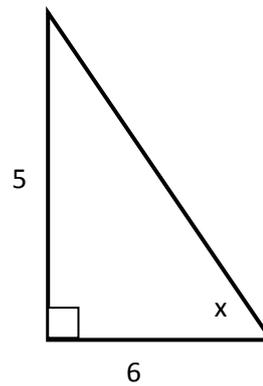
(2)



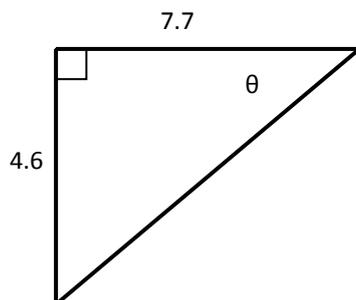
(3)



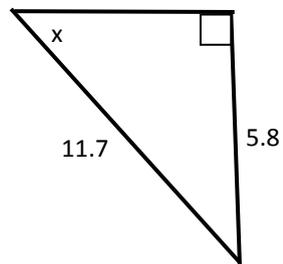
(4)



(5)



(6)



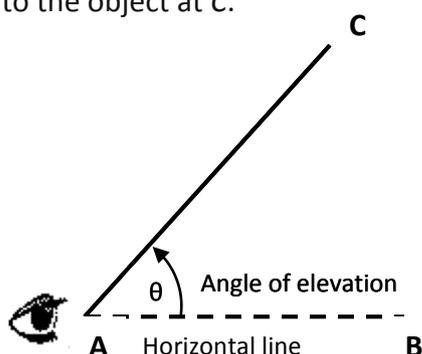


11.5.2.3 Angles of Elevation and Depression

We will now consider some practical applications of trigonometry in the calculation of angles of elevation and angles of depression.

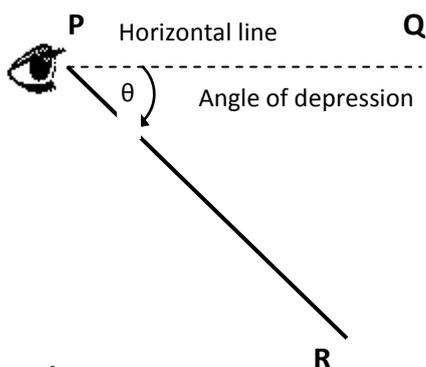
The **angle of elevation** is the angle between a horizontal line from the observer and the line of sight to an object that is above the horizontal line.

In the diagram below, AB is the horizontal line. θ is the angle of elevation from the observer at A to the object at C .



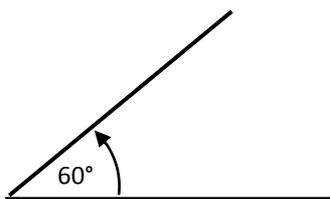
The **angle of depression** is the angle between a horizontal line from the observer and the line of sight to an object that is below the horizontal line.

In the diagram below, PQ is the horizontal line. θ is the angle of depression from the observer at P to the object at R .

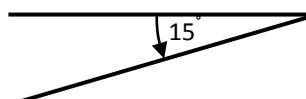


Example 1

An angle of elevation of 60° .



An angle of depression of 15° .

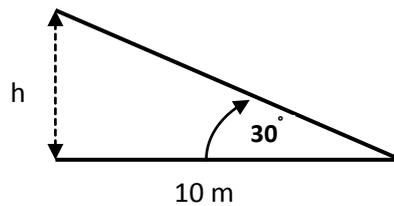


**Example 2**

The angle of elevation of the top of a tree from a point 10 m from its base is 30° . How high is the tree?

Solution

Draw a diagram of the problem and label the parts of the right triangle.



The tangent ratio is used to solve the missing side (height)

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan 30^\circ = \frac{h}{10}$$

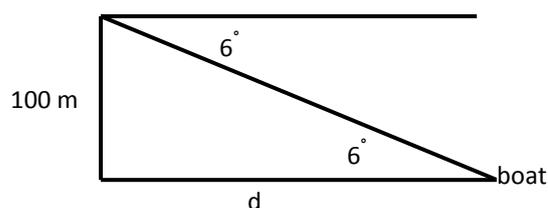
$$h = 10 \tan 30$$

$$h = 10 \times \frac{1}{\sqrt{10}}$$

$$h = 5.77 \text{ (2 decimal places)}$$

Example 3

Find the distance of a boat from a lighthouse if the lighthouse is 100 meters tall, and the angle of depression is 6° .

Solution



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan \theta = \frac{100}{d}$$

$$d \times \tan \theta = 100 \text{ (by cross multiplication)}$$

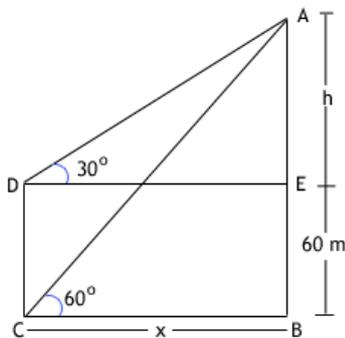
$$d \times \tan 6 = 100 \text{ (dividing both sides by } \tan 6)$$

$$d = \frac{100}{\tan 6}$$

$$d = 951.4 \text{ meters (1 decimal place)}$$

Example 4

The angles of elevation of the top of a tower from the top and bottom of a 60 m high building are 30° and 60° . What is the height of the tower?



Solution:

The height of the building $DC = 60 \text{ m}$

Angle of elevation of the top of the tower from the top of the building = 30°

Angle of elevation of the top of the tower from the bottom of the building = 60°

Let, x be the distance between building and tower.

$$\tan D = \frac{\text{opposite side}}{\text{adjacent side}}$$

Choose an appropriate trigonometric ratio.

$$\text{From } \triangle ADE, \tan 30^\circ = \frac{AE}{DE}$$



$$0.5773 = \frac{h}{x}$$

$$x = \frac{h}{0.5773}$$

Multiply both sides with $x / 0.5773$.

$$\text{From } \triangle ABC, \tan 60^\circ = \frac{h+60}{x}$$

$$1.732 = \frac{h+60}{x}$$

Substitute the values.

$$x = \frac{h+60}{1.732}$$

Multiply both sides with $x / 1.732$.

$$\text{From the work out above } x = \frac{h}{0.5773} ; x = \frac{h+60}{1.732}$$

Solving simultaneously,

$$\frac{h}{0.5773} = \frac{h+60}{1.732}$$

$$1.732 h = 0.5773(h + 60)$$

by cross multiplication

$$1.732 h = 0.5773h + 34.638$$

$$1.732 h - 0.5773h = 34.638$$

$$1.1547h = 34.638$$

dividing both sides by 1.1547

$$h = 29.99 = 30$$

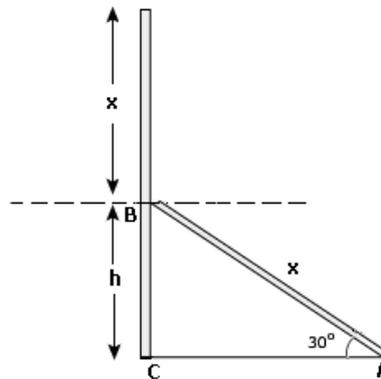
$$\text{The height of the tower} = h + 60$$

$$= 30 + 60 = 90$$

[Substitute and simplify.]

Example 5

The upper part of a pole broken over by wind makes an angle 30° with the ground and touches the ground at a distance 26 feet. What is the height of the pole?

**Solution:**

From the figure, h is the height of the unbroken part of the pole and x is the length of the broken part of the pole.

The distance between the bottom of the pole and the point where top touches the ground, $AC = 26$ feet.

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}}$$

Choose an appropriate trigonometric ratio.

$$\text{From } \triangle ABC, \tan A = \frac{BC}{AC}$$

$$\tan 30^\circ = \frac{h}{26}$$

Substitute

$$0.5773 = \frac{h}{26}$$

$$h = 26 \times 0.5773 = 15.00$$

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}}$$

Choose an appropriate trigonometric ratio.

$$\text{From } \triangle ABC, \sin A = \frac{BC}{AB}$$

$$\sin 30^\circ = \frac{h}{x}$$

Use calculator or table to find the value of $\sin 30^\circ$.

$$0.5 = \frac{h}{x}$$

Substitute.



$$0.5x = h$$

$$0.5x = 15.00 \Rightarrow x = 30$$

The height of the pole = $h + x$

$$= 15.00 + 30 = 45 \quad \text{Substitute and add}$$



Learning Activity 11.5.2.3



20 minutes

Solve the following problems. Draw the diagram and label the parts of the triangle.

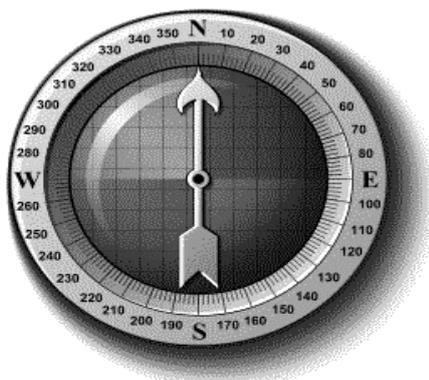
1. A ladder must reach the top of a building. The base of the ladder will be 3 m from the base of the building. The angle of elevation from the base of the ladder to the top of the building is 64° . Find the height of the building (h) and the length of the ladder (m).
2. From the top of a vertical cliff 40 m high, the angle of depression of an object that is level with the base of the cliff is 34° . How far is the object from the base of the cliff?
3. Two poles on horizontal ground are 60 m apart. The shorter pole is 3 m high. The angle of depression of the top of the shorter pole from the top of the longer pole is 20° . Sketch a diagram to represent the situation.



4. A man is 2 m tall and stands on horizontal ground 30 m from a tree. The angle of elevation of the top of the tree from his eyes is 28° . Estimate the height of the tree.
5. A boat is 500 metres from the base of a cliff. Jackie is sitting in the boat, notices that the angle of elevation to the top of the cliff is $32^\circ 15'$. How high is the cliff? Give your answer to the nearest metre.

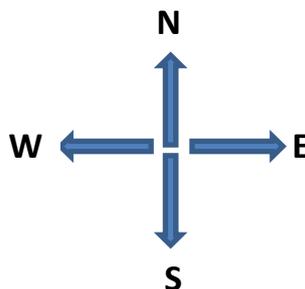
11.5.2.4 Bearings

A **directional compass** is shown below. It is used to find a **direction** or **bearing**.



The four main directions of a compass are known as **cardinal points**. They are north (N), east (E), south (S) and west (W). Sometimes, the half-cardinal points of north-east (NE), north-west (NW), south-east (SE) and south-west (SW) are shown on the compass. The above compass shows degree measurements from 0° to 360° in 10° intervals with:

- north representing 0° or 360°
- east representing 90°
- south representing 180°
- west representing 270°



When using a directional compass, hold the compass so that the point marked north points directly away from you. Note that the magnetic needle always points to the north.

Bearings

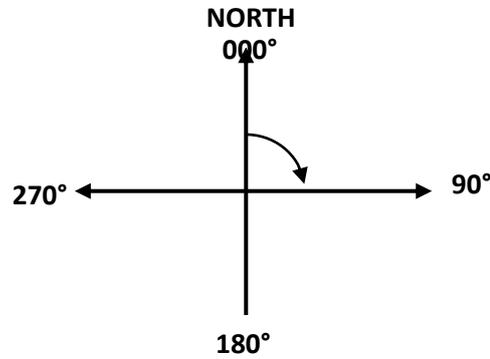
Bearings are a measure of direction, with North taken as a reference.

If you are travelling north, your bearing is 000° , and this is usually represented as straight up



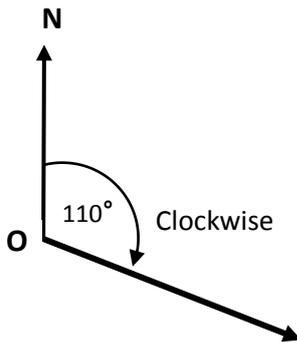
on the page.

If you are travelling in any other direction, your *bearing* is measured clockwise from North.



Example

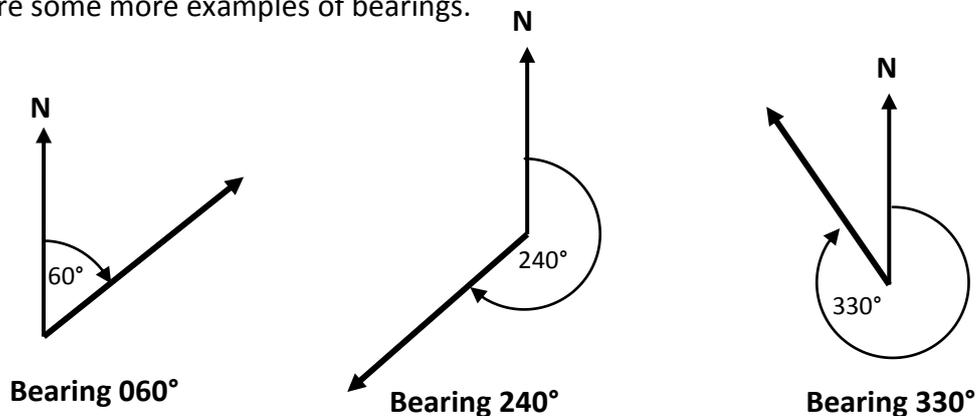
Look at the diagram below:



If you walk from O in the direction shown by the arrow, you are walking on a bearing of 110° .

REMEMBER: Bearings are always measured clockwise from **North** and are given as 3 digits.

Here are some more examples of bearings.



**Example 1**

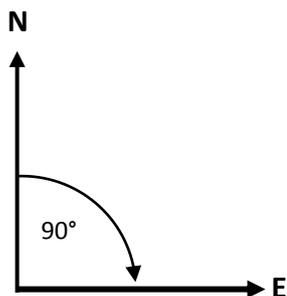
Points of the compass can all be converted into bearings. We already know that north is 000° .

Find the bearings for

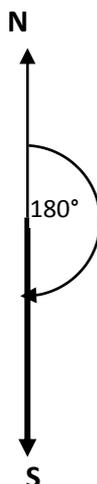
- (a) East (E)
- (b) South (S)
- (c) South-East (SE)

Solution:

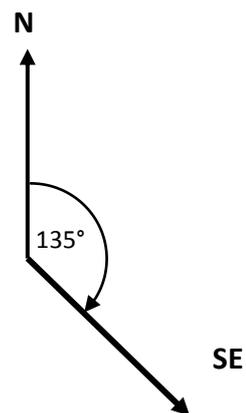
a.



b.

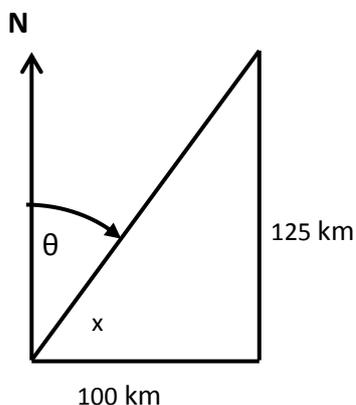


c.

**Example 2**

A plane flies 100 km due east then 125 km due north. Find its bearing from its starting point to the nearest degree.

Solution:



$$\tan x = \frac{\text{opp}}{\text{adj}} = \frac{125}{100}$$

$$\tan x = 1.25$$

$$x = 51^\circ$$

$$\theta + x = 90^\circ \text{ (angle sum, right angle)}$$

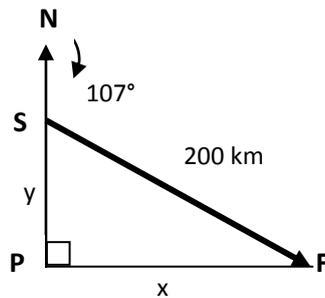
$$\theta = 90^\circ - x$$

$$\theta = 90^\circ - 51^\circ = 39^\circ$$

So the bearing of the plane from the starting point is 039° .

**Example 3**

A boat sails at a bearing of 107° for 200 km. How far south and east does it travel?

**Solution**

$$\angle PSF = 180^\circ - 107^\circ = 73^\circ$$

Since x is the opposite and 200 is the hypotenuse, we can use \sin to find x .

$$\sin 73^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\text{So, } \sin 73^\circ = \frac{x}{200}$$

$$x = 200 \times \sin 73 \quad (\text{By cross multiplication})$$

$$= 200 \times 0.9563$$

$$= 191 \text{ km}$$

Since y is the adjacent and 200 is the hypotenuse, we can use \cos to find y .

$$\cos 73^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\text{So, } \cos 73^\circ = \frac{y}{200}$$

$$y = 200 \times \cos 73^\circ \quad (\text{By cross multiplication})$$

$$= 200 \times 0.2924$$



= 58 km

The boat is 58 km south and 191 km east of its starting point.



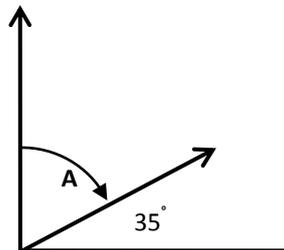
Learning Activity 11.5.2.4



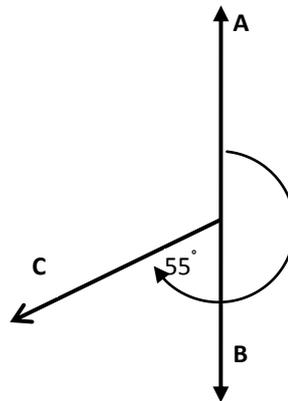
20 minutes

1. Name the bearing of the following:

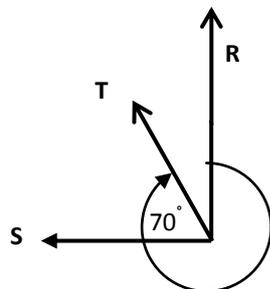
(a)



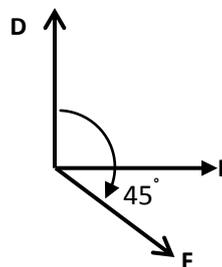
(b)



(c)



d)



2. Sketch the diagram

a. M is on the bearing of 315° .

b. X is on a bearing of 30° from Y.



c. R is on a bearing of 100° from S

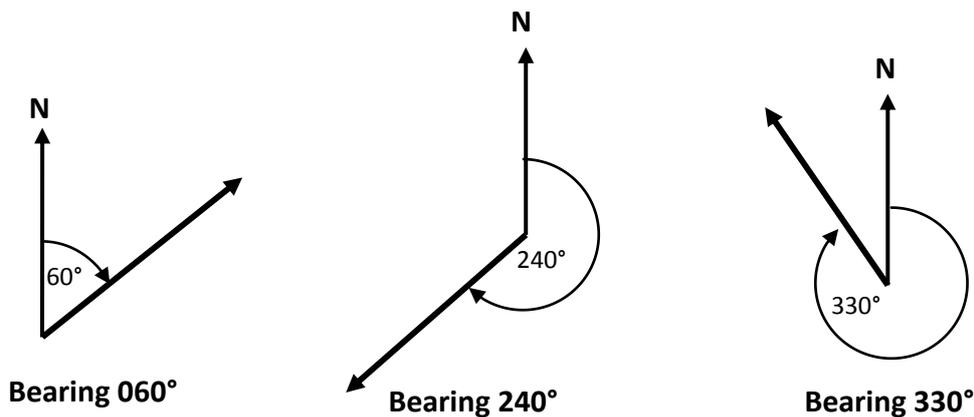
3. After walking due west, turning and walking due south, a man is 900 metres from his starting point and bearing 205° from it. How far did he walk?

- westward?
- southward?

4. A light aircraft takes off flying due north then turns and flies 11 000 metres due west. The plane then has a bearing of 340° from its starting point. For what distance did it fly due north?

11.5.2.5 Back Bearings

Note that the first two bearings below are directly in opposite directions to each other.



They have different bearings, but they are exactly 180° apart as they are in opposite directions.

A line in the opposite direction to the third bearing above would have a bearing of 150° because $330^\circ - 180^\circ = 150^\circ$.

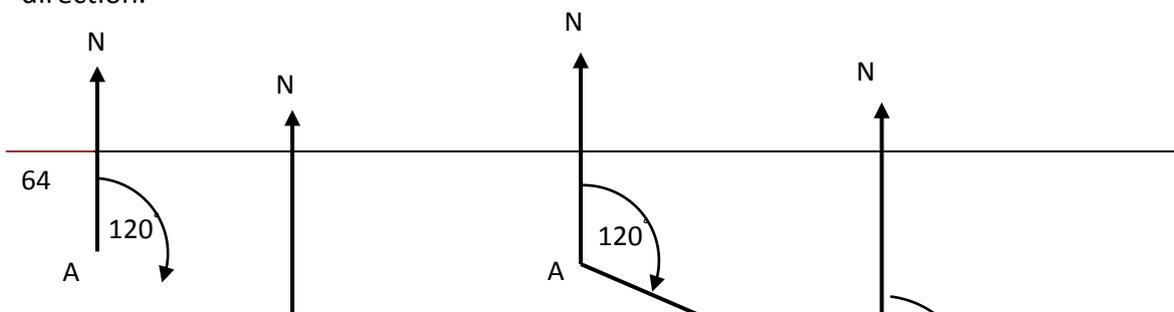
These bearings in the opposite direction are called **back bearings or reciprocal bearings**.

Example 1

A ship sails from A to B on a bearing of 120° . On what bearing will it have to sail to return from B to A?

Solution

We can extend the line from A to B, then rotate through 180° to head in the opposite direction.



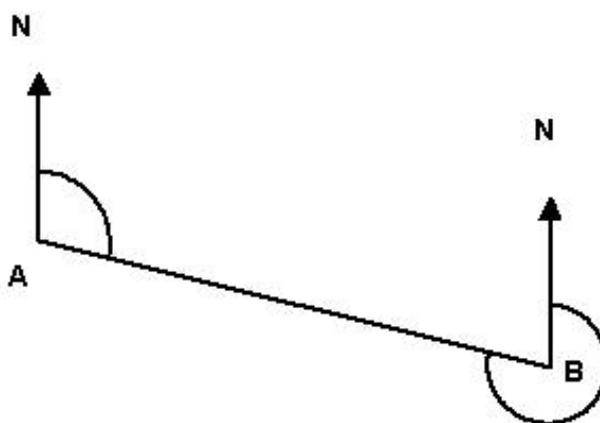


We can see from the diagram above that the bearing from B to A is 300° , because $120^\circ + 180^\circ = 300^\circ$.

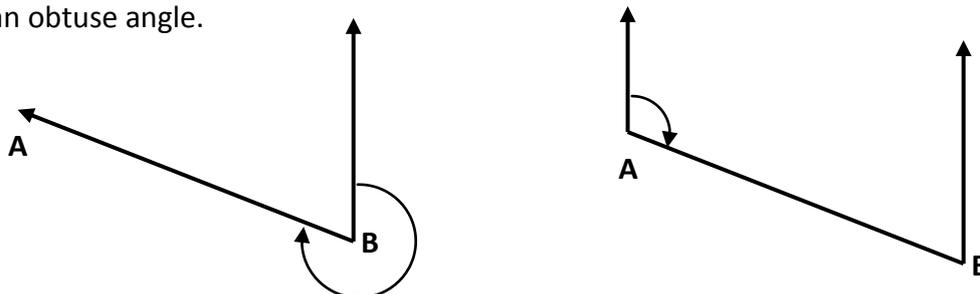
Finding the back (Reverse) Bearing

If we know the bearing of a first object from another second object, then we will want to find the bearing of the second object from the first object.

For instance if I know that the bearing of an aircraft is from an airport, then the pilot of the aircraft will want to know what bearing he or she must fly on in order to head towards the airport – and these two bearings are not the same.



Here we can see that the bearing of A from B is a reflex angle and the bearing of B from A is an obtuse angle.



$\angle B$ is a reflex angle

(Reflex angle is more than 180° but less than 270° .)

$\angle A$ is an obtuse angle

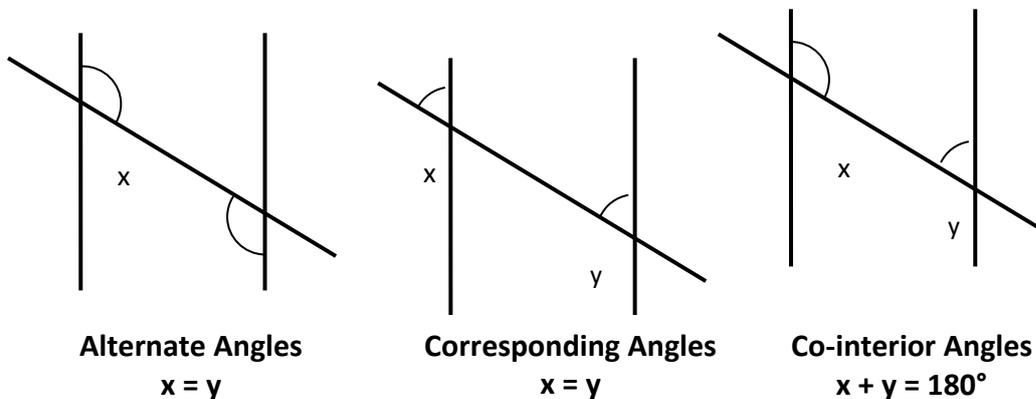
(Obtuse angle is more than 90° but less than 180° .)



How can we find these **back** bearings or reverse bearings?

Here are the three basic theorems, with the parallels drawn vertically. You have learned in Grade 11 about parallel lines being cut by a transversal.

When a transversal cuts two lines, it forms pairs of angles. When the two lines are parallel, these pairs of angles have special properties.



- If lines are parallel, then alternate angles are equal.
- If the lines are parallel, then corresponding angles are equal.
- If the lines are parallel, co-interior angles are supplementary (their sum is 180°)

The most useful one will be the **interior angles** theorem. Also we need to remember that angles around a point make 360° , a full turn.

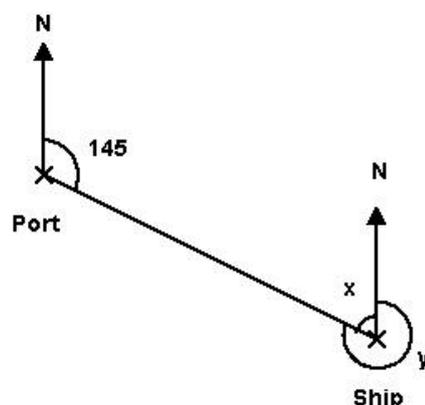
Here are two worked examples.

Example 1

A ship is on a bearing of 145° from a port. What is the bearing of the port from the ship? (In other words what heading should the ship be on to make port?)

Solution: First draw

a diagram.





We want to find the angle y . To do this we first find the angle x , which is **an interior angle with 145**. **cointerior to 145**.

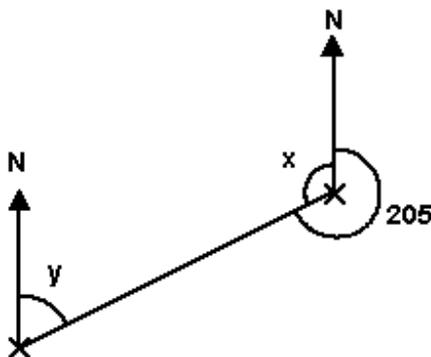
$$\text{So } x = 180^\circ - 145^\circ = 35^\circ$$

We can now take this away from 360 to find the reflex angle y

$$\text{So } y = 360^\circ - 35^\circ = \mathbf{325^\circ}$$

Example 2

An aeroplane is measured as being on a bearing of 205° from a control tower at an airfield. What **heading** /**direction** must the aircraft fly on to get to the control tower?



Solution: In order to find y , we first find x $x = 360^\circ - 205^\circ = 155^\circ$

Now we use interior angles to find y . $y = 180^\circ - 155^\circ = 25^\circ$

BUT the bearing is 025°, not 25°!

In summary, to calculate back bearings:

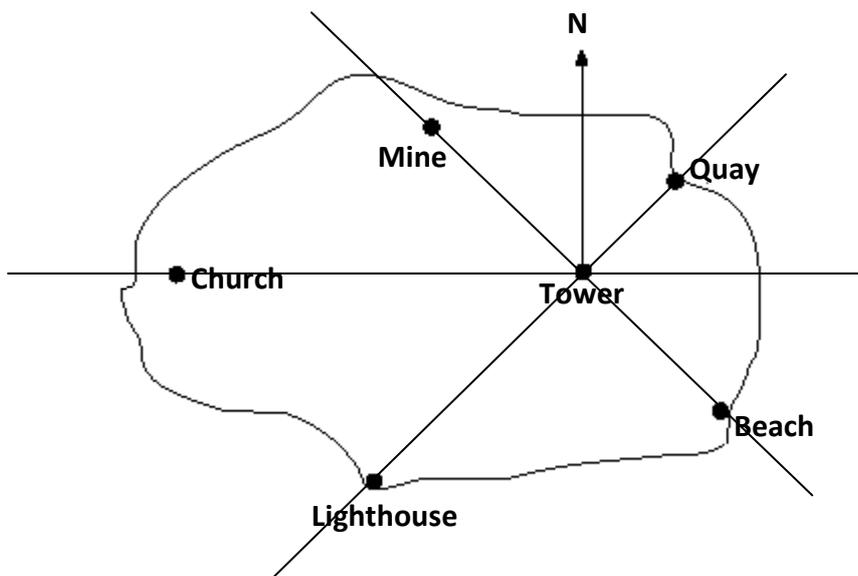


1. Subtract 180 from the compass bearing to your destination if the bearing is greater than or equal to 180° . The result is the back bearing.
For example, if the bearing to your destination is 200° , the back bearing is 20°
2. Add 180° to the compass bearing to your destination if the bearing is less than 180° . The result is the back bearing.
For example, if the bearing to your destination is 50 degrees, the back bearing is 230° .
3. Check the accuracy of your calculations by reversing them. For example, if you added 180° , now subtract it from the back bearing. If the result is the original bearing, then your calculations were good and the back bearing is correct.

**Learning Activity 11.5.2.5**

20 minutes

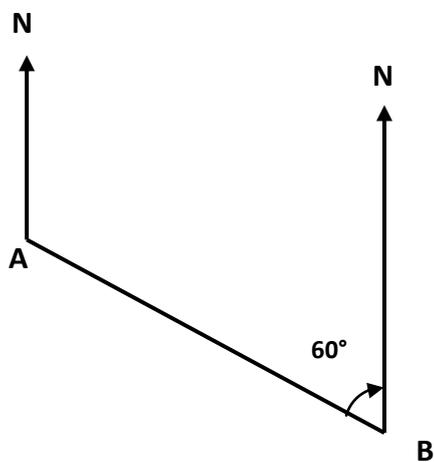
1. On what bearing is a ship sailing if it is heading:
 - (a) West?
 - (b) North-East?
 - (c) South-West?
 - (d) North-west
 - (e) North
 - (f) South –east
2. What is the back bearing or reciprocal bearing to each of the bearings below?
 - (a) 045°
 - (b) 200°
 - (c) 180°
 - (d) 300°
 - (e) 235°
 - (f) 330°
3. An aeroplane flies from Port Moresby to Lae on a bearing of 044° .
On what bearing should the pilot fly, to return to Port Moresby from Lae?
4. The map of an island has is shown below. A faint set of compass points has been put on the map to help you.



What is the bearing of:

- a) the Beach from the Tower?
- b) the Tower from the Church?
- c) the Mine from the Tower?
- d) the Quay from the Tower?
- e) the Tower from the Lighthouse?

5. The diagram shows the positions of two ships, A and B.



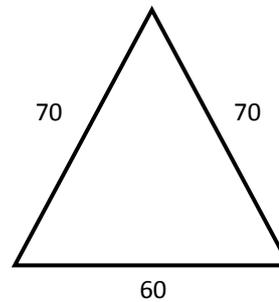
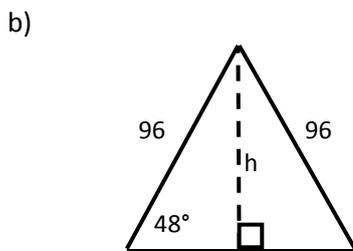
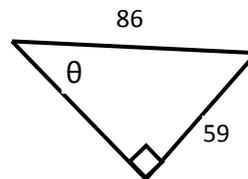
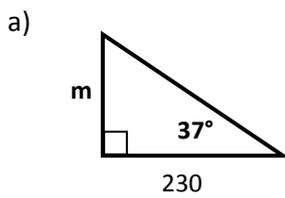
What is the bearing

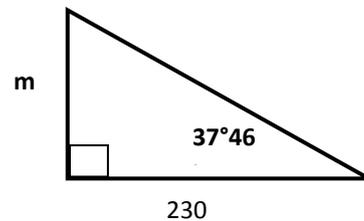
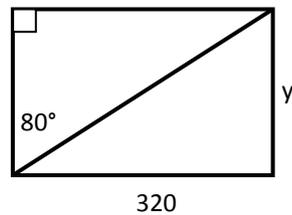


- a) From ship B to ship A?
- b) From ship A to ship B?

SUMMATIVE TASK 2 : Solution of Right Triangles

1. Find the value of the unknown in each diagram. All distances given are in millimetres.





Problem Solving: Start each question by drawing a large, clear diagram.

3. A ladder of length 6 m leans against a vertical wall so that the base of the ladder is 2 m from the wall. Calculate the angle between the ladder and the wall.
4. A ladder of length 8 m rests against a wall so that the angle between the ladder and the wall is 31° . How far is the base of the ladder from the wall?
5. From a point the angle of elevation of a tower is 30° . If the tower is 20 distant from the point, what is the height of the tower?



6. A man 1.5 m tall is 15 m away from a tower 20 m high. What is the angle of elevation of the top of the tower from his eyes?

7. A man lying down on top of hill 40 m high observes the angle of depression of a boat to be 20° . If he is in line with the boat, calculate the distance between the boat and the foot of the cliff.

11.5.3 SOLUTIONS OF OBLIQUE TRIANGLES

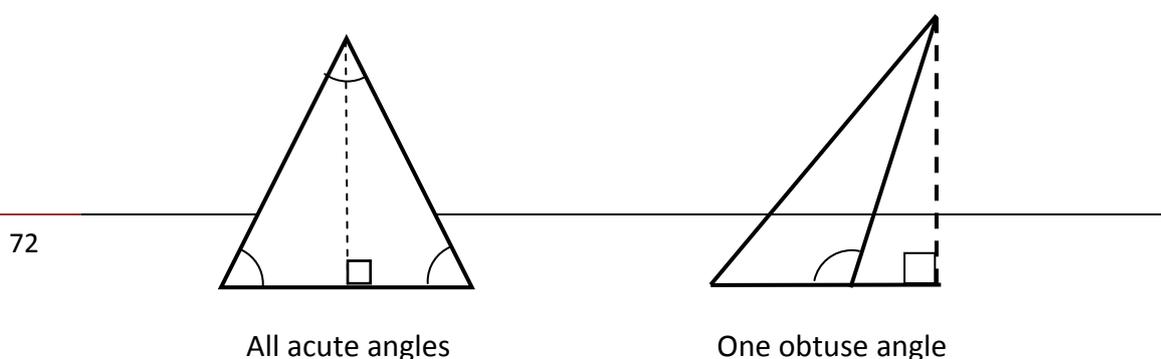
An oblique triangle is a triangle that contains no right angles. It is a triangle whose angles are all acute or a triangle with one obtuse angle.

Oblique triangles are not as easy to solve as right triangles because three parts of the triangle must be known in order to solve the triangle. Also, with oblique triangles, we can no longer use two of the simplest techniques for solving right triangles - the Pythagorean Theorem and the fact that two acute angles are complementary. Instead, two new techniques are needed: the **Law of Sines** and the **Law of Cosines**. These laws are formulas that relate the parts of triangles to each other. They work for all triangles, including right triangles, but since right triangles have special properties we use simpler methods to solve them.

In the following lessons, we will review oblique triangles and learn the Law of Sines and the Law of Cosines. In addition, we will also see a new way to find the area of a triangle.

11.5.3.1 The Oblique Triangles

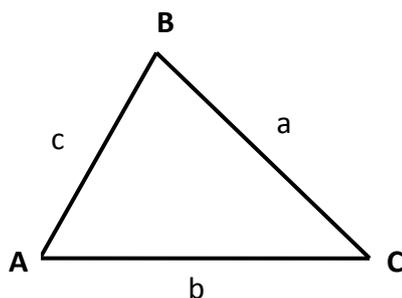
An **oblique triangle** is a **triangle** with no right angle. An **oblique triangle** has either three acute angles, or one obtuse angle and two acute angles. In any case, as in any **triangle**, the sum of all three angles is equal to 180 degrees.





The Standard Notation for a Triangle

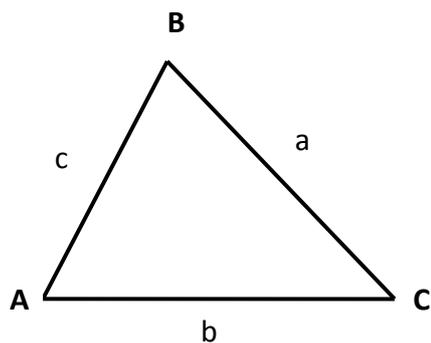
In triangle ABC the angles are denoted by the capital letters as shown in the diagram below. The side a lies opposite the angle A , side b opposite the angle B and side c opposite the angle C . This is the standard notation for a triangle.



To solve oblique triangles, use the laws of sine and cosine. There are four different potential scenarios: Given

1. one side and two adjacent angles.
2. two sides and an included angle.
3. two sides and the opposite angle.
4. two sides and the opposite angle.

The solution for an oblique triangle can be done with the application of the Law of Sine and Law of Cosine, simply called the Sine and Cosine Rules.



SINE RULE	COSINE RULE
-----------	-------------



<p>For finding sides</p> $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ <p>For finding angles</p> $\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$	<p>For finding sides</p> $a^2 = b^2 + c^2 - 2bc \cos A$ <p>For finding angles</p> $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
---	--

11.5.3.2. The Sine Rule

The Sine Rule can be used in any triangle (not just right-angled triangles) where a side and its opposite angle are known.

The Sine Rule states that the sides of a triangle are proportional to the sines of the opposite angles. In symbols,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The Sine Rule is used in the following cases: Given

CASE 1: two angles and one side (AAS or ASA)

CASE 2: two sides and a non-included angle (SSA)

Finding Sides



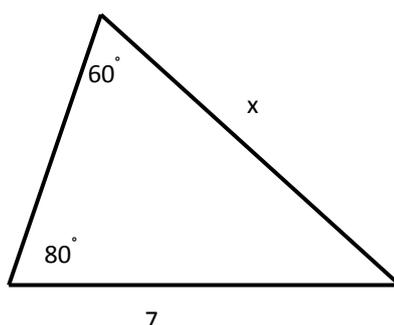
If you need to find the length of a side, you need to use the version of the Sine Rule where the lengths are on the top.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

You will only ever need two parts of the Sine Rule formula, not all three.

You will need to know at least one pair of a side with its opposite angle to use the Sine Rule.

Example 1



Step 1 Start by writing out the Sine Rule formula for finding sides.

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)}$$

Step 2 Fill in the values you know, and the unknown length.

$$\frac{x}{\sin(80^\circ)} = \frac{7}{\sin(60^\circ)}$$

Remember that each fraction in the Sine Rule formula should contain a side and its opposite angle.

Step 3 Solve the resulting equation to find the unknown side, giving your answer to 3 significant figures.

$$\frac{x}{\sin(80^\circ)} = \frac{7}{\sin(60^\circ)} \quad (\text{Multiply by } \sin 80^\circ \text{ on both sides})$$

$$x = \frac{7}{\sin(60^\circ)} \times \sin(80^\circ)$$

$$x = 7.96 \text{ (accurate to 3 significant figures)}$$

Note that you should try and keep full accuracy until the end of your calculation to avoid errors.



Finding Angles

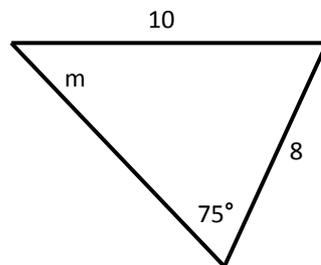
If you need to find the size of an angle, you need to use the version of the Sine Rule where the angles are on the top:

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

As before, you will only need two parts of the Sine Rule, and you still need at least a side and its opposite angle.

Example 2

Work out angle m° in the diagram below.



Step 1 Start by writing out the Sine Rule formula for finding angles.

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

Step 2 Fill in the values you know, and the unknown angle.

$$\frac{\sin(m^\circ)}{8} = \frac{\sin(75^\circ)}{10}$$

Remember that each fraction in the Sine Rule formula should contain a side and its opposite angle.

Step 3 Solve the resulting equation to find the sine of the unknown angle.

$$\frac{\sin(m^\circ)}{8} = \frac{\sin(75^\circ)}{10} \quad (\text{multiply by 8 on both sides})$$

$$\sin(m^\circ) = \frac{\sin(75^\circ)}{10} \times 8$$

$$\sin(m^\circ) = 0.773 \text{ (3 significant figures)}$$

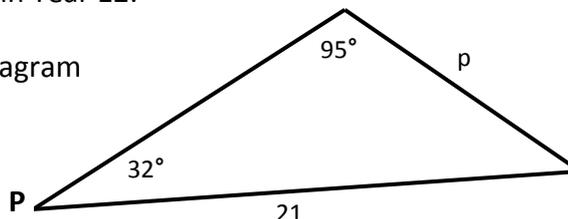
Step 4 Use the inverse-sine function (\sin^{-1}) to find the angle.

$$m^\circ = \sin^{-1}(0.773) = 50.6^\circ \text{ (3 significant figures)}$$



You may be aware that sometimes Sine Rule questions can have two solutions (only when you are finding angles). You do not need to know about these additional solutions at this time but you will learn more about them in Year 12.

Example 3 Find the missing side in the diagram below.



Solution

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{p}{\sin 32^\circ} = \frac{21}{\sin 95^\circ}$$

Cross multiply.

$$p \times \sin 95^\circ = 21 \sin 32^\circ$$

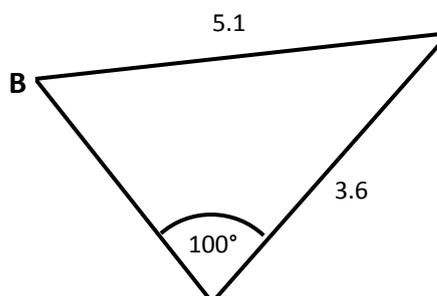
Divide both sides by $\sin 95^\circ$

$$p = \frac{21 \times \sin 32^\circ}{\sin 95^\circ}$$

$$p = 11.2$$

Example 4

Find the missing angle in the diagram below.



Solution:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$
$$\frac{\sin 100^\circ}{5.1} = \frac{\sin B}{3.6}$$

Cross multiply

$$\sin B \times 5.1 = \sin 100^\circ \times 3.6 \quad \text{Divide both sides by 5.1}$$

$$\sin B = \frac{\sin B \times 3.6}{5.1}$$

$$\sin B = 0.695$$

$$B = \sin^{-1} 0.695$$

$$B = 44.0$$

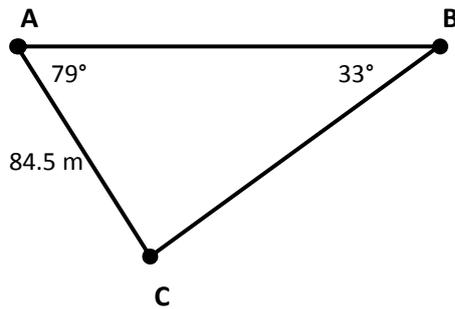
Solving Problems Involving Oblique Triangles Using
The Law of Sines



The following examples show how the law of sines is used to solve word problems involving oblique triangles.

Example 1

Points A and B are on opposite sides of a lake. A point C is 84.5 meters from A. The measure of angle BAC is $79^{\circ}20'$, and the measure of angle ACB is determined to be $33^{\circ}10'$. Find the distance between points A and B (to the nearest meter).



Solution

Before we can use sine rule, we need to find the measure of angle C.

$$\angle A + \angle B + \angle C = 180$$

$$79 + 33 + \angle C = 180$$

$$\angle C = 180 - 112$$

$$\angle C = 68$$

Now using sine rule,

$$\frac{AB}{\sin C} = \frac{AC}{\sin B}$$

$$\frac{AB}{\sin 68} = \frac{84.5}{\sin 33}$$

$$AB \sin 33 = 84.5 \sin 68$$

$$AB = \frac{84.5 \sin 68}{\sin 33}$$

$$AB = 131.57\text{m}$$

**Example 2**

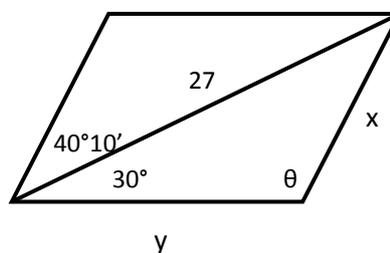
The longer diagonal of a parallelogram is 27 cm. Find the perimeter of the parallelogram if the angles between the sides and the diagonal are 30° and $40^\circ 10'$.

Solution

The perimeter of any parallelogram is twice the sum of the lengths of the two consecutive sides.

Let x and y be the consecutive sides as shown in the figure. The angle opposite x is $40^\circ 10'$. Hence, the angle θ between x and y is

$$\begin{aligned}30^\circ + 40^\circ 10' + \theta &= 180^\circ \\ \theta &= 180^\circ - (30^\circ + 40^\circ 10') \\ \theta &= 180^\circ - 70^\circ 10' \\ \theta &= 109^\circ 50'\end{aligned}$$



To solve for x :

$$\begin{aligned}\frac{x}{\sin 40^\circ 10'} &= \frac{27}{\sin 109^\circ 50'} \\ x \sin 109^\circ 50' &= 27 \sin 40^\circ 10' \\ x &= \frac{27(0.64501)}{0.94068}\end{aligned}$$

$$x = 18.51 \text{ cm}$$

To solve for y :

$$\begin{aligned}\frac{y}{\sin 30^\circ} &= \frac{27}{\sin 109^\circ 50'} \\ y \sin 109^\circ 50' &= 27 \sin 30^\circ \\ y &= \frac{27(0.5)}{0.94068} \\ y &= 14.35 \text{ cm}\end{aligned}$$

The perimeter P of the parallelogram is

$$\begin{aligned}P &= 2(x + y) \\ P &= 2(18.51 + 14.35)\end{aligned}$$



$P = 2(32.86)$
 $P = 65.72 \text{ cm}$

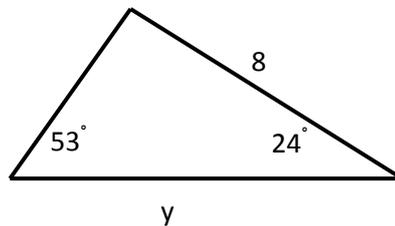


Learning Activity 11.5.3.2

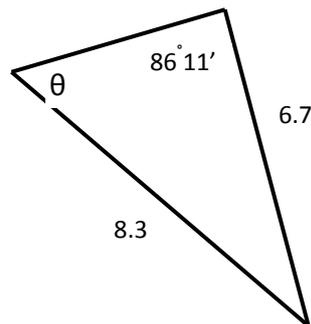


20 minutes

1. Find the value of y to the nearest whole number.

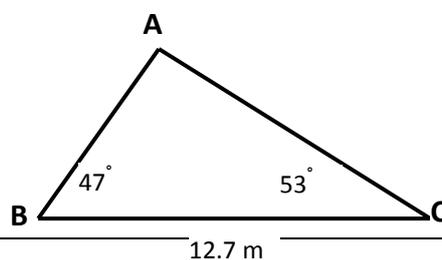


2. Find the value of θ in degrees and minutes.



3. Find the value of $\angle A$ to the nearest degree if $AB = 3.2 \text{ cm}$, $BC = 4.6 \text{ cm}$ and $\angle ACB = 33^\circ 47'$.

4. Triangle ABC has $BC = 12.7 \text{ m}$, $\angle ABC = 47^\circ$ and $\angle ACB = 53^\circ$ as shown. Find the lengths of AB and AC .





5. The shorter diagonal of a parallelogram is 5.2 m. Find the perimeter of the parallelogram if the angles between the sides and the diagonal are 40° and $30^\circ 10'$.

11.5.3.3. The Cosine Rule

The Cosine Rule can be used in any triangle where you are trying to relate all three sides to one angle.

The Cosine Rule states that the square of the length of any side of a triangle equals the sum of the squares of the length of the other sides minus twice their product multiplied by the cosine of their included angle.

Finding sides

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Finding angles

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The Cosine Rule is used in the following cases given

Case 1: two sides and an included angle (SAS)

Case 2: three sides (SSS)



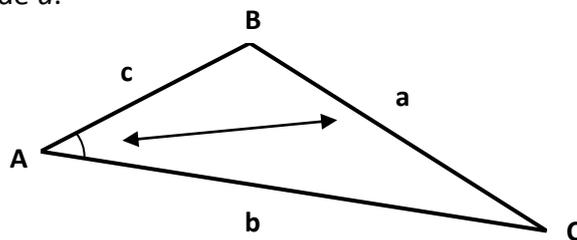
Finding Sides

If you need to find the length of a side, you need to know the other two sides and the opposite angle.

You need to use the version of the Cosine Rule where a^2 is the subject of the formula.

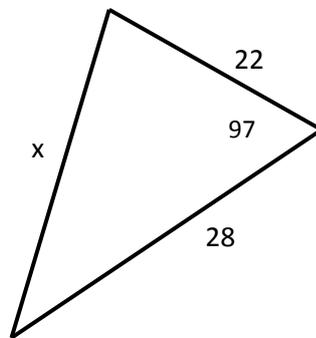
$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

Side a is the one you are trying to find. Sides b and c are the other two sides, and angle A is the angle opposite side a .



Example 1

Work out the length of x in the diagram below.



Step 1 Start by writing out the Cosine Rule formula for finding sides.

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

Step 2 Fill in the values you know, and the unknown length.

$$x^2 = 22^2 + 28^2 - 2 \times 22 \times 28 \times \cos(97^\circ)$$

It does not matter which way around you put sides b and c . It will work both ways.

Step 3 Evaluate the right-hand-side and then square-root to find the length.

$$x^2 = 22^2 + 28^2 - 2 \times 22 \times 28 \times \cos(97^\circ) \text{ (evaluate the right hand side)}$$



$$x^2 = 1418.143..... \quad (\text{square-root both sides})$$

$$x = 37.7 \text{ (accurate to 3 significant figures)}$$

As with the Sine Rule you observe full accuracy until the end of your calculation to avoid errors.

Finding Angles

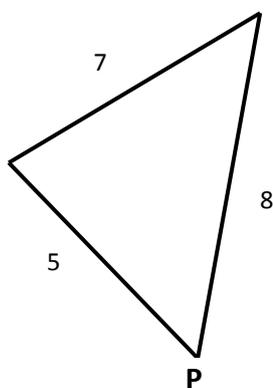
If you need to find the size of an angle, you need to use the version of the Cosine Rule where the $\cos(A)$ is on the left.

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

It is very important to get the terms on the top in the correct order; b and c are either side of angle A which you are trying to find and these can be either way around, but side a must be the side opposite angle A .

Example 2

Work out angle P° in the diagram below.



Step 1 Start by writing out the Cosine Rule formula for finding angles.

$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Step 2 Fill in the values you know, and the unknown length.

$$\cos(P^\circ) = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8}$$



Remember to make sure that the terms on top of the fraction are in the correct order.

Step3 Evaluate the right-hand-side and then use inverse-cosine (\cos^{-1}) to find the angle.

$$\cos(P^\circ) = \frac{5^2 + 8^2 - 7^2}{2 \times 5 \times 8} \quad (\text{the right evaluate -hand side})$$

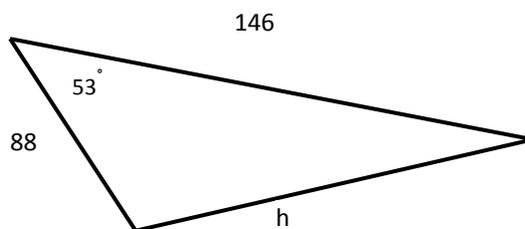
$$\cos(P^\circ) = 0.5 \quad (\text{do the inverse-cosine of both sides})$$

$$P = \cos^{-1}(0.5) = 60^\circ \quad (3 \text{ significant figures})$$

If you know two sides and an angle which is not in between them then you can use the Cosine Rule to find the other side, but it is easier to use the Sine Rule in this situation. You should always use the Sine Rule if you have an angle and its opposite side.

Example 3

Find the missing side in the diagram below.



$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

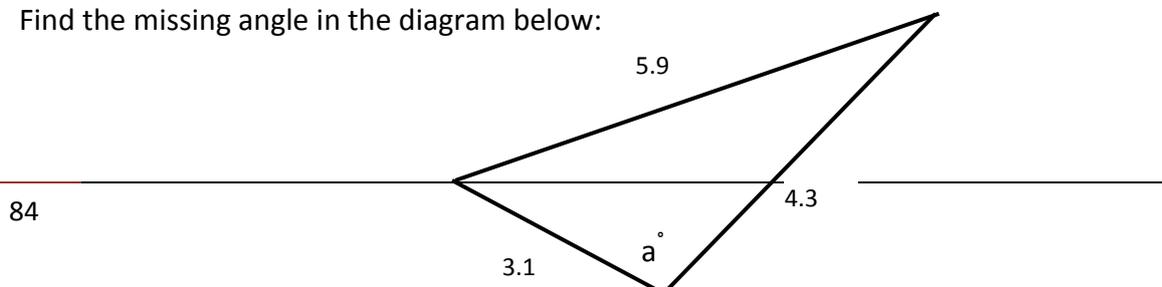
$$h^2 = 88^2 + 146^2 - 2 \times 88 \times 146 \times \cos(53^\circ)$$

$$h^2 = 13595.761$$

$$h = 117 \quad (\text{accurate to 3 significant figures})$$

Example 4

Find the missing angle in the diagram below:





$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{3.1^2 + 4.3^2 - 5.9^2}{2 \times 3.1 \times 4.3}$$

$$\cos A = -0.252$$

$$A = \cos^{-1}(-0.252)$$

$$A = 105^\circ \text{ (accurate to 3 significant figures)}$$

Solving Problems Involving Oblique Triangles Using The Law of Sines

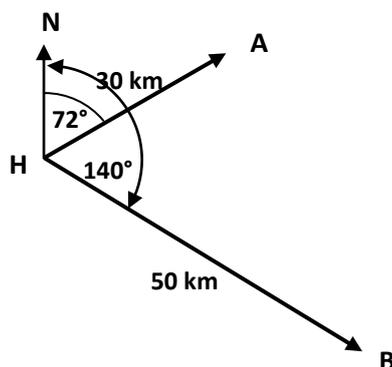
The following examples show how the law of cosines is used to solve word problems involving oblique triangles.

Example 1

Two boats leave from village H. Boat A sails on a bearing of 072° for 30 kilometres and stops. Boat B sails on a bearing of 140° for 50 kilometres and stops.

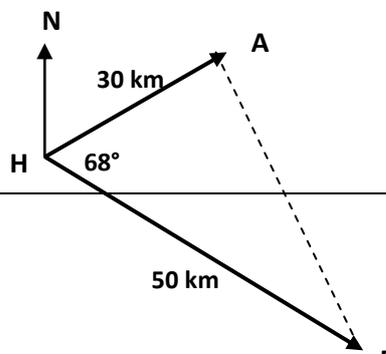
How far apart are the two boats when they have both stopped?

Solution



From the diagram above, the triangle is

$$\angle H = 140 - 72 = 68$$





To find AB, we use cosine rule

$$AB^2 = 30^2 + 50^2 - 2(30)(50) \cos 68$$

$$AB^2 = 900 + 2500 - 3000(.3746)$$

$$AB = \sqrt{2276.2}$$

$$AB = 47.7 \text{ kilometres}$$

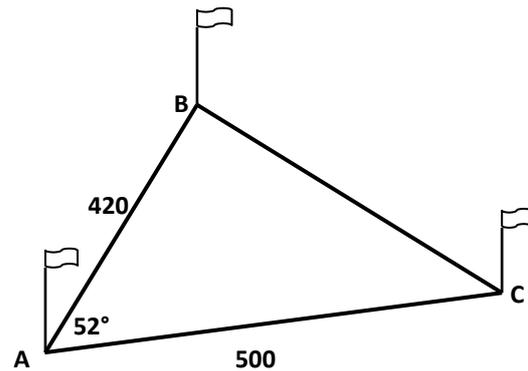
Example 2

The diagram shows part of a golf course.

The distance AB is 420 metres.

The distance AC is 500 metres
and angle BAC = 52° .

Calculate the distance BC.



Solution

$$BC^2 = 420^2 + 500^2 - 2(420)(500) \cos 52^\circ$$

$$BC^2 = 1764000 + 250000 - 420\,000 (.6157)$$

$$BC = \sqrt{167822}$$

$$BC = 409.66$$

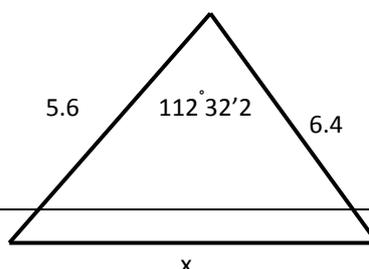


Learning Activity 11.5.3.3



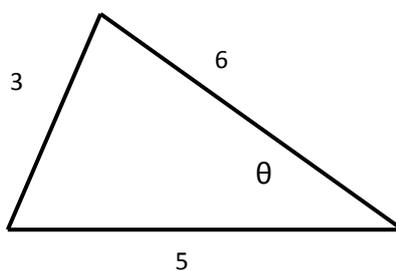
20 minutes

1. Find the value of x , correct to the nearest whole number.

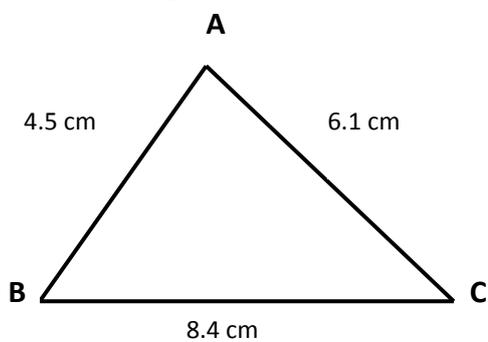




2. Find θ in degrees and minutes.



3. Evaluate $\angle BAC$ in degrees and minutes.

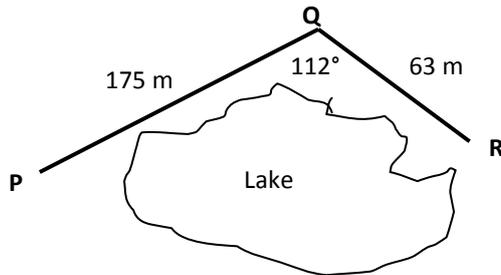


4. An aircraft flies 74 km in a direction 038° and then 63 km in a direction 160° . Find the distance of the aircraft from its starting point? 67.11 km



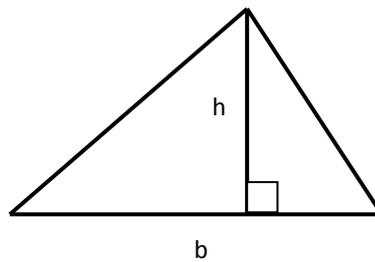
5. To get from P to R a park worker had to walk along a path to Q and then to R as shown.

What is the distance in straight line form P to R?



11.5.3.4. Areas of Triangles

To find the area of a triangle, you need to find its perpendicular height (h).



Knowing Base and Height

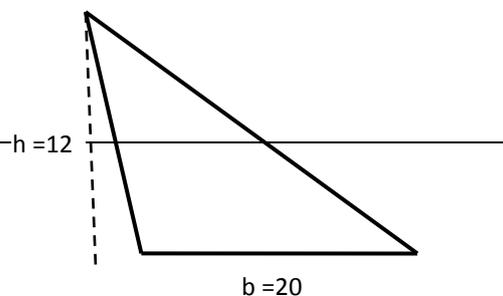
When we know the base and height, it is easy.
It is simply **half of b times h**.

$$A = \frac{1}{2} \times (\text{base}) \times (\text{height})$$

$$A = \frac{1}{2}bh$$

Example What is the area of this triangle?

Solution Height = h = 12





$$\text{Base} = b = 20$$

$$\begin{aligned}\text{Area} &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 20 \times 12 \\ &= 120 \text{ units}^2\end{aligned}$$

There are several ways to find the area of a triangle. Trigonometry allows us to find the height in terms of one of the angles in the triangle. The Area of triangles without right angles can be found given

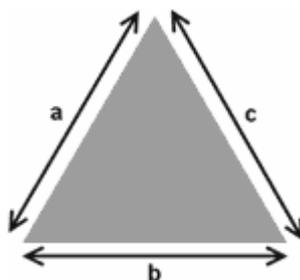
- a. three sides
- b. two sides and the included angle.

Knowing Three Sides

There is also a formula to find the area of any triangle when we know the lengths of all three of its sides.

This can be found by using the **Heron's Formula**, a formula that has been known for nearly 2000 years. This formula is named after Hero of Alexandria.

Given the triangle with sides a, b and c , just use this two-step process to find the area.



Step 1: Calculate " s " (half of the triangles perimeter) using the formula : $s = \frac{a+b+c}{2}$

Step 2: Then calculate the **Area** using the formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$

**Example 1**

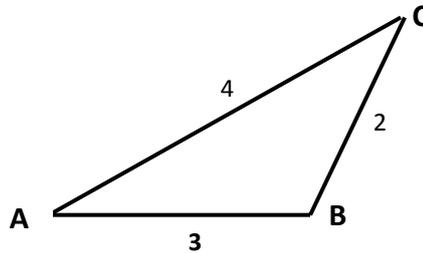
What is the area of a triangle where every side is 5 cm long?

$$\text{Step 1: } s = \frac{5+5+5}{2} = 7.5$$

$$\begin{aligned}\text{Step 2: } A &= \sqrt{7.5 \times 2.5 \times 2.5 \times 2.5} \\ &= \sqrt{(117.1875)} = 10.825\end{aligned}$$

Therefore the area is **11 cm²** (to the nearest unit)

Example 2 Use Heron's Formula to find the area of triangle ABC,



If $AB = 3$, $BC = 2$ and $CA = 4$.

Solution

Step 1: Calculate the semi perimeter, S

$$S = \frac{(3+4+2)}{2}$$

$$S = \frac{9}{2} = 4.5$$

Step 2: Substitute S into the formula. Round answer to the nearest tenth.

$$A = \sqrt{4.5(4.5-2)(4.5-3)(4.5-4)}$$

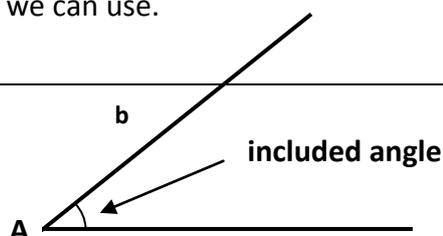
$$A = \sqrt{4.5(2.4)(1.5)(0.5)}$$

$$A = \sqrt{8.4375}$$

$$A = 2.9 \text{ units}^2$$

Knowing Two Sides and the Included Angle

When we know two sides and the included angle (SAS), there is another formula (in fact, three equivalent formulas) we can use.





Depending on which sides and angles we know, the formula can be written in three ways:

$$\text{Either: Area} = \frac{1}{2}ab\sin C$$

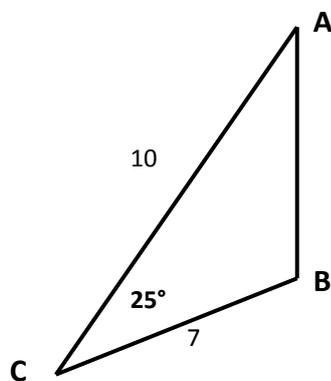
$$\text{Or: Area} = \frac{1}{2}bc\sin A$$

$$\text{Or: Area} = \frac{1}{2}ac\sin B$$

They are really the same formula, just with the sides and angle changed.

Example

Find the area of this triangle:



First let us identify the given information we can use.

We know angle $C = 25^\circ$, and sides $a = 7$ and $b = 10$.

So let us get going.

$$\text{Start with: Area} = \frac{1}{2}ab\sin C$$



Put in the values we know: $\text{Area} = \frac{1}{2} \times 7 \times 10 \times \sin(25^\circ)$

Do some calculator work: $\text{Area} = 35 \times 0.4226\dots$

$\text{Area} = \mathbf{14.8}$ to one decimal place

How to Remember

Just think "abc": $\text{Area} = \frac{1}{2} ab \sin C$

How does it work?

Well, we know that we can find an area when we know a base and height.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

In this triangle:

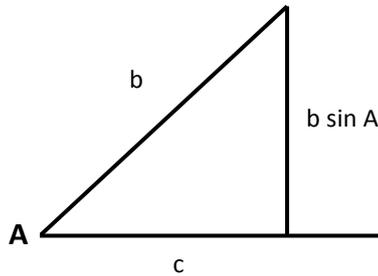
- The base is c
- The height is $b \sin A$

Putting that together gets us.

$$\text{Area} = \frac{1}{2} \times (c) \times (b \times \sin A)$$

This is (more simply)

$$\text{Area} = \frac{1}{2} bc \sin A$$



By changing the labels on the triangle we can also get

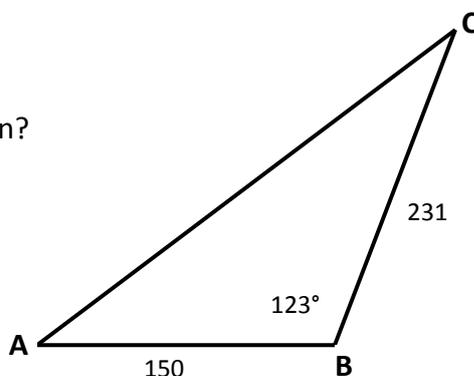
- $\text{Area} = \frac{1}{2} ab \sin C$
- $\text{Area} = \frac{1}{2} ca \sin B$



Example: Find how much land.

Farmer owns a triangular piece of land. The length of the fence AB is 150 m. The length of the fence BC is 231 m. The angle between fence AB and fence BC is 123° .

How much land does farmer own?



Solution:

First of all we must decide which lengths and angles we know.

- $AB = c = 150$ m,
- $BC = a = 231$ m,
- and angle $B = 123^\circ$

So we use:

$$\text{Area} = \frac{1}{2} ca \sin B$$

Start with: $\text{Area} = \frac{1}{2} ca \sin B$

$$\text{Put in the values we know: Area} = \frac{1}{2} \times 150 \times 231 \times \sin(123^\circ) \text{ m}^2$$

$$\text{Do some calculator work: Area} = 17,325 \times 0.838... \text{ m}^2$$

$$\text{Area} = 14,530 \text{ m}^2$$

Farmer has **14,530 m²** of land



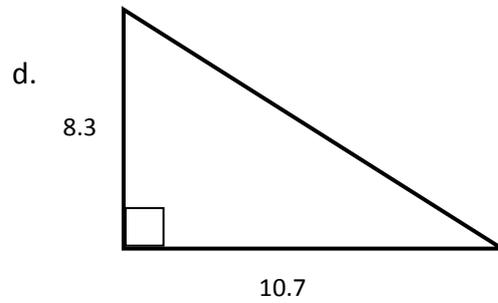
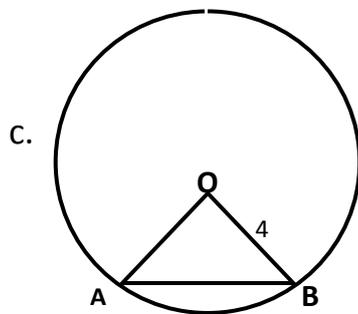
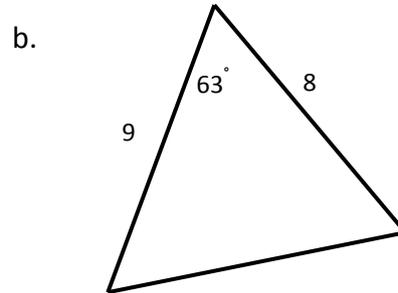
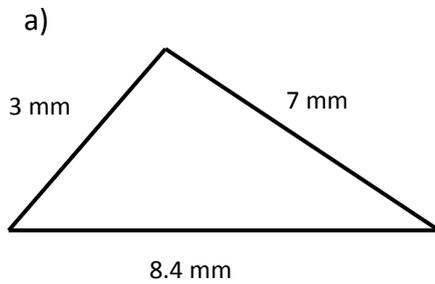
Learning Activity 11.5.3.4



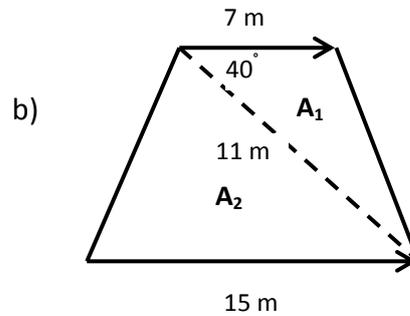
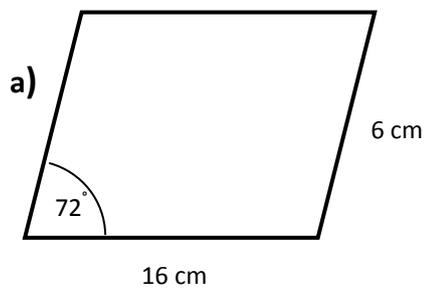
20 minutes



1. Find the area of each triangle correct to one decimal place.

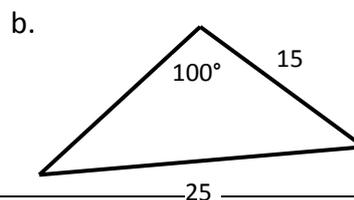
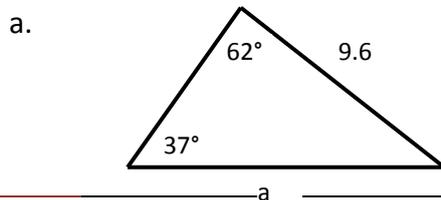


2. Find the area of each shape below.



SUMMATIVE TASK 3: Solution of Oblique Triangles

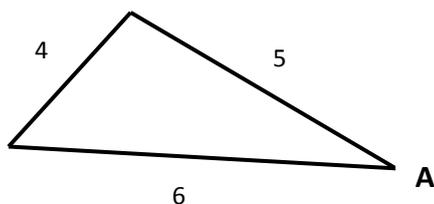
1. Use sine rule to solve the missing side and angle.



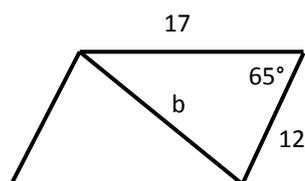


2. Use cosine rule to solve the missing side and angle.

a.



b.



3.

a) In triangle LMN, $\angle M = 84^\circ$, $LM = 7\text{ m}$ and $MN = 9\text{ m}$. Find LN.

b) In triangle PQR, $\angle Q = 72^\circ$, $\angle R = 32^\circ$ and $PR = 12\text{ cm}$. Find PQ.

c) In triangle PQR, $p = 8$, $q = 14$ and $r = 7$. Find $\angle Q$.

4. Solve the following problems. Start each question by drawing a large, clear diagram.

a) Two honeybees A and B leave the hive H at the same time. Honeybee A flies 27 m due South and B flies 9 m on a bearing of 111° . How far apart are they?



b) From A, B lies 11 km away on a bearing of 041° and C lies 8 km away on a bearing of 341° . Find

i. The distance between B and C.

ii. The bearing of B from C.

5. A ship sails 35 km on a bearing of 042° .

a. How far north has it travelled?

b. How far east has it travelled?

11.5.4 MAPS, CONTOURS AND VECTORS

A **scale drawing** is an enlarged or reduced drawing of an object that is similar to an actual object. Maps and floor plans are smaller than the actual size. A scale drawing of a human cell is larger than the actual size.

On a map, the equal sign in 1 inch = 30 mile does not mean that the two quantities are equal, as it would in an equation.

A scale is the ratio that compares a length in a drawing to the corresponding length in the actual object. If a 30-mile road is 1 inch long on a map, you can write the scale of the map in different ways.

There is a special map called a **contour map**. Contour means shape.

A contour map shows the true shape of the land. It also shows elevations and changes in elevations. A contour map shows shape and elevation by means of contour lines. A contour line is a line that connects points that are at the same elevation.

In the physical world, some quantities, such as mass, length, age, and value, can be represented only by magnitude. Other quantities, such as speed and force, also involve



direction. You can use **vectors** to represent those quantities that involve both magnitude and direction. One common use of vectors involves finding the actual speed and direction of an aircraft given its air speed and direction and the speed and direction of a tailwind. Another common use of vectors involves finding the resulting force on an object being acted upon by several separate forces.

11.5.4.1. MAP SCALES

A map cannot be of the same size as the area it represents. So, the measurements are **scaled down** to make the map of a size that can be conveniently used by users such as motorists, cyclists and bushwalkers. A scale drawing of a building (or bridge) has the same shape as the real building (or bridge) that it represents but of a different size. Builders use scaled drawings to make buildings and bridges.

A ratio is used in scale drawings of maps and buildings that is,

$$\text{The scale of a drawing} = \text{drawing length} : \text{Actual length}$$

Likewise, we have:

$$\text{Map scale} = \text{Map distance} : \text{Actual distance}$$

A scale is usually expressed in one of two ways:

- **As a ratio**, that is without explicitly mentioning units as in 1: 100 000 or $\frac{1}{100\,000}$.

A scale of 1: 100 000 means that the real distance is 100 000 times the length of 1 unit on the map or drawing.

No units are involved when scale is expressed in this way. Any distance measured off the map represents 100 000 times this distance on the ground.

- **As a scale**, using units as in 1 cm to 1 km . This means that 1 cm on the map represents an actual horizontal distance of 1 kilometre.

Example 1 Write the scale 1 cm to 1 m in ratio form.

Solution

$$\begin{aligned} 1 \text{ cm to } 1 \text{ m} &= 1 \text{ cm} : 100 \text{ cm} \\ &= 1 : 100 \end{aligned}$$

Example 2 Simplify the scale 5 mm : 1 m

$$\begin{aligned} 5 \text{ mm} : 1 \text{ m} &= 5 \text{ mm} : 100 \text{ cm} \\ &= 5 \text{ mm} : 1000 \text{ mm} \\ &= 5 : 1000 \\ &= 1 : 200 \end{aligned}$$

**Solution****Calculating the Actual Distance using the Scale**

If the scale is 1 : x , then **MULTIPLY** the map distance by x to calculate the actual distance.

Example

A particular map shows a scale of 1 : 5000. What is the actual distance if the map distance is 8 cm?

Solution

Conversion:
100 cm = 1 m

Given: Scale: 1 : 5000 = 1 cm : 5000 cm

Map distance = 8 cm

Actual distance = 8 x 5000 = 40 000 cm

$$\begin{aligned}\text{Actual distance} &= \frac{40000}{100} \\ &= 40\,000 \text{ cm} \times \frac{1\text{m}}{100 \text{ cm}} \\ &= 400 \text{ m}\end{aligned}$$

Alternative Solution:

Map distance = 8 cm

Scale: 1 : 5000 = 1 cm : 5000 cm

Map distance : Actual distance

1 : 5000

1 x 8 : 5000 x 8

8 : 40 000

∴ Actual distance = 40 000



Finding the map distance

If the scale is 1 : x , then **DIVIDE** the map distance by x to calculate the actual distance.

Example

A particular map shows a scale of 1 cm : 5 km. What would the map distance (in cm) be if the actual distance is 14 km?

Solution:

Scale : 1 cm : 5 km

Map distance: Actual Distance = 1 cm : 5 km

Actual distance = 14 km

Let the map distance = x cm

$$\frac{x \text{ cm}}{14 \text{ km}} = \frac{1 \text{ cm}}{5 \text{ km}}$$

$$5x = 14$$

$$\frac{5x}{5} = \frac{14}{5}$$

$$x = 2.8 \text{ cm}$$

Alternative Way:

Scale is 1cm : 5 km

∴ Scale factor = 5 km

Actual distance = 14 km

$$\text{Map distance} = \frac{\text{actual distance}}{\text{scale factor}} = \frac{14}{5}$$

So the map distance is 2.8 cm



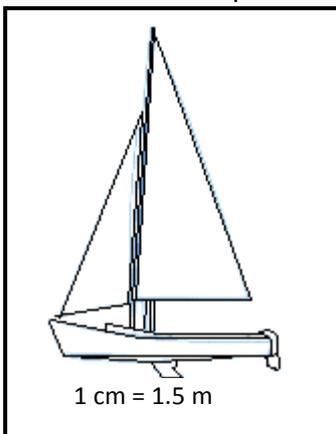
Applications to real world

Example 1

You have a scale drawing of a boat. The length of the boat on the drawing is 3 cm. What is the actual length of the boat?

Write the scale of the drawing, 1 cm = 1.5 m. Then write a proportion in which each ratio compares centimetres to metres.

Solution: Let n represent the actual length of the boat.



$$\frac{\text{drawing(cm)} \rightarrow 1}{\text{actual (km)} \rightarrow 1.5} = \frac{3 \rightarrow \text{drawing(cm)}}{n \rightarrow \text{actual (m)}}$$

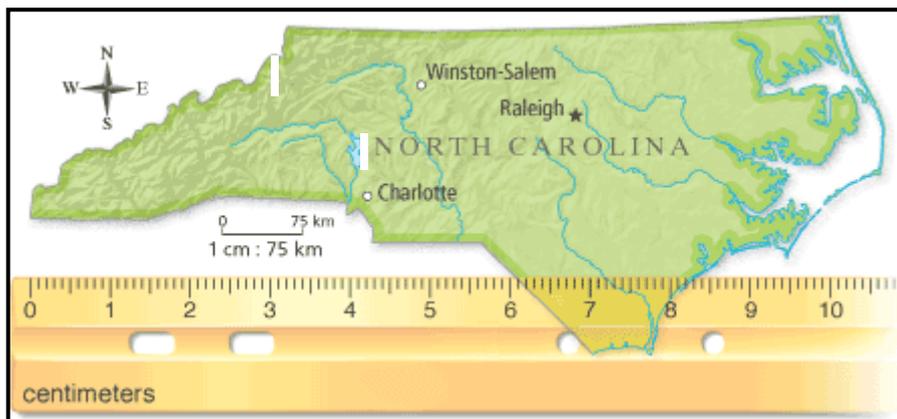
$$1(n) = 1.5(3) \quad \text{Write to cross products}$$

$$n = 4.5 \quad \text{Simplify}$$

The actual length of the boat is 4.5 m.

You can use proportions and a map's scale to find actual distances.

Example 2 Geography



Find the actual distance from Charlotte to Winston-Salem.

Solution

1. Use a centimetre ruler to find the map distance from Charlotte to Winston-Salem. The map distance is about 1.6 cm.



2. Use a proportion to find the actual distance. Let n represent the actual distance.

$$\frac{\text{map(cm)} \rightarrow 1}{\text{actua(km)} \rightarrow 75} = \frac{1.6 \leftarrow \text{map}}{n \leftarrow \text{actua(m)}}$$

$$\frac{1}{75} = \frac{1.6}{n}$$

$$1(n) = 75(1.6) \quad \text{Write cross products}$$

$$n = 120 \quad \text{Simplify}$$

The actual distance from Charlotte to Winston-Salem is about 120 km.



Learning Activity 11.5.4.1



20 minutes

1. Express as a simple ratio scale :
- 16 mm to 4 cm
 - 0.5 km to 1 cm
2. Two cities known to be 20 km apart measure 5 centimetres apart on a map. What is the ratio scale of the map?
3. The scale of a map is 1: 250 000. What actual distance does 7 cm on the map represent?
4. The scale of a map is 1 cm: 3.6 km. Find the actual distance for each map distance. Round your answer to the nearest tenth, if necessary.
- 12 cm
 - 2 cm
 - 28 mm



c. 18 cm

d. 52 mm

5. A map is drawn to a scale of 1 to 50 000. Calculate the length of a road which appears as 3 cm long on the map.

11.5.4.2. Contours

There are many kinds of maps. The maps we use most often are surface maps. A surface map is a drawing that shows all or a part of the earth's surface. The main problem with a surface map is that it is flat. The earth's surface is not at all flat. It has hills, mountains, plateaus, valleys and oceans. We cannot see those on a surface map.

The different parts of the earth's surface are at different heights or **elevations**. The elevation of the ocean is 0. This is called sea level.





The rest of the earth's surface is measured from sea level. This means that all elevations are measured in feet or meters above or below sea level.



The question is, how can a flat map show elevations? Some maps use colors. Some use shading. However, such maps give only a general idea about the surface.

There is a special map called a **contour map**. Contour means **shape**. A contour map shows the true shape of the land. It also shows elevations and changes in elevations.

A contour map shows shape and elevation by means of contour lines. A **contour line** is a line that connects points that are at the same elevation.

Every point on a contour line is at the same elevation

Several contour lines make up a contour map.

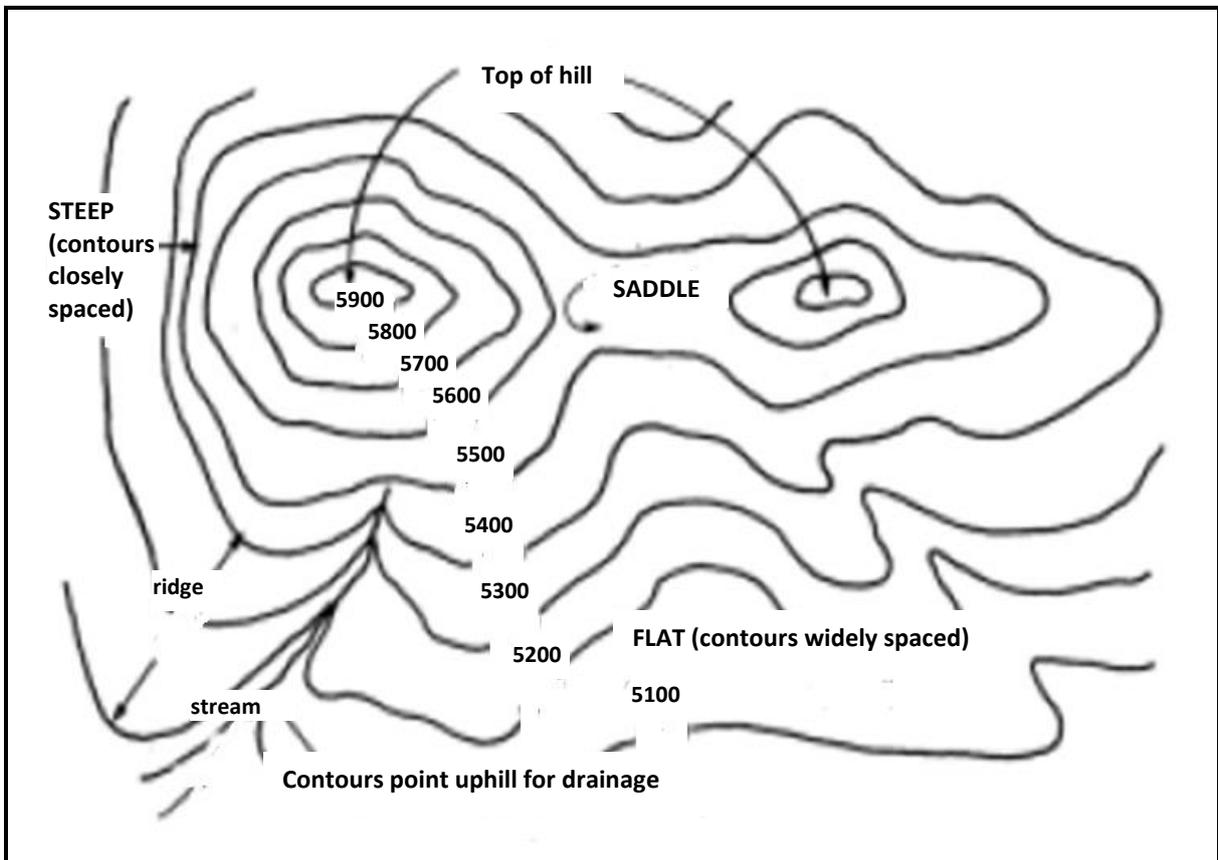
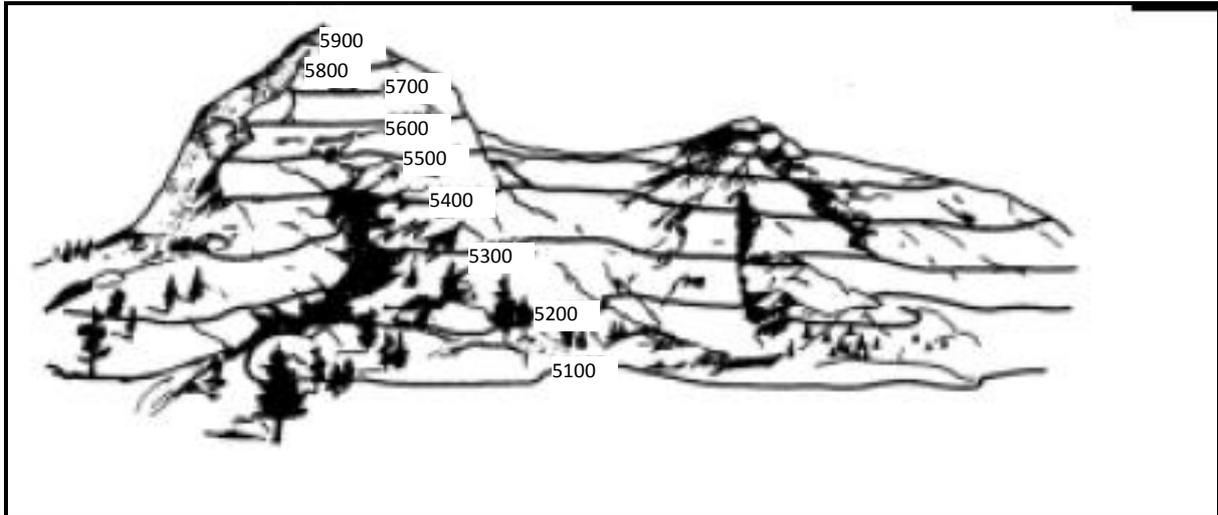
The difference in elevation between two neighboring contour lines is called the **contour interval** of the map.

Contour Lines and Intervals

A **contour line** is a line drawn on a topographic map to indicate ground elevation or depression.

A **contour interval** is the vertical distance or difference in elevation between contour lines. Index contours are bold or thicker lines that appear at every fifth contour line.

The figure below illustrates various topographic features. Notice how a mountain saddle, a ridge, a stream, a steep area, and a flat area are shown with contour lines.



If the numbers associated with specific contour lines are **increasing**, the **elevation** of the terrain is also **increasing**. If the numbers associated with the contour lines are **decreasing**, there is a **decrease in elevation**.

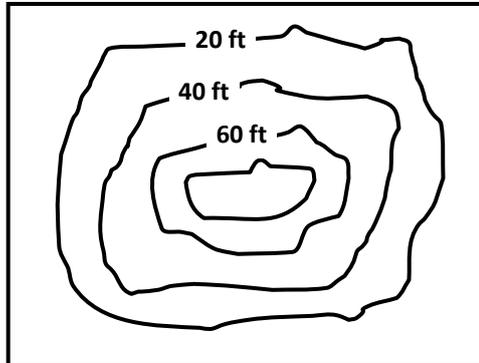
Contour lines can show how much the land slopes.

- Contour lines that are **far apart** indicate that the land is fairly flat, or has a **gentle** slope.
- Contour lines that are **close together** show that the land is hilly, or has a **steep** slope.



Example

In the graphic below, what is the vertical distance between the contour lines?



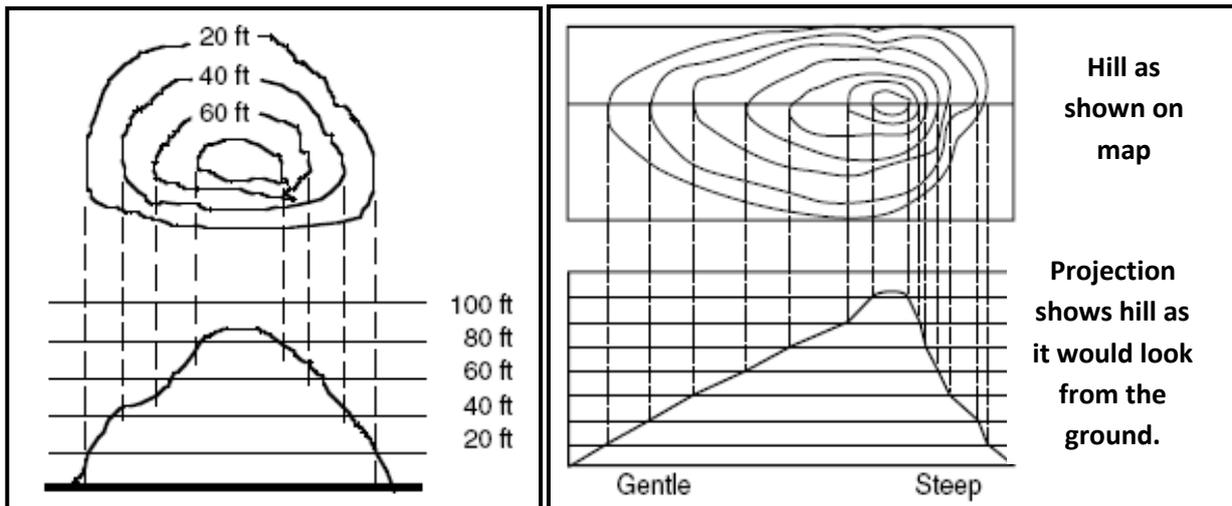
Solution

Pick two contour lines that are next to each other and find the difference in associated numbers. $40 \text{ feet} - 20 \text{ feet} = 20 \text{ feet}$

The contour lines in this figure are equally spaced. The even spacing indicates the hill has a uniform slope. From the contour map, a profile can be drawn of the terrain.

Example

Draw a profile showing the elevations of the contours.



Solution

The intervals are increasing; therefore, the contours indicate a hill. The peak is normally considered to be located at half the interval distance.



Widely separated contour lines indicate a gentle slope. Contour lines that are very close together indicate a steep slope.

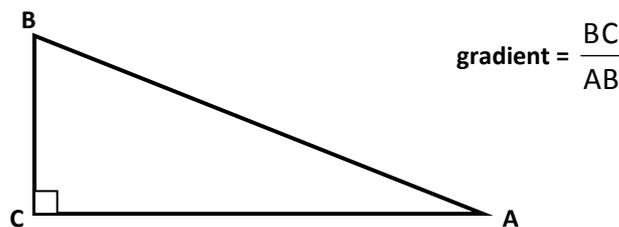
Gradients /Slope

Contour maps also have a horizontal map scale so that you can find the horizontal distance between two points. This makes it possible to calculate the **AVERAGE SLOPE** between two points on a contour map.

1. Measure the distance between the two points.
2. Use contour lines to find the height difference between the two points.
3. Now just use the formula for slope that you learned in class!

$$\text{Slope formula : } m = \frac{y_2 - y_1}{x_2 - x_1}$$

When used in connection with roads and railways the gradient is used to denote the ratio of the vertical distance to the corresponding distance measured along the slope.



The gradient is usually expressed as ratio. If a road has a gradient of 1 in 20, we mean that for every 20 m along the slope the road rises 1 m vertically.

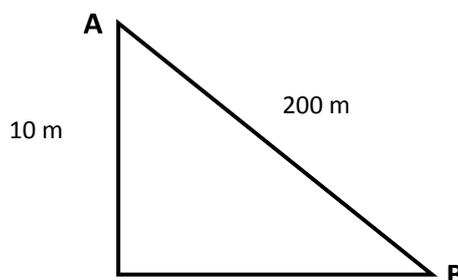
Example 1

Two points P and Q are linked by a straight road with a uniform gradient. If A is 10 m higher than B and the distance AB measured along the road is 200 m, what is the gradient of the road?

Solution

$$\text{Gradient} = \frac{10}{200} = \frac{1}{20}$$

The gradient of the road is 1 in 20.

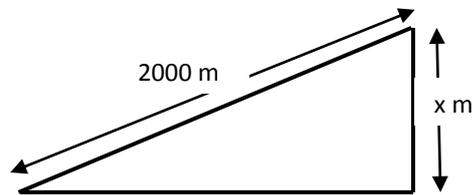


Example 2



A man walks 2 km up a road whose gradient is 1 in 10. How much higher is he than when he started?

Solution



$$\text{gradient of road} = \frac{x}{2000}$$

$$\frac{x}{2000} = \frac{1}{10}$$

$$x = \frac{2000}{10} = 200 \text{ m}$$

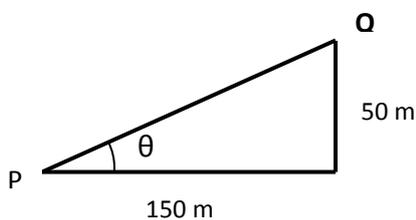
The man is 200 m higher when he started.

Example 3

Two points P and Q whose heights differ by 50 m are shown 0.60 cm apart on a map whose scale is 1 : 25 000. Calculate the angle of elevation of the line PQ.

Solution

0.60 cm on the map represents $0.60 \times 25\,000$ cm on the ground = 150 m



To find angle of elevation we use tan ratio

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$



$$\tan \theta = \frac{50}{150} = 0.3333$$

$$\theta = 18.42^\circ = 18^\circ 26'$$

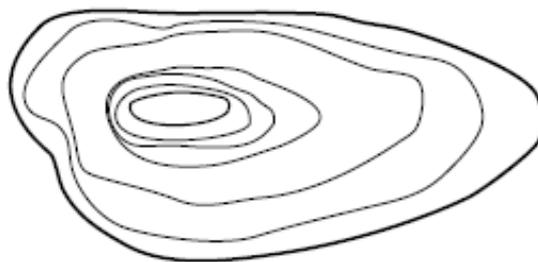


Learning Activity 11.5.4.2

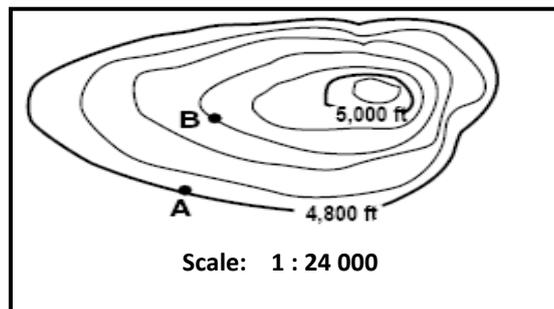


20 minutes

1. Is the figure shown below steeper at the top of the hill or the bottom?



2. On this topographic map, the contour interval is not indicated. What contour interval can you determine based on the available information?



3. Two points P and Q whose heights differ by 50 m are connected by a straight road 2 km long. What is the gradient of the road?

4. A man walks up a road whose gradient is 1 in 30. If he walks a distance of 900 m how much higher is he than when he first started?



5. Two points A and B whose heights differ by 100 m are shown 1.2 cm apart on a map whose scale is 1: 10 000. Calculate the angle of elevation of the line AB.

11.5.4.3. Vector and Scalar Quantities

Definition of Vectors and Scalar Quantities

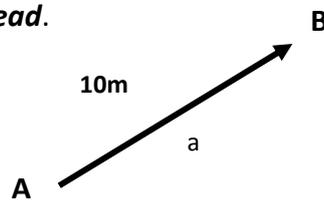
Vector quantities have both magnitude (size) and direction	Scalar quantities have only magnitude (size).
<ul style="list-style-type: none">• displacement• velocity• acceleration• force• weight• momentum	<ul style="list-style-type: none">• distance• speed• time• power• energy• area
You use vectors in almost every activity you do. Examples of everyday activities that involve vectors include: <ul style="list-style-type: none">• Breathing (Your diaphragm muscles exert a force that has a magnitude and direction.)• Walking (You walk at a velocity of around 6 km/h in the direction of the bathroom.)• Going to school (The bus has a length of about 20 m and is headed towards your school.)• Lunch (The displacement from your class room to the canteen is about 40 m in a northerly direction.)	<ul style="list-style-type: none">• 12 metres• 60 km/hr• 5 o'clock• 30 m²• 60 kilos• 3 kilometres• 20 years old
Vector quantities change when their <ul style="list-style-type: none">• magnitude changes• direction changes• magnitude and direction both change	Scalar quantities change when their magnitude changes.



--	--

Vector Notation

A vector is represented by a **directed line segment**. The direction of the vector is indicated by an arrow pointing from the **tail** to the **head**.



If the tail is at point A and the head is at point B , the vector from A to B is written \overrightarrow{AB} .

The **length** (magnitude) of a vector \mathbf{v} is written $|\mathbf{v}|$. The length is always a non-negative real number.

The vectors can be denoted by \overrightarrow{AB} or \mathbf{AB} or \underline{a} or \mathbf{a} .

A is called the **initial point** and B is called the **terminal point** of \overrightarrow{AB} .

The **magnitude** of a vector is the length of the corresponding segment. The magnitude of \overrightarrow{AB} is denoted by $|\overrightarrow{AB}|$.



Learning Activity 11.5.4.3



20 minutes

1. Identify if the following is a vector or a scalar quantity.

a. A speed of 35 km/h is a _____ quantity.

b. The weight of a 7 kg mass is a _____ quantity.

c. The train is going 80 km/hr towards Sydney is a _____ quantity.



- d. The force on the bridge is 50 N acting downwards is a _____ quantity.
- e. A temperature of 100°C is a _____ quantity.
- f. The sum of K300 is a _____ quantity.

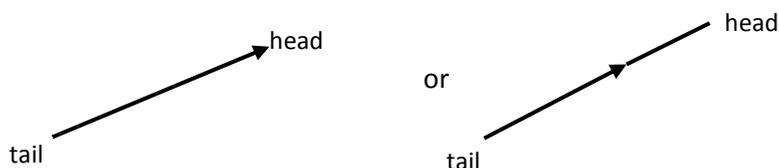
2. Draw the following vectors and label each correctly.

- a. \vec{AB} 5 m due North
- b. \vec{XY} 8 km due South
- c. \vec{PQ} 7 km due West
- d. \vec{MN} 10 meters due East
- e. \vec{RS} 6 meters South –East

11.6.4.4. Types of Vectors

Vectors as directed line segments

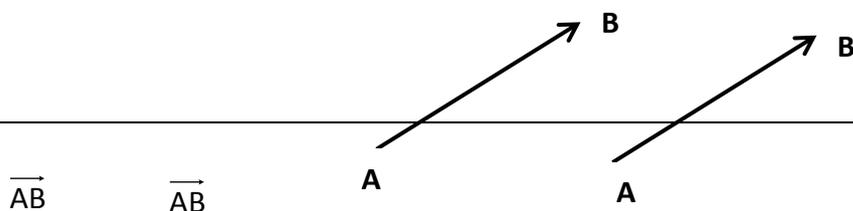
Vectors are represented in a natural way by directed line segments, the length of the segment measuring the magnitude of the vector and the arrowhead indicating the direction of the vector. The arrowhead is usually placed at one end (or near the end) of the line segment. This end is called the head of the vector and the other end is called the tail of the vector. Such vectors are called **free vectors** because they are distinguished only by their length and direction, and not by their position in space.



Equal Vectors

Two vectors are equal if they have the same magnitude and direction.

If two arrows are used to represent vectors, then equal vectors are parallel and equal in length.

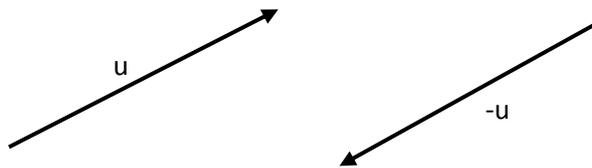




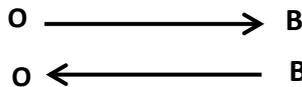
=

Negative Vectors

The vector with the same magnitude as \mathbf{u} but opposite direction, the **negative** of \mathbf{u} , is labelled $-\mathbf{u}$.



We note that the **magnitude** of each vector is the same, but they are acting in **opposite directions**. In such a case, we indicate the opposite directions by use of a **negative sign**.



So we write : $OA = -OB$

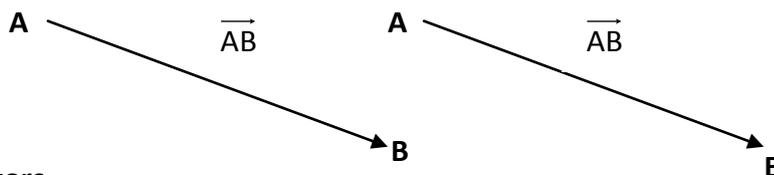
Zero Vectors

The zero vector (a vector with zero length and no direction) is written $\mathbf{0}$. A **zero vector** has magnitude of 0. A vector may have zero magnitude at an instance in time.

$$\mathbf{OA} + \mathbf{OB} = \mathbf{0}$$

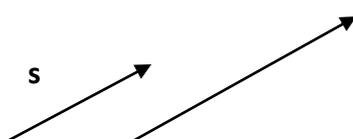
Position Vectors and Displacement Vectors

The line segment \overrightarrow{AB} , where A and B are points in space, is called the **position vector** of B relative to A . It is also known as the vector from A to B as well as the **displacement** of B from A .



Parallel Vectors

Two vectors \mathbf{u} and \mathbf{v} are said to be **parallel** if they have either the same direction or opposite direction. This means that each is a scalar multiple of the other: for some non-zero scalar s .





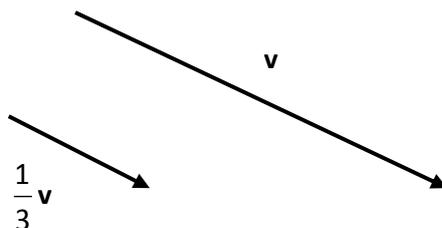
$$\mathbf{v} = s\mathbf{u} \text{ and so } \mathbf{u} = \frac{1}{s}\mathbf{v}.$$

Unit Vectors

It is often useful to be able to find a **unit vector** (a vector of length 1) in the direction of a given non-zero vector \mathbf{v} . A common notation used for this unit vector is $\hat{\mathbf{v}}$.

Example

If \mathbf{v} has length 3, then $\hat{\mathbf{v}} = \frac{1}{3}\mathbf{v}$ has length 1 and is in the same direction as \mathbf{v} .

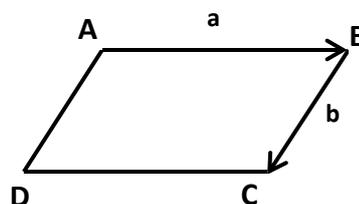


Learning Activity 11.5.4.4



20 minutes

1. ABCD is a parallelogram in which $\vec{AB} = \mathbf{a}$ and $\vec{BC} = \mathbf{b}$.

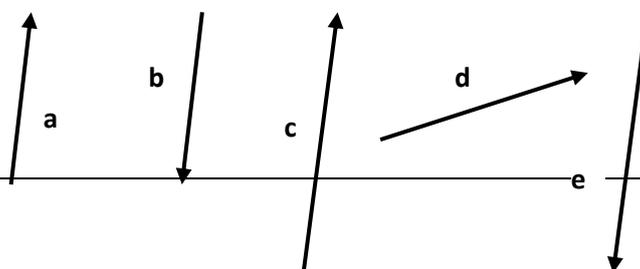


Find vector expressions for :

- a. \vec{BA} b. \vec{CB} c. \vec{AD} d. \vec{CD}

2. State the vectors which are:

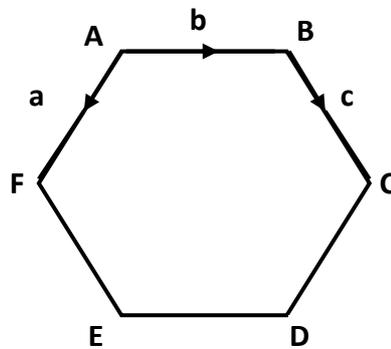
- a. Equal in magnitude
b. In the same direction





- c. Negatives of one another
- d. Parallel
- e. Equal

3. The figure below is a regular hexagon. If $AF = a$, $AB = b$ and $BC = c$, express each of the following in terms of a , b and c .



- a. DC
- b. DE
- c. FE
- d. FC
- e. AE
- f. AD

11.5.4.5. The Addition and Subtraction of Vectors

Because non-zero vectors have direction as well as magnitude, adding vectors involves more than simply adding numbers. The sum of two vectors is another vector, and so the definition of addition must give a process for determining both the magnitude and the direction of the sum vector.

Addition of Vectors

There are two equivalent procedures for addition of vectors, called the **parallelogram rule** and the **triangle rule**.

The parallelogram rule for addition

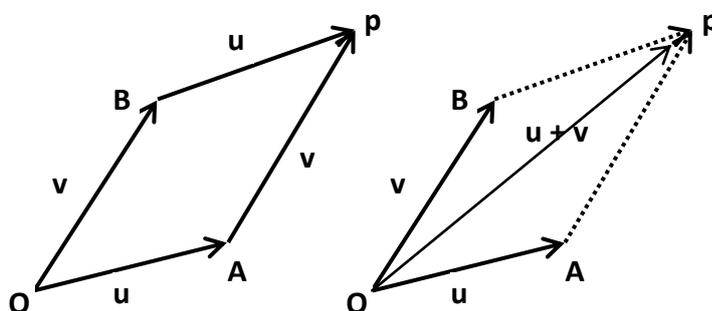
Suppose u and v are two vectors (in the plane or in space), translate them so that they are tail-to-tail at point O .





From the head of each vector, draw a copy of the other vector to complete a parallelogram $OAPB$.

In this parallelogram, $u = OA = BP$ and $v = OB = AP$



The vector $u + v$ is defined to be the vector OP .

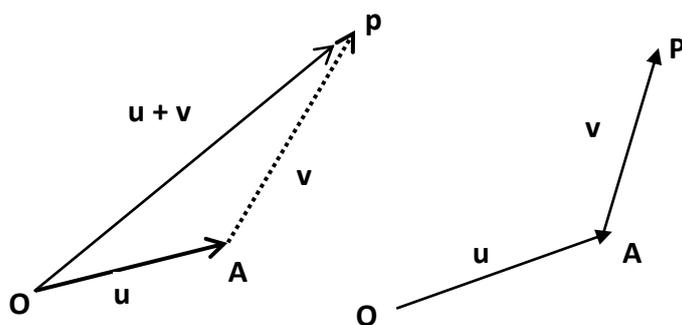
$$OP = u + v$$

The triangle rule for addition

Another way to define addition of two vectors is by a **head-to-tail** construction that creates two sides of a triangle. **The third side of the triangle determines the sum of the two vectors**, as shown below.

Place the tail of the vector v at the head of the vector u . That is, $u = OA$ and $v = AP$.

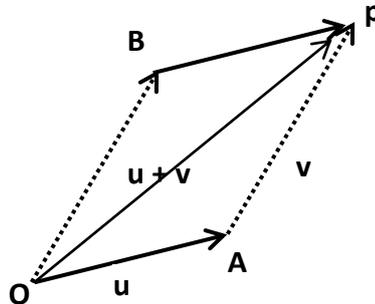
Now construct the vector OP to complete the third side of the triangle OAP .



The vector $u + v$ is defined to be the vector OP .



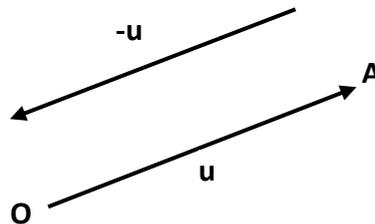
This method is equivalent to the parallelogram law of addition, as can be easily seen by drawing a copy of v tail-to-tail with u , to obtain the same parallelogram as before.



Both the triangle and the parallelogram rules of addition are procedures that are independent of the order of the vectors; that is, using either rule, it is always true that $u + v = v + u$ for all vectors u and v . This is known as the **commutative law of addition**. There are other rules like this one, and they are discussed in the component Vector Algebra.

Adding a vector to its negative

To add u to $-u$, place the tail of $-u = AO$ at the head of $u = OA$



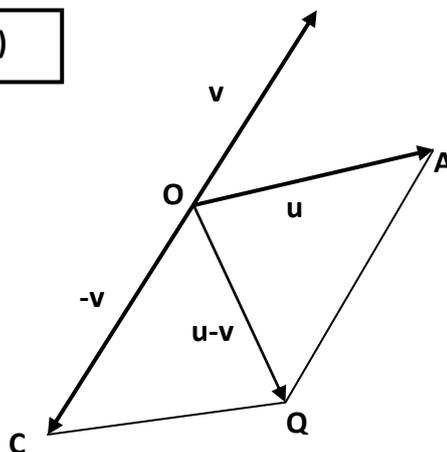
Then $u + (-u) = \mathbf{0}$, the zero vector. By the commutative law,

$u + (-u) = -u + u = \mathbf{0}$

Subtraction of Vectors

Subtraction of two vectors is just a special case of addition. The vector $u - v$ is defined to be

$u - v = u + (-v)$

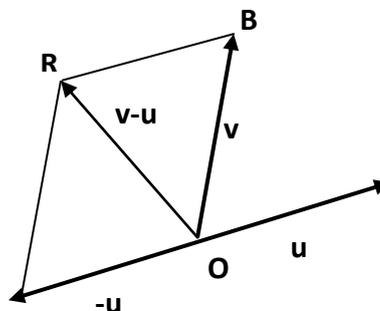




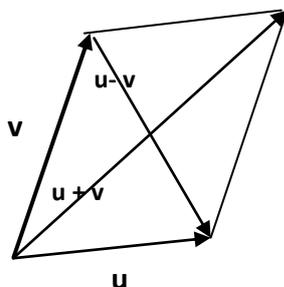
Using the parallelogram law of addition, $u - v = u + (-v) = OA + OC = OQ$

The vector $v - u$ can be found in the same way.

$$v - u = v + (-u) = OB + OD = OR$$



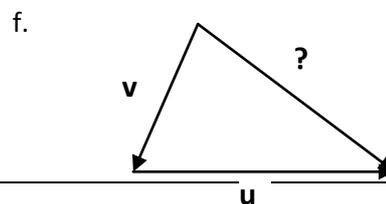
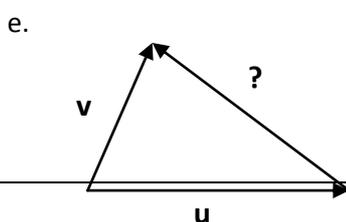
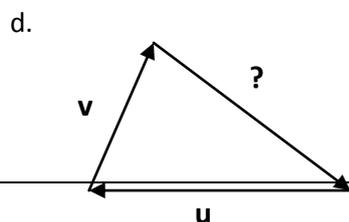
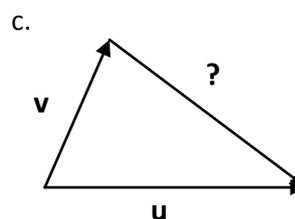
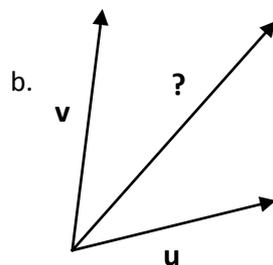
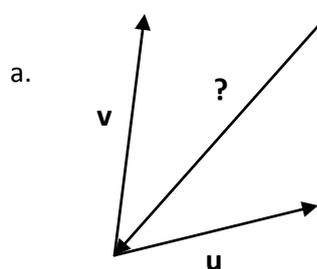
The vectors $u + v$ and $u - v$ are both diagonals of the parallelogram formed by u and v .



Adding the zero vector

Adding the zero vectors 0 to any vector v gives the vector v again. That is, $0 + v = v + 0 = v$ for all vectors v .

Example 1 Name the vectors with question mark (?)



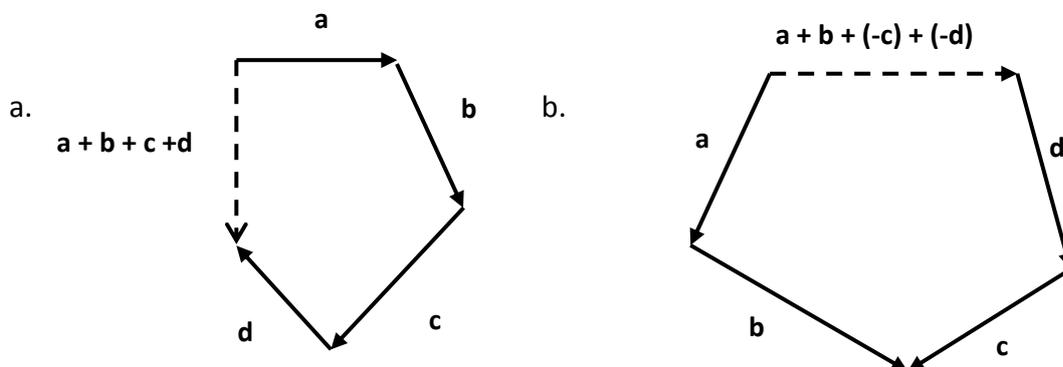


Solution:

The vectors are 1st row : a. $-u - v$ b. $u + v$ c. $-v + u$

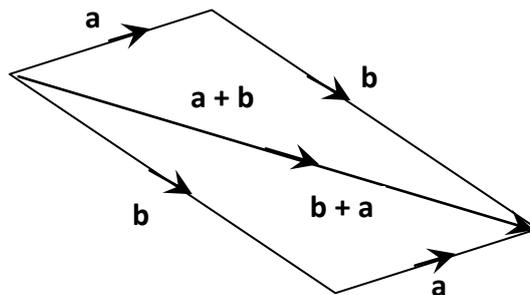
2nd row: d. $-v - u$ e. $-u + v$ f. $v + u$

Example 2 The addition of more than one vector is done in the same manner.



Vector Addition is Commutative

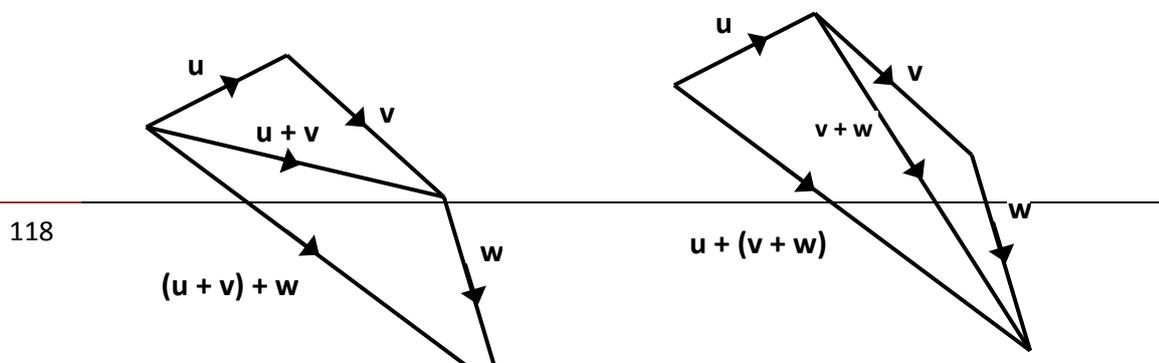
We will find that vector addition is commutative, that is $a + b = b + a$. This can be illustrated in the diagram below.



Vector Addition is Associative

We also find that vector addition is associative, that is $(u + v) + w = u + (v + w)$.

This can be illustrated in the following two diagrams. Notice that $(u + v) + w$ and $u + (v + w)$ have the same magnitude and direction and so they are equal.





Learning Activity 11.5.4.5



20 minutes

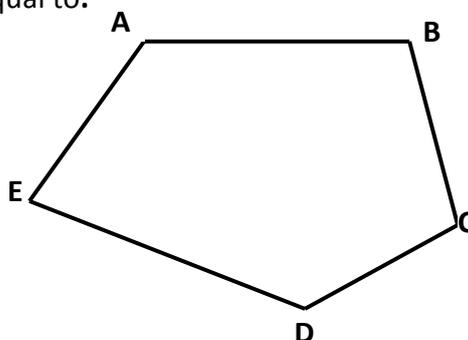
1. Find a single vector which is equal to.

a. $BC + CA =$

b. $BA + AE + EC =$

c. $AB + BC =$

d. $AB + BC + CD + DE =$

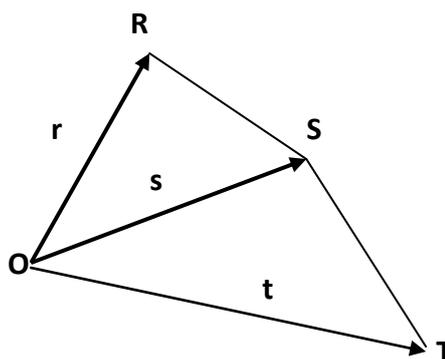


2. Find in terms of r, s and t .

a. $RS =$

b. $SR =$

c. $ST =$

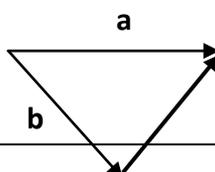


3. Find the highlighted vector.

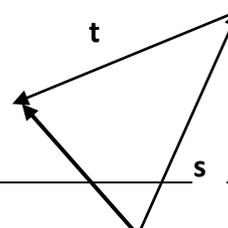
a.



b.



c.

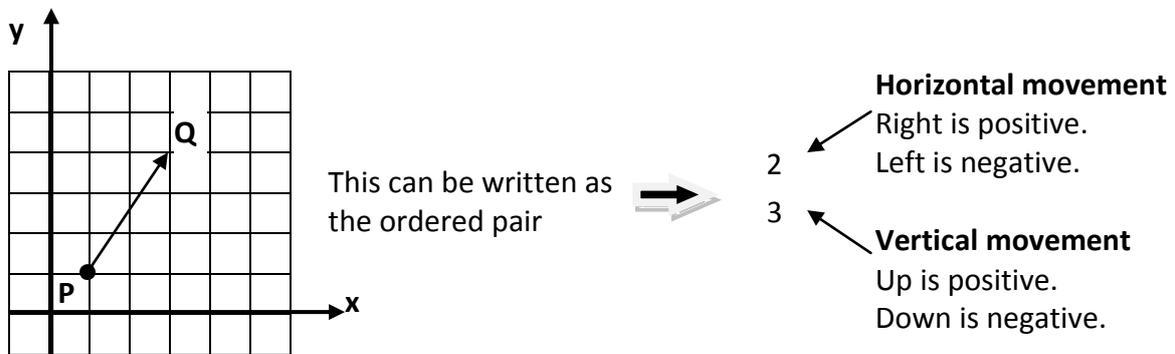




11.5.4.6. Addition and Subtraction of Column Vectors

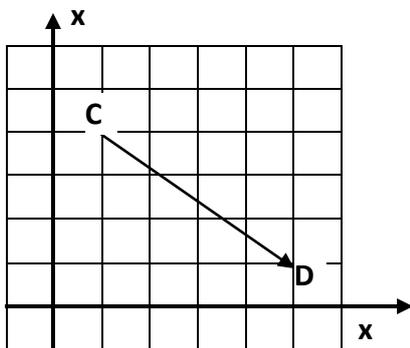
A vector can be written as an ordered pair called a **column vector**.

Consider the line PQ in the diagram. The line represents the translation of P to Q , which is 2 right and 3 up.



Example:

Express CD as a column vector.



Solution:

The translation of C to D is 4 right and 3 down.

$$CD = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

Equality Of Column Vectors

If two vectors are equal then their vector columns are equal.

Example:



The column vectors \mathbf{p} and \mathbf{q} are defined by $\mathbf{p} = \begin{pmatrix} 8-x \\ 6-y \end{pmatrix}$; $\mathbf{q} = \begin{pmatrix} x-4 \\ y+2 \end{pmatrix}$ given that $\mathbf{p} = \mathbf{q}$

a) Find the values of x and y .

b) Find the values of \mathbf{p} and \mathbf{q} .

Solution:

a. $\mathbf{p} = \mathbf{q}$

$$\mathbf{p} = \begin{pmatrix} 8-x \\ 6-y \end{pmatrix}; \quad \mathbf{q} = \begin{pmatrix} x-4 \\ y+2 \end{pmatrix}$$

$$8-x = x-4 \qquad 6-y = y+2$$

$$-2x = -12 \qquad -2y = -4$$

$$x = 6 \qquad y = 2$$

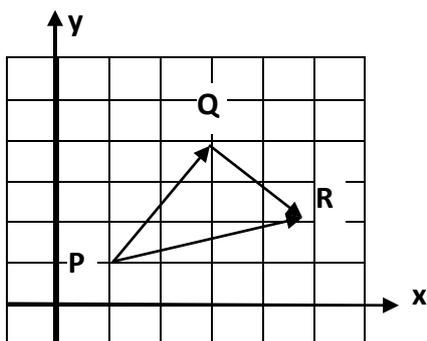
Using the values of $x = 6$ and $y = 2$

b. $\mathbf{p} = \begin{pmatrix} 8-x \\ 6-y \end{pmatrix} = \begin{pmatrix} 8-6 \\ 6-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \mathbf{p} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$$\mathbf{q} = \begin{pmatrix} x-4 \\ y+2 \end{pmatrix} = \begin{pmatrix} 6-4 \\ 2+2 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \quad \mathbf{q} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

Example 1

Given that $\mathbf{PQ} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{QR} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$, find the sum of the vectors.



**Solution:**

The sum of vectors **PQ** and **QR** is the same as the vector **PR**

Example , **PQ + QR = PR**

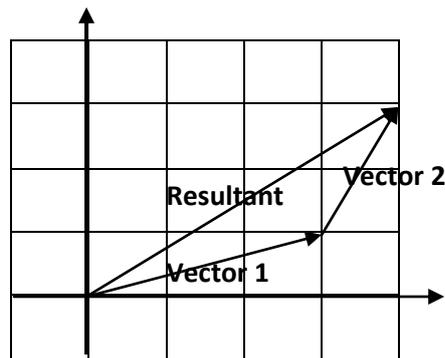
In column vector form, we **add** the corresponding components of the vectors

$$\mathbf{PQ} + \mathbf{QR} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \mathbf{PR}$$

Example 2

Find the sum of the of $\begin{pmatrix} 3 \\ -3 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$$\text{Solution } \begin{pmatrix} 3 \\ -3 \end{pmatrix} + \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

The Resultant Vector

The **resultant** vector is the vector that 'results' from adding two or more vectors together.

$$\text{Vector 1} + \text{Vector 2} = \text{Resultant Vector}$$

There are two different ways to calculate the resultant vector.

- The head to tail method to calculate a resultant which involves lining up the head of the one vector with the tail of the other.



- The parallelogram method to calculate resultant vector. This method involves properties of parallelograms, but in the end boils down to a simple formula.

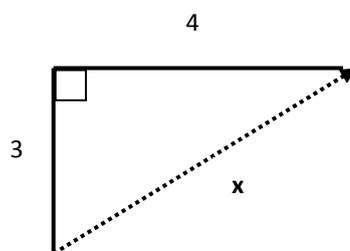
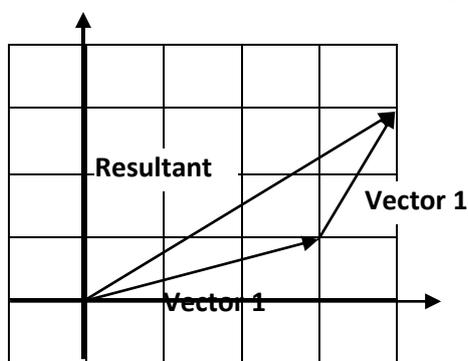
1. Head to Tail Method

The head to tail method is way to find the resultant vector. The steps are quite straight forward. The head to tail method considers the head of a vector to be the end with the arrow, or the 'pointy end'. The tail of the vector is where the vector begins.

Head to Tail Method to calculate the magnitude resultant vector

Find the sum of each pair of vectors (the magnitude of the resultant vector).

To find the resultant vector's magnitude, use the Pythagorean Theorem.



$$c^2 = a^2 + b^2$$

$$x^2 = 3^2 + 4^2$$

$$x^2 = 9 + 16$$

$$x = \sqrt{25}$$

$$x = 5$$

2. Parallelogram Method to Calculate Resultant

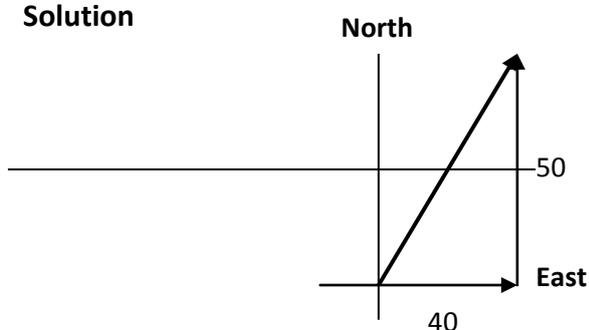
Before tackling the parallelogram method for solving resultant vectors, you should be comfortable with the following topics

- SOHCAHTOA (basic sine, cosine, tangent)
- Law of cosines
- Law of sines
- The following properties of parallelograms
 - Opposite sides of parallelograms are congruent
 - Opposite angles of parallelograms are congruent

Example 1

You left your house to visit a friend. You got in your car drove 40 miles east, then got on a highway and went 50 miles north. Draw a vector from the beginning of your journey, your home, and the end, your friend's house.

Solution





$$c^2 = a^2 + b^2$$

$$x^2 = 40^2 + 50^2$$

$$x^2 = 1600 + 2500$$

$$x = \sqrt{4100}$$

$$x = 64$$

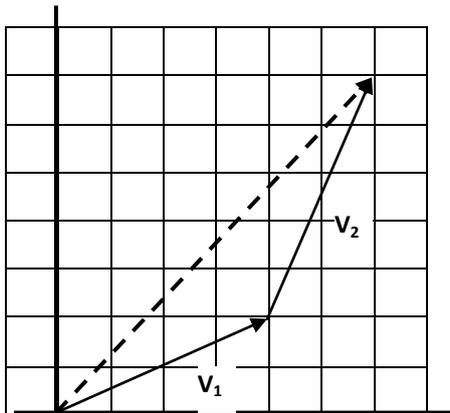
Draw Resultant

Example 2

What is the sum of the two vectors? Use the head to tail method to calculate the resultant vector in the picture on the right

Solution

Draw Resultant Vector



$$c^2 = a^2 + b^2$$

$$x^2 = 7^2 + 6^2$$

$$x^2 = 49 + 36$$

$$x = \sqrt{84}$$

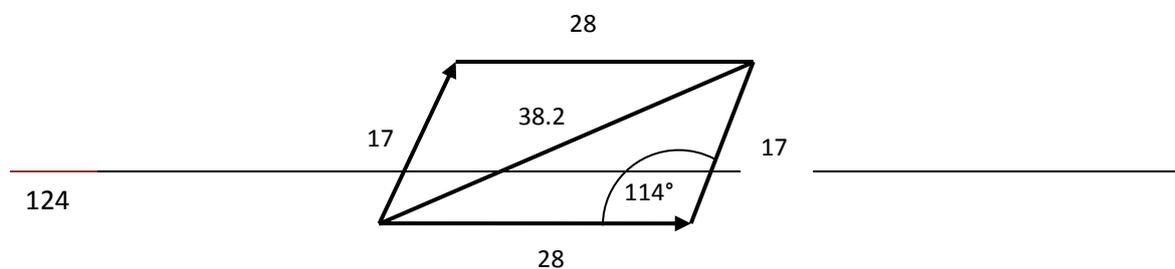
$$x = 9.2$$

Example 3

Find the magnitude of the resultant of the vectors 17 and 28 respectively. The angle between them is 66° .

Solution:

To best understand how the parallelogram method works, let us examine the two vectors below. The vectors have magnitudes of 17 and 28 and the angle between them is 66° . Our goal is to use the parallelogram method to determine the magnitude of the resultant.



**Step 1**

Draw a parallelogram based on the two vectors that you already have. These vectors will be two sides of the parallelogram (not the opposite sides since they have the angle between them).

Step 2

We now have a parallelogram and know two angles (opposite angles of parallelograms are congruent). We can also figure out the other pair of angles since the other pair is congruent and all four angles must add up to 360° .

Step 3

Draw the parallelogram's diagonal. This diagonal is **the resultant vector**.

Use the law of cosines to determine the length of the resultant.

Use the law of cosines to calculate the resultant.

$$c^2 = a^2 + b^2 - 2(a)(b)\cos C$$

$$x^2 = 17^2 + 28^2 - 2(17)(28)\cos(114)$$

$$x^2 = 1460.213284$$

$$x = \sqrt{1460.213284}$$

$$x = 38.2$$

Finding the direction of the resultant vector

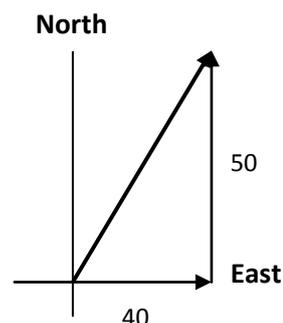
To find the direction of a resultant vector, we use the basic trigonometric ratios (sin, cos and tan) and the sine and cosine laws.

Example

Find the direction of the following resultant vectors.

Solution

We use Soh Cah Toa because the triangle is right.





$$\text{Using } \tan A = \frac{\text{opp}}{\text{adj}} = \frac{50}{40} = 1.25$$

To find $\angle A$, use $\tan^{-1} 1.25 = \angle A$

$$\angle A = 51^\circ$$

But bearing is from North direction, so the direction is

$$90^\circ - 51^\circ = 39^\circ.$$



Learning Activity 11.5.4.6



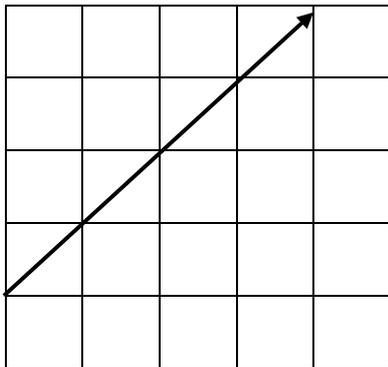
20 minutes

1. Draw arrow diagrams to represent the vectors

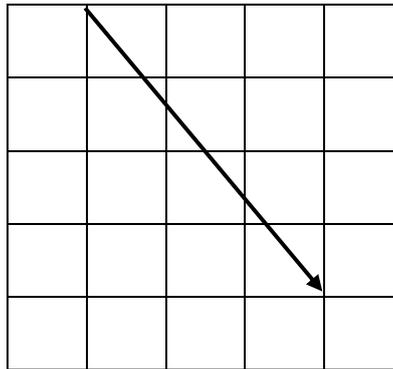
a. $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ b. $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ c. $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ d. $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ e. $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$

2. a. Write the illustrated vector in component form:

a.



b.



4. If $a = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $b = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$, $c = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$

Find: a. $a + b$

b. $b + c$

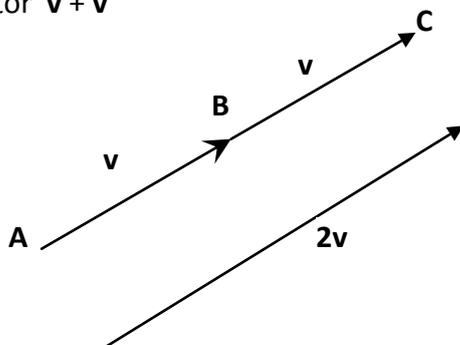
c. $a + c$

d. $a + a$

e. $a + b + c$

**11.5.4.7. Multiplying by a Scalar**

A scalar quantity has a magnitude but no direction. (e.g. mass, volume, temperature). Ordinary numbers are scalars. Using a head-to-tail procedure, a vector \mathbf{v} can be added to itself to give the vector $\mathbf{v} + \mathbf{v}$

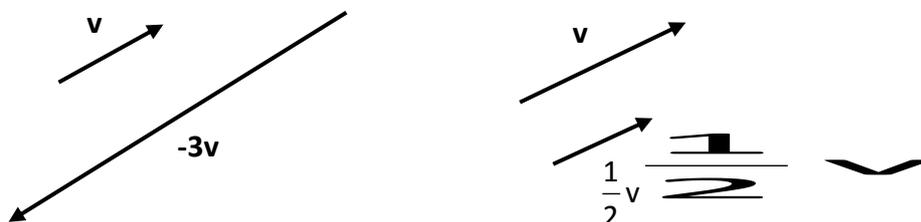


We have $\mathbf{v} = \mathbf{AB}$ and $\mathbf{v} = \mathbf{BC}$. It is natural to write $\mathbf{AB} + \mathbf{BC} = \mathbf{AC}$ as $2\mathbf{v}$.

Example 1

$-3\mathbf{v}$ has three times the magnitude of \mathbf{v} but points in the opposite direction;

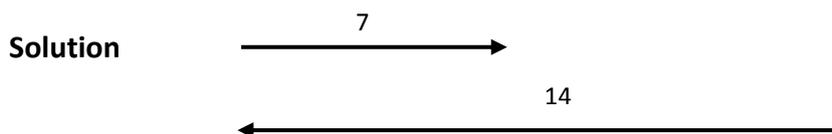
$\frac{1}{2}\mathbf{v}$ has the same direction as \mathbf{v} .



Note that $-1\mathbf{v}$ has the same magnitude as \mathbf{v} but has the opposite direction, and so is the same vector as the negative of \mathbf{v} , that is, $-1\mathbf{v} = -\mathbf{v}$.

Example 2

If \mathbf{v} is a vector of length 7, what is the length of the vector $-2\mathbf{v}$?



The vector $-2\mathbf{v}$ is twice the length of \mathbf{v} but points in the opposite direction. Therefore, its length is 14.



In summary:

When you multiply a vector by a scalar, the result is a vector.

Geometrically speaking, scalar multiplication achieves the following:

- Scalar multiplication by a positive number other than 1 changes the magnitude of the vector but not its direction.
- Scalar multiplication by -1 reverses its direction but does not change its magnitude.
- Scalar multiplication by any other negative number both reverses the direction of the vector and changes its magnitude.

Scalar multiplication can change the magnitude of a vector by either increasing it or decreasing it.

- Scalar multiplication by a number greater than 1 or less than -1 *increases* the magnitude of the vector.
- Scalar multiplication by a fraction between -1 and 1 *decreases* the magnitude of the vector.

Scalar multiplication of a vector changes its magnitude and/or its direction.

For example, the vector $2\mathbf{p}$ is twice as long as \mathbf{p} , the vector $\frac{1}{2}\mathbf{p}$ is half as long as \mathbf{p} , and the vector $-\mathbf{p}$ is the same length as \mathbf{p} but extends in the opposite direction from the origin.

Multiplication of column vectors by a scalar

If k is a scalar, then $k\mathbf{a} = \begin{pmatrix} k a_1 \\ k a_2 \end{pmatrix}$

Example 1

$$\text{If } \mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \text{then } 2\mathbf{a} = 2 \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$$

Each component is multiplied by the number 2.

$$\text{If } \mathbf{b} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \quad \text{then } \frac{1}{2}\mathbf{b} = \frac{1}{2} \times \begin{pmatrix} -4 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$



Each component is multiplied by the number $\frac{1}{2}$

Example 2

For $p = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ and $q = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

find a. $3q$ b. $p + 3q$ c. $\frac{1}{2}p - 3q$

Solution

$$\text{a. } 3q = 3 \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 9 \end{pmatrix}$$

$$\text{b. } p + 3q = \begin{pmatrix} 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ 9 \end{pmatrix} = \begin{pmatrix} 4+6 \\ 1+9 \end{pmatrix} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$

$$\text{c. } \frac{1}{2}p - 3q = \frac{1}{2} \begin{pmatrix} 4 \\ 1 \end{pmatrix} - 3 \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(4) - 3(2) \\ \frac{1}{2}(1) - 3(3) \end{pmatrix} = \begin{pmatrix} 2 - 6 \\ \frac{1}{2} - 9 \end{pmatrix} = \begin{pmatrix} -4 \\ -8\frac{1}{2} \end{pmatrix}$$

Example 3

Perform the following multiplications.

$$\text{a) } 3 \times \begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$$

$$\text{b) } 5 \times \begin{pmatrix} 2 & -4 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 10 & -20 \\ -5 & -5 \end{pmatrix}$$

$$\text{c) } 2 \begin{pmatrix} 4 \\ 1 \\ 5 \end{pmatrix} = \begin{pmatrix} 8 \\ 2 \\ 10 \end{pmatrix}$$



$$d) \frac{1}{3} \times \begin{pmatrix} 6 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

**Learning Activity 11.5.4.7**

20 minutes

Solve the following resultant vectors:

For $\mathbf{p} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$ find

a. $-3\mathbf{p}$

e. $\mathbf{p} - \frac{1}{2}\mathbf{pr}$

b. $\frac{1}{2}\mathbf{q}$

f. $2\mathbf{p} + 3\mathbf{r}$

c. $2\mathbf{p} + \mathbf{q}$

g. $2\mathbf{q} - 3\mathbf{r}$

d. $\mathbf{p} - 2\mathbf{q}$

h. $2\mathbf{p} - \mathbf{q} + \mathbf{r}$

$$\frac{1}{2}\mathbf{q}$$

SUMMATIVE TASK 4: MAPS, CONTOURS AND VECTORS

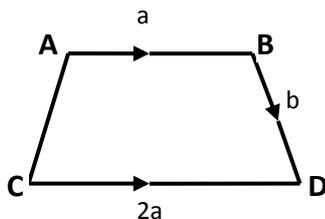
1. The scale of a map is 1: 20 000. What distance does 5 cm on the map represent?
2. The scale of a map is 1: 100 000. What distance does 28 cm on the map represent?



3. The scale of a map is 1 cm = 5 km. Express this scale as a ratio.
4. The scale of a map is 1 cm = 100 km. Express this scale as a ratio.
5. The scale of a map is 1 cm: 2 km.
 - a. What distance does 3.5 cm on the map represent?
 - b. What is the scale expressed as a ratio?

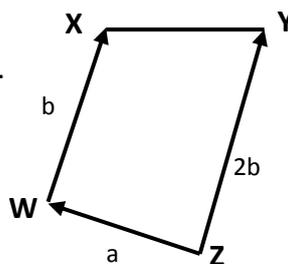
6. Write each vector in terms of **a** and/ or **b**.

- a. \overrightarrow{BA}
- b. \overrightarrow{AC}
- c. \overrightarrow{DB}
- d. \overrightarrow{AD}



7. Write each vector in terms of **a** and/ or **b**.

- a. \overrightarrow{ZX}
- b. \overrightarrow{YW}
- c. \overrightarrow{XY}
- d. \overrightarrow{XZ}



8. If $a = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $b = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$ $c = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ $d = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$e = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ $f = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ $g = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $h = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$

Find the following vectors in component form:



- a. $b + h$ b. $f + g$ c. $e - b$ d. $2a + 3c$
e. $3f + 2d$ g. $a + b + c$ h. $3f - a + c$

9. Refer to the following vectors:

$$a = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad b = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \quad c = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \quad d = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \quad e = \begin{pmatrix} -4 \\ -3 \end{pmatrix} \quad f = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$$

Find the following:

- a. $|a|$ b. $|d|$ c. $|a+b|$ d. $|c-d|$ e. $|f+2b|$

SUMMARY:

SOLVING RIGHT TRIANGLES

- One of the most important applications of trigonometric functions is to “solve” a right triangle. By now you should know that every right triangle has five unknowns: the lengths of its three sides and the measures of its two acute angles. Solving the triangle means finding the values of these unknowns.

General Rules of Solving Right Triangles

We have just shown you two of the different paths you can take when solving a right triangle. The actual solution will depend on the specific problem, but the same tools are always used.

1. The trigonometric functions
2. The Pythagorean Theorem

The knowledge that the sum of the angles of a triangle is 180° .

SOLVING OBLIQUE TRIANGLES

- An oblique triangle has no right angles. Yet trigonometry—a subject whose rules are generally based on right triangles—can still be used to solve a non-right triangle. You need different tools, though. Enter the laws of sines and cosines.
- In an oblique triangle, there are six unknowns: the three angle measures and the three side lengths. Every triangle has 3 sides and 3 angles for a total of 6 parts. In these problems we are given 3 of the 6 parts of a triangle and must solve for the remaining 3 parts. To solve an oblique triangle you need one of the following sets of information:



1. Two sides and an angle opposite one of the known sides
2. Two angles and any side
3. Two sides and their included angle
4. All three sides

1. Law Of Sines

To use this we must know an angle and its opposite side.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

2. Law of Cosines

Finding sides

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$a^2 = c^2 + b^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

Finding Angles

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos A = \frac{c^2 + b^2 - a^2}{2bc}$$

This must be used when you do not know an angle and its opposite side.

MAPS, CONTOURS AND VECTORS

CONTOUR MAPS

- Contour or topographic maps are two-dimensional representations of the three-dimensional surface features of a given area. Topographic maps have contour lines which connect points of identical elevation above sea level.
- Contour lines run next to each other and adjacent contour lines are separated by a constant difference in elevation, usually noted on the map. Topographic maps have a horizontal scale to indicate horizontal distances. Topographic maps help users see how the land changes in elevation.



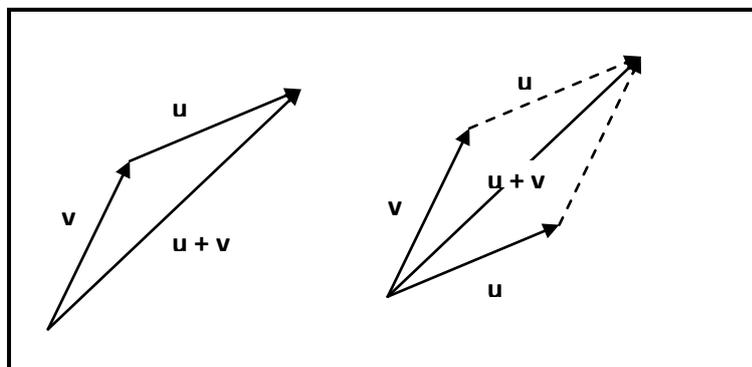
- Many people use topographic maps to locate surface features in a given area, to find their way through a particular area, and to determine the direction of water flow in a given area.

VECTORS

- Vector quantities are extremely useful in physics. The important characteristic of a vector quantity is that it has both a magnitude (and size) and a direction. Both of these properties must be given in order to specify a vector completely.
- A quantity with magnitude alone, but no direction, is not a vector. It is called a scalar instead.

Vector addition using the tail-tip rule

- To add vectors \mathbf{v} and \mathbf{u} , translate vector \mathbf{u} so that the initial point of \mathbf{u} is at the terminal point of \mathbf{v} . The resulting vector from the initial point of \mathbf{v} to the terminal point of \mathbf{u} is the vector $\mathbf{v} + \mathbf{u}$ and is called the **resultant**. The vectors \mathbf{v} and \mathbf{u} are called the **components** of the vector $\mathbf{v} + \mathbf{u}$. If the two vectors to be added are not parallel, then the **parallelogram rule** can also be used. In this case, the initial points of the vectors are the same, and the resultant is the diagonal of the parallelogram formed by using the two vectors as adjacent sides of the parallelogram.



In order to multiply a vector \mathbf{u} by a real number q , multiply the length of \mathbf{u} by $|q|$ and reverse the direction of \mathbf{u} if $q < 0$. This is called **scalar multiplication**. If a vector \mathbf{u} is multiplied by -1 , the resulting vector is designated as $-\mathbf{u}$. It has the same magnitude as \mathbf{u} but opposite direction.

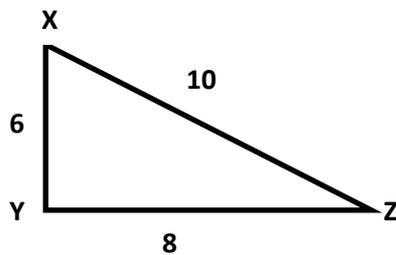


**ANSWERS TO LEARNING ACTIVITIES 11.5****11.5.1 TRIGONOMETRIC IDEAS****11.5.1.1 The Right Triangles**

1. a. DF b. EF c. DE 2. a. $\angle T$ b. TS, TR

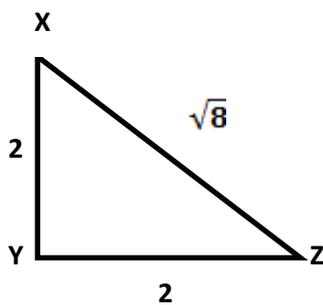
3. a. BC b. AB c. AC

4.



5. a. XZ b. XY c. XY

5.



$$c^2 = a^2 + b^2$$

$$(\sqrt{8})^2 = 2^2 + 2^2$$

$$8 = 4 + 4$$

$$8 = 8$$

\therefore the triangle is an isosceles right angled triangle.

6. $x = 65, y = 34, z = 56$

7. a. AE = 13 cm
b. 90°
c. 45°
d. 45

11.5.1.2 Pythagorean Formula

1. a. 6.40 units b. 5.67 units

2. a. $\sqrt{61}$ units b. $\sqrt{41}$ units

3. By using Pythagorean Formula

$$c^2 = a^2 + b^2$$

$$15^2 = 9^2 + 12^2$$

$$225 = 81 + 144$$

$$225 = 225$$



4. 6.3 m
5. 10.8
6. 16.8 m
7. 5 000 m
8. 12 km
9. 8 km
10. $10\sqrt{5}$ m or 22.36m

11.5.1.3 The Trigonometric Ratios

1. Find length of rectangle ABCD

$$AB = 15 \quad \text{a. } \frac{8}{15} \quad \text{b. } \frac{8}{17} \quad \text{c. } \frac{15}{17}$$

2. Find hypotenuse = 17

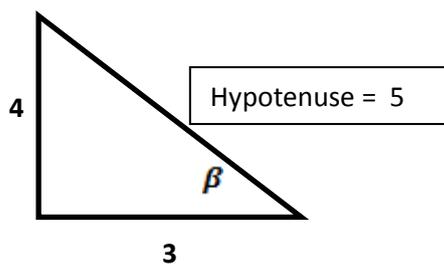
$$\text{a. } \sin A = \frac{8}{17} \quad \text{b. } \cos A = \frac{15}{17} \quad \text{c. } \tan A = \frac{8}{15}$$

$$\text{d. } \csc A = \frac{17}{8} \quad \text{e. } \sec A = \frac{17}{8} \quad \text{f. } \cot A = \frac{15}{8}$$

3. Find hypotenuse = $\sqrt{74}$

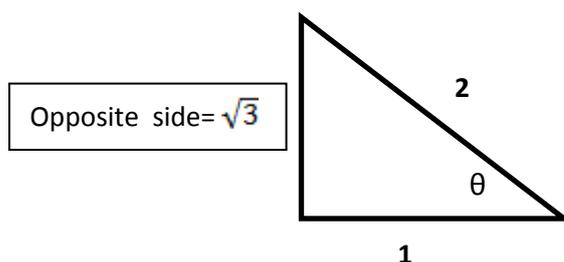
$$\text{a. } \sin \beta = \frac{7}{\sqrt{74}} \quad \text{b. } \cos \beta = \frac{5}{\sqrt{74}} \quad \text{c. } \tan \beta = \frac{7}{5}$$

- 4.



$$\text{a. } \sin \beta = \frac{4}{5} \quad \text{b. } \sin \beta = \frac{3}{5}$$

- 5.





$$\cos \theta = \frac{1}{2}; \theta = 60^\circ$$

$$\tan \theta = \frac{\sqrt{3}}{1} = \sqrt{3}$$

11.5.1.4 The Reciprocal Trigonometric Ratios (page 20)

1. a. $\sin \theta = \frac{12}{13}$

b. $\csc \theta = \frac{13}{12}$

c. $\cos \theta = \frac{5}{13}$

d. $\sec \theta = \frac{13}{5}$

e. $\tan \theta = \frac{12}{5}$

f. $\cot \theta = \frac{5}{12}$

2. a. $\sin \theta = \frac{4}{5}$

b. $\csc \theta = \frac{5}{4}$

c. $\cos \theta = \frac{3}{5}$

d. $\sec \theta = \frac{5}{3}$

e. $\tan \theta = \frac{4}{3}$

f. $\cot \theta = \frac{3}{4}$

3. given $\tan \theta = \frac{3}{5}$

missing side: hypotenuse = $\sqrt{34}$

$\sin \theta = \frac{3}{\sqrt{34}}$

$\csc \theta = \frac{\sqrt{34}}{3}$

$\cos \theta = \frac{5}{\sqrt{34}}$

$\sec \theta = \frac{\sqrt{34}}{5}$

$\cot \theta = \frac{4}{3}$

4. given $\sin \theta = \frac{1}{6}$

Missing side: adjacent side to $\theta = \sqrt{35}$

$\csc \theta = 6$ $\cos \theta = \frac{\sqrt{35}}{6}$

$\sec \theta = \frac{6}{\sqrt{35}}$

$\tan \theta = \frac{1}{\sqrt{35}}$

$\cot \theta = \frac{\sqrt{35}}{1} = \sqrt{35}$

5. Missing side: adjacent side = $\sqrt{51}$

$\sin \theta = \frac{7}{10}$

$\cos \theta = \frac{\sqrt{51}}{10}$

$\csc \theta = \frac{10}{7}$

$\cos \theta = \frac{10}{\sqrt{51}}$

$\tan \theta = \frac{7}{\sqrt{51}}$

$\sin \theta = \frac{\sqrt{51}}{7}$

11.5.1.5 The Trigonometric Functions for 30° , 45° and 60°

1. a. $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}}$

b. $\frac{1}{2} + \frac{1}{2} = 1$

c. $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$



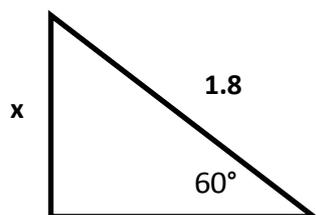
2. a. $2x \left(\frac{\sqrt{3}}{2} \right) = \sqrt{3}$

b. $\left(\frac{1}{\sqrt{2}} \right)^2 = \frac{1}{2}$

c. $\frac{\sqrt{3}}{\frac{1}{2}} = 2\sqrt{3}$

d. $2(2) = 4$

3.



$$\sin 60 = \frac{\sqrt{3}}{2}$$

$$\sin 60 = \frac{\text{opp}}{\text{hyp}}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{1.8}$$

$$2x = 1.8(\sqrt{3})$$

$$x = \frac{1.8 \times \sqrt{3}}{2} = 1.56 \text{ (2 decimal places)}$$

11.5.1.6 The Trigonometric Functions of Complementary Angles

1. a. $\sin 75^\circ$ b. $\cot 58^\circ$ c. $\csc 48^\circ$ d. $\cot \emptyset$ e. $\csc (90 - \emptyset)$

2. a. $x = 80^\circ$ b. $t = 20^\circ$ 3. $y = 68^\circ$ 4. $B = 20^\circ$ 5. $T = 20^\circ$

11.5.2. SOLUTION OF RIGHT TRIANGLES

11.5.2.1 Finding the unknown sides of the right triangle

1. $x = 6.2$ 2. $x = 13.5$ 3. $y = 9.9$ 4. $x = 5.1$ 5. $P = 28.2$

11.5.2.2 Finding the unknown sides of the right triangle

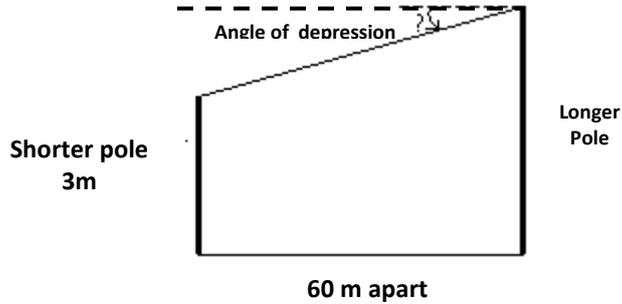
1. $\theta = 38^\circ 54$ 2. $x = 44^\circ 50$ 3. $45^\circ 3$
4. $39^\circ 48$ 5. $30^\circ 15$ 6. $29^\circ 43$

11.5.2.3 Angles of Elevation and Depression



1. $h = 51$ (nearest metre) $m = 57$

2. $x = 59.30$



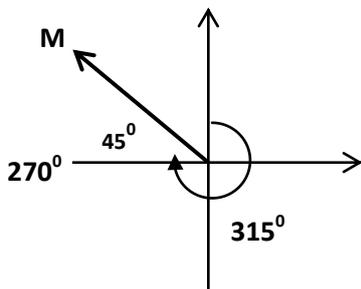
4. Approximately 17.95 m

5. 315 m

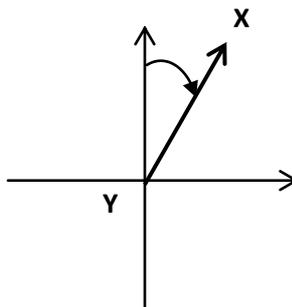
11.5.2.3 Bearings

1. a. 055° b. 235° c. 340° d. 135°

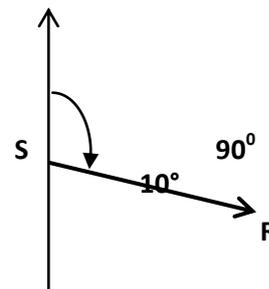
2. a.



b.



c.



3. a. 380 m b. 816 m

4. 30 222 m

11.5.2.4 Back Bearings

1a. 270° b. 045° c. 225°

2. a. 225° b. 240° c. 180°

3. 224° 4. a. 135° b. 090° c. 315° d. 045° e. 025°

5. a. 300° b. 120°

11.5.3 SOLUTIONS OF OBLIQUE TRIANGLES

11.5.3.1 The Sine Rule

1. $x = 9.8$

2. $\theta = 53^\circ 39'$

3. $\angle B = 22^\circ 45'$

4. $\angle A = 80^\circ$ $AB = 10$ m $AC = 9$ m 5. 12.66 m

**11.5.3.2 The Cosine Rule**

1. 10 2. $\theta = 29^\circ 56'$ 3. $\theta = 103^\circ 48'$ 4. 67.11 km 5. 207

11.5.3.3 Areas of Triangles

1. a. $A = 10 \text{ mm}^2$ b. $A = 7.5 \text{ units}^2$ 2. c. $A = 32.1 \text{ units}^2$ d. 44.4 units^2
 3. a. $A = 91.3 \text{ cm}^2$ b. $A(\text{total}) = 77.8 \text{ m}^2$

11.6.4. MAPS, CONTOURS AND VECTORS**11.6.4.1 Map Scales**

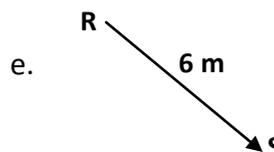
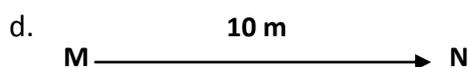
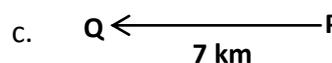
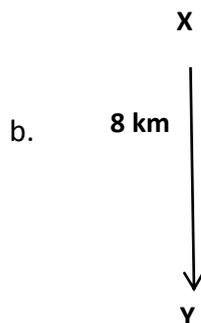
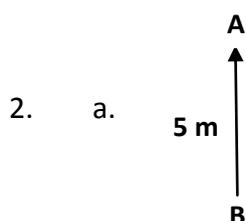
1. 2 : 5 2. 1 : 400 000 3. 17.5 km 4. a. 43.2 km b. 7.2 km c. 64.8 km d. 18.72 km
 e. 10.08 km 5. 15 km

11.6.4.2 Contours

1. steeper at the top of the hill 2. 50 ft 3. 1 in 40 4. 30 m 5. $39^\circ 48'$

11.6.4.3 Vectors (page 73)

1. a. scalar b. scalar c. vector d. vector e. scalar f. scalar

**11.6.4.4 Type of Vectors**

1. a. $-a$ b. $-b$ c. b d. $-a$
 2. a. a and b ; c and e b. a and c ; b and e



c. a and b ; c and e d. a, b, c and e e. none

3. a. $-a$ b. $-b$ c. c d. $-a+b+c$ e. $a+c$ f. $b+c+a$

11.6.4.5 Addition and Subtraction of Vectors (page 80)

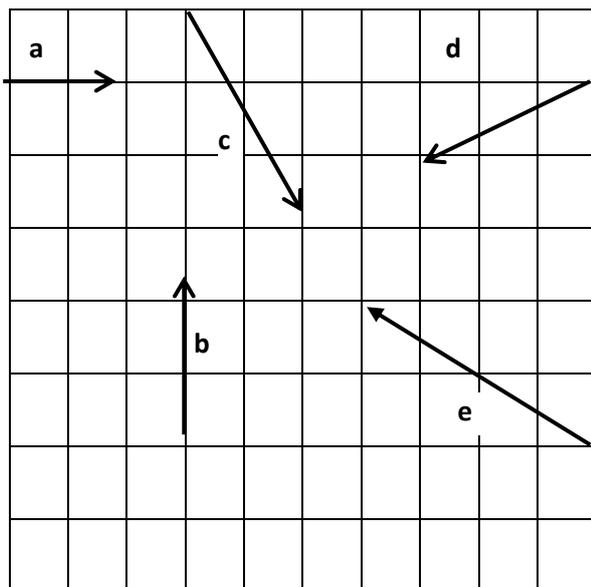
1. a. \vec{BA} b. \vec{BC} c. \vec{AC} d. \vec{AE}

2. a. $-r+s$ b. $-(-r+s)$ c. $-s+t$

3. a. $w+x$ b. $-b+a$ c. $-s+t$

11.6.4.6 Addition and Subtraction of Column Vectors

1.



2. a. $\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ b. $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ 3.a. $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ b. $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$ c. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ d. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

e. $\begin{pmatrix} -1 \\ -8 \end{pmatrix}$ f. $\begin{pmatrix} -1 \\ -8 \end{pmatrix}$

11.6.4.7 Multiplying by a Scalar



$$\text{a. } \begin{pmatrix} -3 \\ -15 \end{pmatrix} \quad \text{b. } \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \text{c. } \begin{pmatrix} 0 \\ 4 \end{pmatrix} \quad \text{d. } \begin{pmatrix} 5 \\ -3 \end{pmatrix} \quad \text{e. } \begin{pmatrix} -3 \\ 2 \\ 5 \\ 2 \end{pmatrix} \quad \text{f. } \begin{pmatrix} -7 \\ 7 \end{pmatrix} \quad \text{g. } \begin{pmatrix} 5 \\ 5 \end{pmatrix} \quad \text{h. } \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

Answers to Summative Task

Summative Task 1

1. a 2. c 3. b 4. b 5. B 6. a 7. c 8. a.
9. a 10. A 11. a. 10 11 b. i. $\frac{3}{5}$ ii. $\frac{4}{5}$ iii. $\frac{4}{3}$
- iv. $\frac{4}{5}$ v. $\frac{5}{3}$ vi. $\frac{4}{3}$ 12. a. $x = 70^\circ$ b. $x = 25^\circ$ c. $x = x = 55^\circ$
13. a. 2 b. $\sqrt{3}$ c. 1 d. $\frac{1}{\sqrt{3}}$ 14. $\sin \theta = \frac{\sqrt{8}}{3}$ $\tan \theta = \frac{\sqrt{8}}{1}$
15. $\cos \theta = \frac{2}{3}$ $\tan \theta = \frac{\sqrt{5}}{2}$

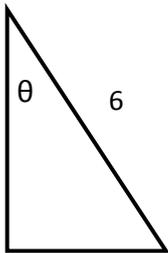
Summative Task 2

<p>1. a. $\tan 37^\circ = \frac{m}{230}$</p> <p>$m = 230 \times \tan 37$</p> <p>$m = 173.3$</p>	<p>1.d $\sin \theta = \frac{59}{86}$</p> <p>$\sin \theta = 0.6512$</p> <p>$\theta = \sin^{-1} 0.6512$</p> <p>$\theta = 40.63^\circ$ or $40^\circ 37'$</p>
<p>1.b. $\sin 48^\circ = \frac{h}{96}$</p> <p>$h = 96 \times \sin 48^\circ$</p> <p>$h = 71.3$</p>	<p>1.e. $\cos \theta = \frac{30}{70}$</p> <p>$\cos \theta = 0.4286$</p> <p>$\theta = 64.62^\circ$ or $64^\circ 37'$</p>
<p>1.c. $\tan 80^\circ = \frac{320}{y}$</p> <p>$y \tan 80^\circ = 320$</p>	<p>1.f $\tan 37^\circ 46' = \frac{m}{230}$</p>



$y = \frac{320}{\tan 80^\circ}$ $y = 56.4$	$0.7747 = \frac{m}{230}$ $m = 230 \times 0.7747$ $m = 178.19$
--	---

2. i.

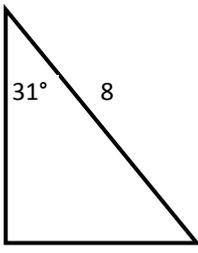


$$\sin \theta = \frac{2}{6}$$

$$\theta = \sin^{-1} 0.3333$$

$$\theta = 19^\circ 28'$$

ii.

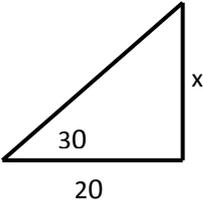


$$\sin \theta = \frac{x}{8}$$

$$x = 8 (\sin 31^\circ)$$

$$x = 4.1$$

iii.

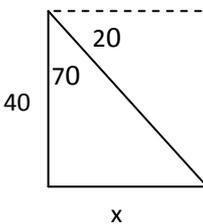


$$\tan 30 = \frac{x}{20}$$

$$x = 20 (\tan 30)$$

$$x = 11.5$$

iv.



$$\tan 70 = \frac{x}{40}$$

$$x = 40 \tan 70$$

$$x = 109.9$$

Summative Task 3

1. Sine Rule

a.
$$\frac{9}{\sin 37} = \frac{a}{\sin 62}$$

$$a \times \sin 37 = 9 \times \sin 62$$

$$a = \frac{9 \times \sin 62}{\sin 37}$$

$$a = 13.2$$

b.
$$\frac{\sin \theta}{15} = \frac{\sin 100}{50}$$

$$50 \times \sin \theta = 15 \sin 100$$

$$\sin \theta = \frac{15 \times \sin 100}{50}$$

$$\sin \theta = 0.5908$$

$$\theta = 36^\circ 13'$$

2. Cosine rule

a. Find $\angle A$

$$4^2 = 6^2 + 5^2 - 2(6)(5)\cos A$$

$$16 = 36 + 25 - 60\cos A$$

$$60\cos A = 61 - 16$$

$$60\cos A = 45$$

$$\cos A = \frac{45}{60}$$

$$\cos A = 0.75$$

$$A = 41^\circ 24'$$

b.

$$b^2 = 17^2 + 12^2 - 2(17)(12)\cos 65$$

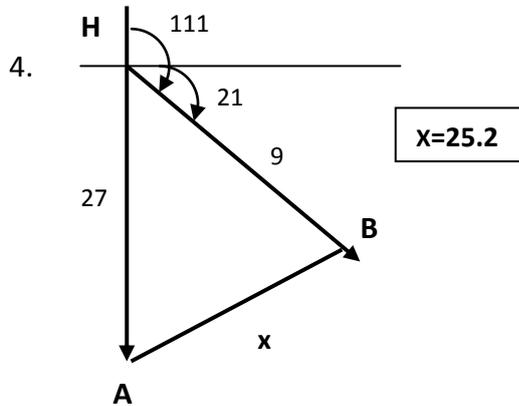
$$b^2 = 289 + 144 - 402\cos 65$$

$$b^2 = 260.57$$

$$b = \sqrt{260.57} = 16.14$$

3. a. LN = 10.

b. x = 6.7

c. $\angle Q = 137^\circ 49'$ **Summative Task 4**

1. 1 km 2. 28 km 3. 1 : 500 000 4. 1 : 10 000 000 5a. 7 km b. 1 : 200 000

6. a. $-a$ b. b c. $-b$ d. $a + b$ 7. a. $a + b$ b. $-2b + a$ c. $b - a$ d. $-b - a$ 8. a. $\frac{11}{9}$ 9a. 5 b. $\sqrt{17}$ c. $\sqrt{74}$ d. 10e. $\sqrt{89}$



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