PRACTICE EXAMINATION NO. 5
May 2005 Course FM Examination

1. Which of the following expressions does NOT represent a definition for $a_{\overline{n|}}$?

A. $v^n \cdot \dfrac{(1+i)^n - 1}{i}$ B. $\dfrac{1-v^n}{i}$ C. $v + v^2 + \ldots + v^n$

D. $v \cdot \dfrac{1-v^n}{1-v}$ E. $\dfrac{s_{\overline{n|}}}{(1+i)^{n-1}}$

2. Lori borrows 10,000 for 10 years at an annual effective interest rate of 9%. At the end of each year, she pays the interest on the loan and deposits the level amount necessary to repay the principal to a sinking fund earning an annual effective rate of 8%. The total of all payments made by Lori over the 10-year period is X. Calculate X.

A. 15803 B. 15853 C. 15903 D. 15953 E. 16003

3. A bond will pay a coupon of 100 at the end of each of the next three years and will pay the face amount of 1000 at the end of the three-year period. The bond's duration (Macaulay duration) when valued using an annual effective interest rate of 20% is X. Calculate X.

A. 2.61 B. 2.70 C. 2.77 D. 2.89 E. 3.00

4. An estate provides a perpetuity with payments of X at the end of each year. Seth, Susan, and Lori share the perpetuity such that Seth receives the payments of X for the first n years and Susan receives the payments of X for the next m years, after which Lori receives all the remaining payments of X. Which of the following represents the difference between the present value of Seth's and Susan's payments using a constant rate of interest?

A. $X\left(a_{\overline{n}|} - v^n \cdot a_{\overline{m}|}\right)$ B. $X\left(\ddot{a}_{\overline{n}|} - v^n \cdot \ddot{a}_{\overline{m}|}\right)$ C. $X\left(a_{\overline{n}|} - v^{n+1} \cdot a_{\overline{m}|}\right)$
D. $X\left(a_{\overline{n}|} - v^{n-1} \cdot a_{\overline{m}|}\right)$ E. $X\left(v \cdot a_{\overline{n}|} - v^{n+1} \cdot a_{\overline{m}|}\right)$

5. Susan can buy a zero coupon bond that will pay 1000 at the end of 12 years and is currently selling for 624.60. Instead she purchases a 6% bond with coupons payable semi-annually that will pay 1000 at the end of 10 years. If she pays *X* she will earn the same annual effective interest rate as the zero coupon bond. Calculate *X*.

A. 1164 B. 1167 C. 1170 D. 1173 E. 1176

6. John purchased three bonds to form a portfolio as follows:
- Bond A has semi-annual coupons at 4%, a duration of 21.46 years, and was purchased for 980.
- Bond B is a 15-year bond with a duration of 12.35 years and was purchased for 1015.
- Bond C has a duration of 16.67 years and was purchased for 1000.

Calculate the duration of the portfolio at the time of purchase.

A. 16.62 years B. 16.67 years C. 16.72 years D. 16.77 years E. 16.82 years

7. Mike receives cash flows of 100 today, 200 in one year, and 100 in two years. The present value of these cash flows is 364.46 at an annual effective rate of interest i. Calculate i.

A. 10% B. 11% C. 12% D. 13% E. 14%

8. A loan is being repaid with 25 annual payments of 300 each. With the 10th payment, the borrower pays an extra 1000, and then repays the balance over 10 years with a revised annual payment. The effective rate of interest is 8%. Calculate the amount of the revised annual payment.

A. 157 B. 183 C. 234 D. 257 E. 383

9. The present value of a series of 50 payments starting at 100 at the end of the first year and increasing by 1 each year thereafter is equal to X. The annual effective rate of interest is 9%. Calculate X.

A. 1165 B. 1180 C. 1195 D. 1210 E. 1225

10. Yield rates to maturity for zero coupon bonds are currently quoted at 8.5% for one-year maturity, 9.5% for two-year maturity, and 10.5% for three-year maturity. Let i be the one-year forward rate for year two implied by current yields of these bonds. Calculate i.

A. 8.5% B. 9.5% C. 10.5% D. 11.5% E. 12.5%

11. A 1000 par value bond pays annual coupons of 80. The bond is redeemable at par in 30 years, but is callable any time from the end of the 10th year at 1050. Based on her desired yield rate, an investor calculates the following potential purchase prices, P:
• Assuming the bond is called at the end of the 10th year, $P = 957$.
• Assuming the bond is held until maturity, $P = 897$.
The investor buys the bond at the highest price that guarantees she will receive at least her desired yield rate regardless of when the bond is called. The investor holds the bond for 20 years, after which time the bond is called. Calculate the annual yield rate the investor earns.

A. 8.56% B. 9.00% C. 9.24% D. 9.53% E. 9.99%

12. Which of the following are characteristics of all perpetuities?
I. The present value is equal to the first payment divided by the annual effective rate.
II. Payments continue forever.
III. Each payment is equal to the interest earned on the principal.

A. I only B. II only C. III only D. I, II and III
E. The correct answer is not given by A, B, C or D.

13. At a nominal interest rate of i convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year. Calculate i.

A. 2.75% B. 2.77% C. 2.79% D. 2.81% E. 2.83%

14. An annuity-immediate pays 20 per year for 10 years, then decreases by 1 per year for 19 years. At an annual effective interest rate of 6%, the present value is equal to *X*. Calculate *X*.

A. 200 B. 205 C. 210 D. 215 E. 220

15. An insurance company accepts an obligation to pay 10,000 at the end of each year for 2 years. The insurance company purchases a combination of the following two bonds at a total cost of *X* in order to exactly match its obligation:
(i) 1-year 4% annual coupon bond with a yield rate of 5%.
(ii) 2-year 6% annual coupon bond with a yield rate of 5%.
Calculate *X*.

A. 18564 B. 18574 C. 18584 D. 18594 E. 18604

16. At the beginning of the year, an investment fund was established with an initial deposit of 1000. A new deposit of 1000 was made at the end of 4 months. Withdrawals of 200 and 500 were made at the end of 6 months and 8 months, respectively. The amount in the fund at the end of the year is 1560. Calculate the dollar-weighted (money-weighted) yield rate earned by the fund during the year.

A. 18.57% B. 20.00% C. 22.61% D. 26.00% E. 28.89%

17. At an annual effective interest rate of i, the present value of a perpetuity-immediate starting with a payment of 200 in the first year and increasing by 50 each year thereafter is 46,530. Calculate i.

A. 3.25% B. 3.50% C. 3.75% D. 4.00% E. 4.25%

18. A store is running a promotion during which customers have two options for payment. Option one is to pay 90% of the purchase price two months after the date of sale. Option two is to deduct *X*% off the purchase price and pay cash on the date of sale. A customer wishes to determine *X* such that he is indifferent between the two options when valuing them using an effective annual interest rate of 8%. Which of the following equations of value would the customer need to solve?

A. $\frac{X}{100}\cdot\left(1+\frac{0.08}{6}\right)=0.90$

B. $\left(1-\frac{X}{100}\right)\cdot\left(1+\frac{0.08}{6}\right)=0.90$

C. $\frac{X}{100}\cdot 1.08^{\frac{1}{6}}=0.90$

D. $\frac{X}{100}\cdot\frac{1.08}{1.06}=0.90$

E. $\left(1-\frac{X}{100}\right)\cdot 1.08^{\frac{1}{6}}=0.90$

19. Calculate the nominal rate of discount convertible monthly that is equivalent to a nominal rate of interest of 18.9% per year convertible monthly.

A. 18.0% B. 18.3% C. 18.6% D. 18.9% E. 19.2%

20. An investor wishes to accumulate 10,000 at the end of 10 years by making level deposits at the beginning of each year. The deposits earn a 12% annual effective rate of interest paid at the end of each year. The interest is immediately reinvested at an annual effective interest rate of 8%. Calculate the level deposit.

A. 541 B. 572 C. 598 D. 615 E. 621

21. A discount electronics store advertises the following financing arrangement: “We don’t offer you confusing interest rates. We’ll just divide

your total cost by 10 and you can pay us that amount each month for a year." The first payment is due on the date of sale and the remaining eleven payments at monthly intervals thereafter. Calculate the effective annual interest rate the store's customers are paying on their loans.

A. 35.1%   B. 41.3%   C. 42.0%   D. 51.2%   E. 54.9%

22. On January 1, 2004, Karen sold stock A short for 50 with a margin requirement of 80%. On December 31, 2004, the stock paid a dividend of 2, and an interest amount of 4 was credited to the margin account. On January 1, 2005, Karen covered the short sale at a price of X, earning a 20% return. Calculate X.

A. 40   B. 44   C. 48   D. 52   E. 56

23. The stock of Company X sells for 75 per share assuming an annual effective interest rate of i. Annual dividends will be paid at the end of each year forever. The first dividend is 6, with each subsequent dividend 3% greater than the previous year's dividend. Calculate i.

A. 8%   B. 9%   C. 10%   D. 11%   E. 12%

24. An annuity pays 1 at the end of each year for n years. Using an annual effective interest rate of i, the accumulated value of the annuity at time $(n+1)$ is 13.776. It is also known that $(1+i)^n = 2.476$. Calculate n.

A. 4   B. 5   C. 6   D. 7   E. 8

25. A bank customer takes out a loan of 500 with a 16% nominal interest rate convertible quarterly. The customer makes payments of 20 at the end of each quarter. Calculate the amount of principal in the fourth payment.

A. 0.0   B. 0.9   C. 2.7   D. 5.2
E. There is not enough information to calculate the amount of principal

SOLUTIONS to PRACTICE EXAMINATION NO. 5
(May 2005 Course FM Examination)

1. **May 2005 Course FM Examination, Problem No. 1**
Which of the following expressions does NOT represent a definition for $a_{\overline{n}|}$?

A. $v^n \cdot \dfrac{(1+i)^n - 1}{i}$ B. $\dfrac{1-v^n}{i}$ C. $v + v^2 + \ldots + v^n$

D. $v \cdot \dfrac{1-v^n}{1-v}$ E. $\dfrac{s_{\overline{n}|}}{(1+i)^{n-1}}$

Solution.
We have

A: $$v^n \cdot \frac{(1+i)^n - 1}{i} = \frac{v^n \cdot (1+i)^n - v^n \cdot 1}{i} = \frac{1-v^n}{i} = a_{\overline{n}|},$$

B: $$\frac{1-v^n}{i} = a_{\overline{n}|},$$

C: $$v + v^2 + \ldots + v^n = a_{\overline{n}|},$$

D: $$v \cdot \frac{1-v^n}{1-v} = \frac{1}{1+i} \cdot \frac{1-v^n}{d} = \frac{1-v^n}{i} = a_{\overline{n}|},$$

E: $$\frac{s_{\overline{n}|}}{(1+i)^{n-1}} = v^{n-1} \cdot s_{\overline{n}|} = v^{-1} \cdot v^n \cdot s_{\overline{n}|} = (1+i) \cdot a_{\overline{n}|} = \ddot{a}_{\overline{n}|} \neq a_{\overline{n}|}.$$

Answer E.

2. **May 2005 Course FM Examination, Problem No. 2**
Lori borrows 10,000 for 10 years at an annual effective interest rate of 9%. At the end of each year, she pays the interest on the loan and deposits the level amount necessary to repay the principal to a sinking fund earning an annual effective rate of 8%. The total of all payments made by Lori over the 10-year period is *X*. Calculate *X*.

A. 15803 B. 15853 C. 15903 D. 15953 E. 16003

Solution.
Annual interest paid to the lender is $10000 \cdot 0.09 = 900$. Annual sinking fund deposit is

$$\frac{10000}{s_{\overline{10}|8\%}} \approx 690.29.$$

Total paid over 10 years is

$$10 \cdot (900 + 690.29) = 15903.$$

Answer C.

3. **May 2005 Course FM Examination, Problem No. 3**
A bond will pay a coupon of 100 at the end of each of the next three years and will pay the face amount of 1000 at the end of the three-year period. The bond's duration (Macaulay duration) when valued using an annual effective interest rate of 20% is *X*. Calculate *X*.

A. 2.61 B. 2.70 C. 2.77 D. 2.89 E. 3.00

Solution.
The Macaulay duration is

$$\frac{1 \cdot 100 v_{20\%}^{1} + 2 \cdot 100 v_{20\%}^{2} + 3 \cdot 1100 v_{20\%}^{3}}{100 v_{20\%}^{1} + 100 v_{20\%}^{2} + 1100 v_{20\%}^{3}} \approx \frac{2131.94}{789.35} \approx 2.70087977.$$

Answer B.

4. **May 2005 Course FM Examination, Problem No. 4**
An estate provides a perpetuity with payments of *X* at the end of each year. Seth, Susan, and Lori share the perpetuity such that Seth receives the payments of *X* for the first *n* years and Susan receives the payments of *X* for the next *m* years, after which Lori receives all the remaining payments of *X*. Which of the following represents the difference between the present value of Seth's and Susan's payments using a constant rate of interest?

A. $X\left(a_{\overline{n}|} - v^{n} \cdot a_{\overline{m}|}\right)$ B. $X\left(\ddot{a}_{\overline{n}|} - v^{n} \cdot \ddot{a}_{\overline{m}|}\right)$ C. $X\left(a_{\overline{n}|} - v^{n+1} \cdot a_{\overline{m}|}\right)$
D. $X\left(a_{\overline{n}|} - v^{n-1} \cdot a_{\overline{m}|}\right)$ E. $X\left(v \cdot a_{\overline{n}|} - v^{n+1} \cdot a_{\overline{m}|}\right)$

Solution.
The present value of Seth's payments is $X \cdot a_{\overline{n}|}$ and the present value of Susan's payments is $X \cdot v^n \cdot a_{\overline{m}|}$. The difference is $X\left(a_{\overline{n}|} - v^n \cdot a_{\overline{m}|}\right)$.
Answer A.

5. **May 2005 Course FM Examination, Problem No. 5**
Susan can buy a zero coupon bond that will pay 1000 at the end of 12 years and is currently selling for 624.60. Instead she purchases a 6% bond with coupons payable semi-annually that will pay 1000 at the end of 10 years. If she pays *X* she will earn the same annual effective interest rate as the zero coupon bond. Calculate *X*.

A. 1164 B. 1167 C. 1170 D. 1173 E. 1176

Solution.
Let *j* be the six-month effective rate of interest. Then

$$1000 v_j^{24} = 624.60,$$

and this solves to $j \approx 1.9803702\%$. Therefore

$$X = 30 a_{\overline{20}|j} + 1000 v_j^{20} \approx 1167.03.$$

Answer B.

6. **May 2005 Course FM Examination, Problem No. 6**
John purchased three bonds to form a portfolio as follows:
• Bond A has semi-annual coupons at 4%, a duration of 21.46 years, and was purchased for 980.
• Bond B is a 15-year bond with a duration of 12.35 years and was purchased for 1015.
• Bond C has a duration of 16.67 years and was purchased for 1000.
Calculate the duration of the portfolio at the time of purchase.

A. 16.62 years B. 16.67 years C. 16.72 years
D. 16.77 years E. 16.82 years

Solution.

Duration of a portfolio (be it Macaulay duration or duration) is a market-value-weighted average of the durations of its individual pieces, thus the duration of this portfolio is:

$$\frac{980}{980+1015+1000}\cdot 21.46+\frac{1015}{980+1015+1000}\cdot 12.35+$$
$$+\frac{1000}{980+1015+1000}\cdot 16.67\approx 16.7733.$$

Answer D.

7. **May 2005 Course FM Examination, Problem No. 7**
Mike receives cash flows of 100 today, 200 in one year, and 100 in two years. The present value of these cash flows is 364.46 at an annual effective rate of interest i. Calculate i.

A. 10% B. 11% C. 12% D. 13% E. 14%

Solution.
We set up the equation of value

$$100+200v+100v^2=364.46,$$

which gives

$$100v^2+200v-264.46=0.$$

This is a quadratic equation, and its solution is:

$$v=\frac{-200\pm\sqrt{200^2+4\cdot 100\cdot 264.46}}{200}=$$
$$=-1\pm\frac{1}{2}\sqrt{2^2+4\cdot 2.6446}=\begin{cases}-2.9090835,\\ 0.9090835.\end{cases}$$

The negative solution is unacceptable, thus $v = 0.9090836$ and $i\approx 10\%$.
Answer A.

8. **May 2005 Course FM Examination, Problem No. 8**
A loan is being repaid with 25 annual payments of 300 each. With the 10th payment, the borrower pays an extra 1000, and then repays the balance over 10 years with a revised annual payment. The effective rate of interest is 8%. Calculate the amount of the revised annual payment.

A. 157 B. 183 C. 234 D. 257 E. 383

Solution.
After the regular 10th payment, the remaining balance of the loan is

$$300a_{\overline{15}|8\%} \approx 2567.84.$$

After the borrower pays an extra 1000, the balance drops to 1567.84. The new payment is therefore

$$\frac{1567.84}{a_{\overline{10}|8\%}} \approx 233.65.$$

Answer C.

9. **May 2005 Course FM Examination, Problem No. 9**
The present value of a series of 50 payments starting at 100 at the end of the first year and increasing by 1 each year thereafter is equal to *X*. The annual effective rate of interest is 9%. Calculate *X*.

A. 1165 B. 1180 C. 1195 D. 1210 E. 1225

Solution.
The present value of the cash flows is

$$99a_{\overline{50}|9\%} + (Ia)_{\overline{50}|9\%} = 99a_{\overline{50}|9\%} + \frac{\ddot{a}_{\overline{50}|9\%} - 50v_{9\%}^{50}}{0.09} \approx$$

$$\approx 1085.21 + \frac{11.95 - 0.67}{0.09} \approx 1210.52.$$

Answer D.

10. **May 2005 Course FM Examination, Problem No. 10**
Yield rates to maturity for zero coupon bonds are currently quoted at 8.5% for one-year maturity, 9.5% for two-year maturity, and 10.5% for three-year maturity. Let i be the one-year forward rate for year two implied by current yields of these bonds. Calculate i.

A. 8.5% B. 9.5% C. 10.5% D. 11.5% E. 12.5%

Solution.
Using the notation s_1, s_2, and s_3 for the spot rates and f_1, f_2, and f_3 for the forward rates we have

$$\left(1+s_1\right)\left(1+f_2\right)=\left(1+s_2\right)^2,$$

so that

$$1.085\cdot\left(1+f_2\right)=1.095^2,$$

$$f_2=\frac{1.095^2}{1.085}-1\approx 10.5\%.$$

Answer C.

11. **May 2005 Course FM Examination, Problem No. 11**

A 1000 par value bond pays annual coupons of 80. The bond is redeemable at par in 30 years, but is callable any time from the end of the 10th year at 1050. Based on her desired yield rate, an investor calculates the following potential purchase prices, *P*:

- Assuming the bond is called at the end of the 10th year, $P = 957$.
- Assuming the bond is held until maturity, $P = 897$.

The investor buys the bond at the highest price that guarantees she will receive at least her desired yield rate regardless of when the bond is called. The investor holds the bond for 20 years, after which time the bond is called. Calculate the annual yield rate the investor earns.

A. 8.56% B. 9.00% C. 9.24% D. 9.53% E. 9.99%

Solution.

The bond is a par value bond, so that its redemption value is 1000. Since the price assuming holding to maturity is less than 1000, the bond was purchased at a discount. For such a bond, its coupon is below current market level of a coupon payment, and the earlier the bond is called, the better for the investor. The least favorable call date from the perspective of the investor is the maturity date. Thus, we need to assure the investor of her desired yield if the bond is held to maturity. The price assuming that is 897. Let j be the annual yield for the investor. We must have

$$897=80a_{\overline{20}|j}+1050v_j^{20},$$

and using a calculator, we determine $j\approx 9.24\%$.

Answer C.

12. **May 2005 Course FM Examination, Problem No. 12**

Which of the following are characteristics of all perpetuities?

I. The present value is equal to the first payment divided by the annual effective rate.
II. Payments continue forever.
III. Each payment is equal to the interest earned on the principal.

A. I only B. II only C. III only D. I, II and III
E. The correct answer is not given by A, B, C or D.

Solution.
I is only true for a level perpetuity-immediate, and false in general. II is true by definition, a perpetuity is defined as a set of cash flows that continue forever. III may be true for some level perpetuities, but it is clearly not true for an increasing perpetuity – for example, an increasing perpetuity due has a present value of $\frac{1}{d^2}$ and its first payment is 1, clearly not the interest on its present value, even if we calculate the interest as the discount rate applied to the present value.
Answer B.

13. **May 2005 Course FM Examination, Problem No. 13**
At a nominal interest rate of i convertible semi-annually, an investment of 1000 immediately and 1500 at the end of the first year will accumulate to 2600 at the end of the second year. Calculate i.

A. 2.75% B. 2.77% C. 2.79% D. 2.81% E. 2.83%

Solution.
The equation of value is

$$1000 \cdot \left(1+\frac{i}{2}\right)^4 + 1500 \cdot \left(1+\frac{i}{2}\right)^2 = 2600.$$

Let us write

$$x = \left(1+\frac{i}{2}\right)^2.$$

We get the following quadratic equation

$$10x^2 + 15x - 26 = 0.$$

Therefore,

$$x = \frac{-15 \pm \sqrt{15^2 + 40 \cdot 26}}{20} \approx \begin{cases} -2.52834192, \\ 1.02834192. \end{cases}$$

The negative solution is not feasible, so that

$$x = \left(1 + \frac{i}{2}\right)^2 = 1.02834192,$$

and

$$i = 2 \cdot \left(\sqrt{1.02834192} - 1\right) \approx 2.81439\%.$$

Answer D.

14. **May 2005 Course FM Examination, Problem No. 14**

An annuity-immediate pays 20 per year for 10 years, then decreases by 1 per year for 19 years. At an annual effective interest rate of 6%, the present value is equal to *X*. Calculate *X*.

A. 200 B. 205 C. 210 D. 215 E. 220

Solution.

The present value of this annuity is

$$20a_{\overline{10}|6\%} + v_{6\%}^{10} \cdot (Da)_{\overline{19}|6\%} = 20a_{\overline{10}|6\%} + v_{6\%}^{10} \cdot \frac{19 - a_{\overline{10}|6\%}}{0.06} \approx 220.20.$$

Answer E.

15. **May 2005 Course FM Examination, Problem No. 15**

An insurance company accepts an obligation to pay 10,000 at the end of each year for 2 years. The insurance company purchases a combination of the following two bonds at a total cost of *X* in order to exactly match its obligation:

(i) 1-year 4% annual coupon bond with a yield rate of 5%.

(ii) 2-year 6% annual coupon bond with a yield rate of 5%.

Calculate *X*.

A. 18564 B. 18574 C. 18584 D. 18594 E. 18604

Solution.

Since both the one-year and two-year yield is 5%, this is the interest rate that applies to all cash flows at time 1 and 2, including the liabilities cash flows. Thus the present value of the liabilities, and also the present value of the assets matching them, is

$$\frac{10000}{1.05}+\frac{10000}{1.05^2}=18594.10.$$

Yet another application of the Strawberry Principle: cash flows that match each other have the same present value.
Answer D.

16. **May 2005 Course FM Examination, Problem No. 16**
At the beginning of the year, an investment fund was established with an initial deposit of 1000. A new deposit of 1000 was made at the end of 4 months. Withdrawals of 200 and 500 were made at the end of 6 months and 8 months, respectively. The amount in the fund at the end of the year is 1560. Calculate the dollar-weighted (money-weighted) yield rate earned by the fund during the year.

A. 18.57% B. 20.00% C. 22.61% D. 26.00% E. 28.89%

Solution.
Recall that all such problems assume simple interest. The interest earned during the year is:

$$1560 + 200 + 500 - 1000 - 1000 = 260.$$

The dollar-weighted yield rate is:

$$\frac{260}{1000\cdot 1+1000\cdot\frac{8}{12}-200\cdot\frac{6}{12}-500\cdot\frac{8}{12}}=$$

$$=\frac{260}{1000+\frac{2000}{3}-100-\frac{500}{3}}=\frac{260}{900+\frac{1500}{3}}=\frac{3\cdot 260}{4200}\approx 18.57\%.$$

Answer A.

17. **May 2005 Course FM Examination, Problem No. 17**
At an annual effective interest rate of i, the present value of a perpetuity-immediate starting with a payment of 200 in the first year and increasing by 50 each year thereafter is 46,530. Calculate i.

A. 3.25% B. 3.50% C. 3.75% D. 4.00% E. 4.25%

Solution.
The present value of a perpetuity-immediate starting with a payment of 200 in the first year and increasing by 50 each year thereafter is

$$\frac{150}{i}+\frac{50}{di},$$

and this equals 46,530. We arrive at the following equation

$$\frac{150}{i}+\frac{50(1+i)}{i^2}=46530,$$

or (by multiplying by i^2, rearranging the terms, and dividing by 10)

$$4653i^2-20i-5=0.$$

This is a quadratic equation that solves to

$$i=\frac{20\pm\sqrt{400+20\cdot 4653}}{2\cdot 4653}=\frac{10\pm\sqrt{100+5\cdot 4653}}{4653}\approx\begin{cases}0.03500025,\\ -0.03070194.\end{cases}$$

The negative solution is unacceptable, so that $i\approx 3.500025\%$.
Answer B.

18. **May 2005 Course FM Examination, Problem No. 18**
A store is running a promotion during which customers have two options for payment. Option one is to pay 90% of the purchase price two months after the date of sale. Option two is to deduct *X*% off the purchase price and pay cash on the date of sale. A customer wishes to determine *X* such that he is indifferent between the two options when valuing them using an effective annual interest rate of 8%. Which of the following equations of value would the customer need to solve?

A. $\frac{X}{100}\cdot\left(1+\frac{0.08}{6}\right)=0.90$

B. $\left(1-\frac{X}{100}\right)\cdot\left(1+\frac{0.08}{6}\right)=0.90$

C. $\frac{X}{100}\cdot 1.08^{\frac{1}{6}}=0.90$

D. $\frac{X}{100}\cdot\frac{1.08}{1.06}=0.90$

E. $\left(1-\frac{X}{100}\right)\cdot 1.08^{\frac{1}{6}} = 0.90$

Solution.

The fraction of the full price paid now under option two is $1-\frac{X}{100}$. Over the next two months, the accumulation factor is $1.08^{\frac{1}{6}}$. If the full price is P then option two amount paid is $P\left(1-\frac{X}{100}\right)$, and this amount will accumulate to $P\left(1-\frac{X}{100}\right)\cdot 1.08^{\frac{1}{6}}$. This is supposed to be equivalent to $0.90P$. Thus,

$$P\left(1-\frac{X}{100}\right)\cdot 1.08^{\frac{1}{6}} = 0.90P.$$

Answer E.

19. **May 2005 Course FM Examination, Problem No. 19**

Calculate the nominal rate of discount convertible monthly that is equivalent to a nominal rate of interest of 18.9% per year convertible monthly.

A. 18.0% B. 18.3% C. 18.6% D. 18.9% E. 19.2%

Solution.

Let $d^{(12)}$ be the nominal rate of discount convertible monthly sought. Then

$$\left(1-\frac{d^{(12)}}{12}\right)^{-12} = \left(1+\frac{0.189}{12}\right)^{12},$$

so that

$$\frac{d^{(12)}}{12} = 1-\frac{1}{1+\frac{0.189}{12}},$$

and

$$d^{(12)} = 12-\frac{12}{1+\frac{0.189}{12}} = 12-\frac{144}{12.189} \approx 18.60694\%.$$

Answer C.

20. **May 2005 Course FM Examination, Problem No. 20**
An investor wishes to accumulate 10,000 at the end of 10 years by making level deposits at the beginning of each year. The deposits earn a 12% annual effective rate of interest paid at the end of each year. The interest is immediately reinvested at an annual effective interest rate of 8%. Calculate the level deposit.

A. 541   B. 572   C. 598   D. 615   E. 621

Solution.
Let x be the level deposit sought. Then we have

$$0.12x \cdot (Is)_{\overline{10}|8\%} + 10x = 10000,$$

so that

$$x = \frac{10000}{0.12 \cdot (Is)_{\overline{10}|8\%} + 10} = \frac{10000}{0.12 \cdot \dfrac{\ddot{s}_{\overline{10}|8\%} - 10}{0.08} + 10} =$$

$$= \frac{10000}{1.5 \cdot \left(\ddot{s}_{\overline{10}|8\%} - 10\right) + 10} = \frac{10000}{1.5\ddot{s}_{\overline{10}|8\%} - 5} \approx \frac{10000}{18.468232} \approx 541.47035.$$

Answer A.

21. **May 2005 Course FM Examination, Problem No. 21**
A discount electronics store advertises the following financing arrangement: "We don't offer you confusing interest rates. We'll just divide your total cost by 10 and you can pay us that amount each month for a year." The first payment is due on the date of sale and the remaining eleven payments at monthly intervals thereafter. Calculate the effective annual interest rate the store's customers are paying on their loans.

A. 35.1%   B. 41.3%   C. 42.0%   D. 51.2%   E. 54.9%

Solution.
Since all amounts are proportional to the purchase price, we can simply assume that the purchase price is 100. The customer effectively borrows 100 and pays off that loan in twelve monthly payments of 10 at the beginning of each monthly period. Let j be the effective monthly interest rate. Then

$$100 = 10\ddot{a}_{\overline{12}|j}.$$

Using a financial calculator, we determine that $j \approx 0.03503153$, and the effective annual interest rate is $(1+0.03503153)^{12} - 1 \approx 51.16211\%$. Ouch.
Answer D.

22. **May 2005 Course FM Examination, Problem No. 22**
On January 1, 2004, Karen sold stock A short for 50 with a margin requirement of 80%. On December 31, 2004, the stock paid a dividend of 2, and an interest amount of 4 was credited to the margin account. On January 1, 2005, Karen covered the short sale at a price of *X*, earning a 20% return. Calculate *X*.

A. 40 B. 44 C. 48 D. 52 E. 56

Solution.
Karen's initial deposit was 80% of 50, i.e., 40. She made 20%, so when the final cash flows happened on January 1, 2005, she received a net cash flow of 48. She paid a dividend of 2, and received interest of 4, thus the net from the short covering was 46. Of that, 40 was the return of her deposit, and 6 was the profit on the short sale. Since she sold at 50, and made 6, she covered at 44.
Answer B.

23. **May 2005 Course FM Examination, Problem No. 23**
The stock of Company X sells for 75 per share assuming an annual effective interest rate of i. Annual dividends will be paid at the end of each year forever. The first dividend is 6, with each subsequent dividend 3% greater than the previous year's dividend. Calculate i.

A. 8% B. 9% C. 10% D. 11% E. 12%

Solution.
Using the standard formula for the share price under the constant growth model,

$$\frac{6}{i-0.03} = 75.$$

This implies that

$$i = \frac{6}{75} + 0.03 = 0.08 + 0.03 = 0.11.$$

Answer D.

24. **May 2005 Course FM Examination, Problem No. 24**

An annuity pays 1 at the end of each year for n years. Using an annual effective interest rate of i, the accumulated value of the annuity at time $(n+1)$ is 13.776. It is also known that $(1+i)^n = 2.476.$ Calculate n.

A. 4 B. 5 C. 6 D. 7 E. 8

Solution.

Based on the information given

$$13.776 = \ddot{s}_{\overline{n}|i} = \frac{(1+i)^n - 1}{d} = \frac{2.476 - 1}{d} = \frac{1.476}{d}.$$

Therefore,

$$d = \frac{1.476}{13.776} = \frac{123}{1148}.$$

From this,

$$i = \frac{d}{1-d} = \frac{\frac{123}{1148}}{\frac{1148-123}{1148}} = \frac{123}{1025} = 0.12.$$

Hence, $1.12^n = 2.476,$ and

$$n = \frac{\ln 2.476}{\ln 1.12} \approx 8.000121232.$$

Answer E.

25. **May 2005 Course FM Examination, Problem No. 25**

A bank customer takes out a loan of 500 with a 16% nominal interest rate convertible quarterly. The customer makes payments of 20 at the end of each quarter. Calculate the amount of principal in the fourth payment.

A. 0.0 B. 0.9 C. 2.7 D. 5.2

E. There is not enough information to calculate the amount of principal

Solution.
The effective quarterly interest rate is 4%. That 4% applied to the principal of 500 gives 20 as the interest due every quarter. Thus the entire customer's payment is interest, and there is no principal repaid.
Answer A.