9th Grade Math Packet

(For students entering 9th grade in August 2017)

This packet is OPTIONAL. If you complete it, you will earn BONUS HOMEWORK points.

You will get 2 bonus points for each section you complete up to a *maximum* of 50 points. In order to receive points for a section, you must:

- Show reasonable, clear work on every problem.
- Turn in your packet by September 1st, 2017.

Each section has a *worked example* and then some problems for you to try. If you are still feeling confused, type the section title into any of these websites:

khanacademy.org
ixl.com
Learnzillion.com

You may also email questions to azern@nhcsb.org

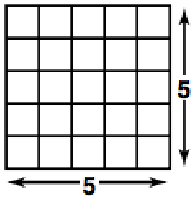
We will *not* be going over the answers to the packets when school begins, but there are answer keys at the school if you want to come check your answers.

You may not have a pencil or pen with you when you check your answers =)

Square Roots

- The *square* of 5 is 25.
  $5 \cdot 5 = 5^2 = 25$
- The *square root* of 25 is 5 because $5^2 = 25$.

$\sqrt{25} = 5$



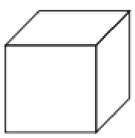
$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
} *perfect squares*

Find each square root. Estimate to the nearest integer if necessary.
Use ≈ to show that a value is estimated.

1. $\sqrt{16}$ ____________ **2.** $\sqrt{85}$ ____________ **3.** $\sqrt{26}$ ____________ **4.** $\sqrt{36}$ ____________

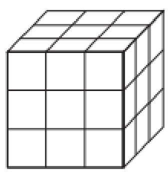
5. $\sqrt{98}$ ____________ **6.** $\sqrt{40}$ ____________ **7.** $\sqrt{100}$ ____________ **8.** $\sqrt{18}$ ____________

Cube Roots



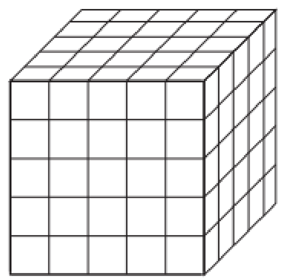
The cube of 1 is 1.

$1 \times 1 \times 1 = 1^3 = 1$



The cube of 3 is 27.

$3 \times 3 \times 3 = 3^3 = 27$



The cube of 5 is 125.

$5 \times 5 \times 5 = 5^3 = 125$

perfect cubes

$1^3 = 1$ $3^3 = 27$ $5^3 = 125$

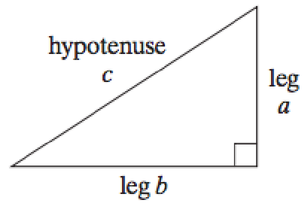
Find the cube root of each number.

1. 729 __________ **2.** 125 __________ **3.** 512 __________

The Pythagorean Theorem

The Pythagorean Theorem

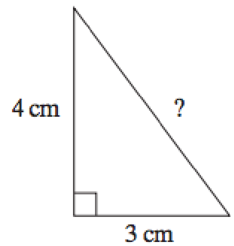
The sum of the squares of the lengths of the *legs* of a right triangle is equal to the square of the length of the *hypotenuse*.



$a^2 + b^2 = c^2$

Example 1: Find the length of the hypotenuse.

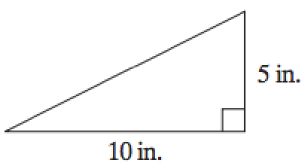
$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$9 + 16 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = c^2$$
$$5 = c$$



The length c of the hypotenuse is 5 cm.

Find the length of the hypotenuse of each triangle. If necessary, round to the nearest tenth.

1.



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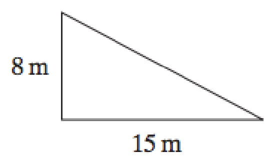


The lengths of the legs of a right triangle are given. Find the length of the hypotenuse.

3. legs: 6 ft and 8 ft
hypotenuse:

____

4. legs: 12 cm and 5 cm
hypotenuse:

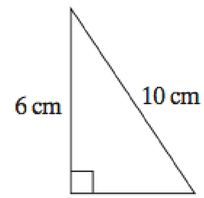
____

The Pythagorean Theorem Continued

You can use the Pythagorean Theorem to find the length of a leg in a right triangle.

Example: Find the length of the unknown side.

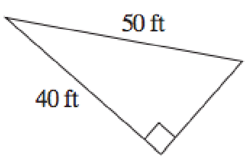
$$a^2 + b^2 = c^2$$
$$6^2 + b^2 = 10^2$$
$$36 + b^2 = 100$$
$$b^2 = 100 - 36$$
$$b^2 = 64$$
$$b = \sqrt{64}$$
$$b = 8$$



The length b of the unknown leg is 8 cm.

Find the missing leg length. If necessary, round to the nearest tenth.

1.



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5. Marcus leans a 12-ft ladder against a wall to clean a window. If the base of the ladder is 3 feet away from the wall, how high up the wall does the ladder reach? If necessary, round to the nearest tenth.

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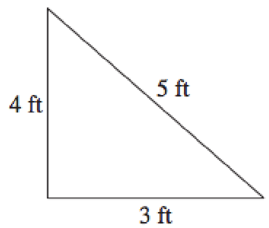
Converse of the Pythagorean Theorem

You can use the Pythagorean Theorem to determine whether a triangle is a right triangle.



$a^2 + b^2 \stackrel{?}{=} c^2$ ← Use the Pythagorean Theorem.

$3^2 + 4^2 \stackrel{?}{=} 5^2$ ← Substitute 3 for *a*, 4 for *b*, and 5 for *c*.

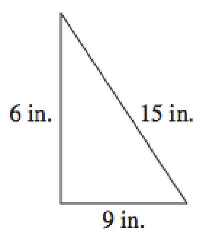
$9 + 16 \stackrel{?}{=} 25$ ← Simplify.

$25 = 25$

The equation is true so the triangle is a right triangle.

Determine whether the given lengths can be side lengths of a right triangle.

1.



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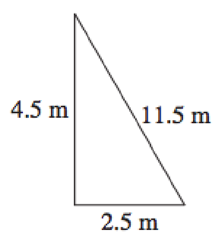
2.



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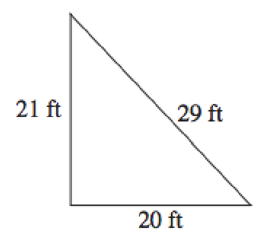
3.



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Solving Two-Step Equations

Follow these steps to solve the two-step equation: $4b + 5 = 17$

① Add or subtract on each side. $4b + 5 - 5 = 17 - 5$

$4b = 12$

② Multiply or divide to isolate the variable. $\frac{4b}{4} = \frac{12}{4}$

$b = 3$ ← Each

③ Check by substituting your answer for the variable. Check: $4b + 5 = 17$

$4 \cdot 3 + 5 \stackrel{?}{=} 17$

$17 = 17$ ✔

Solve each equation.

4. $\frac{x}{-2} + 6 = 4$

$x =$ ______________

5. $14j - 7 = 91$

$j =$ ______________

6. $240a - 3 = 5$

$a =$ ______________

Simplifying Expressions

A *term* is a number, a variable, or the product of a number and variable(s). The two terms in $-2x + 4y$ are $-2x$ and $4y$.

Terms with exactly the same variable factor are called *like terms*. In $-3x + 4y + 5x$, $-3x$ and $5x$ are like terms.

One way to *combine like terms* is by addition or subtraction.

- Add to combine like terms in $4y + y$.

  $4y + y = (4 + 1)y = 5y$

- Subtract to combine like terms in $2m - 5m$.

  $2m - 5m = (2 - 5)m = -3m$

To *simplify* an expression, combine its like terms. Perform as many of its operations as possible.

Simplify: $3a + 5b - a + 2b$
$= (3a - a) + (5b + 2b)$
$= 2a + 7b$

Simplify: $2(x - 4)$
$= 2x - 2(4)$
$= 2x - 8$

Combine like terms.

1. $6x + 2x =$ ____________ **2.** $4c - c =$ ____________ **3.** $-h - h =$ ____________

Simplify each expression.

10. $3(m + 4) - 5m =$ ____________ **11.** $(v - 4)5 =$ ____________

12. $4a + 2 - 8a + 1 =$ ____________ **13.** $6s + 5 - (s - 6) =$ ____________

Solving Multi-Step Equations

Combining terms can help solve equations.

Solve: $5n + 6 + 3n = 22$

$5n + 3n + 6 = 22$ ← Commutative Property

$8n + 6 = 22$

$8n + 6 - 6 = 22 - 6$

$8n = 16$

$\frac{8n}{8} = \frac{16}{8}$

$n = 2$

Check: $5n + 6 + 3n = 22$

$5(2) + 6 + 3(2) \stackrel{?}{=} 22$

$22 = 22$ ✔

Sometimes you need to distribute a term in order to simplify.

Solve: $4(x + 2) = 28$

$4x + 8 = 28$ ← Distributive Property

$4x = 20$

$\frac{4x}{4} = \frac{20}{4}$

$x = 5$

Check: $4(n + 2) = 28$

$4(5 + 2) \stackrel{?}{=} 28$

$28 = 28$ ✔

Solve each equation. Check the solution.

1. $a - 4a = 36$

$a =$ ________________

2. $3b - 5 - 2b = 5$

$b =$ ________________

3. $5n + 4 - 8n = -5$

$n =$ ________________

4. $12k + 6 = 10$

$k =$ ________________

5. $3(x - 4) = 15$

$x =$ ________________

6. $y - 8 + 2y = 10$

$y =$ ________________

Solving Equations with Variables on Both Sides

When an equation has a variable on both sides, add or subtract to get the variable on one side.

Solve: $-6m + 45 = 3m$

$-6m + 6m + 45 = 3m + 6m$ ← Add 6m to each side.

$45 = 9m$

$\frac{45}{9} = \frac{9m}{9}$

$5 = m$

Check: $-6m + 45 = 3m$

$-6(5) + 45 \stackrel{?}{=} 3(5)$

$15 = 15$ ✔

Sometimes you need to distribute a term in order to simplify.

Solve: $5(x - 3) = 32 - 2$

$5x - 15 = 32 - 2$ ← Distributive Property

$5x - 15 = 30$

$5x = 45$

$\frac{5x}{5} = \frac{45}{5}$

$x = 9$

Check: $5(x - 3) = 32 - 2$

$5(9 - 3) = 32 - 2$

$30 = 30$ ✔

Solve each equation. Check the solution.

1. $9j + 35 = 4j$

   $j =$ ________________

2. $13s = 2s - 66$

   $s =$ ________________

3. $2(5t - 4) = 12t$

   $t =$ ________________

4. $6q = 6(4q + 1)$

   $q =$ ________________

5. $7(t - 2) - t = 4$

   $t =$ ________________

6. $6w + 4 = 4w + 1$

   $w =$ ________________

Types of Solutions

If an equation is true for all values of *x*:		**If an equation is true for one value of *x*:**	
$a = a$		$x = a$	
infinitely many solutions		**one solution**	
$4x + 8 = 4(x + 2)$		$5x - 3 = -3x + 5$	
$4x + 8 = 4x + 8$	Distributive Property	$5x - 3 + 3 = -3x + 5 + 3$	Add
$4x + 8 - 4x = 4x + 8 - 4x$	Subtract	$5x = -3x + 8$	Simplify
$8 = 8$	Simplify	$5x + 3x = -3x + 3x + 8$	Add
		$8x = 8$	Divide
		$x = 1$	

If an equation is not true for any values of *x*:

$a = b$

no solutions

$6x + 2 = 6(x - 1)$	
$6x + 2 = 6x - 6$	Distributive Property
$6x - 6x + 2 = 6x - 6x - 6$	Subtract
$2 = -6$	

Tell whether each equation has one solution, infinitely many solutions, or no solution.

1. $3x - 2 = x + 6$ ________________

2. $5x - 10 = 5(x - 2)$ ________________

3. $6x - 1 = 6(x + 2)$ ________________

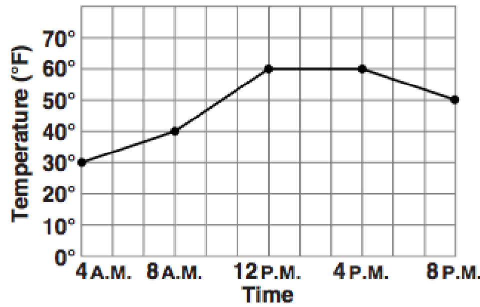
4. $8(x + 2) = 8x + 16$ ________________

5. $2(x - 3) = 2x + 4$ ________________

6. $x + 4 = 3(x - 2)$ ________________

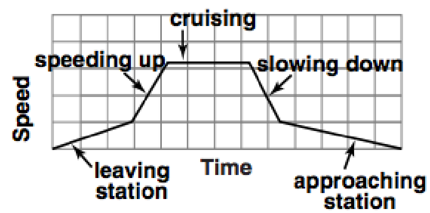
Reading Graphs

The graph at the right shows the outside temperature during 16 hours of one day.



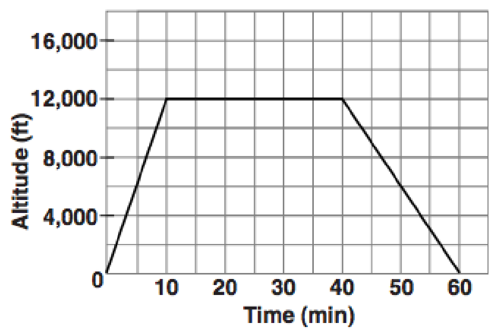
- You can see how the temperature changed throughout the day. *The temperature rose 10°F from 4 A.M. to 8 A.M. The temperature remained at 60°F for 4 hours, from 12 P.M. to 4 P.M.*

The graph at the right shows a train moving between stations. *The train moves slowly while leaving the station. Then it picks up speed until it reaches a cruising speed. It slows down as it approaches the next station and gradually comes to a stop.*



- Since the graph is *sketched* to show relationships, the axes do not need number scales. But the axes and the parts of the graph should have labels to show what they represent.

The graph at the right shows the altitude of an airplane during a flight. Use the graph for Exercises 1–3.



1. What was the airplane's altitude for most of the flight?

    ____

2. How long did it take the airplane to reach an altitude of 12,000 ft?

    ____

3. The third segment in the graph is not as steep as the first segment. What does this mean?

    ____

    ____

Function Rules

A *function* describes the relationship between two variables called the *input* and the *output*. In a function, each input value has only one output value.

Function:

$$y = 2x + 4$$

↑ *output variable y* ↑ *input variable x*

You can list input/output pairs in a table.

$y = 2x + 4$

Input x	Output y
−10	−16
−5	−6
0	4
1	6

To find output y, substitute values for input x into the function equation.

For $x = -10$: $y = 2(-10) + 4$
$y = -16$

You can also show input/output pairs using *function rules*.

Function rule:

$$y = 2x + 4$$
$$y = 2(-10) + 4 = -16$$

↑ *input* ↑ *output*

Find y when $x = 0$.

$$y = 2(0) + 4$$
$$y = 4$$

Use the function rule $y = 3x + 1$. Find each output.

4. y when $x = 0$.
$= 3(\_\_\_\_\_) + 1$
$= \_\_\_\_\_\_\_\_$

5. y when $x = 1$.
$= 3(\_\_\_\_\_) + 1$
$= \_\_\_\_\_\_\_\_$

6. y when $x = 5$.
____

7. y when $x = -6$.
____

Proportional Relationships

A proportional relationship is a relationship between inputs and outputs in which the ratio of inputs and outputs is always the same.

Gallons of Gas	Cost ($)	
1	3	1/3
2	6	2/6 = 1/3
3	9	3/9 = 1/3
4	12	4/12 = 1/3

← Write the ratio of each input to its corresponding output.

Then simplify.

The ratios are all the same, so the relationship is proportional.

Determine if the relationship is proportional.

1.

x	y
−3	−9
−1	−3
2	6
4	12

____

2.

m	n
6	8
15	20
24	32
36	48

____

Linear Functions

A function is linear if the relationship between the changes in variables is constant.

+1 +1 +2

x	1	2	3	5
y	3	6	9	15

+3 +3 +6

$\frac{1}{3}$ $\frac{1}{3}$ $\frac{2}{6} = \frac{1}{3}$

A function is not linear if the relationship between the changes in variables is not constant.

+2 +2 +2

x	2	4	6	8
y	4	6	10	16

+2 +4 +6

$\frac{2}{2} = 1$ $\frac{2}{4} = \frac{1}{2}$ $\frac{2}{6} = \frac{1}{3}$

Determine if the function represented in the table is linear.

x	−4	−2	1	5
y	−2	0	3	7

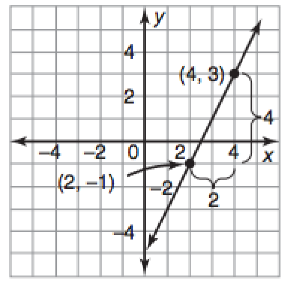
____

x	−2	−1	1	2
y	7	4	4	7

____

Finding Slope

The *slope of a line* is $\frac{\text{change in } y}{\text{change in } x}$, found by using two points on the line.



Find the slope of the line that passes through these two points: (4, 3) and (2, −1).

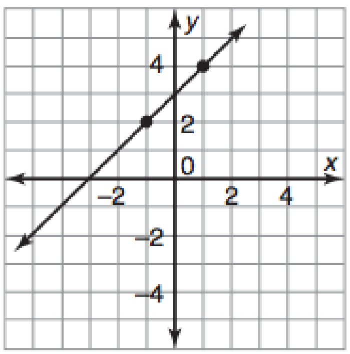
- To find the change in *y*, subtract one *y*-coordinate from the other: (3 − (−1)) = 4.
- To find the change in *x*, subtract one *x*-coordinate from the other: (4 −2) = 2.

When you find the slope of a line, the *y*-coordinate you use first for the rise must belong to the same point as the *x*-coordinate you use first for the run.

The slope of the line is: $\frac{\text{change in } y}{\text{change in } x} = \frac{3 - (-1)}{4 - 2} = \frac{4}{2} = 2$

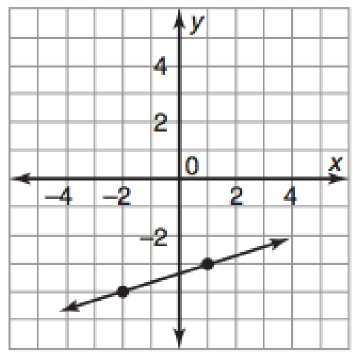
Find the slope of each line.

1.



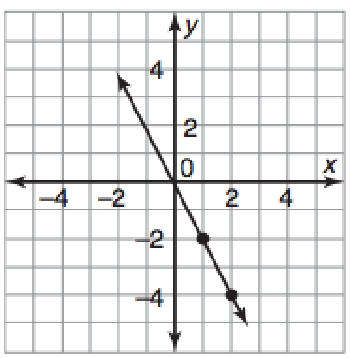
slope = ______________

2.



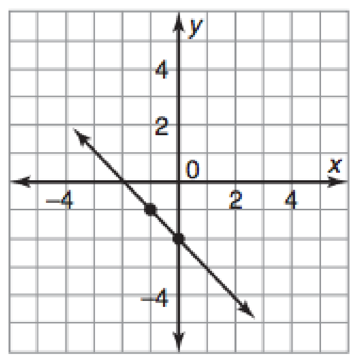
slope = ______________

3.



slope = ______________

4.



slope = ______________

Finding Slope Continued

Find the slope to compare the rate of change.

Use two values from a table.

x	1	2	3	4
y	1	4	7	10

(2, 4) and (4, 10)

slope $= \frac{10 - 4}{4 - 2} = \frac{6}{2} = 3$

Use the equation $y = mx + b$.

$y = 2x + 5$

$y = mx + b$.

↓ The slope is m, which is 2.

$y = 2x + 5$

$3 > 2$, so the function in the table has the greater rate of change.

For Questions 1–4, match each linear function with its rate of change.

1. Austin pays a registration fee of $10 plus $1 for every audiobook he borrows.

2.

x	1	3	4	6
y	5	9	11	15

3. (1, 5), (2, 9)

4. $y = 3x - 1$

A. 4

B. 3

C. 2

D. 1

Graphing Linear Functions

You can graph a function in the coordinate plane. To plot points for the graph, use *input* as x-values (x-axis) and *output* as y-values (y-axis).

output as y-values *input as x-values*

↓ ↓

$$y = 2x + 4$$

This function has the form of a linear equation and is called a *linear function.* To draw its graph, use

slope and y-intercept: $y = 2x + 4$

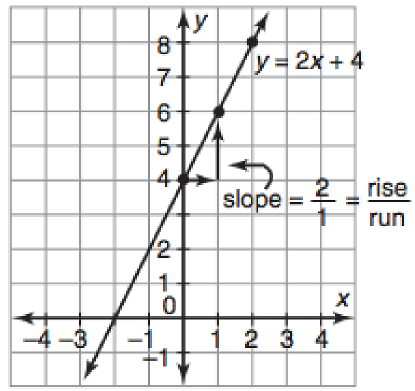
slope $= 2$

y-intercept $= 4$

or

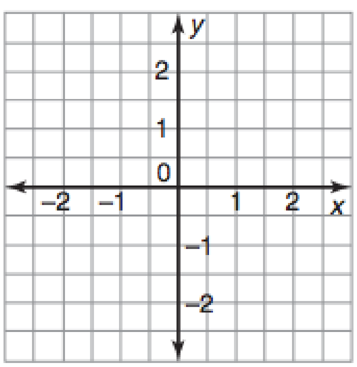
plot points from a table and connect them in a line.

x	*y*
0	4
1	6
2	8

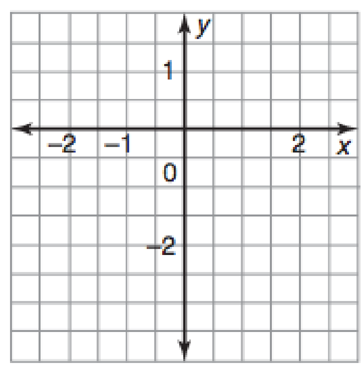


Graph each linear function.

1. $y = 3x$

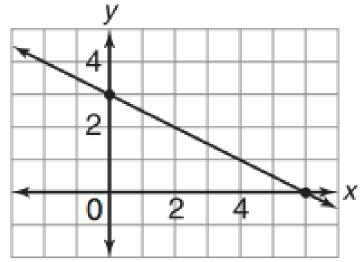


2. $y = 2x - 2$



Writing an Equation of a Line

You can use the graph of a linear function to write its function rule.
First, you need to find the slope and the y-intercept.



① From the graph, the slope (m) is $-\frac{1}{2}$.

② The point $(0, 3)$ is on the graph so the y-intercept (b) is 3.

③ Substitute in the slope-intercept form.

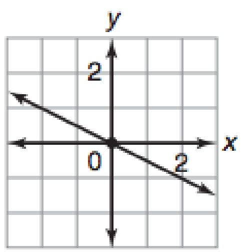
$$y = mx + b$$

$$y = -\frac{1}{2}x + 3$$

The function rule is $y = -\frac{1}{2}x + 3$.

Identify the slope and y-intercept of each graph. Then write a linear

3.



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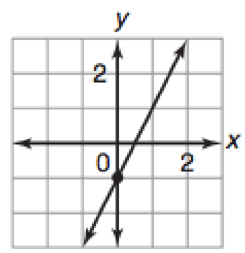
4.



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Solving Systems of Equations by Substitution

You can solve systems of equation by substitution.

$$-2x + 4y = 2$$
$$x + y = 8$$

Step 1	**Solve one of the equations for one of the variables.**
$x + y = 8$	← Write the second equation.
$y = -x + 8$	← Subtract x from both sides.

Step 2	**Substitute $-x + 8$ for y in the other equation.**
$-2x + 4y = 2$	← Write the first equation.
$-2x + 4(-x + 8) = 2$	← Substitute $-x + 8$ for y.
$-2x - 4x + 32 = 2$	← Use the Distributive Property.
$-6x + 32 = 2$	← Simplify.
$-6x = -30$	← Subtract 32 from each side.
$x = 5$	← Divide each side by -6.

Step 3	**Substitute 5 for x in either equation and solve for y.**
$x + y = 8$	← Write either equation.
$5 + y = 8$	← Substitute 5 for x.
$y = 3$	← Subtract 5 from both sides.
The solution is (5, 3).	

Solve each system by substitution. Check your answer.

1. $y = -x + 1$
$-2x - y = 2$

2. $2x + y = 6$
$6x - y = 2$

Solving Systems of Equations by Elimination

You can solve some systems of equations by adding.

Step 1: Eliminate one variable.

$2x + 3y = 12$

$\underline{x - 3y = -3}$

$3x + 0 = 9$ ← Add

$x = 3$ ← Solve for x.

Step 2: Substitute the value you found into one equation.

$2x + 3y = 12$ ← Write either equation.

$2(3) + 3y = 12$ ← Substitute 3 for x.

$6 + 3y = 12$ ← Simplify.

$3y = 6$ ← Divide by 3.

$y = 2$ ← Solve for y.

The solution is (3, 2).

Solve each system of equations by elimination. Check your solution.

1. $x + y = 9$

$x - y = 1$

____

2. $3x + 2y = 2$

$x - 2y = 6$

____

Scientific Notation

To write a number such as 67,000 in *scientific notation,* move the decimal point to form a number between 1 and 10. The number of places moved shows which power of 10 to use.

- Write 67,000 in scientific notation.

  6.7 is between 1 and 10. So, move the decimal point in 67,000 to the left 4 places and multiply by 10^4.

  $67{,}000 = 6.7 \times 10^4$

To write scientific notation in *standard form,* look at the exponent. The exponent shows the number of places and the direction to move the decimal point.

- Write 8.5×10^5 in standard form.

  The exponent is positive 5, so move the decimal point 5 places to the right.

  $8.5 \times 10^5 = 850{,}000$

Write each number in scientific notation.

1. 6,500 ______________________

2. 65,000 ______________________

3. 6,520 ______________________

Write each number in standard form.

10. 4×10^4 ______________________

11. 4×10^5 ______________________

Multiplying Exponents

- To multiply numbers or variables with the same base, add the exponents.

Simplify $3^2 \cdot 3^4$

$3^2 \cdot 3^4 = 3^{(2+4)}$

$= 3^6$

Simplify $n^3 \cdot n^4$

$n^3 \cdot n^4 = n^{(3+4)}$

$= n^7$

Simplify $(-4)^3 \cdot (-4)^5$

$(-4)^3 \cdot (-4)^5 = (-4)^{(3+5)}$

$= (-4)^8$

- You can also simplify expressions with exponents.

$6x^2 \cdot -2x^5 = 6 \cdot -2 \cdot x^2 \cdot x^5$ ← Use the Commutative Property of Multiplication

$= -12x^{(2+5)}$ ← Add the exponents.

$= -12x^7$ ← Simplify.

Write each expression using a single exponent.

1. $5^3 \cdot 5^4$ ______________________

2. $a^2 \cdot a^5$ ______________________

3. $(-8)^4 \cdot (-8)^5$ ______________________

Find each product. Write the answer in scientific notation.

10. $2x^3 \cdot x^2$ ______________________

11. $-4x^3 \cdot 2x^4$ ______________________

Multiplying Scientific Notation

- To multiply numbers in scientific notation.

Find the product $(5 \times 10^4)(7 \times 10^5)$. Write the result in scientific notation.

$(5 \times 10^4)(7 \times 10^5)$

$(5 \cdot 7)(10^4 \cdot 10^5)$ ← Use the Associative and Commutative properties.

$35 \times (10^4 \cdot 10^5)$ ← Multiply 5 and 7.

$35 \times 10^{4+5}$ ← Add the exponents for the powers of 10.

35×10^9

$3.5 \times 10^1 \times 10^9$ ← Write 35 in scientific notation.

3.5×10^{10} ← Add the exponents.

Find each product. Write the answer in scientific notation.

1. $(3 \times 10^4)(5 \times 10^3)$ ________________

2. $(2 \times 10^3)(7 \times 10^6)$ ________________

3. $(8 \times 10^2)(5 \times 10^2)$ ________________

4. $(9 \times 10^4)(7 \times 10^4)$ ________________

Dividing Exponents

To divide powers with the same base, subtract exponents.

$$\frac{8^6}{8^4} = 8^{6-4} = 8^2 = 64 \qquad \frac{a^5}{a^3} = a^{5-3} = a^2$$

- For any nonzero number a, $a^0 = 1$.

$3^0 = 1$ $\qquad (-6)^0 = 1$ $\qquad 4t^0 = 4(1) = 4$

- For any nonzero number a and any integer n, $a^{-n} = \frac{1}{a^n}$.

$$2^{-4} = \frac{1}{2^4} = \frac{1}{16} \qquad 3c^{-2} = \frac{3}{c^2} \qquad \frac{5^3}{5^6} = 5^{3-6} = 5^{-3} = \frac{1}{5^3} = \frac{1}{125} \qquad \frac{10z^3}{5z} = 2z^{3-1} = 2z^2$$

Simplify each expression.

1. $\frac{6^5}{6^3} =$ ______________
2. $(-4)^5 \div (-4)^3 =$ ________
3. $(-3)^{-2} =$ ______________
4. $\frac{2^5}{2^7} =$ ______________
5. $(-8)^0 =$ ______________
6. $\frac{5^0}{5^2} =$ ______________

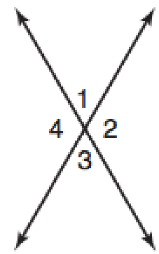
Simplify each expression. Write your answer using only positive exponents.

10. $w^8 \div w^3 =$ ____________
11. $x^6 \div x^1 =$ ____________
12. $\frac{d^7}{d^3} =$ ______________

Angles

- *Vertical angles* are pairs of opposite angles formed by two intersecting lines. They are congruent.

  Example 1: ∠1 and ∠3, ∠4 and ∠2



- *Adjacent angles* have a common vertex and a common side, but no common interior points.

  Example 2: ∠1 and ∠2, ∠1 and ∠4

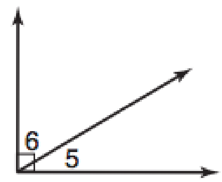
- Two *supplementary angles* form a 180° angle.

  Example 3: ∠1 and ∠4 are supplementary angles.
  ∠3 is also a supplement of ∠4.

If you know the measure of one supplementary angle, you can find the measure of the other. → If $m\angle 4$ is 120°, then $m\angle 1$ is 180° − 120°, or 60°.

- Two *complementary angles* form a 90° angle.

  Example 4: ∠5 and ∠6 are complementary angles.
  ∠6 is a complement of ∠5.



If you know the measure of one complementary angle, you can find the measure of the other. → If $m\angle 5$ is 30°, then $m\angle 6$ is 90° − 30°, or 60°.

Use the diagrams at the right for Exercises 1–5.

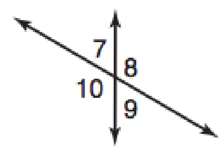
1. Vertical angles: ∠7 and ______________________

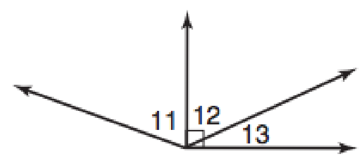
2. Adjacent angles: ∠10 and ____________________

3. Supplementary angles: ∠8 and _______________

4. Complementary angles: ∠12 and _____________

5. Vertical angles: ∠8 and ______________________





Find the measure of the supplement of each angle.

6. 38° ______________________

7. 65° ______________________

Find the measure of the complement of each angle.

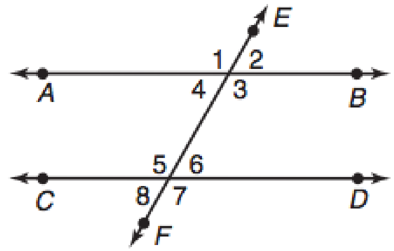
9. 25° ______________________

10. 18° ______________________

Parallel Lines and Angles

Look at the figure at the right.

- Line $\overleftrightarrow{AB}$ is parallel to line $\overleftrightarrow{CD}$ ($\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$)
- Line $\overleftrightarrow{EF}$ is a *transversal.*



Alternate interior angles lie within a pair of lines and on opposite sides of the transversal.

Example 1: $\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$

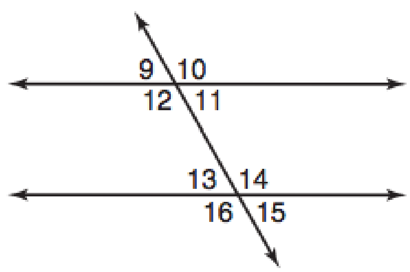
Alternate interior angles are congruent. If $m\angle 4$ is 60°, then $m\angle 6$ is also 60°.

Corresponding angles lie on the same side of the transversal and in corresponding positions.

Example 2: $\angle 1$ and $\angle 5$, $\angle 3$ and $\angle 7$

Corresponding angles are congruent. If $m\angle 1$ is 120°, then $m\angle 5$ is also 120°.

Use the diagram at the right to complete Exercises 1–2.



1. Name the alternate interior angles.

a. $\angle 11$ and $\angle$ _?_

____

b. $\angle 12$ and $\angle$ _?_

____

2. Name the corresponding angles.

a. $\angle 16$ and $\angle$ _?_

____

b. $\angle 14$ and $\angle$ _?_

____

c. $\angle 9$ and $\angle$ _?_

____

d. $\angle 11$ and $\angle$ _?_

____

In the diagram at the right, $\ell \parallel m$. Find the measure of each angle.

3. $\angle 1$

____

4. $\angle 3$

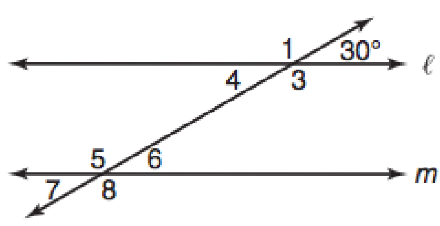
____

5. $\angle 6$

____

6. $\angle 5$

____



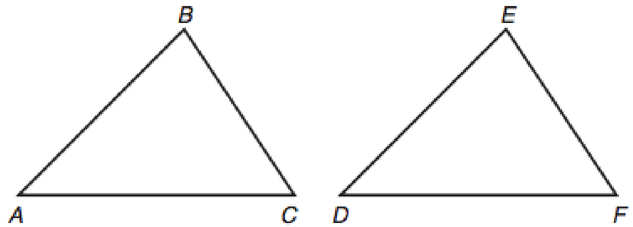
Congruence

Congruence statements reveal corresponding parts. $\triangle ABC \cong \triangle DEF$

Example 1: $\overline{AB}$ corresponds to $\overline{DE}$.
$\angle C$ corresponds to $\angle F$.

Corresponding parts are congruent ($\cong$).

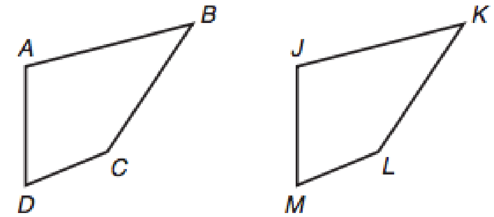
Example 2: $\overline{AB} \cong \overline{DE}$
$\angle C \cong \angle F$



In the diagram at the right, *ABCD* ≅ *JKLM*.
Complete the following.

1. $\angle A \cong$ ______________ **2.** $\overline{KL} \cong$ ______________

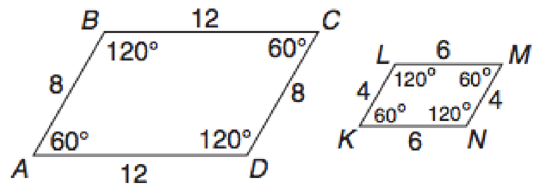
3. $\angle M \cong$ ______________ **4.** $\overline{DC} \cong$ ______________



Similarity

Similar polygons have congruent corresponding angles and corresponding sides that are in proportion.
The symbol ~ means *is similar to*.

Example: Is parallelogram
ABCD ~ parallelogram *KLMN*?



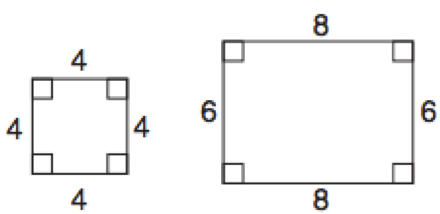
① Check corresponding angles. $\angle A \cong \angle K$, $\angle B \cong \angle L$, $\angle C \cong \angle M$, and $\angle D \cong \angle N$

② Compare corresponding sides. $\frac{AB}{KL} = \frac{8}{4} = \frac{2}{1}$ $\frac{BC}{LM} = \frac{12}{6} = \frac{2}{1}$

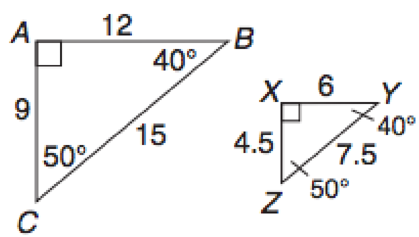
$\frac{CD}{MN} = \frac{8}{4} = \frac{2}{1}$ $\frac{DA}{NK} = \frac{12}{6} = \frac{2}{1}$

Tell whether each pair of polygons is similar. Explain why or why not.

1.



2.



3

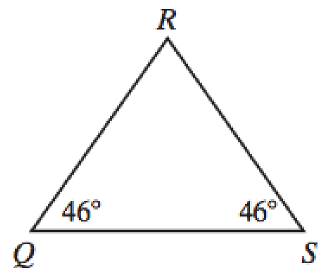
______________________________ ______________________________

______________________________ ______________________________

Angles in Triangles

The angles of a triangle add to 180°.

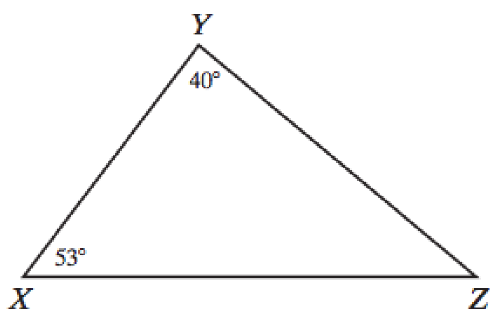
You can use the angle sum to find a missing angle measure.



$m\angle Q + m\angle R + m\angle S = 180°$	← Angle sum.
$46° + m\angle R + 46° = 180°$	← Substitute.
$92° + m\angle R = 180°$	← Simplify.
$92° - 92° + m\angle R = 180° - 92°$	← Subtract.
$m\angle R = 88°$	← Simplify.

Determine the unknown angle measure in each triangle.

1.



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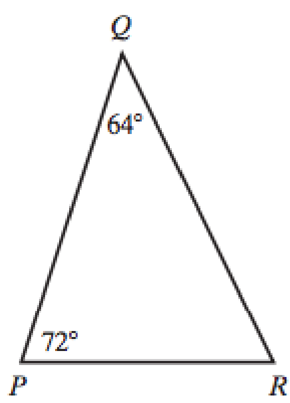
2.



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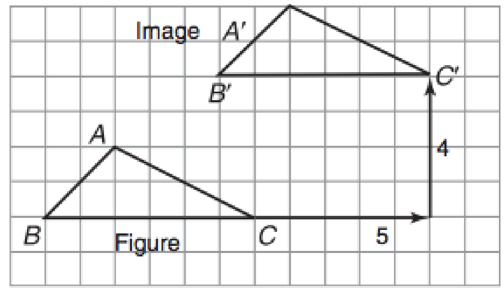
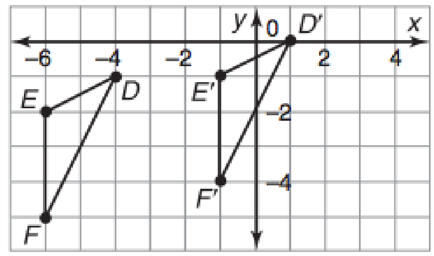
Translations

A *translation* moves every point of a figure the same distance in the same direction.



Triangle ABC is translated 5 units to the right and 4 units up. The *image* of $\triangle ABC$ is $\triangle A'B'C'$.

You can write a rule to describe a translation in the coordinate plane.



To get the translation of $\triangle DEF$, you have to add 5 to each x-coordinate and add 1 to each y-coordinate.

$D(-4, -1) \rightarrow D'(1, 0)$

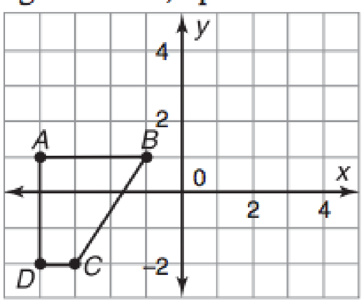
$E(-6, -2) \rightarrow E'(-1, -1)$

$F(-6, -5) \rightarrow F'(-1, -4)$

$(x, y) \rightarrow (x + 5, y + 1)$

Copy each figure. Then graph the image after the given translation. Name the coordinates of the image.

1. right 5 units, up 1 unit



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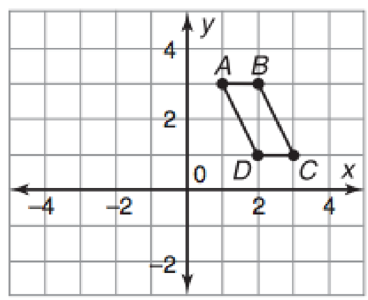
2. left 3 units, down 2 units



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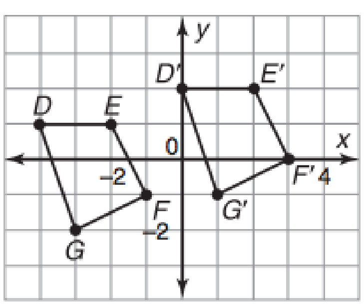
Use arrow notation to write a rule that describes the translation shown on each graph.

3.



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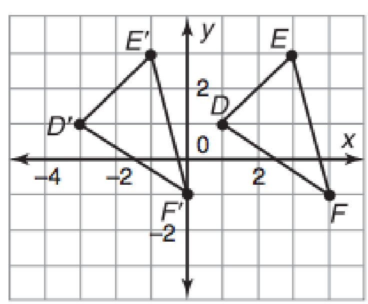
4.



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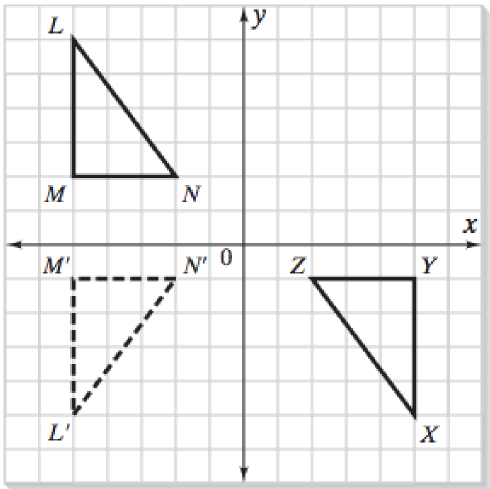
Transformations and Congruence

You can use transformations to determine congruence.

Determine whether the two triangles are congruent. If so, write a congruence statement.

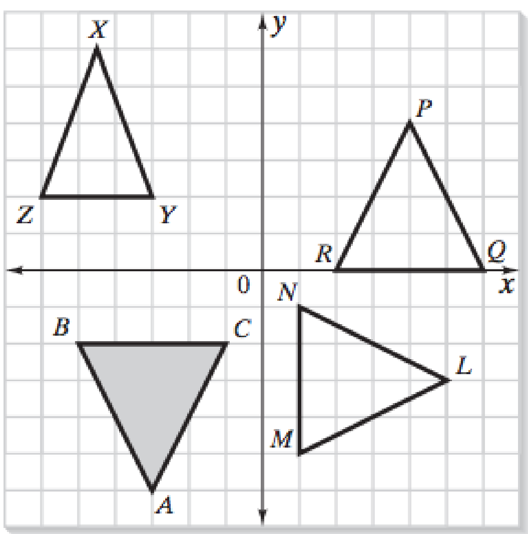


Sample method:
The triangles are on opposite sides of the x-axis. Start by reflecting ΔLMN over the x-axis to get $\Delta L'M'N'$.

$\Delta L'M'N'$ and ΔXYZ are on opposite sides of the y-axis. Reflect $\Delta L'M'N'$ over the x-axis to get ΔXYZ.

A reflection over the x-axis followed by a reflection over the y-axis maps ΔLMN onto ΔXYZ. So $\Delta LMN \cong \Delta XYZ$.

Determine which triangles, if any, are congruent to ΔABC.



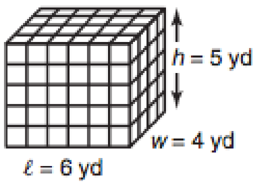
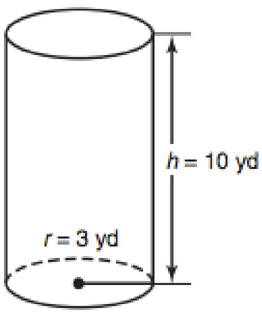
1. ΔPQR Yes ○ No ○

2. ΔXYZ Yes ○ No ○

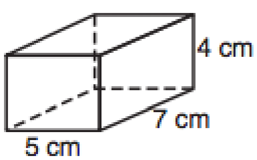
3. ΔPQR Yes ○ No ○

<u>Volume of Prisms and Cylinders</u>

To find the volume of a prism or a cylinder, multiply the base area B and the height h.

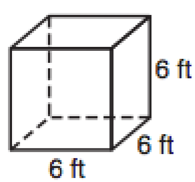
	① Find the base area B.	② Multiply base area B and height h. $V = Bh$
	$B = \ell w$ $= 6 \cdot 4$ $= 24 \text{ yd}^2$	$V = Bh$ $= 24 \cdot 5$ $= 120 \text{ yd}^3$ The volume is 120 yd^3.
	$B = \pi r^2$ $= \pi \cdot 3^2$ $\approx 28.26 \text{ yd}^2$	$V = Bh$ $\approx 28.26 \text{ yd}^2 \times 10$ $\approx 282.6 \text{ yd}^3$ The volume is about 283 yd^3.

Find the base area and volume of each prism.

1. 

$B =$ ______________

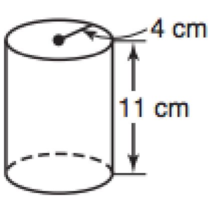
$V =$ ______________

2. 

$B =$ ______________

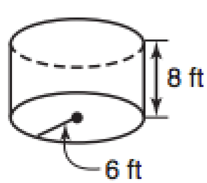
$V =$ ______________

Find the base area of each cylinder to the nearest hundredth. Then find the volume of each cylinder to the nearest cubic unit.

4. 

$B \approx$ ______________

$V \approx$ ______________

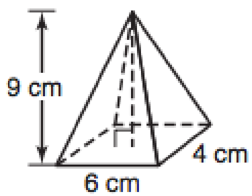
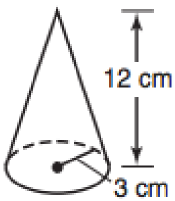
5. 

$B \approx$ ______________

$V \approx$ ______________

Volume of Cones and Pyramids

To find the volume of a pyramid or cone, multiply $\frac{1}{3}$, the base area B, and the height h.

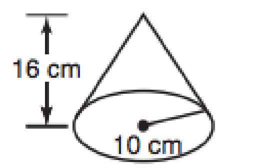
	① Find the base area B.	② Multiply $\frac{1}{3}$, the base area B, and the height h. $V = \frac{1}{3}Bh$
	$B = \ell w$ $= 6 \cdot 4$ $= 24 \text{ cm}^2$	$V = \frac{1}{3}Bh$ $= \frac{1}{3}(24)(9)$ $= 72 \text{ cm}^3$ The volume is 72 cm^3.
	$B = \pi r^2$ $= \pi \cdot 3^2$ $\approx 28.26 \text{ cm}^2$	$V = \frac{1}{3}Bh$ $\approx \frac{1}{3}(28.26)(12)$ $\approx 113.04 \text{ cm}^3$ The volume is about 113 cm^3.

Find the volume of each figure to the nearest whole cubic unit.

1.

____

2.



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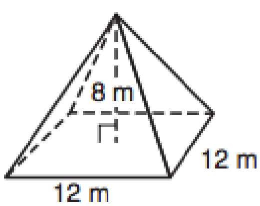
3.



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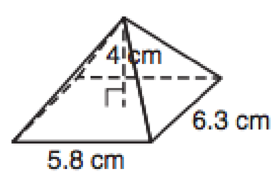
4.



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Spheres

Find the surface area and volume of a beach ball with a radius of 8 inches.

The *surface area* of a sphere is four times the product of π and the square of the radius r.

$$\text{S.A.} = 4\pi r^2 \quad \longleftarrow \text{Surface area of a sphere}$$
$$= 4\pi(8^2) \quad \longleftarrow \text{Substitute.}$$
$$= 256\pi \quad \longleftarrow \text{Simplify.}$$
$$\approx 804 \quad \longleftarrow \text{Use a calculator.}$$

The surface area of the beach ball is about 804 in.^{2}.

The *volume* of a sphere is four-thirds of the product of π and the radius r cubed.

$$V = \tfrac{4}{3}\pi r^3 \quad \longleftarrow \text{Volume of a sphere}$$
$$= \tfrac{4}{3}\pi(8^3) \quad \longleftarrow \text{Substitute.}$$
$$= \tfrac{2{,}048}{3}\pi \quad \longleftarrow \text{Simplify.}$$
$$\approx 2{,}145 \quad \longleftarrow \text{Use a calculator.}$$

The volume of the beach ball is about 2.145 in.^{3}.

A glass blower sells opalescent glass spheres. Find the surface area and volume of each sphere to the nearest whole number.

1. blue: $r = 2$ in.

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____

2. green: $d = 9$ cm

____

____

<u>Scatter Plots</u>

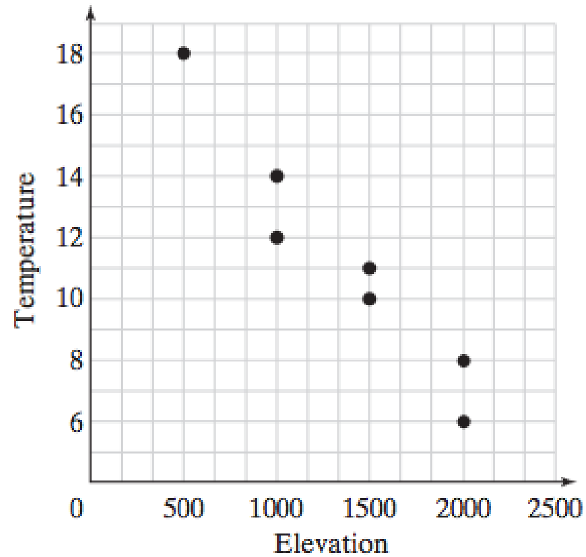
You can make a scatterplot to show data.

Elevation (m)	500	1000	1000	1500	1500	2000	2000
Temperature (°C)	18	14	12	11	10	8	6

Step 1 Use the horizontal axis to represent elevation. The elevation ranges from 500 to 2,000. A reasonable scale is 0 to 2,000 where each grid line increases by 500.

Step 2 Use the vertical axis to represent the temperature. The temperature ranges from 18°C to 6°C. A reasonable scale is 0 to 20 where each grid line increases by 2°C.

Step 3 Plot the data. For example, at an elevation of 500 m, the temperature is 18°C. Plot (500, 18).



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1. What information is shown on the horizontal axis of the scatter plot?

____

2. What information is shown on the vertical axis of the scatter plot?

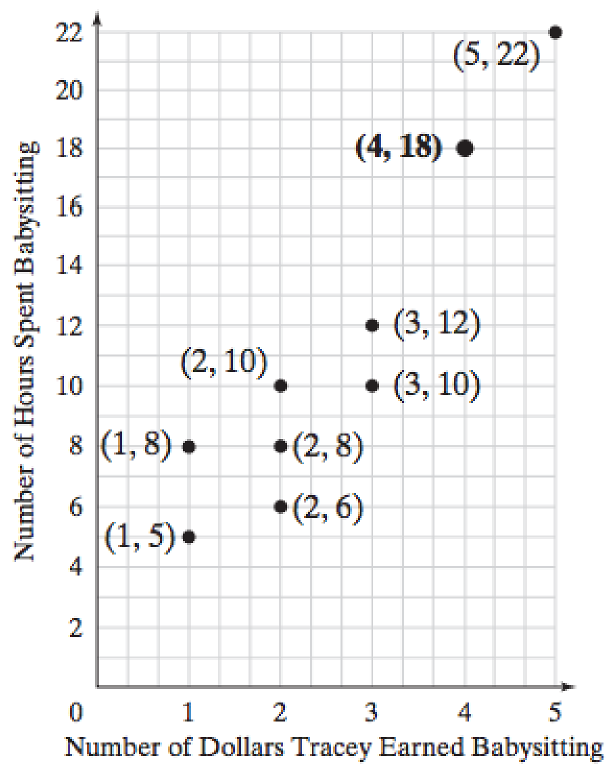
____

3. What does the highlighted point represent?

____

4. How many hours did she have to babysit to earn $22?

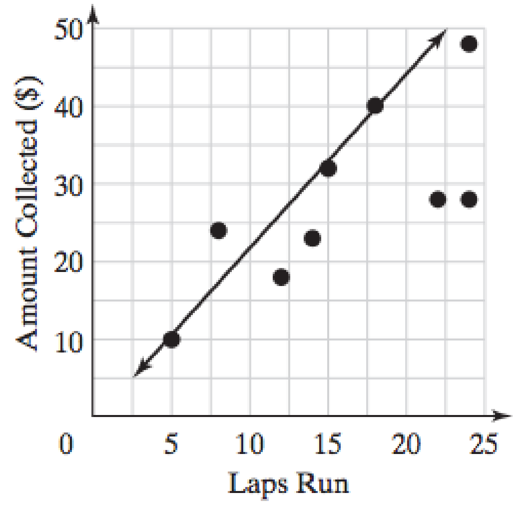
____



Linear Models

A trend line is a line you draw on a graph to approximate the relationship between data sets.

- To find the trend line, first plot the data.
- Then look for a trend. Draw a line that has a slope with the same trend.
- Make sure there are about as many points above the line as below it.
- You can use two points on the trend line to calculate its slope. Then you can use the slope and estimate the y-intercept to write an equation to describe the line.
- You can use a trend line equation to estimate values and make predictions.



Number of Laps	5	8	12	15	18	22	24	14
Amount Collected ($)	10	24	18	32	40	28	48	23

Plot the data and label the graph.

Seeding Height (cm)	9	14	16	20	38	42	54	62
Day	5	8	12	16	22	25	28	30

1. Draw a trend line to represent the data.
2. Find the slope of the trend line.

   ____

3. Estimate the y-intercept of the trend line.

   ____

4. Write an equation to describe the trend line.

   ____

5. Use the equation to predict the seedling height on day 45.

   ____

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