GLENCOE MATHEMATICS

Geometry

Chapter 7
Resource Masters

Glencoe

New York, New York
Columbus, Ohio
Chicago, Illinois
Peoria, Illinois
Woodland Hills, California

Consumable Workbooks

Many of the worksheets contained in the Chapter Resource Masters booklets are available as consumable workbooks.

Study Guide and Intervention Workbook	0-07-860191-6
Skills Practice Workbook	0-07-860192-4
Practice Workbook	0-07-860193-2
Reading to Learn Mathematics Workbook	0-07-861061-3

ANSWERS FOR WORKBOOKS The answers for Chapter 7 of these workbooks can be found in the back of this Chapter Resource Masters booklet.

Glencoe

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Geometry
Chapter 7 Resource Masters

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Contents

Teacher's Guide to Using the Chapter 7 Resource Masters

The ***Fast File*** Chapter Resource system allows you to conveniently file the resources you use most often. The *Chapter 7 Resource Masters* includes the core materials needed for Chapter 7. These materials include worksheets, extensions, and assessment options. The answers for these pages appear at the back of this booklet.

All of the materials found in this booklet are included for viewing and printing in the *Geometry TeacherWorks* CD-ROM.

Vocabulary Builder Pages vii–viii include a student study tool that presents up to twenty of the key vocabulary terms from the chapter. Students are to record definitions and/or examples for each term. You may suggest that students highlight or star the terms with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 7-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to add definitions and examples as they complete each lesson.

Vocabulary Builder Pages ix–x include another student study tool that presents up to fourteen of the key theorems and postulates from the chapter. Students are to write each theorem or postulate in their own words, including illustrations if they choose to do so. You may suggest that students highlight or star the theorems or postulates with which they are not familiar.

WHEN TO USE Give these pages to students before beginning Lesson 7-1. Encourage them to add these pages to their Geometry Study Notebook. Remind them to update it as they complete each lesson.

Study Guide and Intervention
Each lesson in *Geometry* addresses two objectives. There is one Study Guide and Intervention master for each objective.

WHEN TO USE Use these masters as reteaching activities for students who need additional reinforcement. These pages can also be used in conjunction with the Student Edition as an instructional tool for students who have been absent.

Skills Practice There is one master for each lesson. These provide computational practice at a basic level.

WHEN TO USE These masters can be used with students who have weaker mathematics backgrounds or need additional reinforcement.

Practice There is one master for each lesson. These problems more closely follow the structure of the Practice and Apply section of the Student Edition exercises. These exercises are of average difficulty.

WHEN TO USE These provide additional practice options or may be used as homework for second day teaching of the lesson.

Reading to Learn Mathematics
One master is included for each lesson. The first section of each master asks questions about the opening paragraph of the lesson in the Student Edition. Additional questions ask students to interpret the context of and relationships among terms in the lesson. Finally, students are asked to summarize what they have learned using various representation techniques.

WHEN TO USE This master can be used as a study tool when presenting the lesson or as an informal reading assessment after presenting the lesson. It is also a helpful tool for ELL (English Language Learner) students.

Enrichment There is one extension master for each lesson. These activities may extend the concepts in the lesson, offer an historical or multicultural look at the concepts, or widen students' perspectives on the mathematics they are learning. These are not written exclusively for honors students, but are accessible for use with all levels of students.

WHEN TO USE These may be used as extra credit, short-term projects, or as activities for days when class periods are shortened.

Assessment Options

The assessment masters in the *Chapter 7 Resources Masters* offer a wide range of assessment tools for intermediate and final assessment. The following lists describe each assessment master and its intended use.

Chapter Assessment

CHAPTER TESTS

- *Form 1* contains multiple-choice questions and is intended for use with basic level students.
- *Forms 2A and 2B* contain multiple-choice questions aimed at the average level student. These tests are similar in format to offer comparable testing situations.
- *Forms 2C and 2D* are composed of free-response questions aimed at the average level student. These tests are similar in format to offer comparable testing situations. Grids with axes are provided for questions assessing graphing skills.
- *Form 3* is an advanced level test with free-response questions. Grids without axes are provided for questions assessing graphing skills.

  All of the above tests include a free-response Bonus question.
- The **Open-Ended Assessment** includes performance assessment tasks that are suitable for all students. A scoring rubric is included for evaluation guidelines. Sample answers are provided for assessment.
- A **Vocabulary Test**, suitable for all students, includes a list of the vocabulary words in the chapter and ten questions assessing students' knowledge of those terms. This can also be used in conjunction with one of the chapter tests or as a review worksheet.

Intermediate Assessment

- Four free-response **quizzes** are included to offer assessment at appropriate intervals in the chapter.
- A **Mid-Chapter Test** provides an option to assess the first half of the chapter. It is composed of both multiple-choice and free-response questions.

Continuing Assessment

- The **Cumulative Review** provides students an opportunity to reinforce and retain skills as they proceed through their study of Geometry. It can also be used as a test. This master includes free-response questions.
- The **Standardized Test Practice** offers continuing review of geometry concepts in various formats, which may appear on the standardized tests that they may encounter. This practice includes multiple-choice, grid-in, and short-response questions. Bubble-in and grid-in answer sections are provided on the master.

Answers

- Page A1 is an answer sheet for the Standardized Test Practice questions that appear in the Student Edition on pages 398–399. This improves students' familiarity with the answer formats they may encounter in test taking.
- The answers for the lesson-by-lesson masters are provided as reduced pages with answers appearing in red.
- Full-size answer keys are provided for the assessment masters in this booklet.

7 Reading to Learn Mathematics

Vocabulary Builder

This is an alphabetical list of the key vocabulary terms you will learn in Chapter 7. As you study the chapter, complete each term's definition or description. Remember to add the page number where you found the term. Add these pages to your Geometry Study Notebook to review vocabulary at the end of the chapter.

Vocabulary Term	Found on Page	Definition/Description/Example
ambiguous case		
angle of depression		
angle of elevation		
cosine		
geometric mean		
Law of Cosines		
Law of Sines		
Pythagorean identity puh·thag·uh·REE·ahn		

(continued on the next page)

NAME ____ DATE ____________ PERIOD _____

7

Reading to Learn Mathematics

Vocabulary Builder *(continued)*

Vocabulary Term	Found on Page	Definition/Description/Example
Pythagorean triple		
reciprocal identity ri·SIP·ruh·kuhl		
sine		
solve a triangle		
tangent		
trigonometric identity trig·uh·nuh·MET·rik		
trigonometric ratio		
trigonometry		

NAME ______________________________ DATE __________ PERIOD _____

7 Learning to Read Mathematics

Proof Builder

Proof Builder

This is a list of key theorems and postulates you will learn in Chapter 7. As you study the chapter, write each theorem or postulate in your own words. Include illustrations as appropriate. Remember to include the page number where you found the theorem or postulate. Add this page to your Geometry Study Notebook so you can review the theorems and postulates at the end of the chapter.

Theorem or Postulate	Found on Page	Description/Illustration/Abbreviation
Theorem 7.1		
Theorem 7.2		
Theorem 7.3		
Theorem 7.4 *Pythagorean Theorem*		
Theorem 7.5 *Converse of the Pythagorean Theorem*		
Theorem 7.6		
Theorem 7.7		

NAME ____ DATE ____________ PERIOD _____

7-1 Study Guide and Intervention

Geometric Mean

Geometric Mean The **geometric mean** between two numbers is the square root of their product. For two positive numbers a and b, the geometric mean of a and b is the positive number x in the proportion $\frac{a}{x} = \frac{x}{b}$. Cross multiplying gives $x^2 = ab$, so $x = \sqrt{ab}$.

Example **Find the geometric mean between each pair of numbers.**

a. 12 and 3

Let x represent the geometric mean.

$\frac{12}{x} = \frac{x}{3}$ Definition of geometric mean

$x^2 = 36$ Cross multiply.

$x = \sqrt{36}$ or 6 Take the square root of each side.

b. 8 and 4

Let x represent the geometric mean.

$\frac{8}{x} = \frac{x}{4}$

$x^2 = 32$

$x = \sqrt{32}$

≈ 5.7

Exercises

Find the geometric mean between each pair of numbers.

1. 4 and 4

2. 4 and 6

3. 6 and 9

4. $\frac{1}{2}$ and 2

5. $2\sqrt{3}$ and $3\sqrt{3}$

6. 4 and 25

7. $\sqrt{3}$ and $\sqrt{6}$

8. 10 and 100

9. $\frac{1}{2}$ and $\frac{1}{4}$

10. $\frac{2\sqrt{2}}{5}$ and $\frac{3\sqrt{2}}{5}$

11. 4 and 16

12. 3 and 24

The geometric mean and one extreme are given. Find the other extreme.

13. $\sqrt{24}$ is the geometric mean between a and b. Find b if $a = 2$.

14. $\sqrt{12}$ is the geometric mean between a and b. Find b if $a = 3$.

Determine whether each statement is *always*, *sometimes*, or *never* true.

15. The geometric mean of two positive numbers is greater than the average of the two numbers.

16. If the geometric mean of two positive numbers is less than 1, then both of the numbers are less than 1.

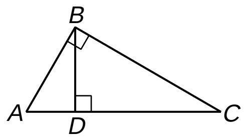
Lesson 7-1

7-1 Study Guide and Intervention *(continued)*

Geometric Mean

Altitude of a Triangle

In the diagram, $\triangle ABC \sim \triangle ADB \sim \triangle BDC$. An altitude to the hypotenuse of a right triangle forms two right triangles. The two triangles are similar and each is similar to the original triangle.



Example 1 **Use right $\triangle ABC$ with $\overline{BD} \perp \overline{AC}$. Describe two geometric means.**

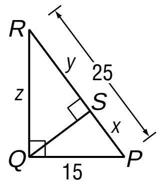
a. $\triangle ADB \sim \triangle BDC$ **so** $\frac{AD}{BD} = \frac{BD}{CD}$.

In $\triangle ABC$, the altitude is the geometric mean between the two segments of the hypotenuse.

b. $\triangle ABC \sim \triangle ADB$ **and** $\triangle ABC \sim \triangle BDC$, **so** $\frac{AC}{AB} = \frac{AB}{AD}$ **and** $\frac{AC}{BC} = \frac{BC}{DC}$.

In $\triangle ABC$, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Example 2 **Find x, y, and z.**



$\frac{PR}{PQ} = \frac{PQ}{PS}$

$\frac{25}{15} = \frac{15}{x}$ $\quad$ $PR = 25$, $PQ = 15$, $PS = x$

$25x = 225$ $\quad$ Cross multiply.

$x = 9$ $\quad$ Divide each side by 25.

Then

$y = PR - SP$

$= 25 - 9$

$= 16$

$\frac{PR}{QR} = \frac{QR}{RS}$

$\frac{25}{z} = \frac{z}{y}$ $\quad$ $PR = 25$, $QR = z$, $RS = y$

$\frac{25}{z} = \frac{z}{16}$ $\quad$ $y = 16$

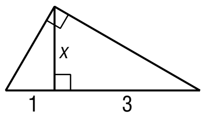
$z^2 = 400$ $\quad$ Cross multiply.

$z = 20$ $\quad$ Take the square root of each side.

Exercises

Find x, y, and z to the nearest tenth.

1.



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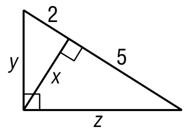

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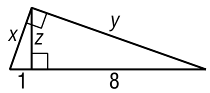

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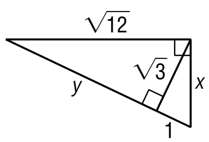

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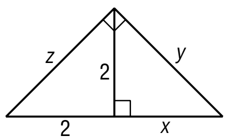

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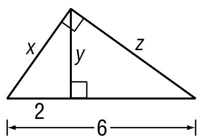


7-1 Skills Practice

Geometric Mean

Find the geometric mean between each pair of numbers. State exact answers and answers to the nearest tenth.

1. 2 and 8

2. 9 and 36

3. 4 and 7

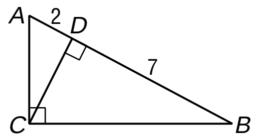
4. 5 and 10

5. $2\sqrt{2}$ and $5\sqrt{2}$

6. $3\sqrt{5}$ and $5\sqrt{5}$

Find the measure of each altitude. State exact answers and answers to the nearest tenth.

7.



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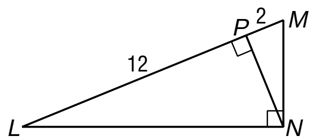

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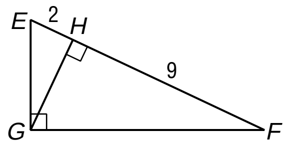

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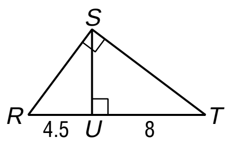
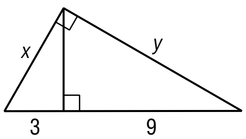


Find *x* and *y*.

11.



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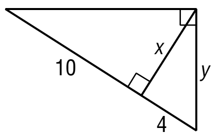

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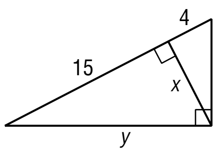

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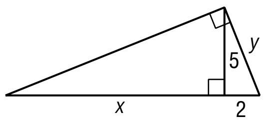


NAME _______________________________________ DATE ___________ PERIOD _____

7-1 Practice

Geometric Mean

Find the geometric mean between each pair of numbers to the nearest tenth.

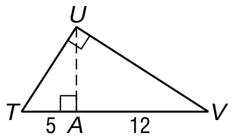
1. 8 and 12

2. $3\sqrt{7}$ and $6\sqrt{7}$

3. $\frac{4}{5}$ and 2

Find the measure of each altitude. State exact answers and answers to the nearest tenth.

4.



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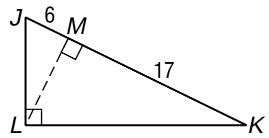
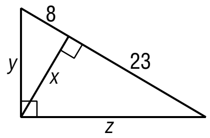


Find *x*, *y*, and *z*.

6.



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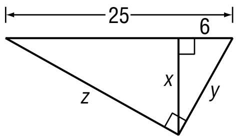

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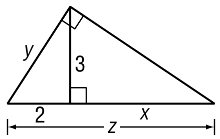

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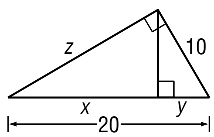


10. CIVIL ENGINEERING An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 6.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth.

NAME ______________________________ DATE ____________ PERIOD _____

7-1 Reading to Learn Mathematics

Geometric Mean

Pre-Activity **How can the geometric mean be used to view paintings?**

Read the introduction to Lesson 7-1 at the top of page 342 in your textbook.

- What is a disadvantage of standing too close to a painting?
- What is a disadvantage of standing too far from a painting?

Lesson 7-1

Reading the Lesson

1. In the past, when you have seen the word *mean* in mathematics, it referred to the *average* or *arithmetic mean* of the two numbers.

a. Complete the following by writing an algebraic expression in each blank.

If a and b are two positive numbers, then the geometric mean between a and b is

________ and their arithmetic mean is ________.

b. Explain in words, without using any mathematical symbols, the difference between the geometric mean and the algebraic mean.

2. Let r and s be two positive numbers. In which of the following equations is z equal to the geometric mean between r and s?

A. $\frac{s}{z} = \frac{z}{r}$ **B.** $\frac{r}{z} = \frac{s}{z}$ **C.** $s:z = z:r$ **D.** $\frac{r}{z} = \frac{z}{s}$ **E.** $\frac{z}{r} = \frac{z}{s}$ **F.** $\frac{z}{s} = \frac{r}{z}$

3. Supply the missing words or phrases to complete the statement of each theorem.

a. The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the ________________ between the measures of the two segments of the ________________.

b. If the altitude is drawn from the vertex of the ________________ angle of a right triangle to its hypotenuse, then the measure of a ________________ of the triangle is the ________________ between the measure of the hypotenuse and the segment of the ________________ adjacent to that leg.

c. If the altitude is drawn from the ________________ of the right angle of a right triangle to its ________________, then the two triangles formed are ________________ to the given triangle and to each other.

Helping You Remember

4. A good way to remember a new mathematical concept is to relate it to something you already know. How can the meaning of *mean* in a proportion help you to remember how to find the geometric mean between two numbers?

7-1 Enrichment

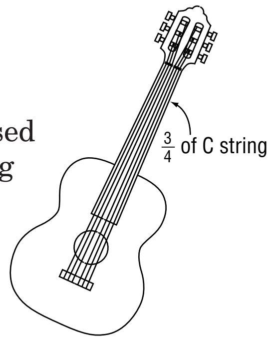
Mathematics and Music

Pythagoras, a Greek philosopher who lived during the sixth century B.C., believed that all nature, beauty, and harmony could be expressed by whole-number relationships. Most people remember Pythagoras for his teachings about right triangles. (The sum of the squares of the legs equals the square of the hypotenuse.) But Pythagoras also discovered relationships between the musical notes of a scale. These relationships can be expressed as ratios.

C	D	E	F	G	A	B	C′
$\frac{1}{1}$	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

When you play a stringed instrument, you produce different notes by placing your finger on different places on a string. This is the result of changing the length of the vibrating part of the string.

The C string can be used to produce F by placing a finger $\frac{3}{4}$ of the way along the string.



Suppose a C string has a length of 16 inches. Write and solve proportions to determine what length of string would have to vibrate to produce the remaining notes of the scale.

1. D

2. E

3. F

4. G

5. A

6. B

7. C′

8. Complete to show the distance between finger positions on the 16-inch C string for each note. For example, $C(16) - D\left(14\frac{2}{9}\right) = 1\frac{7}{9}$.

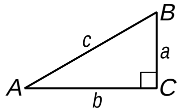
C $\underline{1\frac{7}{9}\text{ in.}}$ D ________ E ________ F ________ G ________ A ________ B ________ C′

9. Between two consecutive musical notes, there is either a whole step or a half step. Using the distances you found in Exercise 8, determine what two pairs of notes have a half step between them.

7-2 Study Guide and Intervention

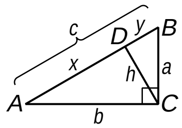
The Pythagorean Theorem and Its Converse

The Pythagorean Theorem In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.



$\triangle ABC$ is a right triangle, so $a^2 + b^2 = c^2$.

Example 1 **Prove the Pythagorean Theorem.**



With altitude $\overline{CD}$, each leg a and b is a geometric mean between hypotenuse c and the segment of the hypotenuse adjacent to that leg.

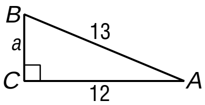
$\frac{c}{a} = \frac{a}{y}$ and $\frac{c}{b} = \frac{b}{x}$, so $a^2 = cy$ and $b^2 = cx$.

Add the two equations and substitute $c = y + x$ to get

$a^2 + b^2 = cy + cx = c(y + x) = c^2$.

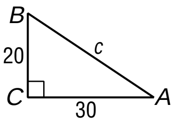
Example 2

a. Find *a*.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$a^2 + 12^2 = 13^2$	$b = 12$, $c = 13$
$a^2 + 144 = 169$	Simplify.
$a^2 = 25$	Subtract.
$a = 5$	Take the square root of each side.

b. Find *c*.

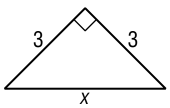


$a^2 + b^2 = c^2$	Pythagorean Theorem
$20^2 + 30^2 = c^2$	$a = 20$, $b = 30$
$400 + 900 = c^2$	Simplify.
$1300 = c^2$	Add.
$\sqrt{1300} = c$	Take the square root of each side.
$36.1 \approx c$	Use a calculator.

Exercises

Find *x*.

1.



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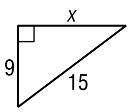

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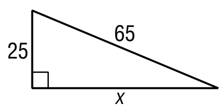
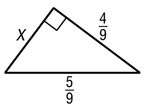


4.



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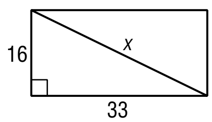

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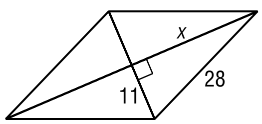
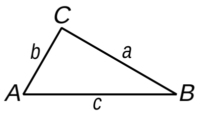


Lesson 7-2

7-2 Study Guide and Intervention *(continued)*

The Pythagorean Theorem and Its Converse

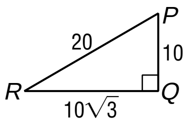
Converse of the Pythagorean Theorem If the sum of the squares of the measures of the two shorter sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.



If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.

If the three whole numbers a, b, and c satisfy the equation $a^2 + b^2 = c^2$, then the numbers a, b, and c form a **Pythagorean triple**.

Example **Determine whether $\triangle PQR$ is a right triangle.**



$a^2 + b^2 \stackrel{?}{=} c^2$ Pythagorean Theorem

$10^2 + (10\sqrt{3})^2 \stackrel{?}{=} 20^2$ $a = 10$, $b = 10\sqrt{3}$, $c = 20$

$100 + 300 \stackrel{?}{=} 400$ Simplify.

$400 = 400$✓ Add.

The sum of the squares of the two shorter sides equals the square of the longest side, so the triangle is a right triangle.

Exercises

Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

1. 30, 40, 50

2. 20, 30, 40

3. 18, 24, 30

4. 6, 8, 9

5. $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}$

6. 10, 15, 20

7. $\sqrt{5}, \sqrt{12}, \sqrt{13}$

8. $2, \sqrt{8}, \sqrt{12}$

9. 9, 40, 41

A *family* of Pythagorean triples consists of multiples of known triples. For each Pythagorean triple, find two triples in the same family.

10. 3, 4, 5

11. 5, 12, 13

12. 7, 24, 25

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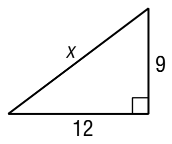
7-2 Skills Practice

The Pythagorean Theorem and Its Converse

Find x.

1.



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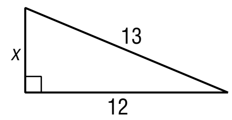

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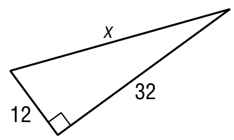

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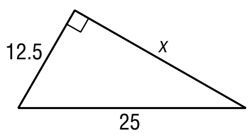

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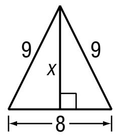

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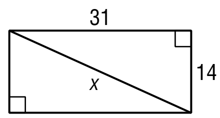


Determine whether $\triangle STU$ is a right triangle for the given vertices. Explain.

7. $S(5, 5)$, $T(7, 3)$, $U(3, 2)$

8. $S(3, 3)$, $T(5, 5)$, $U(6, 0)$

9. $S(4, 6)$, $T(9, 1)$, $U(1, 3)$

10. $S(0, 3)$, $T(-2, 5)$, $U(4, 7)$

11. $S(-3, 2)$, $T(2, 7)$, $U(-1, 1)$

12. $S(2, -1)$, $T(5, 4)$, $U(6, -3)$

Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

13. 12, 16, 20

14. 16, 30, 32

15. 14, 48, 50

16. $\frac{2}{5}, \frac{4}{5}, \frac{6}{5}$

17. $2\sqrt{6}, 5, 7$

18. $2\sqrt{2}, 2\sqrt{7}, 6$

Lesson 7-2

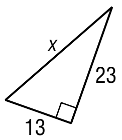
NAME ______________________________ DATE ____________ PERIOD _____

7-2 Practice

The Pythagorean Theorem and Its Converse

Find x.

1.



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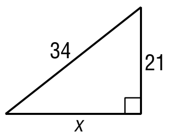

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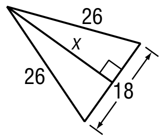

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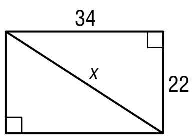

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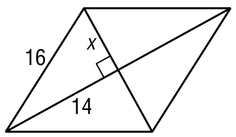

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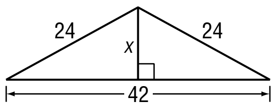


Determine whether $\triangle GHI$ is a right triangle for the given vertices. Explain.

7. $G(2, 7)$, $H(3, 6)$, $I(-4, -1)$

8. $G(-6, 2)$, $H(1, 12)$, $I(-2, 1)$

9. $G(-2, 1)$, $H(3, -1)$, $I(-4, -4)$

10. $G(-2, 4)$, $H(4, 1)$, $I(-1, -9)$

Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

11. 9, 40, 41

12. 7, 28, 29

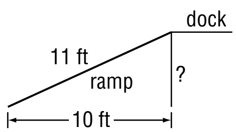
13. 24, 32, 40

14. $\frac{9}{5}, \frac{12}{5}, 3$

15. $\frac{1}{3}, \frac{2\sqrt{2}}{3}, 1$

16. $\frac{\sqrt{4}}{7}, \frac{2\sqrt{3}}{7}, \frac{4}{7}$

17. CONSTRUCTION The bottom end of a ramp at a warehouse is 10 feet from the base of the main dock and is 11 feet long. How high is the dock?



7-2 Reading to Learn Mathematics

The Pythagorean Theorem and Its Converse

Pre-Activity **How are right triangles used to build suspension bridges?**

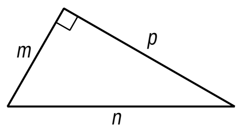
Read the introduction to Lesson 7-2 at the top of page 350 in your textbook.

Do the two right triangles shown in the drawing appear to be similar? Explain your reasoning.

Reading the Lesson

1. Explain in your own words the difference between how the Pythagorean Theorem is used and how the Converse of the Pythagorean Theorem is used.

2. Refer to the figure. For this figure, which statements are true?



A. $m^2 + n^2 = p^2$ **B.** $n^2 = m^2 + p^2$

C. $m^2 = n^2 + p^2$ **D.** $m^2 = p^2 - n^2$

E. $p^2 = n^2 - m^2$ **F.** $n^2 - p^2 = m^2$

G. $n = \sqrt{m^2 + p^2}$ **H.** $p = \sqrt{m^2 - n^2}$

3. Is the following statement true or false?

A Pythagorean triple is any group of three numbers for which the sum of the squares of the smaller two numbers is equal to the square of the largest number. Explain your reasoning.

4. If x, y, and z form a Pythagorean triple and k is a positive integer, which of the following groups of numbers are also Pythagorean triples?

A. $3x, 4y, 5z$ **B.** $3x, 3y, 3z$ **C.** $-3x, -3y, -3z$ **D.** kx, ky, kz

Helping You Remember

5. Many students who studied geometry long ago remember the Pythagorean Theorem as the equation $a^2 + b^2 = c^2$, but cannot tell you what this equation means. A formula is useless if you don't know what it means and how to use it. How could you help someone who has forgotten the Pythagorean Theorem remember the meaning of the equation $a^2 + b^2 = c^2$?

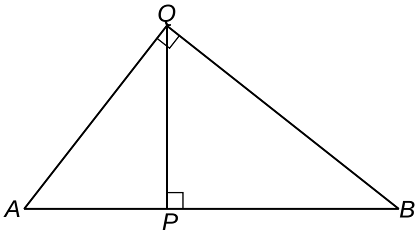
Lesson 7-2

7-2 Enrichment

Converse of a Right Triangle Theorem

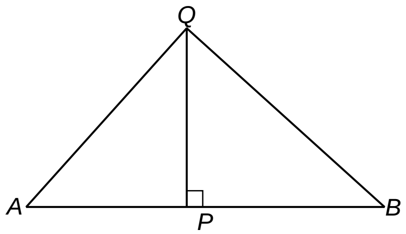
You have learned that the measure of the altitude from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. Is the converse of this theorem true? In order to find out, it will help to rewrite the original theorem in if-then form as follows.

If $\triangle ABQ$ is a right triangle with right angle at Q, then QP is the geometric mean between AP and PB, where P is between A and B and $\overline{QP}$ is perpendicular to $\overline{AB}$.



1. Write the converse of the if-then form of the theorem.

2. Is the converse of the original theorem true? Refer to the figure at the right to explain your answer.



You may find it interesting to examine the other theorems in Chapter 7 to see whether their converses are true or false. You will need to restate the theorems carefully in order to write their converses.

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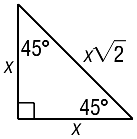
7-3 Study Guide and Intervention

Special Right Triangles

Properties of 45°-45°-90° Triangles The sides of a 45°-45°-90° right triangle have a special relationship.

Example 1 **If the leg of a 45°-45°-90° right triangle is x units, show that the hypotenuse is $x\sqrt{2}$ units.**



Using the Pythagorean Theorem with $a = b = x$, then

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= x^2 + x^2 \\ &= 2x^2 \\ c &= \sqrt{2x^2} \\ &= x\sqrt{2} \end{aligned}$$

Example 2 **In a 45°-45°-90° right triangle the hypotenuse is $\sqrt{2}$ times the leg. If the hypotenuse is 6 units, find the length of each leg.**

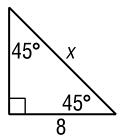
The hypotenuse is $\sqrt{2}$ times the leg, so divide the length of the hypotenuse by $\sqrt{2}$.

$$\begin{aligned} a &= \frac{6}{\sqrt{2}} \\ &= \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}} \\ &= \frac{6\sqrt{2}}{2} \\ &= 3\sqrt{2} \text{ units} \end{aligned}$$

Exercises

Find x.

1.



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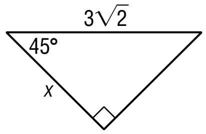

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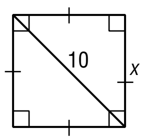

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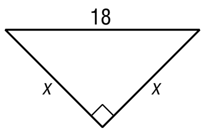

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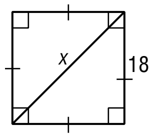

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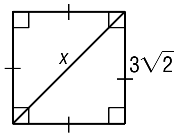


7. Find the perimeter of a square with diagonal 12 centimeters.

8. Find the diagonal of a square with perimeter 20 inches.

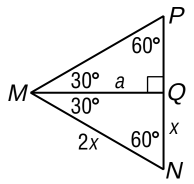
9. Find the diagonal of a square with perimeter 28 meters.

7-3 Study Guide and Intervention *(continued)*

Special Right Triangles

Properties of 30°-60°-90° Triangles The sides of a 30°-60°-90° right triangle also have a special relationship.

Example 1 **In a 30°-60°-90° right triangle, show that the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg.**



△*MNP* is an equilateral triangle.

△*MNQ* is a 30°-60°-90° right triangle.

△*MNQ* is a 30°-60°-90° right triangle, and the length of the hypotenuse $\overline{MN}$ is two times the length of the shorter side $\overline{NQ}$.
Using the Pythagorean Theorem,

$$a^2 = (2x)^2 - x^2$$
$$= 4x^2 - x^2$$
$$= 3x^2$$
$$a = \sqrt{3x^2}$$
$$= x\sqrt{3}$$

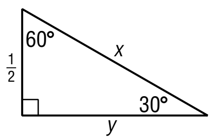
Example 2 **In a 30°-60°-90° right triangle, the hypotenuse is 5 centimeters. Find the lengths of the other two sides of the triangle.**

If the hypotenuse of a 30°-60°-90° right triangle is 5 centimeters, then the length of the shorter leg is half of 5 or 2.5 centimeters. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg, or $(2.5)(\sqrt{3})$ centimeters.

Exercises

Find *x* and *y*.

1.



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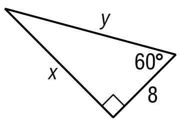

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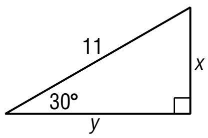

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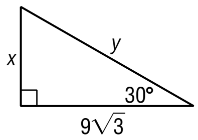

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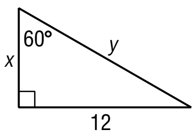

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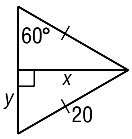

7. The perimeter of an equilateral triangle is 32 centimeters. Find the length of an altitude of the triangle to the nearest tenth of a centimeter.

8. An altitude of an equilateral triangle is 8.3 meters. Find the perimeter of the triangle to the nearest tenth of a meter.

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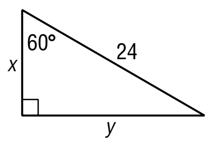
7-3 Skills Practice

Special Right Triangles

Find x and y.

1.



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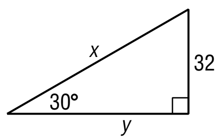

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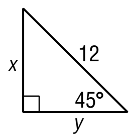

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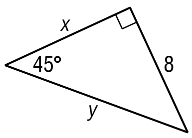

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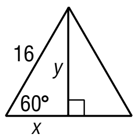

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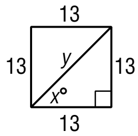
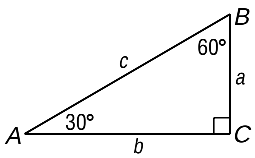


For Exercises 7–9, use the figure at the right.

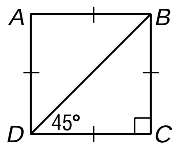


7. If $a = 11$, find b and c.

8. If $b = 15$, find a and c.

9. If $c = 9$, find a and b.

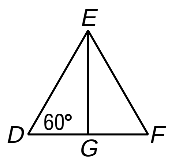
For Exercises 10 and 11, use the figure at the right.



10. The perimeter of the square is 30 inches. Find the length of $\overline{BC}$.

11. Find the length of the diagonal $\overline{BD}$.

12. The perimeter of the equilateral triangle is 60 meters. Find the length of an altitude.



13. $\triangle GEC$ is a 30°-60°-90° triangle with right angle at E, and $\overline{EC}$ is the longer leg. Find the coordinates of G in Quadrant I for $E(1, 1)$ and $C(4, 1)$.

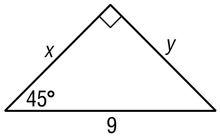
NAME ______________________________ DATE ____________ PERIOD _____

7-3 Practice

Special Right Triangles

Find x and y.

1.



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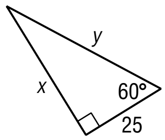

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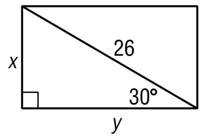

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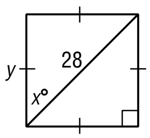

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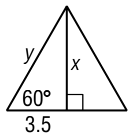

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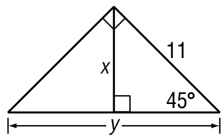
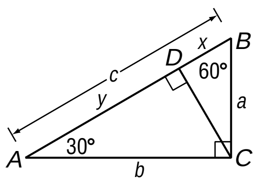


For Exercises 7–9, use the figure at the right.



7. If $a = 4\sqrt{3}$, find b and c.

8. If $x = 3\sqrt{3}$, find a and CD.

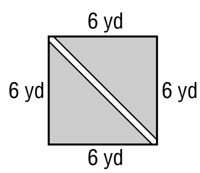
9. If $a = 4$, find CD, b, and y.

10. The perimeter of an equilateral triangle is 39 centimeters. Find the length of an altitude of the triangle.

11. $\triangle MIP$ is a 30°-60°-90° triangle with right angle at I, and $\overline{IP}$ the longer leg. Find the coordinates of M in Quadrant I for $I(3, 3)$ and $P(12, 3)$.

12. $\triangle TJK$ is a 45°-45°-90° triangle with right angle at J. Find the coordinates of T in Quadrant II for $J(-2, -3)$ and $K(3, -3)$.

13. BOTANICAL GARDENS One of the displays at a botanical garden is an herb garden planted in the shape of a square. The square measures 6 yards on each side. Visitors can view the herbs from a diagonal pathway through the garden. How long is the pathway?



7-3 Reading to Learn Mathematics

Special Triangles

Pre-Activity **How is triangle tiling used in wallpaper design?**

Read the introduction to Lesson 7-3 at the top of page 357 in your textbook.

- How can you most completely describe the larger triangle and the two smaller triangles in tile 15?
- How can you most completely describe the larger triangle and the two smaller triangles in tile 16? (Include angle measures in describing all the triangles.)

Reading the Lesson

1. Supply the correct number or numbers to complete each statement.

a. In a 45°-45°-90° triangle, to find the length of the hypotenuse, multiply the length of a leg by _______.

b. In a 30°-60°-90° triangle, to find the length of the hypotenuse, multiply the length of the shorter leg by _______.

c. In a 30°-60°-90° triangle, the longer leg is opposite the angle with a measure of _______.

d. In a 30°-60°-90° triangle, to find the length of the longer leg, multiply the length of the shorter leg by _______.

e. In an isosceles right triangle, each leg is opposite an angle with a measure of _______.

f. In a 30°-60°-90° triangle, to find the length of the shorter leg, divide the length of the longer leg by _______.

g. In 30°-60°-90° triangle, to find the length of the longer leg, divide the length of the hypotenuse by _______ and multiply the result by _______.

h. To find the length of a side of a square, divide the length of the diagonal by _______.

2. Indicate whether each statement is *always*, *sometimes*, or *never* true.

a. The lengths of the three sides of an isosceles triangle satisfy the Pythagorean Theorem.

b. The lengths of the sides of a 30°-60°-90° triangle form a Pythagorean triple.

c. The lengths of all three sides of a 30°-60°-90° triangle are positive integers.

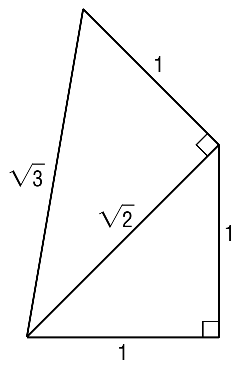
Helping You Remember

3. Some students find it easier to remember mathematical concepts in terms of specific numbers rather than variables. How can you use specific numbers to help you remember the relationship between the lengths of the three sides in a 30°-60°-90° triangle?

7-3 Enrichment

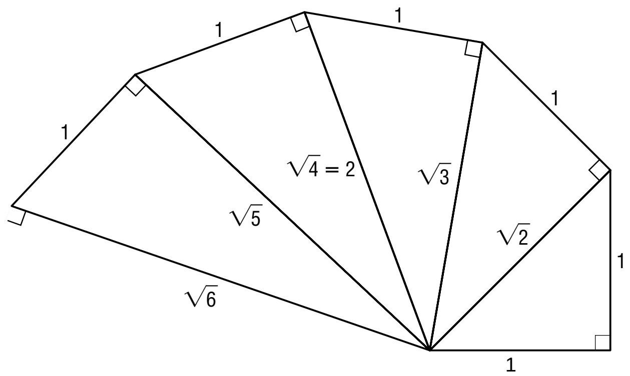
Constructing Values of Square Roots

The diagram at the right shows a right isosceles triangle with two legs of length 1 inch. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{2}$ inches. By constructing an adjacent right triangle with legs of $\sqrt{2}$ inches and 1 inch, you can create a segment of length $\sqrt{3}$.



By continuing this process as shown below, you can construct a "wheel" of square roots. This wheel is called the "Wheel of Theodorus" after a Greek philosopher who lived about 400 B.C.

Continue constructing the wheel until you make a segment of length $\sqrt{18}$.

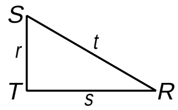


NAME ______________________________ DATE ____________ PERIOD _____

7-4 Study Guide and Intervention

Trigonometry

Trigonometric Ratios The ratio of the lengths of two sides of a right triangle is called a **trigonometric ratio**. The three most common ratios are **sine**, **cosine**, and **tangent**, which are abbreviated *sin*, *cos*, and *tan*, respectively.

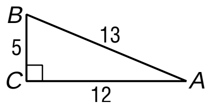


$$\sin R = \frac{\text{leg opposite } \angle R}{\text{hypotenuse}} = \frac{r}{t}$$

$$\cos R = \frac{\text{leg adjacent to } \angle R}{\text{hypotenuse}} = \frac{s}{t}$$

$$\tan R = \frac{\text{leg opposite } \angle R}{\text{leg adjacent to } \angle R} = \frac{r}{s}$$

Example **Find sin *A*, cos *A*, and tan *A*. Express each ratio as a decimal to the nearest thousandth.**



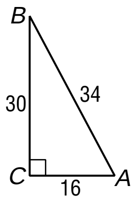
$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{5}{13} \approx 0.385$$

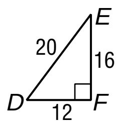
$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \approx 0.923$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \approx 0.417$$

Exercises

Find the indicated trigonometric ratio as a fraction and as a decimal. If necessary, round to the nearest ten-thousandth.





1. sin *A*

2. tan *B*

3. cos *A*

4. cos *B*

5. sin *D*

6. tan *E*

7. cos *E*

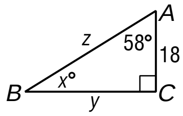
8. cos *D*

7-4 Study Guide and Intervention *(continued)*

Trigonometry

Use Trigonometric Ratios In a right triangle, if you know the measures of two sides or if you know the measures of one side and an acute angle, then you can use trigonometric ratios to find the measures of the missing sides or angles of the triangle.

Example **Find x, y, and z. Round each measure to the nearest whole number.**



a. Find x.

$$x + 58 = 90$$
$$x = 32$$

b. Find y.

$$\tan A = \frac{y}{18}$$
$$\tan 58° = \frac{y}{18}$$
$$y = 18 \tan 58°$$
$$y \approx 29$$

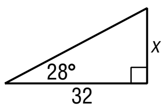
c. Find z.

$$\cos A = \frac{18}{z}$$
$$\cos 58° = \frac{18}{z}$$
$$z \cos 58° = 18$$
$$z = \frac{18}{\cos 58°}$$
$$z \approx 34$$

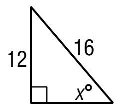
Exercises

Find x. Round to the nearest tenth.

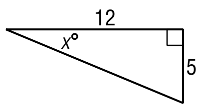
1.



2.



3.



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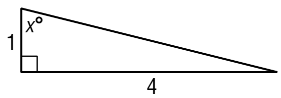

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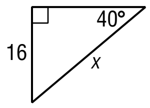

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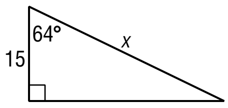

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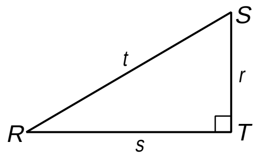
7-4 Skills Practice

Trigonometry

Use △*RST* to find sin *R*, cos *R*, tan *R*, sin *S*, cos *S*, and tan *S*. Express each ratio as a fraction and as a decimal to the nearest hundredth.



1. $r = 16, s = 30, t = 34$

2. $r = 10, s = 24, t = 26$

Use a calculator to find each value. Round to the nearest ten-thousandth.

3. sin 5

4. tan 23

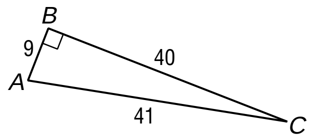
5. cos 61

6. sin 75.8

7. tan 17.3

8. cos 52.9

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.



9. tan C

10. sin A

11. cos C

Find the measure of each acute angle to the nearest tenth of a degree.

12. $\sin B = 0.2985$

13. $\tan A = 0.4168$

14. $\cos R = 0.8443$

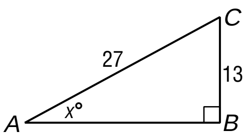
15. $\tan C = 0.3894$

16. $\cos B = 0.7329$

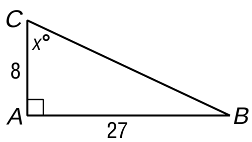
17. $\sin A = 0.1176$

Find *x*. Round to the nearest tenth.

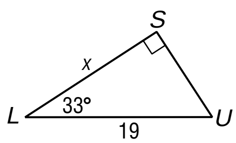
18.



19.



20.



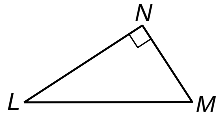
Lesson 7-4

NAME ______________________________ DATE __________ PERIOD _____

7-4 Practice

Trigonometry

Use $\triangle LMN$ to find sin L, cos L, tan L, sin M, cos M, and tan M. Express each ratio as a fraction and as a decimal to the nearest hundredth.



1. $\ell = 15, m = 36, n = 39$

2. $\ell = 12, m = 12\sqrt{3}, n = 24$

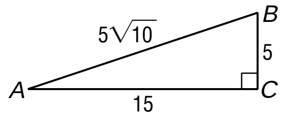
Use a calculator to find each value. Round to the nearest ten-thousandth.

3. sin 92.4

4. tan 27.5

5. cos 64.8

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.



6. cos A

7. tan B

8. sin A

Find the measure of each acute angle to the nearest tenth of a degree.

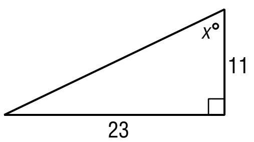
9. sin $B = 0.7823$

10. tan $A = 0.2356$

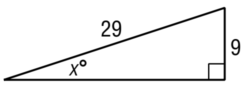
11. cos $R = 0.6401$

Find x. Round to the nearest tenth.

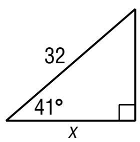
12.



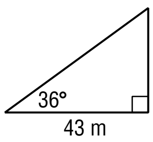
13.



14.



15. GEOGRAPHY Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to the top of the formation. The distance was 43 meters and the angle was 36 degrees. What is the height of the formation to the nearest meter?



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7-4 Reading to Learn Mathematics

Trigonometry

Pre-Activity **How can surveyors determine angle measures?**

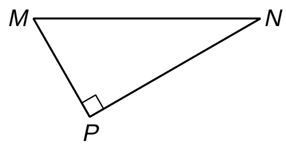
Read the introduction to Lesson 7-4 at the top of page 364 in your textbook.

- Why is it important to determine the relative positions accurately in navigation? (Give two possible reasons.)

- What does *calibrated* mean?

Reading the Lesson

1. Refer to the figure. Write a ratio using the side lengths in the figure to represent each of the following trigonometric ratios.



A. sin N **B.** cos N

C. tan N **D.** tan M

E. sin M **F.** cos M

2. Assume that you enter each of the expressions in the list on the left into your calculator. Match each of these expressions with a description from the list on the right to tell what you are finding when you enter this expression.

a. sin 20	**i.** the degree measure of an acute angle whose cosine is 0.8
b. cos 20	**ii.** the ratio of the length of the leg adjacent to the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle
c. $\sin^{-1} 0.8$	**iii.** the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the adjacent leg is 0.8
d. $\tan^{-1} 0.8$	**iv.** the ratio of the length of the leg opposite the 20° angle to the length of the leg adjacent to it in a 20°-70°-90° triangle
e. tan 20	**v.** the ratio of the length of the leg opposite the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle
f. $\cos^{-1} 0.8$	**vi.** the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the hypotenuse is 0.8

Helping You Remember

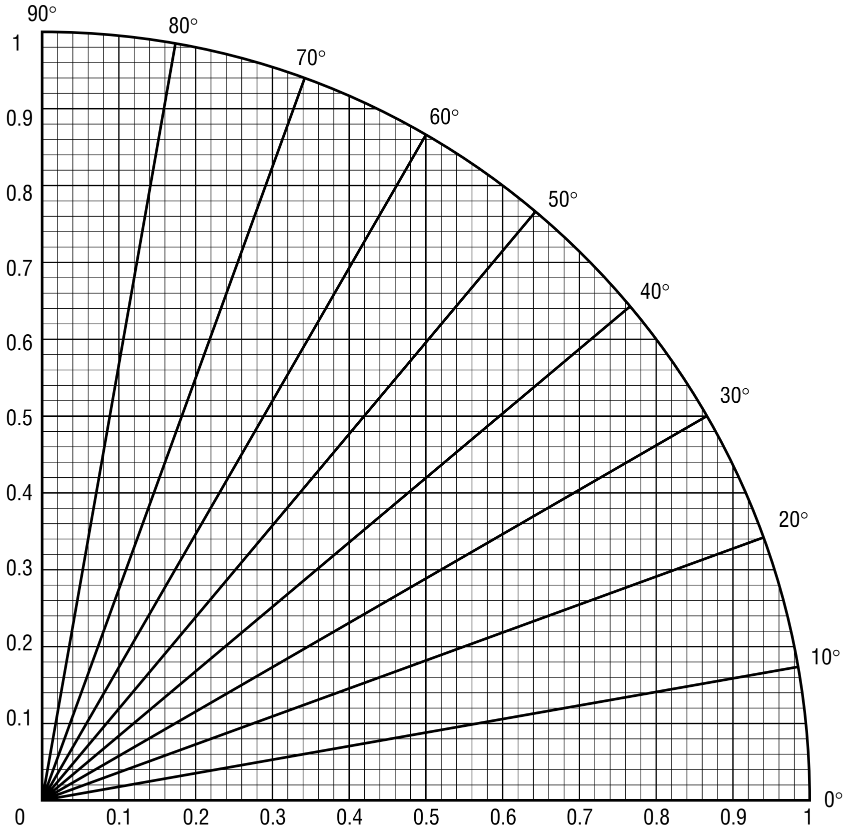
3. How can the *co* in *cosine* help you to remember the relationship between the sines and cosines of the two acute angles of a right triangle?

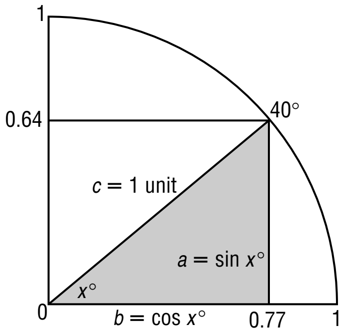
Lesson 7-4

7-4 Enrichment

Sine and Cosine of Angles

The following diagram can be used to obtain approximate values for the sine and cosine of angles from 0° to 90°. The radius of the circle is 1. So, the sine and cosine values can be read directly from the vertical and horizontal axes.





Example **Find approximate values for sin 40° and cos 40°. Consider the triangle formed by the segment marked 40°, as illustrated by the shaded triangle at right.**

$\sin 40° = \frac{a}{c} \approx \frac{0.64}{1}$ or 0.64 $\qquad \cos 40° = \frac{b}{c} \approx \frac{0.77}{1}$ or 0.77

1. Use the diagram above to complete the chart of values.

$x°$	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
sin $x°$					0.64					
cos $x°$					0.77					

2. Compare the sine and cosine of two complementary angles (angles whose sum is 90°). What do you notice?

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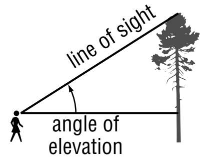
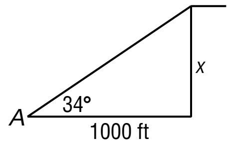
7-5 Study Guide and Intervention

Angles of Elevation and Depression

Angles of Elevation Many real-world problems that involve looking up to an object can be described in terms of an **angle of elevation**, which is the angle between an observer's line of sight and a horizontal line.



Example **The angle of elevation from point A to the top of a cliff is 34°. If point A is 1000 feet from the base of the cliff, how high is the cliff?**



Let x = the height of the cliff.

$\tan 34° = \frac{x}{1000}$ $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$1000(\tan 34°) = x$ Multiply each side by 1000.

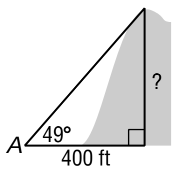
$674.5 = x$ Use a calculator.

The height of the cliff is about 674.5 feet.

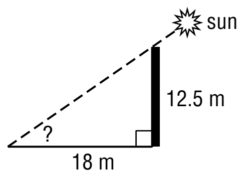
Exercises

Solve each problem. Round measures of segments to the nearest whole number and angles to the nearest degree.

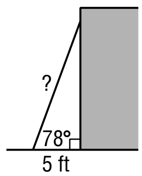
1. The angle of elevation from point A to the top of a hill is 49°. If point A is 400 feet from the base of the hill, how high is the hill?



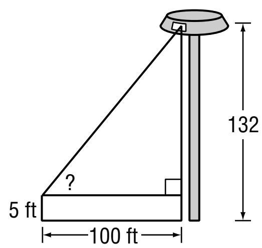
2. Find the angle of elevation of the sun when a 12.5-meter-tall telephone pole casts a 18-meter-long shadow.



3. A ladder leaning against a building makes an angle of 78° with the ground. The foot of the ladder is 5 feet from the building. How long is the ladder?



4. A person whose eyes are 5 feet above the ground is standing on the runway of an airport 100 feet from the control tower. That person observes an air traffic controller at the window of the 132-foot tower. What is the angle of elevation?

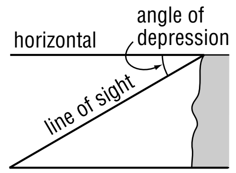


Lesson 7-5

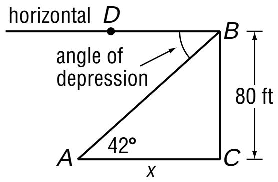
7-5 Study Guide and Intervention *(continued)*

Angles of Elevation and Depression

Angles of Depression When an observer is looking down, the **angle of depression** is the angle between the observer's line of sight and a horizontal line.



Example **The angle of depression from the top of an 80-foot building to point A on the ground is 42°. How far is the foot of the building from point A?**



Let x = the distance from point A to the foot of the building. Since the horizontal line is parallel to the ground, the angle of depression $\angle DBA$ is congruent to $\angle BAC$.

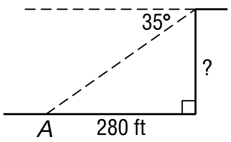
$\tan 42° = \frac{80}{x}$	$\tan = \frac{\text{opposite}}{\text{adjacent}}$
$x(\tan 42°) = 80$	Multiply each side by x.
$x = \frac{80}{\tan 42°}$	Divide each side by tan 42°.
$x \approx 88.8$	Use a calculator.

Point A is about 89 feet from the base of the building.

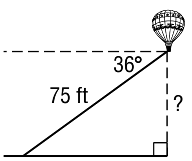
Exercises

Solve each problem. Round measures of segments to the nearest whole number and angles to the nearest degree.

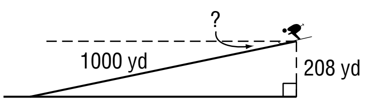
1. The angle of depression from the top of a sheer cliff to point A on the ground is 35°. If point A is 280 feet from the base of the cliff, how tall is the cliff?



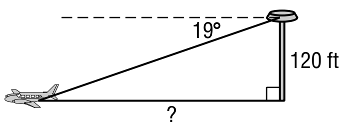
2. The angle of depression from a balloon on a 75-foot string to a person on the ground is 36°. How high is the balloon?



3. A ski run is 1000 yards long with a vertical drop of 208 yards. Find the angle of depression from the top of the ski run to the bottom.



4. From the top of a 120-foot-high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 19°. How far from the base of the tower is the airplane?



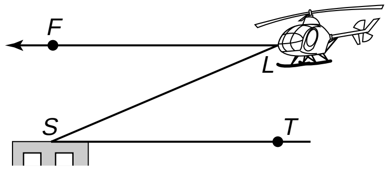
NAME ______________________________________ DATE ____________ PERIOD _____

7-5 Skills Practice

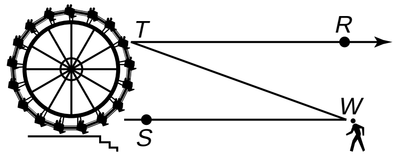
Angles of Elevation and Depression

Name the angle of depression or angle of elevation in each figure.

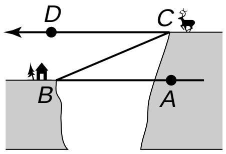
1.



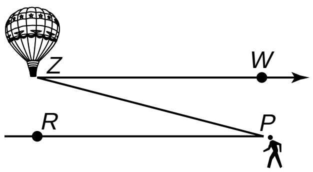
2.



3.

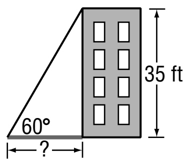


4.



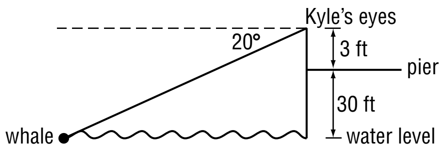
5. MOUNTAIN BIKING On a mountain bike trip along the Gemini Bridges Trail in Moab, Utah, Nabuko stopped on the canyon floor to get a good view of the twin sandstone bridges. Nabuko is standing about 60 meters from the base of the canyon cliff, and the natural arch bridges are about 100 meters up the canyon wall. If her line of sight is five feet above the ground, what is the angle of elevation to the top of the bridges? Round to the nearest tenth degree.

6. SHADOWS Suppose the sun casts a shadow off a 35-foot building. If the angle of elevation to the sun is 60°, how long is the shadow to the nearest tenth of a foot?



7. BALLOONING From her position in a hot-air balloon, Angie can see her car parked in a field. If the angle of depression is 8° and Angie is 38 meters above the ground, what is the straight-line distance from Angie to her car? Round to the nearest whole meter.

8. INDIRECT MEASUREMENT Kyle is at the end of a pier 30 feet above the ocean. His eye level is 3 feet above the pier. He is using binoculars to watch a whale surface. If the angle of depression of the whale is 20°, how far is the whale from Kyle's binoculars? Round to the nearest tenth foot.



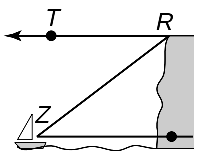
NAME ____ DATE ____________ PERIOD _____

7-5 Practice

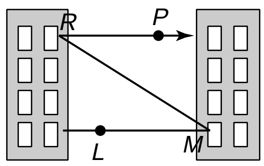
Angles of Elevation and Depression

Name the angle of depression or angle of elevation in each figure.

1.



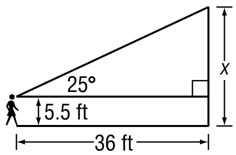
2.



3. WATER TOWERS A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth degree.

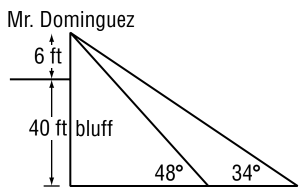
4. CONSTRUCTION A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55°, how far is the ladder from the base of the wall? Round your answer to the nearest foot.

5. TOWN ORDINANCES The town of Belmont restricts the height of flagpoles to 25 feet on any property. Lindsay wants to determine whether her school is in compliance with the regulation. Her eye level is 5.5 feet from the ground and she stands 36 feet from the flagpole. If the angle of elevation is about 25°, what is the height of the flagpole to the nearest tenth foot?



6. GEOGRAPHY Stephan is standing on a mesa at the Painted Desert. The elevation of the mesa is about 1380 meters and Stephan's eye level is 1.8 meters above ground. If Stephan can see a band of multicolored shale at the bottom and the angle of depression is 29°, about how far is the band of shale from his eyes? Round to the nearest meter.

7. INDIRECT MEASUREMENT Mr. Dominguez is standing on a 40-foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?



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7-5 Reading to Learn Mathematics

Angles of Elevation and Depression

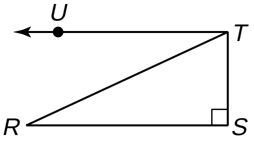
Pre-Activity **How do airline pilots use angles of elevation and depression?**

Read the introduction to Lesson 7-5 at the top of page 371 in your textbook.

What does the angle measure tell the pilot?

Reading the Lesson

1. Refer to the figure. The two observers are looking at one another. Select the correct choice for each question.



a. What is the line of sight?
(i) line RS (ii) line ST (iii) line RT (iv) line TU

b. What is the angle of elevation?
(i) $\angle RST$ (ii) $\angle SRT$ (iii) $\angle RTS$ (iv) $\angle UTR$

c. What is the angle of depression?
(i) $\angle RST$ (ii) $\angle SRT$ (iii) $\angle RTS$ (iv) $\angle UTR$

d. How are the angle of elevation and the angle of depression related?
(i) They are complementary.
(ii) They are congruent.
(iii) They are supplementary.
(iv) The angle of elevation is larger than the angle of depression.

e. Which postulate or theorem that you learned in Chapter 3 supports your answer for part c?
(i) Corresponding Angles Postulate
(ii) Alternate Exterior Angles Theorem
(iii) Consecutive Interior Angles Theorem
(iv) Alternate Interior Angles Theorem

2. A student says that the angle of elevation from his eye to the top of a flagpole is 135°. What is wrong with the student's statement?

Helping You Remember

3. A good way to remember something is to explain it to someone else. Suppose a classmate finds it difficult to distinguish between angles of elevation and angles of depression. What are some hints you can give her to help her get it right every time?

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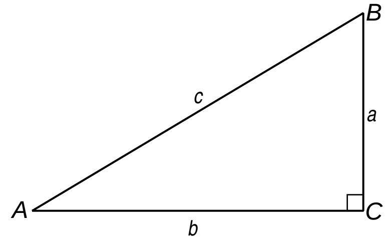
7-5 Enrichment

Reading Mathematics

The three most common trigonometric ratios are **sine**, **cosine**, and **tangent**. Three other ratios are the **cosecant**, **secant**, and **cotangent**. The chart below shows abbreviations and definitions for all six ratios. Refer to the triangle at the right.



Abbreviation	Read as:	Ratio
sin A	the sine of $\angle A$	$\frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$
cos A	the cosine of $\angle A$	$\frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$
tan A	the tangent of $\angle A$	$\frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A} = \frac{a}{b}$
csc A	the cosecant of $\angle A$	$\frac{\text{hypotenuse}}{\text{leg opposite } \angle A} = \frac{c}{a}$
sec A	the secant of $\angle A$	$\frac{\text{hypotenuse}}{\text{leg adjacent to } \angle A} = \frac{c}{b}$
cot A	the cotangent of $\angle A$	$\frac{\text{leg adjacent to } \angle A}{\text{leg opposite } \angle A} = \frac{b}{a}$

Use the abbreviations to rewrite each statement as an equation.

1. The secant of angle A is equal to 1 divided by the cosine of angle A.

2. The cosecant of angle A is equal to 1 divided by the sine of angle A.

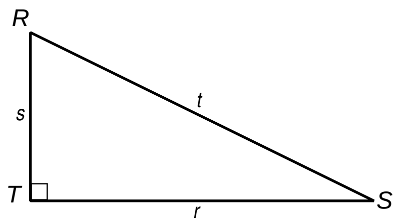
3. The cotangent of angle A is equal to 1 divided by the tangent of angle A.

4. The cosecant of angle A multiplied by the sine of angle A is equal to 1.

5. The secant of angle A multiplied by the cosine of angle A is equal to 1.

6. The cotangent of angle A times the tangent of angle A is equal to 1.

Use the triangle at right. Write each ratio.



7. sec R **8.** csc R **9.** cot R

10. sec S **11.** csc S **12.** cot S

13. If $\sin x° = 0.289$, find the value of $\csc x°$.

14. If $\tan x° = 1.376$, find the value of $\cot x°$.

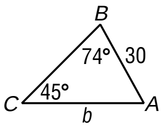
7-6 Study Guide and Intervention

The Law of Sines

The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 1 **In $\triangle ABC$, find b.**



$\frac{\sin C}{c} = \frac{\sin B}{b}$ Law of Sines

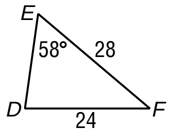
$\frac{\sin 45°}{30} = \frac{\sin 74°}{b}$ $m\angle C = 45$, $c = 30$, $m\angle B = 74$

$b \sin 45° = 30 \sin 74°$ Cross multiply.

$b = \frac{30 \sin 74°}{\sin 45°}$ Divide each side by sin 45°.

$b \approx 40.8$ Use a calculator.

Example 2 **In $\triangle DEF$, find $m\angle D$.**



$\frac{\sin D}{d} = \frac{\sin E}{e}$ Law of Sines

$\frac{\sin D}{28} = \frac{\sin 58°}{24}$ $d = 28$, $m\angle E = 58$, $e = 24$

$24 \sin D = 28 \sin 58°$ Cross multiply.

$\sin D = \frac{28 \sin 58°}{24}$ Divide each side by 24.

$D = \sin^{-1} \frac{28 \sin 58°}{24}$ Use the inverse sine.

$D \approx 81.6°$ Use a calculator.

Exercises

Find each measure using the given measures of $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $c = 12$, $m\angle A = 80$, and $m\angle C = 40$, find a.

2. If $b = 20$, $c = 26$, and $m\angle C = 52$, find $m\angle B$.

3. If $a = 18$, $c = 16$, and $m\angle A = 84$, find $m\angle C$.

4. If $a = 25$, $m\angle A = 72$, and $m\angle B = 17$, find b.

5. If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$, find a.

6. If $a = 30$, $c = 20$, and $m\angle A = 60$, find $m\angle C$.

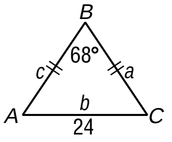
7-6 Study Guide and Intervention *(continued)*

The Law of Sines

Use the Law of Sines to Solve Problems You can use the **Law of Sines** to solve some problems that involve triangles.

Law of Sines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Example **Isosceles $\triangle ABC$ has a base of 24 centimeters and a vertex angle of 68°. Find the perimeter of the triangle.**



The vertex angle is 68°, so the sum of the measures of the base angles is 112 and $m\angle A = m\angle C = 56$.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ Law of Sines

$\frac{\sin 68°}{24} = \frac{\sin 56°}{a}$ $m\angle B = 68$, $b = 24$, $m\angle A = 56$

$a \sin 68° = 24 \sin 56°$ Cross multiply.

$a = \frac{24 \sin 56°}{\sin 68°}$ Divide each side by sin 68°.

≈ 21.5 Use a calculator.

The triangle is isosceles, so $c = 21.5$.

The perimeter is 24 + 21.5 + 21.5 or about 67 centimeters.

Exercises

Draw a triangle to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. One side of a triangular garden is 42.0 feet. The angles on each end of this side measure 66° and 82°. Find the length of fence needed to enclose the garden.

2. Two radar stations A and B are 32 miles apart. They locate an airplane X at the same time. The three points form $\angle XAB$, which measures 46°, and $\angle XBA$, which measures 52°. How far is the airplane from each station?

3. A civil engineer wants to determine the distances from points A and B to an inaccessible point C in a river. $\angle BAC$ measures 67° and $\angle ABC$ measures 52°. If points A and B are 82.0 feet apart, find the distance from C to each point.

4. A ranger tower at point A is 42 kilometers north of a ranger tower at point B. A fire at point C is observed from both towers. If $\angle BAC$ measures 43° and $\angle ABC$ measures 68°, which ranger tower is closer to the fire? How much closer?

7-6 Skills Practice

The Law of Sines

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.

1. If $m\angle A = 35$, $m\angle B = 48$, and $b = 28$, find a.

2. If $m\angle B = 17$, $m\angle C = 46$, and $c = 18$, find b.

3. If $m\angle C = 86$, $m\angle A = 51$, and $a = 38$, find c.

4. If $a = 17$, $b = 8$, and $m\angle A = 73$, find $m\angle B$.

5. If $c = 38$, $b = 34$, and $m\angle B = 36$, find $m\angle C$.

6. If $a = 12$, $c = 20$, and $m\angle C = 83$, find $m\angle A$.

7. If $m\angle A = 22$, $a = 18$, and $m\angle B = 104$, find b.

Solve each $\triangle PQR$ described below. Round measures to the nearest tenth.

8. $p = 27$, $q = 40$, $m\angle P = 33$

9. $q = 12$, $r = 11$, $m\angle R = 16$

10. $p = 29$, $q = 34$, $m\angle Q = 111$

11. If $m\angle P = 89$, $p = 16$, $r = 12$

12. If $m\angle Q = 103$, $m\angle P = 63$, $p = 13$

13. If $m\angle P = 96$, $m\angle R = 82$, $r = 35$

14. If $m\angle R = 49$, $m\angle Q = 76$, $r = 26$

15. If $m\angle Q = 31$, $m\angle P = 52$, $p = 20$

16. If $q = 8$, $m\angle Q = 28$, $m\angle R = 72$

17. If $r = 15$, $p = 21$, $m\angle P = 128$

7-6 Practice

The Law of Sines

Find each measure using the given measures from $\triangle EFG$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.

1. If $m\angle G = 14$, $m\angle E = 67$, and $e = 14$, find g.

2. If $e = 12.7$, $m\angle E = 42$, and $m\angle F = 61$, find f.

3. If $g = 14$, $f = 5.8$, and $m\angle G = 83$, find $m\angle F$.

4. If $e = 19.1$, $m\angle G = 34$, and $m\angle E = 56$, find g.

5. If $f = 9.6$, $g = 27.4$, and $m\angle G = 43$, find $m\angle F$.

Solve each $\triangle STU$ described below. Round measures to the nearest tenth.

6. $m\angle T = 85$, $s = 4.3$, $t = 8.2$

7. $s = 40$, $u = 12$, $m\angle S = 37$

8. $m\angle U = 37$, $t = 2.3$, $m\angle T = 17$

9. $m\angle S = 62$, $m\angle U = 59$, $s = 17.8$

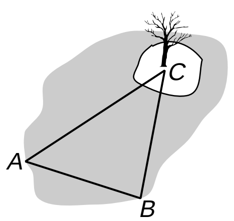
10. $t = 28.4$, $u = 21.7$, $m\angle T = 66$

11. $m\angle S = 89$, $s = 15.3$, $t = 14$

12. $m\angle T = 98$, $m\angle U = 74$, $u = 9.6$

13. $t = 11.8$, $m\angle S = 84$, $m\angle T = 47$

14. INDIRECT MEASUREMENT To find the distance from the edge of the lake to the tree on the island in the lake, Hannah set up a triangular configuration as shown in the diagram. The distance from location A to location B is 85 meters. The measures of the angles at A and B are 51° and 83°, respectively. What is the distance from the edge of the lake at B to the tree on the island at C?



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7-6 Reading to Learn Mathematics

The Law of Sines

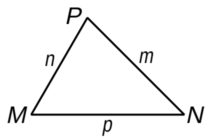
Pre-Activity **How are triangles used in radio astronomy?**

Read the introduction to Lesson 7-6 at the top of page 377 in your textbook.

Why might several antennas be better than one single antenna when studying distant objects?

Reading the Lesson

1. Refer to the figure. According to the Law of Sines, which of the following are correct statements?



A. $\frac{m}{\sin M} = \frac{n}{\sin N} = \frac{p}{\sin P}$

B. $\frac{\sin m}{M} = \frac{\sin n}{N} = \frac{\sin p}{P}$

C. $\frac{\cos M}{m} = \frac{\cos N}{n} = \frac{\cos P}{p}$

D. $\frac{\sin M}{m} + \frac{\sin N}{n} = \frac{\sin P}{p}$

E. $(\sin M)^2 + (\sin N)^2 = (\sin P)^2$

F. $\frac{\sin P}{p} = \frac{\sin M}{m} = \frac{\sin N}{n}$

2. State whether each of the following statements is *true* or *false*. If the statement is false, explain why.

a. The Law of Sines applies to all triangles.

b. The Pythagorean Theorem applies to all triangles.

c. If you are given the length of one side of a triangle and the measures of any two angles, you can use the Law of Sines to find the lengths of the other two sides.

d. If you know the measures of two angles of a triangle, you should use the Law of Sines to find the measure of the third angle.

e. A friend tells you that in triangle RST, $m\angle R = 132$, $r = 24$ centimeters, and $s = 31$ centimeters. Can you use the Law of Sines to solve the triangle? Explain.

Helping You Remember

3. Many students remember mathematical equations and formulas better if they can state them in words. State the Law of Sines in your own words without using variables or mathematical symbols.

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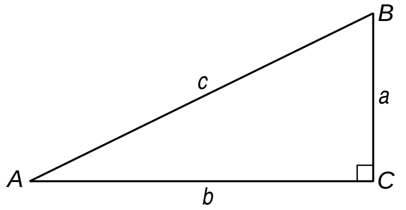
7-6 Enrichment

Identities

An **identity** is an equation that is true for all values of the variable for which both sides are defined. One way to verify an identity is to use a right triangle and the definitions for trigonometric functions.



Example 1 **Verify that $(\sin A)^2 + (\cos A)^2 = 1$ is an identity.**

$$(\sin A)^2 + (\cos A)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$

$$= \frac{a^2 + b^2}{c} = \frac{c^2}{c^2} = 1$$

To check whether an equation *may* be an identity, you can test several values. However, since you cannot test all values, you cannot be *certain* that the equation is an identity.

Example 2 **Test $\sin 2x = 2 \sin x \cos x$ to see if it could be an identity.**

Try $x = 20$. Use a calculator to evaluate each expression.

$\sin 2x = \sin 40$
≈ 0.643

$2 \sin x \cos x = 2 (\sin 20)(\cos 20)$
$\approx 2(0.342)(0.940)$
≈ 0.643

Since the left and right sides seem equal, the equation may be an identity.

Use triangle *ABC* shown above. Verify that each equation is an identity.

1. $\frac{\cos A}{\sin A} = \frac{1}{\tan A}$

2. $\frac{\tan B}{\sin B} = \frac{1}{\cos B}$

3. $\tan B \cos B = \sin B$

4. $1 - (\cos B)^2 = (\sin B)^2$

Try several values for x to test whether each equation could be an identity.

5. $\cos 2x = (\cos x)^2 - (\sin x)^2$

6. $\cos (90 - x) = \sin x$

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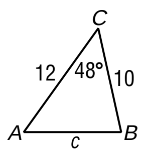
7-7 Study Guide and Intervention

The Law of Cosines

The Law of Cosines Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Example 1 **In $\triangle ABC$, find c.**



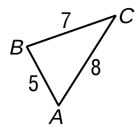
$c^2 = a^2 + b^2 - 2ab \cos C$ Law of Cosines

$c^2 = 12^2 + 10^2 - 2(12)(10)\cos 48°$ $a = 12$, $b = 10$, $m\angle C = 48$

$c = \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48°}$ Take the square root of each side.

$c \approx 9.1$ Use a calculator.

Example 2 **In $\triangle ABC$, find $m\angle A$.**



$a^2 = b^2 + c^2 - 2bc \cos A$ Law of Cosines

$7^2 = 5^2 + 8^2 - 2(5)(8) \cos A$ $a = 7$, $b = 5$, $c = 8$

$49 = 25 + 64 - 80 \cos A$ Multiply.

$-40 = -80 \cos A$ Subtract 89 from each side.

$\frac{1}{2} = \cos A$ Divide each side by -80.

$\cos^{-1}\frac{1}{2} = A$ Use the inverse cosine.

$60° = A$ Use a calculator.

Exercises

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $b = 14$, $c = 12$, and $m\angle A = 62$, find a.

2. If $a = 11$, $b = 10$, and $c = 12$, find $m\angle B$.

3. If $a = 24$, $b = 18$, and $c = 16$, find $m\angle C$.

4. If $a = 20$, $c = 25$, and $m\angle B = 82$, find b.

5. If $b = 18$, $c = 28$, and $m\angle A = 59$, find a.

6. If $a = 15$, $b = 19$, and $c = 15$, find $m\angle C$.

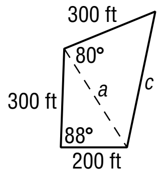
7-7 Study Guide and Intervention *(continued)*

The Law of Cosines

Use the Law of Cosines to Solve Problems You can use the **Law of Cosines** to solve some problems involving triangles.

Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Example **Ms. Jones wants to purchase a piece of land with the shape shown. Find the perimeter of the property.**



Use the Law of Cosines to find the value of a.

$a^2 = b^2 + c^2 - 2bc \cos A$ Law of Cosines

$a^2 = 300^2 + 200^2 - 2(300)(200) \cos 88°$ $b = 300$, $c = 200$, $m\angle A = 88$

$a = \sqrt{130{,}000 - 120{,}000 \cos 88°}$ Take the square root of each side.

≈ 354.7 Use a calculator.

Use the Law of Cosines again to find the value of c.

$c^2 = a^2 + b^2 - 2ab \cos C$ Law of Cosines

$c^2 = 354.7^2 + 300^2 - 2(354.7)(300) \cos 80°$ $a = 354.7$, $b = 300$, $m\angle C = 80$

$c = \sqrt{215{,}812.09 - 212{,}820 \cos 80°}$ Take the square root of each side.

≈ 422.9 Use a calculator.

The perimeter of the land is 300 + 200 + 422.9 + 200 or about 1223 feet.

Exercises

Draw a figure or diagram to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. A triangular garden has dimensions 54 feet, 48 feet, and 62 feet. Find the angles at each corner of the garden.

2. A parallelogram has a 68° angle and sides 8 and 12. Find the lengths of the diagonals.

3. An airplane is sighted from two locations, and its position forms an acute triangle with them. The distance to the airplane is 20 miles from one location with an angle of elevation 48.0°, and 40 miles from the other location with an angle of elevation of 21.8°. How far apart are the two locations?

4. A ranger tower at point A is directly north of a ranger tower at point B. A fire at point C is observed from both towers. The distance from the fire to tower A is 60 miles, and the distance from the fire to tower B is 50 miles. If $m\angle ACB = 62$, find the distance between the towers.

7-7 Skills Practice

The Law of Cosines

In $\triangle RST$, given the following measures, find the measure of the missing side.

1. $r = 5, s = 8, m\angle T = 39$

2. $r = 6, t = 11, m\angle S = 87$

3. $r = 9, t = 15, m\angle S = 103$

4. $s = 12, t = 10, m\angle R = 58$

In $\triangle HIJ$, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.

5. $h = 12, i = 18, j = 7; m\angle H$

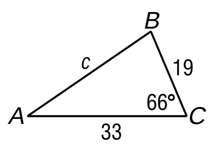
6. $h = 15, i = 16, j = 22; m\angle I$

7. $h = 23, i = 27, j = 29; m\angle J$

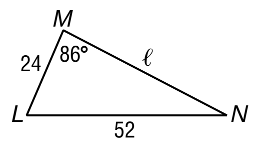
8. $h = 37, i = 21, j = 30; m\angle H$

Determine whether the Law of *Sines* or the Law of *Cosines* should be used first to solve each triangle. Then solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.

9.



10.



11. $a = 10, b = 14, c = 19$

12. $a = 12, b = 10, m\angle C = 27$

Solve each $\triangle RST$ described below. Round measures to the nearest tenth.

13. $r = 12, s = 32, t = 34$

14. $r = 30, s = 25, m\angle T = 42$

15. $r = 15, s = 11, m\angle R = 67$

16. $r = 21, s = 28, t = 30$

Lesson 7-7

NAME ______________________________ DATE __________ PERIOD _____

7-7 Practice

The Law of Cosines

In $\triangle JKL$, given the following measures, find the measure of the missing side.

1. $j = 1.3, k = 10, m\angle L = 77$

2. $j = 9.6, \ell = 1.7, m\angle K = 43$

3. $j = 11, k = 7, m\angle L = 63$

4. $k = 4.7, \ell = 5.2, m\angle J = 112$

In $\triangle MNQ$, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.

5. $m = 17, n = 23, q = 25; m\angle Q$

6. $m = 24, n = 28, q = 34; m\angle M$

7. $m = 12.9, n = 18, q = 20.5; m\angle N$

8. $m = 23, n = 30.1, q = 42; m\angle Q$

Determine whether the Law of Sines or the Law of Cosines should be used first to solve $\triangle ABC$. Then sole each triangle. Round angle measures to the nearest degree and side measure to the nearest tenth.

9. $a = 13, b = 18, c = 19$

10. $a = 6, b = 19, m\angle C = 38$

11. $a = 17, b = 22, m\angle B = 49$

12. $a = 15.5, b = 18, m\angle C = 72$

Solve each $\triangle FGH$ described below. Round measures to the nearest tenth.

13. $m\angle F = 54, f = 12.5, g = 11$

14. $f = 20, g = 23, m\angle H = 47$

15. $f = 15.8, g = 11, h = 14$

16. $f = 36, h = 30, m\angle G = 54$

17. REAL ESTATE The Esposito family purchased a triangular plot of land on which they plan to build a barn and corral. The lengths of the sides of the plot are 320 feet, 286 feet, and 305 feet. What are the measures of the angles formed on each side of the property?

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7-7 Reading to Learn Mathematics

The Law of Cosines

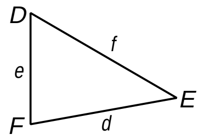
Pre-Activity **How are triangles used in building design?**

Read the introduction to Lesson 7-7 at the top of page 385 in your textbook.

What could be a disadvantage of a triangular room?

Reading the Lesson

1. Refer to the figure. According to the Law of Cosines, which statements are correct for $\triangle DEF$?



A. $d^2 = e^2 + f^2 - ef \cos D$ **B.** $e^2 = d^2 + f^2 - 2df \cos E$

C. $d^2 = e^2 + f^2 + 2ef \cos D$ **D.** $f^2 = d^2 + e^2 - 2ef \cos F$

E. $f^2 = d^2 + e^2 - 2de \cos \text{F}$ **F.** $d^2 = e^2 + f^2$

G. $\frac{\sin D}{d} = \frac{\sin E}{e} = \frac{\sin F}{f}$ **H.** $d = \sqrt{e^2 + f^2 - 2ef \cos\ D}$

2. Each of the following describes three given parts of a triangle. In each case, indicate whether you would use the Law of Sines or the Law of Cosines first in solving a triangle with those given parts. (In some cases, only one of the two laws would be used in solving the triangle.)

a. SSS **b.** ASA

c. AAS **d.** SAS

e. SSA

3. Indicate whether each statement is *true* or *false*. If the statement is false, explain why.

a. The Law of Cosines applies to right triangles.

b. The Pythagorean Theorem applies to acute triangles.

c. The Law of Cosines is used to find the third side of a triangle when you are given the measures of two sides and the nonincluded angle.

d. The Law of Cosines can be used to solve a triangle in which the measures of the three sides are 5 centimeters, 8 centimeters, and 15 centimeters.

Helping You Remember

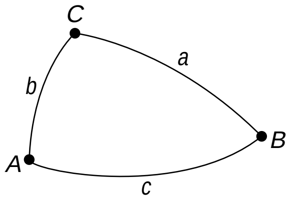
4. A good way to remember a new mathematical formula is to relate it to one you already know. The Law of Cosines looks somewhat like the Pythagorean Theorem. Both formulas must be true for a right triangle. How can that be?

NAME ______________________________ DATE ____________ PERIOD _____

7-7 Enrichment

Spherical Triangles

Spherical trigonometry is an extension of plane trigonometry. Figures are drawn on the surface of a sphere. Arcs of great circles correspond to line segments in the plane. The arcs of three great circles intersecting on a sphere form a spherical triangle. Angles have the same measure as the tangent lines drawn to each great circle at the vertex. Since the sides are arcs, they too can be measured in degrees.



The sum of the sides of a spherical triangle is less than 360°.
The sum of the angles is greater than 180° and less than 540°.
The Law of Sines for spherical triangles is as follows.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

There is also a Law of Cosines for spherical triangles.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Example **Solve the spherical triangle given $a = 72°$, $b = 105°$, and $c = 61°$.**

Use the Law of Cosines.

$$0.3090 = (-0.2588)(0.4848) + (0.9659)(0.8746) \cos A$$
$$\cos A = 0.5143$$
$$A = 59°$$

$$-0.2588 = (0.3090)(0.4848) + (0.9511)(0.8746) \cos B$$
$$\cos B = -0.4912$$
$$B = 119°$$

$$0.4848 = (0.3090)(-0.2588) + (0.9511)(0.9659) \cos C$$
$$\cos C = 0.6148$$
$$C = 52°$$

Check by using the Law of Sines.

$$\frac{\sin 72°}{\sin 59°} = \frac{\sin 105°}{\sin 119°} = \frac{\sin 61°}{\sin 52°} = 1.1$$

Solve each spherical triangle.

1. $a = 56°, b = 53°, c = 94°$

2. $a = 110°, b = 33°, c = 97°$

3. $a = 76°, b = 110°, C = 49°$

4. $b = 94°, c = 55°, A = 48°$

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7 Chapter 7 Test, Form 1

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

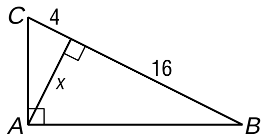
1. Find the geometric mean between 20 and 5. **1.** ______

A. 100 **B.** 50 **C.** 12.5 **D.** 10

2. Find x in $\triangle ABC$. **2.** ______

A. 8 **B.** 10

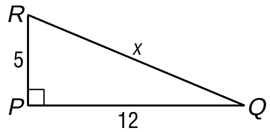
C. $\sqrt{20}$ **D.** 64



3. Find x in $\triangle PQR$. **3.** ______

A. 13 **B.** 15

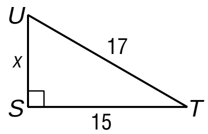
C. 16 **D.** $\sqrt{60}$



4. Find x in $\triangle STU$. **4.** ______

A. 2 **B.** 8

C. $\sqrt{32}$ **D.** $\sqrt{514}$



5. Which set of measures could represent the sides of a right triangle? **5.** ______

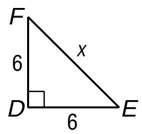
A. 2, 3, 4 **B.** 7, 11, 14

C. 8, 10, 12 **D.** 9, 12, 15

6. Find x in $\triangle DEF$. **6.** ______

A. 6 **B.** $6\sqrt{2}$

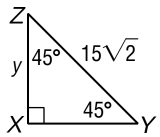
C. $6\sqrt{3}$ **D.** 12



7. Find y in $\triangle XYZ$. **7.** ______

A. $7.5\sqrt{3}$ **B.** $15\sqrt{3}$

C. 15 **D.** 30



8. The length of the sides of a square is 10 meters. Find the length of the diagonal of the square. **8.** ______

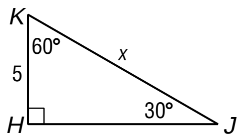
A. 10 m **B.** $10\sqrt{2}$ m

C. $10\sqrt{3}$ m **D.** 20 m

9. Find x in $\triangle HJK$. **9.** ______

A. $5\sqrt{2}$ **B.** $5\sqrt{3}$

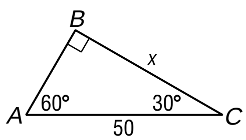
C. 10 **D.** 15



10. Find x in $\triangle ABC$. **10.** ______

A. 25 **B.** $25\sqrt{2}$

C. $25\sqrt{3}$ **D.** 100



Assessments

NAME ______________________________ DATE ____________ PERIOD ______

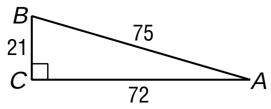
7 Chapter 7 Test, Form 1 *(continued)*

11. In $\triangle QRS$, $\angle R$ is a right angle. Which is the ratio for the tangent of $\angle S$? **11.** ______

A. $\dfrac{\text{measure of leg adjacent to } \angle S}{\text{measure of hypotenuse}}$ **B.** $\dfrac{\text{measure of hypotenuse}}{\text{measure of leg opposite } \angle S}$

C. $\dfrac{\text{measure of leg opposite } \angle S}{\text{measure of hypotenuse}}$ **D.** $\dfrac{\text{measure of leg opposite } \angle S}{\text{measure of leg adjacent to } \angle S}$

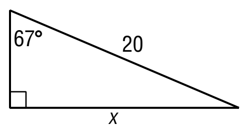
12. Find cos A in $\triangle ABC$. **12.** ______



A. $\frac{7}{24}$ **B.** $\frac{7}{25}$

C. $\frac{25}{24}$ **D.** $\frac{24}{25}$

13. Find x to the nearest tenth. **13.** ______



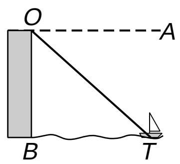
A. 7.3 **B.** 17.3

C. 18.4 **D.** 47.1

14. Find the angle of elevation of the sun when a pole 25 feet tall casts a shadow 42 feet long. **14.** ______

A. 30.8° **B.** 36.5° **C.** 53.5° **D.** 59.2°

15. Which is the angle of depression in the figure at the right? **15.** ______



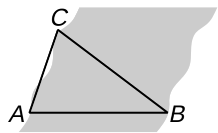
A. $\angle AOT$ **B.** $\angle AOB$

C. $\angle TOB$ **D.** $\angle BTO$

16. Find y in $\triangle XYZ$ to the nearest tenth if $m\angle Y = 36$, $m\angle X = 49$, and $x = 12$. **16.** ______

A. 0.04 **B.** 9.35 **C.** 14.80 **D.** 15.41

17. To find the distance between two points A and B on opposite sides of a river, a surveyor measures the distance from A to C as 200 feet, $m\angle A = 72$, and $m\angle B = 37$. Find the distance from A to B. Round your answer to the nearest tenth. **17.** ______



A. 77.4 ft **B.** 201.2 ft **C.** 250.4 ft **D.** 314.2 ft

18. In $\triangle ABC$, $a = 12$, $b = 8$, and $m\angle A = 40$. Find $m\angle B$ to the nearest tenth. **18.** ______

A. 25.4 **B.** 56.3 **C.** 59.3 **D.** 74.6

19. Find the third side of a triangular garden if two sides are 8 feet and 12 feet and the included angle has a measure 50. **19.** ______

A. 7.8 ft **B.** 9.2 ft **C.** 14.4 ft **D.** 146.3 ft

20. In $\triangle DEF$, $d = 20$, $e = 25$, and $f = 30$. Find $m\angle F$ to the nearest tenth. **20.** ______

A. 82.8 **B.** 75.5 **C.** 55.8 **D.** 47.2

Bonus In $\triangle ABC$, $a = 50$, $b = 48$, and $c = 40$. Find $m\angle A$ to the nearest tenth. **B:** ____________________

NAME ______________________ DATE ____________ PERIOD ______

7 Chapter 7 Test, Form 2A

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Find the geometric mean between 7 and 12. **1.** ______

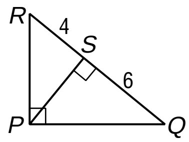
A. 5 **B.** 9.5

C. $\sqrt{19}$ **D.** $\sqrt{84}$

2. In $\triangle PQR$, $RS = 4$ and $QS = 6$. Find PS. **2.** ______

A. 2 **B.** 5

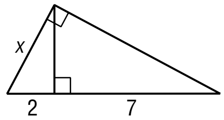
C. $\sqrt{10}$ **D.** $\sqrt{24}$



3. Find x. **3.** ______

A. $\sqrt{18}$ **B.** $\sqrt{14}$

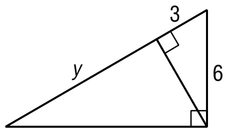
C. 4.5 **D.** 3



4. Find y. **4.** ______

A. 12 **B.** 11

C. 9 **D.** 2



5. Find the length of the hypotenuse of a right triangle whose legs measure 5 and 7. **5.** ______

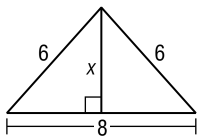
A. 12 **B.** $\sqrt{24}$

C. $\sqrt{35}$ **D.** $\sqrt{74}$

6. Find x. **6.** ______

A. 3 **B.** 4

C. $4\sqrt{3}$ **D.** $2\sqrt{5}$



7. Which of the following could represent sides of a right triangle? **7.** ______

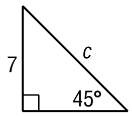
A. 9, 40, 41 **B.** 8, 30, 31

C. 7, 8, 15 **D.** $\sqrt{2}$, $\sqrt{3}$, $\sqrt{6}$

8. Find c. **8.** ______

A. 7 **B.** $7\sqrt{2}$

C. $7\sqrt{3}$ **D.** 14



9. Find the perimeter of a square to the nearest tenth if the length of its diagonal is 12 inches. **9.** ______

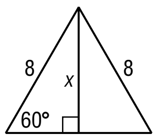
A. 8.5 in. **B.** 33.9 in.

C. 48 in. **D.** 67.9 in.

10. Find x. **10.** ______

A. 4 **B.** $4\sqrt{2}$

C. $4\sqrt{3}$ **D.** $8\sqrt{3}$

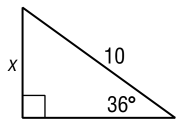


Assessments

NAME ______________________________ DATE ____________ PERIOD ______

7 Chapter 7 Test, Form 2A *(continued)*

11. Find x to the nearest tenth. **11.** ______

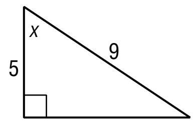


A. 5.8 **B.** 5.9
C. 8.1 **D.** 17.3

12. In right triangle ABC, $a = 12$, $b = 9$, and $c = 15$. Find $\tan \angle B$. **12.** ______

A. $\frac{4}{3}$ **B.** $\frac{5}{4}$ **C.** $\frac{3}{4}$ **D.** $\frac{3}{5}$

13. Find x to the nearest tenth of a degree. **13.** ______



A. 56.3 **B.** 45
C. 33.7 **D.** 29.1

14. If a 20-foot ladder makes a 65° angle with the ground, how many feet up a wall will it reach? Round your answer to the nearest tenth. **14.** ______

A. 8.5 ft **B.** 10 ft **C.** 18.1 ft **D.** 42.9 ft

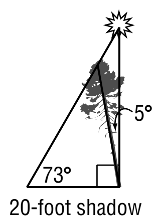
15. A ship's sonar finds that the angle of depression to a wreck on the bottom of the ocean is 12.5°. If a point on the ocean floor is 60 meters directly below the ship, how many meters, to the nearest tenth, is it from that point on the ocean floor to the wreck? **15.** ______

A. 277.2 m **B.** 270.6 m **C.** 61.5 m **D.** 13.3 m

16. To the nearest tenth of a degree, find the angle of elevation of the sun if a building 100 feet tall casts a shadow 150 feet long. **16.** ______

A. 60° **B.** 48.2° **C.** 41.8° **D.** 33.7°

17. When the sun's angle of elevation is 73°, a tree tilted at an angle of 5° from the vertical casts a 20-foot shadow on the ground. Find the length of the tree to the nearest tenth. **17.** ______



A. 6.3 ft **B.** 19.2 ft
C. 51.1 ft **D.** 219.4 ft

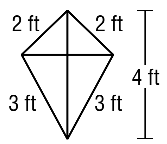
18. In $\triangle CDE$, $m\angle C = 52$, $m\angle D = 17$, and $e = 28.6$. Find c to the nearest tenth. **18.** ______

A. 77.1 **B.** 49.1 **C.** 24.1 **D.** 18.4

19. In $\triangle PQR$, $p = 56$, $r = 17$, and $m\angle Q = 110$. Find q to the nearest tenth. **19.** ______

A. 4076.2 **B.** 63.8 **C.** 52.6 **D.** 3.1

20. Pete is building a kite using the dimensions given in the figure at the right. Find the measure of the angle the 2-foot edge makes with the 3-foot edge. **20.** ______



A. 104.5 **B.** 85.2
C. 60 **D.** 14.5

Bonus From a window 20 feet above the ground, the angle of elevation to the top of another building is 35°. The distance between the buildings is 52 feet. Find the height of the building to the nearest tenth of a foot. **B:** ____________________

NAME ____________________ DATE ____________ PERIOD ______

7 Chapter 7 Test, Form 2B

SCORE ______

Write the letter for the correct answer in the blank at the right of each question.

1. Find the geometric mean between 9 and 11. **1.** ______

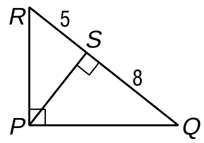
A. $\sqrt{99}$ **B.** $\sqrt{20}$

C. 10 **D.** 2

2. In $\triangle PQR$, $RS = 5$ and $QS = 8$. Find PS. **2.** ______

A. 3 **B.** 6.5

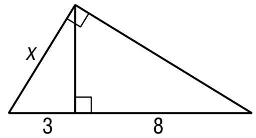
C. $\sqrt{13}$ **D.** $\sqrt{40}$



3. Find x. **3.** ______

A. 5.5 **B.** $\sqrt{11}$

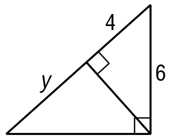
C. $\sqrt{24}$ **D.** $\sqrt{33}$



4. Find y. **4.** ______

A. 4 **B.** 5

C. 8 **D.** 9



5. Find the length of the hypotenuse of a right triangle whose legs measure 6 and 5. **5.** ______

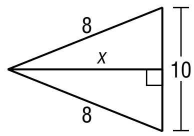
A. 11 **B.** $\sqrt{11}$

C. $\sqrt{30}$ **D.** $\sqrt{61}$

6. Find x. **6.** ______

A. $\sqrt{39}$ **B.** 6

C. $5\sqrt{3}$ **D.** 5



7. Which of the following could represent sides of a right triangle? **7.** ______

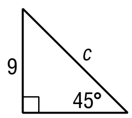
A. $\frac{3}{4}, 1, \frac{5}{4}$ **B.** $\sqrt{3}, \sqrt{5}, \sqrt{15}$

C. 7, 17, 24 **D.** 8, 15, 16

8. Find c. **8.** ______

A. 18 **B.** $9\sqrt{3}$

C. $9\sqrt{2}$ **D.** 9



9. Find the perimeter of a square to the nearest tenth if the length of its diagonal is 16 millimeters. **9.** ______

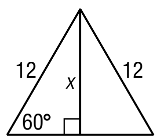
A. 11.3 mm **B.** 45.3 mm

C. 90.5 mm **D.** 128.0 mm

10. Find x. **10.** ______

A. 6 **B.** $6\sqrt{2}$

C. $6\sqrt{3}$ **D.** $12\sqrt{3}$

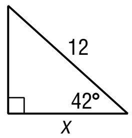


Assessments

7 Chapter 7 Test, Form 2B *(continued)*

11. Find x. **11.** ______

A. 8.0 **B.** 8.9
C. 10.4 **D.** 10.8

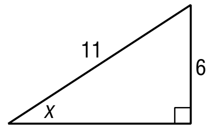


12. In right triangle ABC, $a = 14$, $b = 48$, and $c = 50$. Find tan $\angle A$. **12.** ______

A. $\frac{7}{24}$ **B.** $\frac{7}{25}$ **C.** $\frac{24}{25}$ **D.** $\frac{24}{7}$

13. Find x to the nearest tenth of a degree. **13.** ______

A. 56.9 **B.** 54.5
C. 33.1 **D.** 28.6



14. If a 24-foot ladder makes a 58° angle with the ground, how many feet up a wall will it reach? Round your answer to the nearest tenth. **14.** ______

A. 38.4 ft **B.** 20.8 ft **C.** 20.4 ft **D.** 12.7 ft

15. A ship's sonar finds that the angle of depression to a wreck on the bottom of the ocean is 13.2°. If a point on the ocean floor is 75 meters directly below the ship, how many meters, to the nearest tenth, is it from that point on the ocean floor to the wreck? **15.** ______

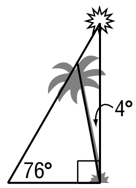
A. 328.4 m **B.** 319.8 m **C.** 77.0 m **D.** 17.6 m

16. To the nearest tenth of a degree, find the angle of elevation of the sun if a building 125 feet tall casts a shadow 196 feet long. **16.** ______

A. 63.8° **B.** 50.4° **C.** 39.6° **D.** 32.5°

17. When the sun's angle of elevation is 76°, a tree tilted at an angle of 4° from the vertical casts a 18-foot shadow on the ground. Find the length of the tree, to the nearest tenth. **17.** ______

A. 250.4 ft **B.** 56.5 ft
C. 17.7 ft **D.** 4.6 ft



18. In $\triangle ABC$, $m\angle A = 46$, $m\angle B = 105$, and $c = 19.8$. Find a to the nearest tenth. **18.** ______

A. 29.4 **B.** 28.5 **C.** 15.7 **D.** 14.7

19. In $\triangle LMN$, $l = 42$, $m = 61$, and $m\angle N = 108$. Find n to the nearest tenth. **19.** ______

A. 7068.4 **B.** 84.1 **C.** 79.2 **D.** 24.7

20. Josephine is planning a triangular garden. If the lengths of the sides are 50 feet, 80 feet, and 100 feet, what is the measure of the largest angle? **20.** ______

A. 7.9° **B.** 82.1° **C.** 89.9° **D.** 97.9°

Bonus From a window 24 feet above the ground, the angle of elevation to the top of another building is 38°. The distance between the buildings is 63 feet. Find the height of the building to the nearest tenth of a foot. **B:** ______________

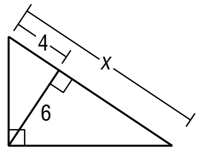
7 Chapter 7 Test, Form 2C

SCORE ______

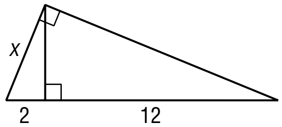
1. Find the geometric mean between $2\sqrt{5}$ and $5\sqrt{2}$. **1.** ________

For Questions 2–5, find *x*.

2.



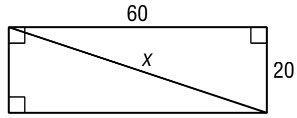
3.



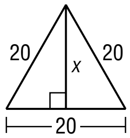
2. ________

3. ________

4.



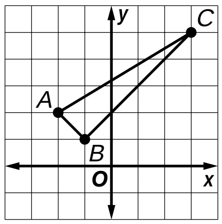
5.



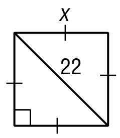
4. ________

5. ________

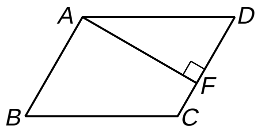
6. Determine whether $\triangle ABC$ is a right triangle. Explain your answer. **6.** ________



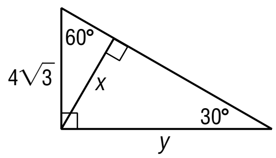
7. Find *x*. **7.** ________



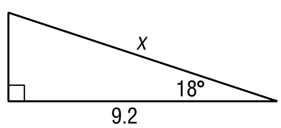
8. In parallelogram *ABCD*, $AD = 4$ and $m\angle D = 60$. Find *AF*. **8.** ________



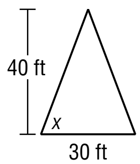
9. Find *x* and *y*. **9.** ________



10. Find *x* to the nearest tenth. **10.** ________



11. An A-frame house is 40 feet high and 30 feet wide. Find the measure of the angle, to the nearest tenth of a degree, that the roof makes with the floor. **11.** ________

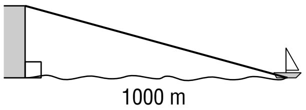


12. A 30-foot tree casts a 12-foot shadow. Find the angle of elevation of the sun to the nearest tenth of a degree. **12.** ________

Assessments

7 Chapter 7 Test, Form 2C *(continued)*

13. A boat is 1000 meters from a cliff. If the angle of depression from the top of the cliff to the boat is 15°, how tall is the cliff? Round your answer to the nearest tenth.

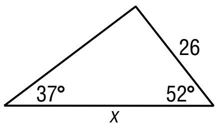


13. ________________

14. A plane flying at an altitude of 10,000 feet begins descending when the end of the runway is below a point 50,000 feet away. Find the angle of descent (depression) to the nearest tenth of a degree.

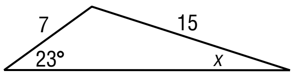
14. ________________

15. Find x to the nearest tenth.



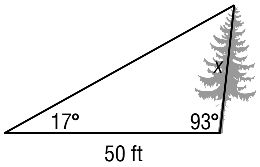
15. ________________

16. Find x to the nearest tenth of a degree.



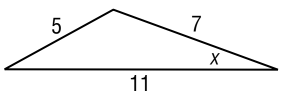
16. ________________

17. A tree grew at a 3° slant from the vertical. At a point 50 feet from the tree, the angle of elevation to the top of the tree is 17°. Find the length of the tree to the nearest tenth of a foot.



17. ________________

18. Find x to the nearest tenth of a degree.

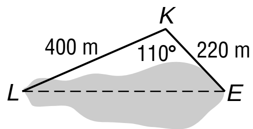


18. ________________

19. In $\triangle XYZ$, $m\angle X = 152$, $y = 15$, and $z = 19$. Find x to the nearest tenth.

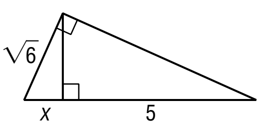
19. ________________

20. To approximate the length of a pond, a surveyor walks 400 meters from point L to point K, then turns and walks 220 meters from point K to point E. If $m\angle LKE = 110$, find the length LE of the pond to the nearest tenth of a meter.



20. ________________

Bonus Find x.



B: ________________

NAME ________________ DATE ________ PERIOD ______

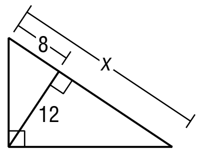
7 Chapter 7 Test, Form 2D

SCORE ______

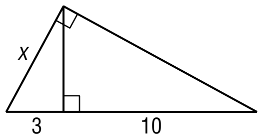
1. Find the geometric mean between $3\sqrt{6}$ and $5\sqrt{6}$. **1.** ________

For Questions 2–5, find *x*.

2.



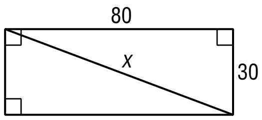
3.



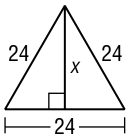
2. ________

3. ________

4.



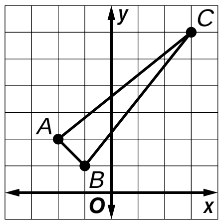
5.



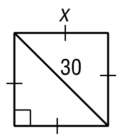
4. ________

5. ________

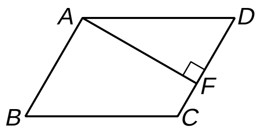
6. Determine whether $\triangle ABC$ is a right triangle. Explain your answer. **6.** ________



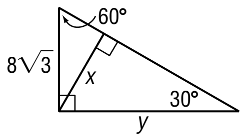
7. Find x. **7.** ________



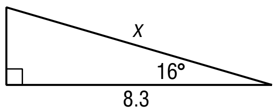
8. In parallelogram $ABCD$, $AD = 14$ and $m\angle D = 60$. Find AF. **8.** ________



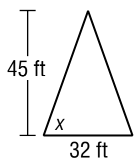
9. Find x and y. **9.** ________



10. Find x to the nearest tenth. **10.** ________



11. An A-frame house is 45 feet high and 32 feet wide. Find the measure of the angle, to the nearest tenth of a degree, that the roof makes with the floor. **11.** ________

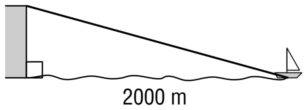


12. A 38-foot tree casts a 16-foot shadow. Find the angle of elevation of the sun to the nearest tenth of a degree. **12.** ________

Assessments

7 Chapter 7 Test, Form 2D *(continued)*

13. A boat is 2000 meters from a cliff. If the angle of depression from the top of the cliff to the boat is 10°, how tall is the cliff? Round your answer to the nearest tenth.

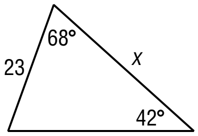


13. ______________

14. A plane flying at an altitude of 10,000 feet begins descending when the end of the runway is below a point 60,000 feet away. Find the angle of descent (depression) to the nearest tenth of a degree.

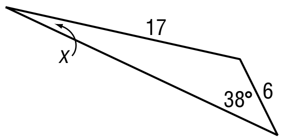
14. ______________

15. Find x to the nearest tenth.



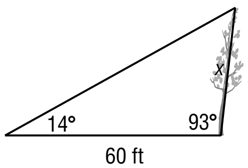
15. ______________

16. Find x to the nearest tenth of a degree.



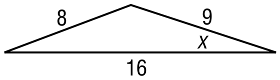
16. ______________

17. A tree grew at a 3° slant from the vertical. At a point 60 feet from the tree, the angle of elevation to the top of the tree is 14°. Find the length of the tree to the nearest tenth of a foot.



17. ______________

18. Find x to the nearest tenth of a degree.

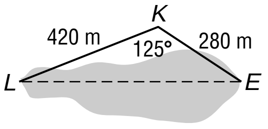


18. ______________

19. In $\triangle XYZ$, $m\angle X = 156$, $y = 18$, and $z = 21$. Find x to the nearest tenth.

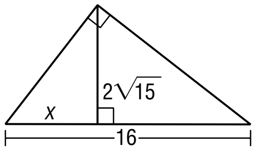
19. ______________

20. To approximate the length of a pond, a surveyor walks 420 meters from point L to point K, then turns and walks 280 meters from point K to point E. If $m\angle LKE = 125$, find the length LE of the pond to the nearest tenth of a meter.



20. ______________

Bonus Find x.



B: ______________

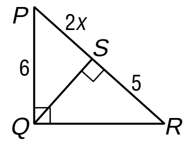
NAME ______________________ DATE __________ PERIOD ______

7 Chapter 7 Test, Form 3

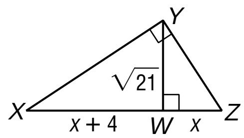
SCORE ______

1. Find the geometric mean between $\frac{2}{9}$ and $\frac{3}{9}$. **1.** ______________

2. Find x in $\triangle PQR$. **2.** ______________



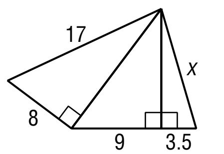
3. Find x in $\triangle XYZ$. **3.** ______________



4. If the length of one leg of a right triangle is three times the length of the other and the hypotenuse is 20, find the length of the shorter leg. **4.** ______________

5. Find the length of the altitude to the hypotenuse of a right triangle with legs of length 3 and 4. **5.** ______________

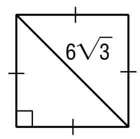
6. Find x. **6.** ______________



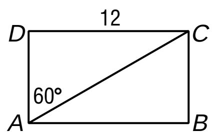
7. Richmond is 200 kilometers due east of Teratown and Hamilton is 150 kilometers directly north of Teratown. Find the shortest distance in kilometers between Hamilton and Richmond. **7.** ______________

8. Is 48, 55, 73 a Pythagorean triple? Show why or why not. **8.** ______________

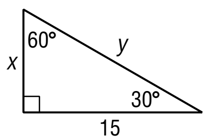
9. Find the perimeter of this square. **9.** ______________



10. Find the perimeter of rectangle $ABCD$. **10.** ______________



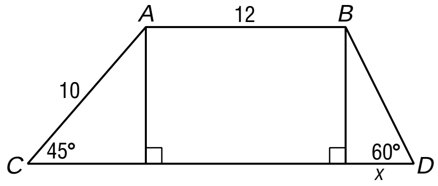
11. Find x and y. **11.** ______________



12. $\triangle ABC$ is a 30°-60°-90° triangle with right angle A and with $\overline{AC}$ as the longer leg. Find the coordinates of C if $A(-4, -2)$ and $B(-4, 6)$. **12.** ______________

7 Chapter 7 Test, Form 3 *(continued)*

13. If $\overline{AB} \parallel \overline{CD}$, find x and the length of $\overline{CD}$.



13. ____________

14. The angle of elevation from a point on the street to the top of a building is 29°. The angle of elevation from another point on the street, 50 feet farther away from the building, to the top of the building is 25°. To the nearest foot, how tall is the building?

14. ____________

15. The angle of depression from the top of a flagpole on top of a lighthouse to a boat on the ocean is 37°, while the angle of depression from the bottom of the flagpole to the boat is 36.8°. If the boat is 1 mile away from shore and the lighthouse is right on the edge of the shore, how tall is the flagpole? Round your answer to the nearest foot.

15. ____________

16. In $\triangle JKL$, $m\angle J = 26.8$, $m\angle K = 19$, and $k = 17$. Find ℓ to the nearest tenth.

16. ____________

17. Solve $\triangle PQR$ for $r = 22$, $p = 51$, and $m\angle Q = 96$. Round answers to the nearest tenth.

17. ____________

18. Don hit a golf ball 250 yards toward the hole. However, due to the wind, his drive is 5° off course. If the angle between the hole and where the ball lands is 97°, how far is it from where Don hit the ball to the hole? Round to the nearest tenth of a yard.

18. ____________

19. In $\triangle HJK$, $m\angle H = 32$, $k = 8$, and $h = 7$. Find $m\angle K$. Round your answer(s) to the nearest tenth of a degree.

19. ____________

20. The distance from Albany to Bethany is 75 miles and from Bethany to Celina 105 miles. If the roads from Bethany to Albany and from Bethany to Celina make an 87° angle, what is the distance from Albany to Celina? Round to the nearest tenth.

20. ____________

Bonus A 50-foot vertical pole that stands on a hillside makes an angle of 10° with the horizontal. Two guy wires extend from the top of the pole to points on the hill 60 feet uphill and downhill from its base. Find the length of each guy wire to the nearest tenth.

B: ____________

NAME ______________________ DATE ____________ PERIOD ______

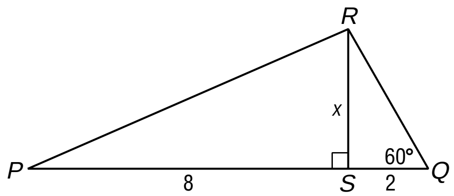
7 Chapter 7 Open-Ended Assessment

SCORE ______

Demonstrate your knowledge by giving a clear, concise solution to each problem. Be sure to include all relevant drawings and justify your answers. You may show your solution in more than one way or investigate beyond the requirements of the problem.

1. If the geometric mean between 10 and x is 6, what is x? Show how you obtained your answer.

2.



a. Max used the following equations to find x in $\triangle PQR$. Is Max correct? Why or why not?

$$\frac{2}{x} = \frac{x}{8}$$
$$x^2 = 2 \cdot 8$$
$$x^2 = 16$$
$$x = 4$$

b. For $\angle PRQ$ to be a right angle, what would the measure of $\overline{PS}$ have to be?

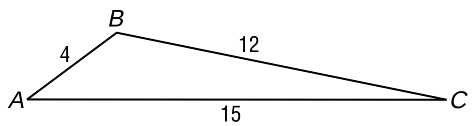
c. Is $\triangle PRS$ a 45°-45°-90° triangle? How do you know?

3. To solve for x in a triangle, when would you use *sin* and when would you use sin^{-1}? Give an example for each type of situation.

4. Draw a diagram showing where the angles of elevation and depression are. How are the measures of these angles related?

5. Draw an obtuse triangle and label the vertices and the measures of two angles and the length of one side. Explain how to solve the triangle.

6. Irina is solving $\triangle ABC$. She plans to first use the Law of Sines to find two of the angles. Is Irina's plan a good one? Why or why not?



NAME ______________________ DATE __________ PERIOD _____

7 Chapter 7 Vocabulary Test/Review

SCORE _____

ambiguous case	geometric mean	reciprocal identities	tangent
angle of depression	Law of Cosines	secant	trigonometric identity
angle of elevation	Law of Sines	sine	trigonometric ratio
cosecant	Pythagorean identity	solving a triangle	trigonometry
cosine	Pythagorean triple		

Choose from the terms above to complete each sentence.

1. The square root of the product of two numbers is the __?__ of the numbers. **1.** __________

2. A group of three whole numbers that satisfy the equation $a^2 + b^2 = c^2$, where c is the greatest number, is called a(n) __?__. **2.** __________

3. A ratio of the lengths of two sides of a right triangle is called a(n) __?__. **3.** __________

4. An angle between the line of sight and the horizontal when an observer looks upward is called a(n) __?__. **4.** __________

5. An angle between the line of sight and the horizontal when an observer looks downward is called a(n) __?__. **5.** __________

6. Three commonly used trigonometric ratios are the __?__, __?__, and __?__. **6.** __________

7. For $\triangle ABC$, the __?__ says $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. **7.** __________

8. For $\triangle ABC$, the __?__ says $a^2 = b^2 + c^2 - 2bc \cos A$. **8.** __________

9. The reciprocal of the sine is called the __?__. **9.** __________

10. The reciprocal of the cosine is called the __?__. **10.** __________

Define each term.

11. solving a triangle **11.** __________

12. Pythagorean Theorem **12.** __________

NAME ______________________ DATE ____________ PERIOD ______

7

Chapter 7 Quiz

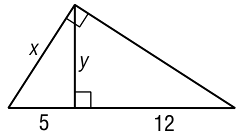
(Lessons 7–1 and 7–2)

SCORE ______

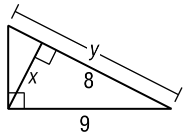
1. Find the geometric mean between 12 and 16. **1.** ____________

For Questions 2 and 3, find *x* and *y*.

2.



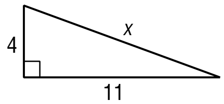
3.



2. ____________

3. ____________

4. Find *x*.



4. ____________

5. Do 19, 15, and 13 form a Pythagorean triple? Why or why not? **5.** ____________

NAME ______________________ DATE ____________ PERIOD ______

7

Chapter 7 Quiz

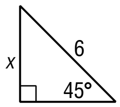
(Lessons 7–3 and 7–4)

SCORE ______

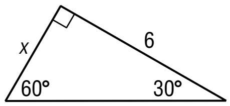
Assessments

For Questions 1 and 2, find *x*.

1.



2.

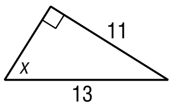


1. ____________

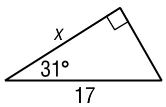
2. ____________

For Questions 3 and 4, find *x* to the nearest tenth.

3.



4.



3. ____________

4. ____________

5. A rectangle has a diagonal 20 inches long that forms angles of 60° and 30° with the sides. Find the perimeter of the rectangle. **5.** ____________

6. Find sin 52°. Round to the nearest ten-thousandth. **6.** ____________

7. If $\cos A = 0.8945$, find $\angle A$ to the nearest tenth of a degree. **7.** ____________

8. The distance along a hill is 24 feet. If the land slopes uphill at an angle of 8°, find the vertical distance from the top to the bottom of the hill. **8.** ____________

9. A surveyor is standing on horizontal ground level with the base of a skyscraper. The angle formed by the line segment from his position to the top of the skyscraper is 31°. The height of the building is 1200 feet. Find the distance from the building to the surveyor to the nearest foot. **9.** ____________

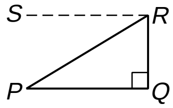
NAME ______________________ DATE ____________ PERIOD ______

7 Chapter 7 Quiz

SCORE ______

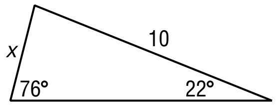
(Lessons 7–5 and 7–6)

1. Name the angle of elevation in the figure.



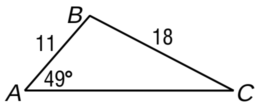
1. ____________

2. Find x to the nearest tenth.



2. ____________

3. Solve $\triangle ABC$. Round your answers to the nearest tenth.



3. ____________

4. A triangular lot has 500 feet of frontage along a river. The other two sides make angles of 48° and 75° with the riverfront side. Find the length of the shortest side to the nearest foot.

4. ____________

5. STANDARDIZED TEST PRACTICE A squirrel 37 feet up in a tree sees a dog 29 feet from the base of the tree. Find the measure of the angle of depression to the nearest tenth of a degree.

A. 38.4 **B.** 51.9 **C.** 45.0 **D.** 128.1

5. ____________

NAME ______________________ DATE ____________ PERIOD ______

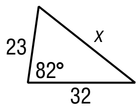
7 Chapter 7 Quiz

SCORE ______

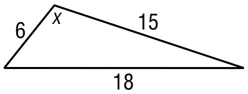
(Lesson 7–7)

For Questions 1 and 2, find x to the nearest tenth.

1.



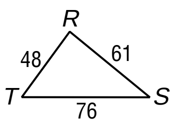
2.



1. ____________

2. ____________

3. Solve $\triangle RST$. Round your answers to the nearest degree.



3. ____________

4. A hiker is 6 miles from a tower and 8 miles from the lodge. She estimates that the angle between her path to the tower and her path to the lodge is 42°. Find the distance from the tower to the lodge to the nearest tenth of a mile.

4. ____________

5. STANDARDIZED TEST PRACTICE For $\triangle ABC$, find a to the nearest tenth if $m\angle A = 96$, $b = 41$, and $c = 50$.

A. 66.3 **B.** 67.9 **C.** 4395.3 **D.** 4609.6

5. ____________

NAME ________________________ DATE ____________ PERIOD ______

7 Chapter 7 Mid-Chapter Test

(Lessons 7–1 through 7–4)

SCORE ______

Part I ***Write the letter for the correct answer in the blank at the right of each question.***

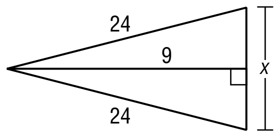
1. Find the geometric mean between 7 and 9. **1.** ______

A. $\sqrt{63}$ **B.** 16 **C.** 8 **D.** 2

2. Find x. **2.** ______

A. $\sqrt{216}$ **B.** $\frac{\sqrt{2}}{5}$

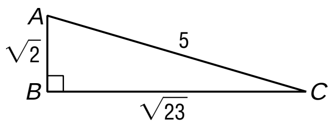
C. $6\sqrt{55}$ **D.** $\frac{\sqrt{23}}{5}$



3. Find sin C. **3.** ______

A. $\sqrt{2}$ **B.** $\frac{\sqrt{2}}{5}$

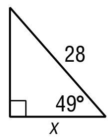
C. $\frac{\sqrt{23}}{\sqrt{2}}$ **D.** $\frac{\sqrt{23}}{5}$



4. Find x to the nearest tenth. **4.** ______

A. 14 **B.** 18.4

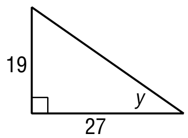
C. 21.1 **D.** 32.2



5. Find y to the nearest tenth of a degree. **5.** ______

A. 144.9 **B.** 60.0

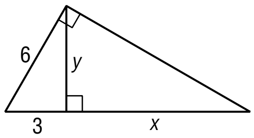
C. 44.7 **D.** 35.1



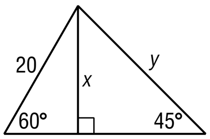
Part II

For Questions 6–8, find *x* and *y*.

6.



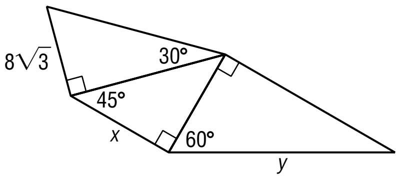
7.



6. ____________________

7. ____________________

8.



8. ____________________

9. Do 56, 90, 106 form a Pythagorean triple? Why or why not? **9.** ____________________

10. Guy wires 80 feet long support a 65-foot tall telephone pole. To the nearest tenth of a degree, what angle will the wires make with the ground? **10.** ____________________

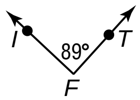
NAME ________________ DATE ________ PERIOD ______

7 Chapter 7 Cumulative Review

(Chapters 1–7)

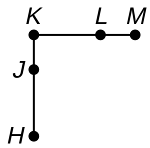
SCORE ______

1. Name the vertex and sides, then classify $\angle IFT$. (Lesson 1-4)



1. ________________

For Questions 2 and 3, complete the following proof. (Lesson 2-7)



Given: $\overline{JK} \cong \overline{LM}$

$\overline{HJ} \cong \overline{KL}$

Prove: $\overline{HK} \cong \overline{KM}$

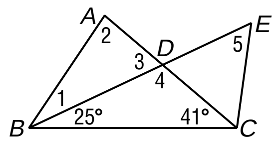
Proof:

Statements	Reasons
1. $\overline{JK} \cong \overline{LM}$, $\overline{HJ} \cong \overline{KL}$	1. Given
2. $JK = LM$, $HJ = KL$	2. (Question 2)
3. (Question 3)	3. Segment Addition Post.
4. $HJ + JK = KL + LM$	4. Substitution Prop.
5. $HK = KM$	5. Substitution Prop.
6. $\overline{HK} \cong \overline{KM}$	6. Def. of $\cong$ segments

2. ________________

3. ________________

For Questions 4 and 5, use the figure at the right.



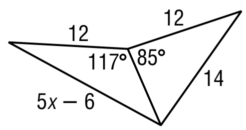
4. Find the measure of the numbered angles if $m\angle ABC = 57$ and $m\angle BCE = 98$. (Lesson 4-2)

4. ________________

5. If $\overline{BD}$ is a median, $AD = 2x - 6$, and $DC = 22.5 - 4x$, find AC. (Lesson 5-1)

5. ________________

6. Write an inequality to describe the possible values of x. (Lesson 5-5)



6. ________________

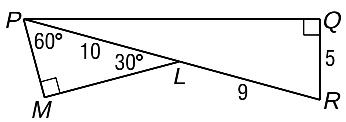
7. A band of sequins that measures 108 inches is cut into two pieces so that their lengths are in a 5:7 ratio. Find the length of each piece. (Lesson 6-1)

7. ________________

8. Stan invests \$1875 in a certificate of deposit that earns 4.5% interest compounded annually. Find the balance of his account after 4 years. (Lesson 6-6)

8. ________________

For Questions 9 and 10, use the figure at the right.



9. Find QP to the nearest tenth. (Lesson 7-2)

9. ________________

10. Find LM and PM. (Lesson 7-3)

10. ________________

NAME ______________________ DATE ____________ PERIOD ______

7 Standardized Test Practice

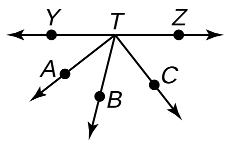
(Chapters 1–7)

SCORE ______

Part 1: Multiple Choice

Instructions: Fill in the appropriate oval for the best answer.

1. If $\overline{TA}$ bisects $\angle YTB$, $\overline{TC}$ bisects $\angle BTZ$, $m\angle YTA = 4y + 6$, and $m\angle BTC = 7y - 4$, find $m\angle CTZ$. (Lesson 1-4)



A. 52 **B.** 38
C. 25 **D.** 8

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. Which statement is *always* true? (Lesson 2-5)

E. If right $\triangle QPR$ has sides q, p, and r, where r is the hypotenuse, then $r^2 = p^2 + q^2$.

F. If $\overline{EF} \parallel \overline{HJ}$, then $EF = HJ$.

G. If lines KL and VT are cut by a transversal, then $\overline{KL} \parallel \overline{VT}$.

H. If $\overline{DR}$ and $\overline{RH}$ are congruent, then R bisects $\overline{DH}$.

2. Ⓔ Ⓕ Ⓖ Ⓗ

3. The equation for $\overleftrightarrow{PT}$ is $y - 2 = 8(x + 3)$. Determine an equation for a line perpendicular to $\overleftrightarrow{PT}$. (Lesson 3-4)

A. $y = \frac{1}{8}x - 7$ **B.** $y = 8x - 13$

C. $y = -\frac{1}{8}x + 2$ **D.** $y = -8x$

3. Ⓐ Ⓑ Ⓒ Ⓓ

4. Angle Y in $\triangle XYZ$ measures 90°. $\overline{XY}$ and $\overline{YZ}$ each measure 16 meters. Classify $\triangle XYZ$. (Lesson 4-1)

E. acute and isosceles

F. equiangular and equilateral

G. right and scalene

H. right and isosceles

4. Ⓔ Ⓕ Ⓖ Ⓗ

5. Two sides of a triangle measure 4 inches and 9 inches. Determine which cannot be the perimeter of the triangle. (Lesson 5-4)

A. 19 in. **B.** 21 in. **C.** 23 in. **D.** 26 in.

5. Ⓐ Ⓑ Ⓒ Ⓓ

6. $\triangle ABC \sim \triangle STR$, so $\frac{AB}{CA} =$ ___?___. (Lesson 6-2)

E. $\frac{AB}{BC}$ **F.** $\frac{ST}{RS}$ **G.** $\frac{TR}{RS}$ **H.** $\frac{RS}{ST}$

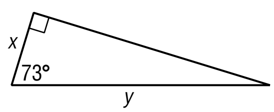
6. Ⓔ Ⓕ Ⓖ Ⓗ

7. The Petronas Towers in Kuala Lumpur, Malaysia, are 452 meters tall. A woman who is 1.75 meters tall stands 120 meters from the base of one tower. Find the angle of elevation between the woman's hat and the top of the tower to the nearest tenth. (Lesson 7-5)

A. 14.8° **B.** 15.4° **C.** 74.5° **D.** 75.1°

7. Ⓐ Ⓑ Ⓒ Ⓓ

8. Which equation can be used to find x? (Lesson 7-4)



E. $x = y \sin 73°$ **F.** $x = y \cos 73°$

G. $x = \frac{y}{\cos 73°}$ **H.** $x = \frac{y}{\sin 73°}$

8. Ⓔ Ⓕ Ⓖ Ⓗ

Assessments

7 Standardized Test Practice *(continued)*

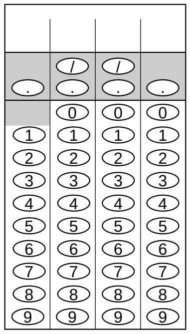
Part 2: Grid In

Instructions: Enter your answer by writing each digit of the answer in a column box and then shading in the appropriate oval that corresponds to that entry.

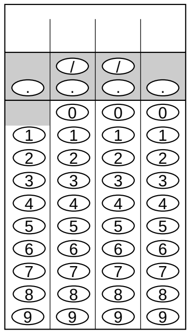
9. Find the measure of the smaller of two complementary angles whose measures differ by 23. (Lesson 1-5)

10. How many counterexamples are necessary to prove that a statement is false? (Lesson 2-3)

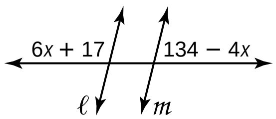
9.



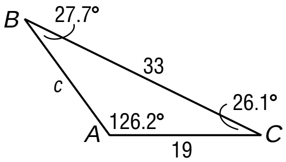
10.



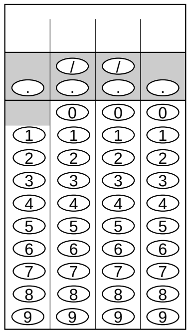
11. Find x so that $\ell \parallel m$. (Lesson 3-5)



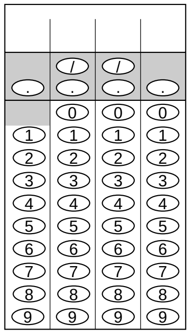
12. Find c to the nearest tenth. (Lesson 7-6)



11.



12.



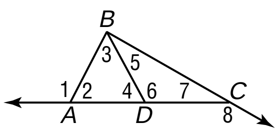
Part 3: Short Response

Instructions: Show your work or explain in words how you found your answer.

13. If $\triangle DEF \cong \triangle HJK$, $m\angle D = 26$, $m\angle J = 3x + 5$, and $m\angle F = 92$, find x. (Lesson 4-3)

13. ____________________

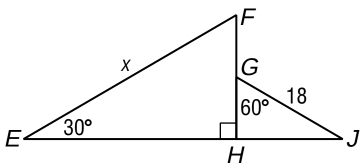
14. Use the Exterior Angle Inequality Theorem to list all of the angles whose measures are less than $m\angle 1$. (Lesson 5-2)



14. ____________________

For Questions 15 and 16, use the figure at the right.

15. Determine whether $\triangle EFH \sim \triangle JGH$. (Lesson 6-3)



15. ____________________

16. If G is the midpoint of $\overline{FH}$, find x. (Lesson 7-3)

16. ____________________

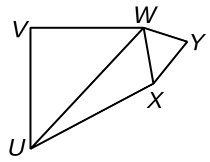
NAME ______________________ DATE ____________ PERIOD ______

7 Unit 2 Review

SCORE ______

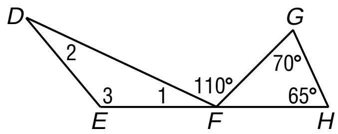
(Chapter 4–7)

1. Use a protractor to classify $\triangle UVW$, $\triangle UWX$, and $\triangle XWY$ as *acute*, *equiangular*, *obtuse*, or *right*.



1. ______________

2. In the figure, $\angle 1 \cong \angle 2$. Find the measures of the numbered angles.



2. ______________

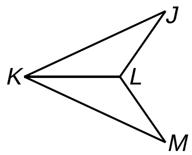
3. Name the corresponding congruent sides for $\triangle AFP \cong \triangle STX$.

3. ______________

4. Determine whether $\triangle ABC \cong \triangle PQR$ given $A(2, -7)$, $B(5, 3)$, $C(-4, 6)$, $P(8, -1)$, $Q(11, 9)$, and $R(2, 12)$.

4. ______________

5. In the figure, $\overline{LK}$ bisects $\angle JKM$ and $\angle KLJ \cong \angle KLM$. Determine which theorem or postulate can be used to prove that $\triangle JKL \cong \triangle MKL$.

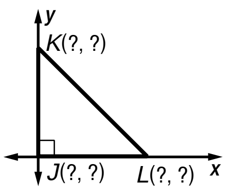


5. ______________

6. Triangle ABC is isosceles with $AB = BC$. Name a pair of congruent angles in this triangle.

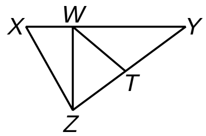
6. ______________

7. Name the missing coordinates for isosceles right $\triangle JKL$ with legs b units long.



7. ______________

For Questions 8 and 9, refer to the figure.



8. Find a and $m\angle ZWT$ if $\overline{ZW}$ is an altitude of $\triangle XYZ$, $m\angle ZWT = 3a + 5$, and $m\angle TWY = 5a + 13$.

8. ______________

9. Determine which angle has the greatest measure: $\angle YWZ$, $\angle WZY$, or $\angle ZYW$.

9. ______________

10. Mr. Ramirez bought a stove and a dishwasher for just over \$1206. State the assumption you would make to start an indirect proof to show that at least one of the appliances cost more than \$603.

10. ______________

Assessments

Unit 2 Review *(continued)*

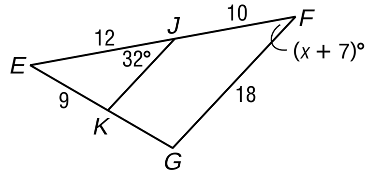
11. Determine whether 128 feet, 136 feet, and 245 feet can be the lengths of the sides of a triangle.

11. ______________

12. Casey has a 13-inch television and a 52-inch television in her home. What is the ratio of the sizes of the smaller and larger TVs?

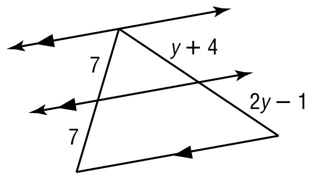
12. ______________

13. If $\triangle EFG \sim \triangle EJK$, find x, JK, KG, and the scale factor relating $\triangle EFG$ to $\triangle EJK$.



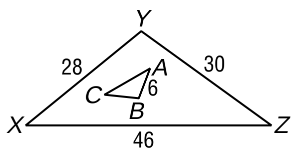
13. ______________

14. Find y.



14. ______________

15. Find the perimeter of $\triangle ABC$ if $\triangle ABC \sim \triangle XYZ$.



15. ______________

16. Alex has $750 in a bank account that earns 2.7% interest. If the interest is compounded annually and he does not make any withdrawals, find the balance of his account after 3 years.

16. ______________

17. Find the geometric mean between 27 and 42 to the nearest tenth.

17. ______________

18. Determine whether 27, 120, and 123 are the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

18. ______________

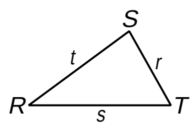
19. The diagonal of a square is 56 centimeters long. Find the perimeter of the square to the nearest tenth.

19. ______________

20. Find $m\angle P$ to the nearest tenth in right $\triangle MNP$ for $M(3, 6)$, $N(3, -8)$, and $P(-5, -8)$.

20. ______________

For Questions 21 and 22, refer to the figure.



21. Find $m\angle S$ if $m\angle T = 68$, $t = 65$, and $s = 33$.

21. ______________

22. Solve $\triangle RST$ if $t = 17$, $s = 11$, and $m\angle R = 40$.

22. ______________

NAME ____________________ DATE ________ PERIOD ______

7 First Semester Test

SCORE ______

(Chapter 1–7)

For Questions 1–7, write the letter for the correct answer in the blank at the right of each question.

1. Angles *AFH* and *HFB* form a linear pair and $m\angle AFH = 83$. Find $m\angle HFB$. **1.** ______

A. 164 **B.** 97 **C.** 83 **D.** 41.5

2. Given $C(2, 5)$, $D(7, 0)$, and $F(13, -6)$, which of the following is a true conjecture? **2.** ______

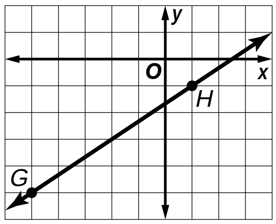
A. $\triangle CDF$ is a right triangle. **B.** $\triangle CDF$ is an isosceles triangle.
C. $\triangle CDF$ is an equilateral triangle. **D.** *C*, *D*, and *F* do not form a triangle.

3. Which is the inverse of the statement *If* $x = 5$, *then* $x + 3 = 8$? **3.** ______

A. If $x + 3 = 8$, then $x = 5$. **B.** If $x \neq 5$, then $x + 3 \neq 8$.
C. If $x = 5$, then $x + 3 = 8$. **D.** If $x + 3 \neq 8$, then $x \neq 5$.

4. Find the slope of a line that is perpendicular to $\overleftrightarrow{GH}$. **4.** ______

A. $\frac{2}{3}$ **B.** $\frac{3}{2}$
C. $-\frac{2}{3}$ **D.** $-\frac{3}{2}$

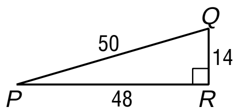


5. Find the distance between parallel lines ℓ and m whose equations are $y = \frac{3}{4}x + 4$ and $y = \frac{3}{4}x - \frac{9}{4}$. **5.** ______

A. 4 **B.** 5 **C.** 9 **D.** $\frac{9}{4}$

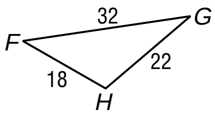
6. Find sin *P*. **6.** ______

A. $\frac{14}{50}$ **B.** $\frac{14}{48}$
C. $\frac{48}{50}$ **D.** 1



7. Find $m\angle G$. **7.** ______

A. 30° **B.** 32°
C. 35° **D.** 55.8°



8. Find *c* and *PK* if *P* is between *L* and *K*, $LP = c + 22$, $PK = 5c$, and $LK = 34$. Does *P* bisect $\overline{LK}$? **8.** ____________

9. Determine the distance between $A(15, -12)$, and $B(-30, 48)$ on a coordinate plane. State the coordinates of the midpoint of $\overline{AB}$. **9.** ____________

Justify each statement with a property or definition.

10. If $\overline{AC} \cong \overline{BD}$, then $AC = BD$. **10.** ____________

11. If $\angle 2$ and $\angle 3$ are complementary, then $m\angle 2 + m\angle 3 = 90$. **11.** ____________

Assessments

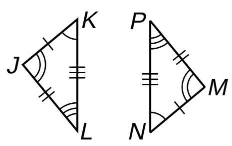
NAME ____________________ DATE __________ PERIOD ______

7 First Semester Test *(continued)*

12. If the measures of two angles of a triangle are 24 and 30, is the triangle *acute*, *obtuse*, or *right*? Explain your reasoning.

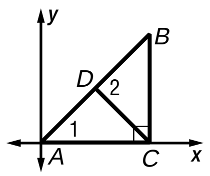
12. ____________________

13. Identify the congruent triangles in the figure.



13. ____________________

For Questions 14 and 15, refer to the figure. Triangle *ABC* is an isosceles right triangle.



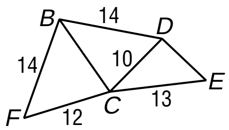
14. If $\overline{CD}$ bisects $\angle C$, find $m\angle 1$ and $m\angle 2$.

14. ____________________

15. Determine the coordinates of A, B, and C, if the triangle has legs n units long.

15. ____________________

For Questions 16–18, refer to the figure.



16. Write a statement using $>$, $<$, or $=$ to describe the measures of $\angle DBC$ and $\angle DCB$.

16. ____________________

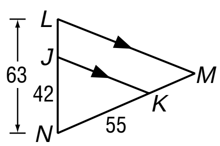
17. Write an inequality to represent the possible measures of $\overline{DE}$.

17. ____________________

18. If $m\angle FBC = 3x + 1$ and $m\angle CBD = 34$, write an inequality to describe the possible values of x.

18. ____________________

19. Identify the similar triangles, find MN, and state the scale factor from the smaller triangle to the larger triangle.



19. ____________________

20. Find the first three iterates of $4(x - 3)$ if x initially equals 0.

20. ____________________

21. A plane is flying at 35,000 feet, and the pilot wants to descend to 22,000 feet over the next 60 miles. What should be his angle of depression to the nearest tenth? (*Hint:* 5280 feet = 1 mile)

21. ____________________

22. Solve $\triangle DEF$ if $DE = 58$, $EF = 62$, and $m\angle E = 49$. Round angle measures to the nearest degree and side measures to the nearest tenth.

22. ____________________

NAME ______________________ DATE ____________ PERIOD ______

7 Standardized Test Practice

Student Record Sheet *(Use with pages 398–399 of the Student Edition.)*

Part 1 *Multiple Choice*

Select the best answer from the choices given and fill in the corresponding oval.

1 (A) (B) (C) (D)

2 (A) (B) (C) (D)

3 (A) (B) (C) (D)

4 (A) (B) (C) (D)

5 (A) (B) (C) (D)

6 (A) (B) (C) (D)

7 (A) (B) (C) (D)

Part 2 *Short Response/Grid In*

Solve the problem and write your answer in the blank.

For Questions 8, 9, 11, and 12, also enter your answer by writing each number or symbol in a box. Then fill in the corresponding oval for that number or symbol.

8 ______________ (grid in)

9 ______________ (grid in)

10 ______________

11 ______________ (grid in)

12 ______________ (grid in)

8

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

9

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

11

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

12

	/	/	
.	.	.	.
	0	0	0
1	1	1	1
2	2	2	2
3	3	3	3
4	4	4	4
5	5	5	5
6	6	6	6
7	7	7	7
8	8	8	8
9	9	9	9

Part 3 *Open-Ended*

Record your answers for Question 13 on the back of this paper.

NAME ______________________ DATE __________ PERIOD _____

7-1 Study Guide and Intervention

Geometric Mean

Geometric Mean The **geometric mean** between two numbers is the square root of their product. For two positive numbers a and b, the geometric mean of a and b is the positive number x in the proportion $\frac{a}{x} = \frac{x}{b}$. Cross multiplying gives $x^2 = ab$, so $x = \sqrt{ab}$.

Example **Find the geometric mean between each pair of numbers.**

a. 12 and 3

Let x represent the geometric mean.

$\frac{12}{x} = \frac{x}{3}$ Definition of geometric mean

$x^2 = 36$ Cross multiply.

$x = \sqrt{36}$ or 6 Take the square root of each side.

b. 8 and 4

Let x represent the geometric mean.

$\frac{8}{x} = \frac{x}{4}$

$x^2 = 32$

$x = \sqrt{32}$

≈ 5.7

Exercises

Find the geometric mean between each pair of numbers.

1. 4 and 4 **4**

2. 4 and 6 **$\sqrt{24} \approx 4.9$**

3. 6 and 9 **$\sqrt{54} \approx 7.3$**

4. $\frac{1}{2}$ and 2 **1**

5. $2\sqrt{3}$ and $3\sqrt{3}$ **$\sqrt{18} \approx 4.2$**

6. 4 and 25 **10**

7. $\sqrt{3}$ and $\sqrt{6}$ **$18^{\frac{1}{4}} \approx 2.1$**

8. 10 and 100 **$\sqrt{1000} \approx 31.6$**

9. $\frac{1}{2}$ and $\frac{1}{4}$ **$\sqrt{\frac{1}{8}} \approx 0.4$**

10. $\frac{2\sqrt{2}}{5}$ and $\frac{3\sqrt{2}}{5}$ **$\sqrt{\frac{12}{25}} \approx 0.7$**

11. 4 and 16 **8**

12. 3 and 24 **$\sqrt{72} \approx 8.5$**

The geometric mean and one extreme are given. Find the other extreme.

13. $\sqrt{24}$ is the geometric mean between a and b. Find b if $a = 2$. **12**

14. $\sqrt{12}$ is the geometric mean between a and b. Find b if $a = 3$. **4**

Determine whether each statement is *always*, *sometimes*, or *never* true.

15. The geometric mean of two positive numbers is greater than the average of the two numbers. **never**

16. If the geometric mean of two positive numbers is less than 1, then both of the numbers are less than 1. **sometimes**

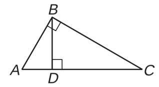
Lesson 7-1

NAME ______________________ DATE __________ PERIOD _____

7-1 Study Guide and Intervention *(continued)*

Geometric Mean

Altitude of a Triangle In the diagram, $\triangle ABC \sim \triangle ADB \sim \triangle BDC$. An altitude to the hypotenuse of a right triangle forms two right triangles. The two triangles are similar and each is similar to the original triangle.



Example 1 **Use right $\triangle ABC$ with $\overline{BD} \perp \overline{AC}$. Describe two geometric means.**

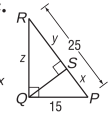
a. $\triangle ADB \sim \triangle BDC$ so $\frac{AD}{BD} = \frac{BD}{CD}$.

In $\triangle ABC$, the altitude is the geometric mean between the two segments of the hypotenuse.

b. $\triangle ABC \sim \triangle ADB$ and $\triangle ABC \sim \triangle BDC$, so $\frac{AC}{AB} = \frac{AB}{AD}$ and $\frac{AC}{BC} = \frac{BC}{DC}$.

In $\triangle ABC$, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Example 2 **Find x, y, and z.**



$\frac{PR}{PQ} = \frac{PQ}{PS}$

$\frac{25}{15} = \frac{15}{x}$ $PR = 25$, $PQ = 15$, $PS = x$

$25x = 225$ Cross multiply.

$x = 9$ Divide each side by 25.

Then

$y = PR - SP$

$= 25 - 9$

$= 16$

$\frac{PR}{QR} = \frac{QR}{RS}$

$\frac{25}{z} = \frac{z}{y}$ $PR = 25$, $QR = z$, $RS = y$

$\frac{25}{z} = \frac{z}{16}$ $y = 16$

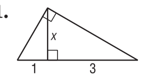
$z^2 = 400$ Cross multiply.

$z = 20$ Take the square root of each side.

Exercises

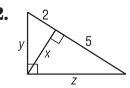
Find x, y, and z to the nearest tenth.

1.



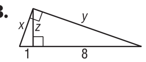
$\sqrt{3} \approx 1.7$

2.



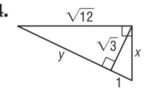
$\sqrt{10} \approx 3.2$; $\sqrt{14} \approx 3.7$; $\sqrt{35} \approx 5.9$

3.



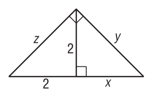
3; $\sqrt{72} \approx 8.5$; $\sqrt{8} \approx 2.8$

4.



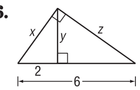
2; 3

5.



2; $\sqrt{8} \approx 2.8$; $\sqrt{8} \approx 2.8$

6.



3.5; $\sqrt{8} \approx 2.8$; $\sqrt{24} \approx 4.9$

NAME ______________________ DATE ________ PERIOD ____

7-1 Skills Practice

Geometric Mean

Find the geometric mean between each pair of numbers. State exact answers and answers to the nearest tenth.

1. 2 and 8
4

2. 9 and 36
18

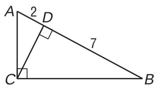
3. 4 and 7
$\sqrt{28} \approx 5.3$

4. 5 and 10
$\sqrt{50} \approx 7.1$

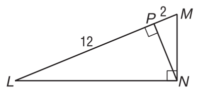
5. $2\sqrt{2}$ and $5\sqrt{2}$
$\sqrt{20} \approx 4.5$

6. $3\sqrt{5}$ and $5\sqrt{5}$
$\sqrt{75} \approx 8.7$

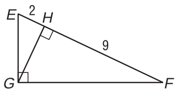
Find the measure of each altitude. State exact answers and answers to the nearest tenth.

7.


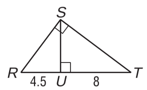
$\sqrt{14} \approx 3.7$

8.


$\sqrt{24} \approx 4.9$

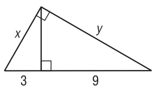
9.


$\sqrt{18} \approx 4.2$

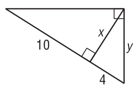
10.


6

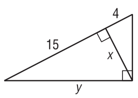
Find x and y.

11.


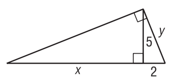
6; $\sqrt{108} \approx 10.4$

12.


$\sqrt{40} \approx 6.3$; $\sqrt{56} \approx 7.5$

13.


$\sqrt{60} \approx 7.7$; $\sqrt{285} \approx 16.9$

14.


12.5; $\sqrt{29} \approx 5.4$

Lesson 7-1

NAME ______________________ DATE ________ PERIOD ____

7-1 Practice (Average)

Geometric Mean

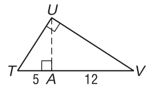
Find the geometric mean between each pair of numbers to the nearest tenth.

1. 8 and 12
$\sqrt{96} \approx 9.8$

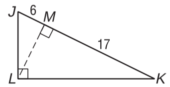
2. $3\sqrt{7}$ and $6\sqrt{7}$
$\sqrt{126} \approx 11.2$

3. $\frac{4}{5}$ and 2
$\sqrt{\frac{8}{5}} \approx 1.3$

Find the measure of each altitude. State exact answers and answers to the nearest tenth.

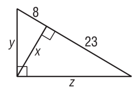
4.


$\sqrt{60} \approx 7.7$

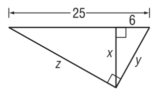
5.


$\sqrt{102} \approx 10.1$

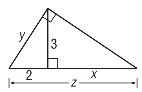
Find x, y, and z.

6.


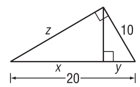
$\sqrt{184} \approx 13.6$; $\sqrt{248} \approx 15.7$; $\sqrt{713} \approx 26.7$

7.


$\sqrt{114} \approx 10.7$; $\sqrt{150} \approx 12.2$; $\sqrt{475} \approx 21.8$

8.


4.5; $\sqrt{13} \approx 3.6$; 6.5

9.


15; 5; $\sqrt{300} \approx 17.3$

10. CIVIL ENGINEERING An airport, a factory, and a shopping center are at the vertices of a right triangle formed by three highways. The airport and factory are 6.0 miles apart. Their distances from the shopping center are 3.6 miles and 4.8 miles, respectively. A service road will be constructed from the shopping center to the highway that connects the airport and factory. What is the shortest possible length for the service road? Round to the nearest hundredth. **2.88 mi**

Answers

NAME ______________________ DATE __________ PERIOD _____

7-1 Reading to Learn Mathematics

Geometric Mean

Pre-Activity **How can the geometric mean be used to view paintings?**

Read the introduction to Lesson 7-1 at the top of page 342 in your textbook.

- What is a disadvantage of standing too close to a painting?
  Sample answer: You don't get a good overall view.
- What is a disadvantage of standing too far from a painting?
  Sample answer: You can't see all the details in the painting.

Reading the Lesson

1. In the past, when you have seen the word *mean* in mathematics, it referred to the *average* or *arithmetic mean* of the two numbers.

a. Complete the following by writing an algebraic expression in each blank.

If a and b are two positive numbers, then the geometric mean between a and b is $\underline{\sqrt{ab}}$ and their arithmetic mean is $\underline{\frac{a+b}{2}}$.

b. Explain in words, without using any mathematical symbols, the difference between the geometric mean and the algebraic mean. **Sample answer: The geometric mean between two numbers is the square root of their product. The arithmetic mean of two numbers is half their sum.**

2. Let r and s be two positive numbers. In which of the following equations is z equal to the geometric mean between r and s? **A, C, D, F**

A. $\frac{s}{z} = \frac{z}{r}$ **B.** $\frac{r}{z} = \frac{s}{z}$ **C.** $s:z = z:r$ **D.** $\frac{r}{z} = \frac{z}{s}$ **E.** $\frac{z}{r} = \frac{z}{s}$ **F.** $\frac{z}{s} = \frac{r}{z}$

3. Supply the missing words or phrases to complete the statement of each theorem.

a. The measure of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the **geometric mean** between the measures of the two segments of the **hypotenuse**.

b. If the altitude is drawn from the vertex of the **right** angle of a right triangle to its hypotenuse, then the measure of a **leg** of the triangle is the **geometric mean** between the measure of the hypotenuse and the segment of the **hypotenuse** adjacent to that leg.

c. If the altitude is drawn from the **vertex** of the right angle of a right triangle to its **hypotenuse**, then the two triangles formed are **similar** to the given triangle and to each other.

Helping You Remember

4. A good way to remember a new mathematical concept is to relate it to something you already know. How can the meaning of *mean* in a proportion help you to remember how to find the geometric mean between two numbers? **Sample answer: Write a proportion in which the two means are equal. This common mean is the geometric mean between the two extremes.**

Lesson 7-1

NAME ______________________ DATE __________ PERIOD _____

7-1 Enrichment

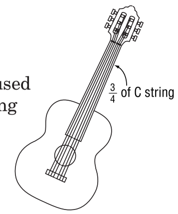
Mathematics and Music

Pythagoras, a Greek philosopher who lived during the sixth century B.C., believed that all nature, beauty, and harmony could be expressed by whole-number relationships. Most people remember Pythagoras for his teachings about right triangles. (The sum of the squares of the legs equals the square of the hypotenuse.) But Pythagoras also discovered relationships between the musical notes of a scale. These relationships can be expressed as ratios.

C	D	E	F	G	A	B	C′
$\frac{1}{1}$	$\frac{8}{9}$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{3}{5}$	$\frac{8}{15}$	$\frac{1}{2}$

When you play a stringed instrument, you produce different notes by placing your finger on different places on a string. This is the result of changing the length of the vibrating part of the string.

The C string can be used to produce F by placing a finger $\frac{3}{4}$ of the way along the string.



Suppose a C string has a length of 16 inches. Write and solve proportions to determine what length of string would have to vibrate to produce the remaining notes of the scale.

1. D $14\frac{2}{9}$ in. **2.** E $12\frac{4}{5}$ in. **3.** F 12 in.

4. G $10\frac{2}{3}$ in. **5.** A $9\frac{3}{5}$ in. **6.** B $8\frac{8}{15}$ in.

7. C′ 8 in.

8. Complete to show the distance between finger positions on the 16-inch C string for each note. For example, $C(16) - D\left(14\frac{2}{9}\right) = 1\frac{7}{9}$.

C $\underline{1\frac{7}{9}\text{ in.}}$ D $\underline{3\frac{1}{5}\text{ in.}}$ E $\underline{4\text{ in.}}$ F $\underline{5\frac{1}{3}\text{ in.}}$ G $\underline{6\frac{2}{5}\text{ in.}}$ A $\underline{7\frac{7}{15}\text{ in.}}$ B $\underline{8\text{ in.}}$ C′

9. Between two consecutive musical notes, there is either a whole step or a half step. Using the distances you found in Exercise 8, determine what two pairs of notes have a half step between them.
E* and *F*, *B* and *C′

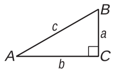
NAME ________________ DATE ________ PERIOD ____

7-2 Study Guide and Intervention

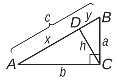
The Pythagorean Theorem and Its Converse

The Pythagorean Theorem In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

$\triangle ABC$ is a right triangle, so $a^2 + b^2 = c^2$.



Example 1 **Prove the Pythagorean Theorem.**



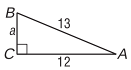
With altitude $\overline{CD}$, each leg a and b is a geometric mean between hypotenuse c and the segment of the hypotenuse adjacent to that leg.

$\frac{c}{a} = \frac{a}{y}$ and $\frac{c}{b} = \frac{b}{x}$, so $a^2 = cy$ and $b^2 = cx$.

Add the two equations and substitute $c = y + x$ to get $a^2 + b^2 = cy + cx = c(y + x) = c^2$.

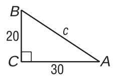
Example 2

a. Find a.



$a^2 + b^2 = c^2$	Pythagorean Theorem
$a^2 + 12^2 = 13^2$	$b = 12$, $c = 13$
$a^2 + 144 = 169$	Simplify.
$a^2 = 25$	Subtract.
$a = 5$	Take the square root of each side.

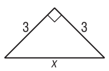
b. Find c.



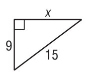
$a^2 + b^2 = c^2$	Pythagorean Theorem
$20^2 + 30^2 = c^2$	$a = 20$, $b = 30$
$400 + 900 = c^2$	Simplify.
$1300 = c^2$	Add.
$\sqrt{1300} = c$	Take the square root of each side.
$36.1 \approx c$	Use a calculator.

Exercises

Find x.

1. 

$\sqrt{18} \approx 4.2$

2. 

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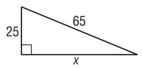
3. 

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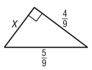
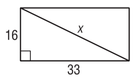
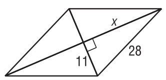
4. 

$\frac{3}{10}$

5. 

$\sqrt{1345} \approx 36.7$

6. 

$\sqrt{663} \approx 25.7$

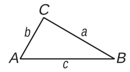
Lesson 7-2

NAME ________________ DATE ________ PERIOD ____

7-2 Study Guide and Intervention *(continued)*

The Pythagorean Theorem and Its Converse

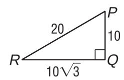
Converse of the Pythagorean Theorem If the sum of the squares of the measures of the two shorter sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.



If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.

If the three whole numbers a, b, and c satisfy the equation $a^2 + b^2 = c^2$, then the numbers a, b, and c form a **Pythagorean triple**.

Example **Determine whether $\triangle PQR$ is a right triangle.**



$a^2 + b^2 \stackrel{?}{=} c^2$	Pythagorean Theorem
$10^2 + (10\sqrt{3})^2 \stackrel{?}{=} 20^2$	$a = 10$, $b = 10\sqrt{3}$, $c = 20$
$100 + 300 \stackrel{?}{=} 400$	Simplify.
$400 = 400$ ✓	Add.

The sum of the squares of the two shorter sides equals the square of the longest side, so the triangle is a right triangle.

Exercises

Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

1. 30, 40, 50 — yes; yes
2. 20, 30, 40 — no; no
3. 18, 24, 30 — yes; yes
4. 6, 8, 9 — no; no
5. $\frac{3}{7}, \frac{4}{7}, \frac{5}{7}$ — yes; no
6. 10, 15, 20 — no; no
7. $\sqrt{5}, \sqrt{12}, \sqrt{13}$ — no; no
8. $2, \sqrt{8}, \sqrt{12}$ — yes; no
9. 9, 40, 41 — yes; yes

A *family* of Pythagorean triples consists of multiples of known triples. For each Pythagorean triple, find two triples in the same family. Sample answers are given.

10. 3, 4, 5 — 30, 40, 50; 12, 16, 20
11. 5, 12, 13 — 10, 24, 26; 15, 36, 39
12. 7, 24, 25 — 14, 48, 50; 21, 72, 75

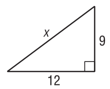
Answers

NAME ______________________ DATE __________ PERIOD _____

7-2 Skills Practice

The Pythagorean Theorem and Its Converse

Find *x*.

1. 

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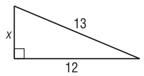
2. 

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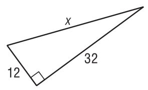
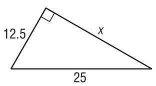
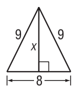
3. 

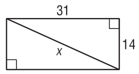
$\sqrt{1168} \approx 34.2$

4. 

$\sqrt{468.75} \approx 21.7$

5. 

$\sqrt{65} \approx 8.1$

6. 

$\sqrt{1157} \approx 34.0$

Determine whether △*STU* is a right triangle for the given vertices. Explain.

7. $S(5, 5)$, $T(7, 3)$, $U(3, 2)$
no; $ST = \sqrt{8}$, $TU = \sqrt{17}$, $US = \sqrt{13}$, $(\sqrt{8})^2 + (\sqrt{13})^2 \neq (\sqrt{17})^2$

8. $S(3, 3)$, $T(5, 5)$, $U(6, 0)$
yes; $ST = \sqrt{8}$, $TU = \sqrt{26}$, $US = \sqrt{18}$, $(\sqrt{8})^2 + (\sqrt{18})^2 = (\sqrt{26})^2$

9. $S(4, 6)$, $T(9, 1)$, $U(1, 3)$
yes; $ST = \sqrt{50}$, $TU = \sqrt{68}$, $US = \sqrt{18}$, $(\sqrt{18})^2 + (\sqrt{50})^2 = (\sqrt{68})^2$

10. $S(0, 3)$, $T(-2, 5)$, $U(4, 7)$
yes; $ST = \sqrt{8}$, $TU = \sqrt{40}$, $US = \sqrt{32}$, $(\sqrt{8})^2 + (\sqrt{32})^2 = (\sqrt{40})^2$

11. $S(-3, 2)$, $T(2, 7)$, $U(-1, 1)$
yes; $ST = \sqrt{50}$, $TU = \sqrt{45}$, $US = \sqrt{5}$, $(\sqrt{45})^2 + (\sqrt{5})^2 = (\sqrt{50})^2$

12. $S(2, -1)$, $T(5, 4)$, $U(6, -3)$
no; $ST = \sqrt{34}$, $TU = \sqrt{50}$, $US = \sqrt{20}$, $(\sqrt{34})^2 + (\sqrt{20})^2 \neq (\sqrt{50})^2$

Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

13. 12, 16, 20
yes, yes

14. 16, 30, 32
no, no

15. 14, 48, 50
yes, yes

16. $\frac{2}{5}, \frac{4}{5}, \frac{6}{5}$
no, no

17. $2\sqrt{6}, 5, 7$
yes, no

18. $2\sqrt{2}, 2\sqrt{7}, 6$
yes, no

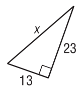
Lesson 7-2

NAME ______________________ DATE __________ PERIOD _____

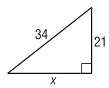
7-2 Practice (Average)

The Pythagorean Theorem and Its Converse

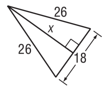
Find *x*.

1. 

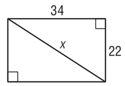
$\sqrt{698} \approx 26.4$

2. 

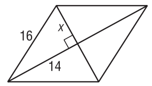
$\sqrt{715} \approx 26.7$

3. 

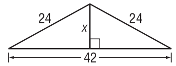
$\sqrt{595} \approx 24.4$

4. 

$\sqrt{1640} \approx 40.5$

5. 

$\sqrt{60} \approx 7.7$

6. 

$\sqrt{135} \approx 11.6$

Determine whether △*GHI* is a right triangle for the given vertices. Explain.

7. $G(2, 7)$, $H(3, 6)$, $I(-4, -1)$
yes; $GH = \sqrt{2}$, $HI = \sqrt{98}$, $IG = \sqrt{100}$, $(\sqrt{2})^2 + (\sqrt{98})^2 = (\sqrt{100})^2$

8. $G(-6, 2)$, $H(1, 12)$, $I(-2, 1)$
no; $GH = \sqrt{149}$, $HI = \sqrt{130}$, $IG = \sqrt{17}$, $(\sqrt{130})^2 + (\sqrt{17})^2 \neq (\sqrt{149})^2$

9. $G(-2, 1)$, $H(3, -1)$, $I(-4, -4)$
yes; $GH = \sqrt{29}$, $HI = \sqrt{58}$, $IG = \sqrt{29}$, $(\sqrt{29})^2 + (\sqrt{29})^2 = (\sqrt{58})^2$

10. $G(-2, 4)$, $H(4, 1)$, $I(-1, -9)$
yes; $GH = \sqrt{45}$, $HI = \sqrt{125}$, $IG = \sqrt{170}$, $(\sqrt{45})^2 + (\sqrt{125})^2 = (\sqrt{170})^2$

Determine whether each set of measures can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

11. 9, 40, 41
yes, yes

12. 7, 28, 29
no, no

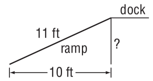
13. 24, 32, 40
yes, yes

14. $\frac{9}{5}, \frac{12}{5}, 3$
yes, no

15. $\frac{1}{3}, \frac{2\sqrt{2}}{3}, 1$
yes, no

16. $\frac{\sqrt{4}}{7}, \frac{2\sqrt{3}}{7}, \frac{4}{7}$
yes, no

17. CONSTRUCTION The bottom end of a ramp at a warehouse is 10 feet from the base of the main dock and is 11 feet long. How high is the dock? **about 4.6 ft high**



NAME ______________________ DATE __________ PERIOD _____

7-2 Reading to Learn Mathematics

The Pythagorean Theorem and Its Converse

Pre-Activity **How are right triangles used to build suspension bridges?**

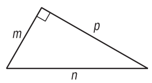
Read the introduction to Lesson 7-2 at the top of page 350 in your textbook.

Do the two right triangles shown in the drawing appear to be similar? Explain your reasoning. **Sample answer: No; their sides are not proportional. In the smaller triangle, the longer leg is more than twice the length of the shorter leg, while in the larger triangle, the longer leg is less than twice the length of the shorter leg.**

Reading the Lesson

1. Explain in your own words the difference between how the Pythagorean Theorem is used and how the Converse of the Pythagorean Theorem is used. **Sample answer: The Pythagorean Theorem is used to find the third side of a right triangle if you know the lengths of any two of the sides. The converse is used to tell whether a triangle with three given side lengths is a right triangle.**

2. Refer to the figure. For this figure, which statements are true? **B, E, F, G**



A. $m^2 + n^2 = p^2$ **B.** $n^2 = m^2 + p^2$

C. $m^2 = n^2 + p^2$ **D.** $m^2 = p^2 - n^2$

E. $p^2 = n^2 - m^2$ **F.** $n^2 - p^2 = m^2$

G. $n = \sqrt{m^2 + p^2}$ **H.** $p = \sqrt{m^2 - n^2}$

3. Is the following statement true or false?

A Pythagorean triple is any group of three numbers for which the sum of the squares of the smaller two numbers is equal to the square of the largest number. Explain your reasoning.

Sample answer: The statement is false because in a Pythagorean triple, all three numbers must be *whole* numbers.

4. If x, y, and z form a Pythagorean triple and k is a positive integer, which of the following groups of numbers are also Pythagorean triples? **B, D**

A. $3x, 4y, 5z$ **B.** $3x, 3y, 3z$ **C.** $-3x, -3y, -3z$ **D.** kx, ky, kz

Helping You Remember

5. Many students who studied geometry long ago remember the Pythagorean Theorem as the equation $a^2 + b^2 = c^2$, but cannot tell you what this equation means. A formula is useless if you don't know what it means and how to use it. How could you help someone who has forgotten the Pythagorean Theorem remember the meaning of the equation $a^2 + b^2 = c^2$?

Sample answer: Draw a right triangle. Label the lengths of the two legs as *a* and *b* and the length of the hypotenuse as *c*.

Lesson 7-2

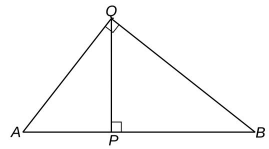
NAME ______________________ DATE __________ PERIOD _____

7-2 Enrichment

Converse of a Right Triangle Theorem

You have learned that the measure of the altitude from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse. Is the converse of this theorem true? In order to find out, it will help to rewrite the original theorem in if-then form as follows.

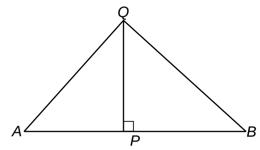
If $\triangle ABQ$ is a right triangle with right angle at Q, then $\overline{QP}$ is the geometric mean between AP and PB, where P is between A and B and $\overline{QP}$ is perpendicular to $\overline{AB}$.



1. Write the converse of the if-then form of the theorem.

If *QP* is the geometric mean between *AP* and *PB*, where *P* is between *A* and *B* and $\overline{QP} \perp \overline{AB}$, then $\triangle ABQ$ is a right triangle with right angle at *Q*.

2. Is the converse of the original theorem true? Refer to the figure at the right to explain your answer.



Yes; $(PQ)^2 = (AP)(PB)$ implies that $\frac{PQ}{AP} = \frac{PB}{PQ}$. Since both $\angle APQ$ and $\angle QPB$ are right angles, they are congruent. Therefore $\triangle APQ \sim \triangle QPB$ by SAS similarity. So $\angle A \cong \angle PQB$ and $\angle AQP \cong \angle B$. But the acute angles of $\triangle AQP$ are complementary and $m\angle AQB = m\angle AQP + m\angle PQB$. Hence $m\angle AQB = 90$ and $\triangle AQB$ is a right triangle with right angle at *Q*.

You may find it interesting to examine the other theorems in Chapter 7 to see whether their converses are true or false. You will need to restate the theorems carefully in order to write their converses.

Answers

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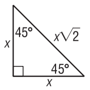
7-3 Study Guide and Intervention

Special Right Triangles

Properties of 45°-45°-90° Triangles The sides of a 45°-45°-90° right triangle have a special relationship.

Example 1 **If the leg of a 45°-45°-90° right triangle is x units, show that the hypotenuse is $x\sqrt{2}$ units.**



Using the Pythagorean Theorem with $a = b = x$, then

$$c^2 = a^2 + b^2 = x^2 + x^2 = 2x^2$$
$$c = \sqrt{2x^2} = x\sqrt{2}$$

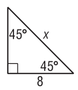
Example 2 **In a 45°-45°-90° right triangle the hypotenuse is $\sqrt{2}$ times the leg. If the hypotenuse is 6 units, find the length of each leg.**

The hypotenuse is $\sqrt{2}$ times the leg, so divide the length of the hypotenuse by $\sqrt{2}$.

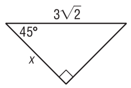
$$a = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2} \text{ units}$$

Exercises

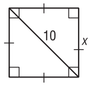
Find x.

1. 

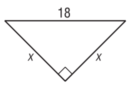
$8\sqrt{2} \approx 11.3$

2. 

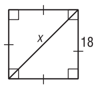
3

3. 

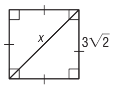
$5\sqrt{2} \approx 7.1$

4. 

$9\sqrt{2} \approx 12.7$

5. 

$18\sqrt{2} \approx 25.5$

6. 

6

7. Find the perimeter of a square with diagonal 12 centimeters. **$24\sqrt{2} \approx 33.9$ cm**

8. Find the diagonal of a square with perimeter 20 inches. **$5\sqrt{2} \approx 7.1$ in.**

9. Find the diagonal of a square with perimeter 28 meters. **$7\sqrt{2} \approx 9.9$ m**

Lesson 7-3

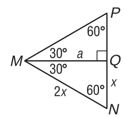
NAME ______________________ DATE __________ PERIOD _____

7-3 Study Guide and Intervention *(continued)*

Special Right Triangles

Properties of 30°-60°-90° Triangles The sides of a 30°-60°-90° right triangle also have a special relationship.

Example 1 **In a 30°-60°-90° right triangle, show that the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg.**



△*MNP* is an equilateral triangle.

△*MNQ* is a 30°-60°-90° right triangle.

$\triangle MNQ$ is a 30°-60°-90° right triangle, and the length of the hypotenuse $\overline{MN}$ is two times the length of the shorter side $\overline{NQ}$.

Using the Pythagorean Theorem,

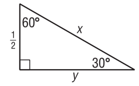
$$a^2 = (2x)^2 - x^2 = 4x^2 - x^2 = 3x^2$$
$$a = \sqrt{3x^2} = x\sqrt{3}$$

Example 2 **In a 30°-60°-90° right triangle, the hypotenuse is 5 centimeters. Find the lengths of the other two sides of the triangle.**

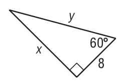
If the hypotenuse of a 30°-60°-90° right triangle is 5 centimeters, then the length of the shorter leg is half of 5 or 2.5 centimeters. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg, or $(2.5)(\sqrt{3})$ centimeters.

Exercises

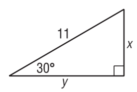
Find x and y.

1. 

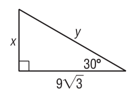
$1; 0.5\sqrt{3} \approx 0.9$

2. 

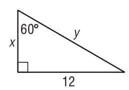
$8\sqrt{3} \approx 13.9; 16$

3. 

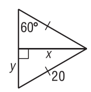
$5.5; 5.5\sqrt{3} \approx 9.5$

4. 

9; 18

5. 

$4\sqrt{3} \approx 6.9; 8\sqrt{3} \approx 13.9$

6. 

$10\sqrt{3} \approx 17.3; 10$

7. The perimeter of an equilateral triangle is 32 centimeters. Find the length of an altitude of the triangle to the nearest tenth of a centimeter. **9.2 cm**

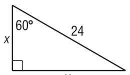
8. An altitude of an equilateral triangle is 8.3 meters. Find the perimeter of the triangle to the nearest tenth of a meter. **28.8 m**

NAME ______________________ DATE __________ PERIOD _____

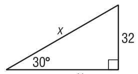
7-3 Skills Practice

Special Right Triangles

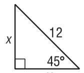
Find *x* and *y*.

1. 

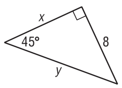
$12, 12\sqrt{3}$

2. 

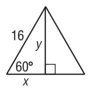
$64, 32\sqrt{3}$

3. 

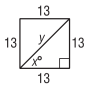
$6\sqrt{2}, 6\sqrt{2}$

4. 

$8, 8\sqrt{2}$

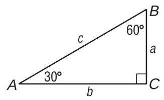
5. 

$8, 8\sqrt{3}$

6. 

$45, 13\sqrt{2}$

For Exercises 7–9, use the figure at the right.

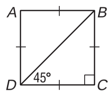


7. If $a = 11$, find b and c.
$b = 11\sqrt{3}$; $c = 22$

8. If $b = 15$, find a and c.
$a = 5\sqrt{3}$; $c = 10\sqrt{3}$

9. If $c = 9$, find a and b.
$a = 4.5$; $b = 4.5\sqrt{3}$

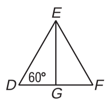
For Exercises 10 and 11, use the figure at the right.



10. The perimeter of the square is 30 inches. Find the length of $\overline{BC}$.
7.5 in.

11. Find the length of the diagonal $\overline{BD}$.
$7.5\sqrt{2}$ in. or about 10.61 in.

12. The perimeter of the equilateral triangle is 60 meters. Find the length of an altitude.
$10\sqrt{3}$ m or about 17.32 m



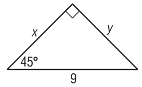
13. $\triangle GEC$ is a 30°-60°-90° triangle with right angle at E, and $\overline{EC}$ is the longer leg. Find the coordinates of G in Quadrant I for $E(1, 1)$ and $C(4, 1)$.
$(1, 1 + \sqrt{3})$ or about (1, 2.73)

NAME ______________________ DATE __________ PERIOD _____

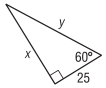
7-3 Practice (Average)

Special Right Triangles

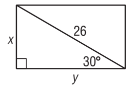
Find *x* and *y*.

1. 

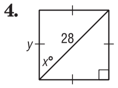
$\frac{9\sqrt{2}}{2}, \frac{9\sqrt{2}}{2}$

2. 

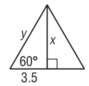
$25\sqrt{3}, 50$

3. 

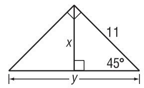
$13, 13\sqrt{3}$

4. 

$45, 14\sqrt{2}$

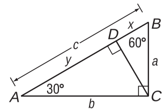
5. 

$3.5\sqrt{3}, 7$

6. 

$\frac{11\sqrt{2}}{2}; 11\sqrt{2}$

For Exercises 7–9, use the figure at the right.



7. If $a = 4\sqrt{3}$, find b and c.
$b = 12$, $c = 8\sqrt{3}$

8. If $x = 3\sqrt{3}$, find a and CD.
$a = 6\sqrt{3}$, $CD = 9$

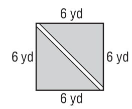
9. If $a = 4$, find CD, b, and y.
$CD = 2\sqrt{3}$, $b = 4\sqrt{3}$, $y = 6$

10. The perimeter of an equilateral triangle is 39 centimeters. Find the length of an altitude of the triangle.
$6.5\sqrt{3}$ in. or about 11.26 in.

11. $\triangle MIP$ is a 30°-60°-90° triangle with right angle at I, and $\overline{IP}$ the longer leg. Find the coordinates of M in Quadrant I for $I(3, 3)$ and $P(12, 3)$.
$(3, 3 + 3\sqrt{3})$ or about (3, 8.19)

12. $\triangle TJK$ is a 45°-45°-90° triangle with right angle at J. Find the coordinates of T in Quadrant II for $J(-2, -3)$ and $K(3, -3)$.
$(-2, 2)$

13. **BOTANICAL GARDENS** One of the displays at a botanical garden is an herb garden planted in the shape of a square. The square measures 6 yards on each side. Visitors can view the herbs from a diagonal pathway through the garden. How long is the pathway?
$6\sqrt{2}$ yd or about 8.48 yd



NAME ______________ DATE ______ PERIOD ____

7-3 Reading to Learn Mathematics

Special Triangles

Pre-Activity **How is triangle tiling used in wallpaper design?**

Read the introduction to Lesson 7-3 at the top of page 357 in your textbook.

- How can you most completely describe the larger triangle and the two smaller triangles in tile 15? **Sample answer: The larger triangle is an isosceles obtuse triangle. The two smaller triangles are congruent scalene right triangles.**
- How can you most completely describe the larger triangle and the two smaller triangles in tile 16? (Include angle measures in describing all the triangles.) **Sample answer: The larger triangle is equilateral, so each of its angle measures is 60. The two smaller triangles are congruent right triangles in which the angle measures are 30, 60, and 90.**

Reading the Lesson

1. Supply the correct number or numbers to complete each statement.

a. In a 45°-45°-90° triangle, to find the length of the hypotenuse, multiply the length of a leg by **$\sqrt{2}$**.

b. In a 30°-60°-90° triangle, to find the length of the hypotenuse, multiply the length of the shorter leg by **2**.

c. In a 30°-60°-90° triangle, the longer leg is opposite the angle with a measure of **60**.

d. In a 30°-60°-90° triangle, to find the length of the longer leg, multiply the length of the shorter leg by **$\sqrt{3}$**.

e. In an isosceles right triangle, each leg is opposite an angle with a measure of **45**.

f. In a 30°-60°-90° triangle, to find the length of the shorter leg, divide the length of the longer leg by **$\sqrt{3}$**.

g. In 30°-60°-90° triangle, to find the length of the longer leg, divide the length of the hypotenuse by **2** and multiply the result by **$\sqrt{3}$**.

h. To find the length of a side of a square, divide the length of the diagonal by **$\sqrt{2}$**.

2. Indicate whether each statement is *always*, *sometimes*, or *never* true.

a. The lengths of the three sides of an isosceles triangle satisfy the Pythagorean Theorem. **sometimes**

b. The lengths of the sides of a 30°-60°-90° triangle form a Pythagorean triple. **never**

c. The lengths of all three sides of a 30°-60°-90° triangle are positive integers. **never**

Helping You Remember

3. Some students find it easier to remember mathematical concepts in terms of specific numbers rather than variables. How can you use specific numbers to help you remember the relationship between the lengths of the three sides in a 30°-60°-90° triangle? **Sample answer: Draw a 30°-60°-90° triangle. Label the length of the shorter leg as 1. Then the length of the hypotenuse is 2, and the length of the longer leg is $\sqrt{3}$. Just remember: 1, 2, $\sqrt{3}$.**

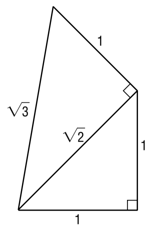
Lesson 7-3

NAME ______________ DATE ______ PERIOD ____

7-3 Enrichment

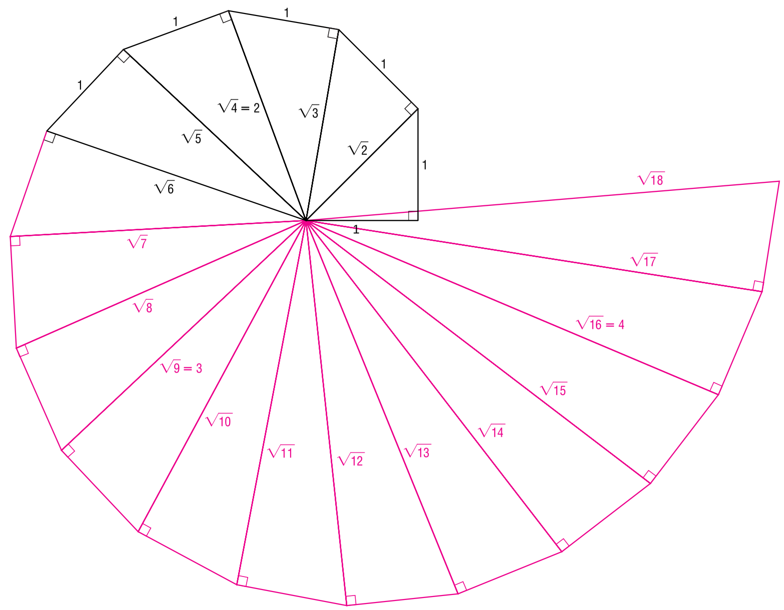
Constructing Values of Square Roots

The diagram at the right shows a right isosceles triangle with two legs of length 1 inch. By the Pythagorean Theorem, the length of the hypotenuse is $\sqrt{2}$ inches. By constructing an adjacent right triangle with legs of $\sqrt{2}$ inches and 1 inch, you can create a segment of length $\sqrt{3}$.



By continuing this process as shown below, you can construct a "wheel" of square roots. This wheel is called the "Wheel of Theodorus" after a Greek philosopher who lived about 400 B.C.

Continue constructing the wheel until you make a segment of length $\sqrt{18}$.

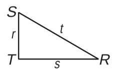


NAME ______________________ DATE __________ PERIOD _____

7-4 Study Guide and Intervention

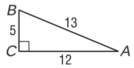
Trigonometry

Trigonometric Ratios The ratio of the lengths of two sides of a right triangle is called a **trigonometric ratio**. The three most common ratios are **sine**, **cosine**, and **tangent**, which are abbreviated *sin*, *cos*, and *tan*, respectively.



$\sin R = \frac{\text{leg opposite } \angle R}{\text{hypotenuse}} = \frac{r}{t}$ $\quad$ $\cos R = \frac{\text{leg adjacent to } \angle R}{\text{hypotenuse}} = \frac{s}{t}$ $\quad$ $\tan R = \frac{\text{leg opposite } \angle R}{\text{leg adjacent to } \angle R} = \frac{r}{s}$

Example **Find sin *A*, cos *A*, and tan *A*. Express each ratio as a decimal to the nearest thousandth.**



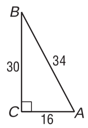
$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{5}{13} \approx 0.385$

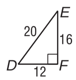
$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \approx 0.923$

$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \approx 0.417$

Exercises

Find the indicated trigonometric ratio as a fraction and as a decimal. If necessary, round to the nearest ten-thousandth.





1. sin *A* $\frac{15}{17}$; 0.8824
2. tan *B* $\frac{8}{15}$; 0.5333
3. cos *A* $\frac{8}{17}$; 0.4706
4. cos *B* $\frac{15}{17}$; 0.8824
5. sin *D* $\frac{4}{5}$; 0.8
6. tan *E* $\frac{3}{4}$; 0.75
7. cos *E* $\frac{4}{5}$; 0.8
8. cos *D* $\frac{3}{5}$; 0.6

Lesson 7-4

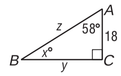
NAME ______________________ DATE __________ PERIOD _____

7-4 Study Guide and Intervention *(continued)*

Trigonometry

Use Trigonometric Ratios In a right triangle, if you know the measures of two sides or if you know the measures of one side and an acute angle, then you can use trigonometric ratios to find the measures of the missing sides or angles of the triangle.

Example **Find *x*, *y*, and *z*. Round each measure to the nearest whole number.**



a. Find *x*.

$x + 58 = 90$

$x = 32$

b. Find *y*.

$\tan A = \frac{y}{18}$

$\tan 58° = \frac{y}{18}$

$y = 18 \tan 58°$

$y \approx 29$

c. Find *z*.

$\cos A = \frac{18}{z}$

$\cos 58° = \frac{18}{z}$

$z \cos 58° = 18$

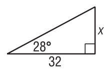
$z = \frac{18}{\cos 58°}$

$z \approx 34$

Exercises

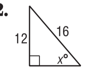
Find *x*. Round to the nearest tenth.

1.



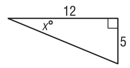
17.0

2.



48.6

3.

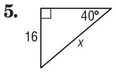


22.6

4.

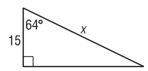
76.0

5.



24.9

6.



34.2

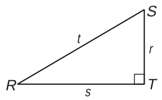
Answers

NAME ______________________ DATE ________ PERIOD _____

7-4 Skills Practice

Trigonometry

Use △RST to find sin R, cos R, tan R, sin S, cos S, and tan S. Express each ratio as a fraction and as a decimal to the nearest hundredth.



1. $r = 16, s = 30, t = 34$

$\sin R = \frac{16}{34} \approx 0.47$;
$\cos R = \frac{30}{34} \approx 0.88$;
$\tan R = \frac{16}{30} \approx 0.53$;
$\sin S = \frac{30}{34} \approx 0.88$;
$\cos S = \frac{16}{34} \approx 0.47$;
$\tan S = \frac{30}{16} \approx 1.88$

2. $r = 10, s = 24, t = 26$

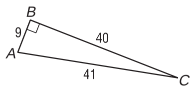
$\sin R = \frac{10}{26} \approx 0.38$;
$\cos R = \frac{24}{26} \approx 0.92$;
$\tan R = \frac{10}{24} \approx 0.42$;
$\sin S = \frac{24}{26} \approx 0.92$;
$\cos S = \frac{10}{26} \approx 0.38$;
$\tan S = \frac{24}{10} = 2.4$

Use a calculator to find each value. Round to the nearest ten-thousandth.

3. sin 5 **0.0872** **4.** tan 23 **0.4245** **5.** cos 61 **0.4848**

6. sin 75.8 **0.9694** **7.** tan 17.3 **0.3115** **8.** cos 52.9 **0.6032**

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.



9. tan C $\frac{9}{40} \approx 0.2250$ **10.** sin A $\frac{40}{41} \approx 0.9756$ **11.** cos C $\frac{40}{41} \approx 0.9756$

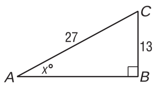
Find the measure of each acute angle to the nearest tenth of a degree.

12. $\sin B = 0.2985$ **17.4** **13.** $\tan A = 0.4168$ **22.6** **14.** $\cos R = 0.8443$ **32.4**

15. $\tan C = 0.3894$ **21.3** **16.** $\cos B = 0.7329$ **42.9** **17.** $\sin A = 0.1176$ **6.8**

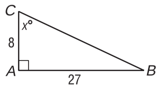
Find x. Round to the nearest tenth.

18.



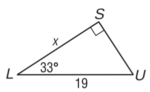
28.8

19.



73.5

20.



15.9

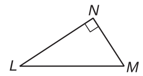
Lesson 7-4

NAME ______________________ DATE ________ PERIOD _____

7-4 Practice (Average)

Trigonometry

Use △LMN to find sin L, cos L, tan L, sin M, cos M, and tan M. Express each ratio as a fraction and as a decimal to the nearest hundredth.



1. $\ell = 15, m = 36, n = 39$

$\sin L = \frac{15}{39} \approx 0.38$;
$\cos L = \frac{36}{39} \approx 0.92$;
$\tan L = \frac{15}{36} \approx 0.42$;
$\sin M = \frac{36}{39} \approx 0.92$;
$\cos M = \frac{15}{39} \approx 0.38$;
$\tan M = \frac{36}{15} = 2.4$

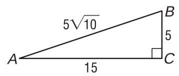
2. $\ell = 12, m = 12\sqrt{3}, n = 24$

$\sin L = \frac{12}{24} = 0.50$;
$\cos L = \frac{12\sqrt{3}}{24} \approx 0.87$;
$\tan L = \frac{12}{12\sqrt{3}} \approx 0.58$;
$\sin M = \frac{12\sqrt{3}}{24} \approx 0.87$;
$\cos M = \frac{12}{24} = 0.50$;
$\tan M = \frac{12\sqrt{3}}{12} \approx 1.73$

Use a calculator to find each value. Round to the nearest ten-thousandth.

3. sin 92.4 **0.9991** **4.** tan 27.5 **0.5206** **5.** cos 64.8 **0.4258**

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.



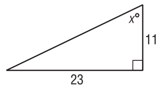
6. cos A $\frac{3\sqrt{10}}{10} \approx 0.9487$ **7.** tan B $\frac{3}{1} = 3.0000$ **8.** sin A $\frac{\sqrt{10}}{10} \approx 0.3162$

Find the measure of each acute angle to the nearest tenth of a degree.

9. $\sin B = 0.7823$ **51.5** **10.** $\tan A = 0.2356$ **13.3** **11.** $\cos R = 0.6401$ **50.2**

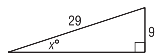
Find x. Round to the nearest tenth.

12.



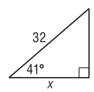
64.4

13.



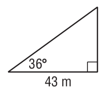
18.1

14.



24.2

15. GEOGRAPHY Diego used a theodolite to map a region of land for his class in geomorphology. To determine the elevation of a vertical rock formation, he measured the distance from the base of the formation to his position and the angle between the ground and the line of sight to the top of the formation. The distance was 43 meters and the angle was 36 degrees. What is the height of the formation to the nearest meter? **31 m**



NAME ______________________ DATE ________ PERIOD ____

7-4 Reading to Learn Mathematics

Trigonometry

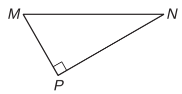
Pre-Activity **How can surveyors determine angle measures?**

Read the introduction to Lesson 7-4 at the top of page 364 in your textbook.

- Why is it important to determine the relative positions accurately in navigation? (Give two possible reasons.) **Sample answers: (1) To avoid collisions between ships, and (2) to prevent ships from losing their bearings and getting lost at sea.**
- What does *calibrated* mean? **Sample answer: marked precisely to permit accurate measurements to be made**

Reading the Lesson

1. Refer to the figure. Write a ratio using the side lengths in the figure to represent each of the following trigonometric ratios.



A. $\sin N$ $\frac{MP}{MN}$ **B.** $\cos N$ $\frac{NP}{MN}$

C. $\tan N$ $\frac{MP}{NP}$ **D.** $\tan M$ $\frac{NP}{MP}$

E. $\sin M$ $\frac{NP}{MN}$ **F.** $\cos M$ $\frac{MP}{MN}$

2. Assume that you enter each of the expressions in the list on the left into your calculator. Match each of these expressions with a description from the list on the right to tell what you are finding when you enter this expression.

a. sin 20 **v** **b.** cos 20 **ii** **c.** $\sin^{-1}$ 0.8 **vi** **d.** $\tan^{-1}$ 0.8 **iii** **e.** tan 20 **iv** **f.** $\cos^{-1}$ 0.8 **i**	**i.** the degree measure of an acute angle whose cosine is 0.8 **ii.** the ratio of the length of the leg adjacent to the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle **iii.** the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the adjacent leg is 0.8 **iv.** the ratio of the length of the leg opposite the 20° angle to the length of the leg adjacent to it in a 20°-70°-90° triangle **v.** the ratio of the length of the leg opposite the 20° angle to the length of hypotenuse in a 20°-70°-90° triangle **vi.** the degree measure of an acute angle in a right triangle for which the ratio of the length of the opposite leg to the length of the hypotenuse is 0.8

Helping You Remember

3. How can the *co* in *cosine* help you to remember the relationship between the sines and cosines of the two acute angles of a right triangle?

Sample answer: The *co* in *cosine* comes from *complement*, as in *complementary* angles. The cosine of an acute angle is equal to the sine of its complement.

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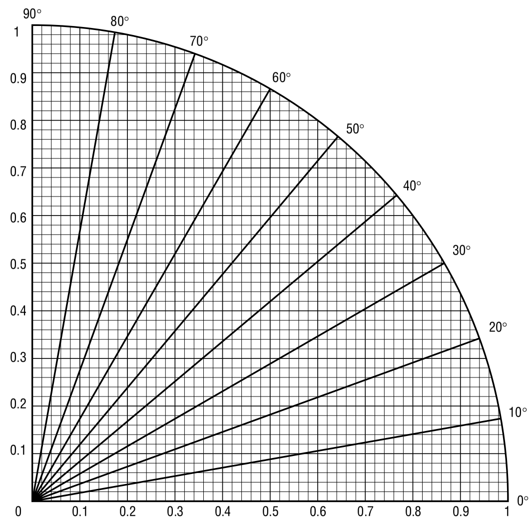
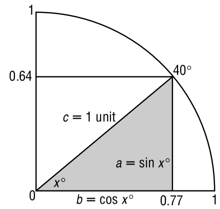
7-4 Enrichment

Sine and Cosine of Angles

The following diagram can be used to obtain approximate values for the sine and cosine of angles from 0° to 90°. The radius of the circle is 1. So, the sine and cosine values can be read directly from the vertical and horizontal axes.



Example **Find approximate values for sin 40° and cos 40°. Consider the triangle formed by the segment marked 40°, as illustrated by the shaded triangle at right.**



$\sin 40° = \frac{a}{c} \approx \frac{0.64}{1}$ or 0.64 $\cos 40° = \frac{b}{c} \approx \frac{0.77}{1}$ or 0.77

1. Use the diagram above to complete the chart of values.

x°	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
sin x°	**0**	**0.17**	**0.34**	**0.5**	0.64	**0.77**	**0.87**	**0.94**	**0.98**	**1**
cos x°	**1**	**0.98**	**0.94**	**0.87**	0.77	**0.64**	**0.5**	**0.34**	**0.17**	**0**

2. Compare the sine and cosine of two complementary angles (angles whose sum is 90°). What do you notice?

The sine of an angle is equal to the cosine of the complement of the angle.

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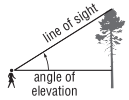
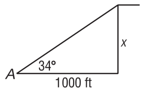
7-5 Study Guide and Intervention

Angles of Elevation and Depression

Angles of Elevation Many real-world problems that involve looking up to an object can be described in terms of an **angle of elevation**, which is the angle between an observer's line of sight and a horizontal line.



Example **The angle of elevation from point A to the top of a cliff is 34°. If point A is 1000 feet from the base of the cliff, how high is the cliff?**



Let x = the height of the cliff.

$\tan 34° = \frac{x}{1000}$ — $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$1000(\tan 34°) = x$ — Multiply each side by 1000.

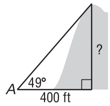
$674.5 = x$ — Use a calculator.

The height of the cliff is about 674.5 feet.

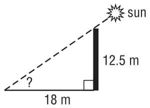
Exercises

Solve each problem. Round measures of segments to the nearest whole number and angles to the nearest degree.

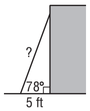
1. The angle of elevation from point A to the top of a hill is 49°. If point A is 400 feet from the base of the hill, how high is the hill?
460 ft



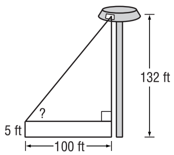
2. Find the angle of elevation of the sun when a 12.5-meter-tall telephone pole casts a 18-meter-long shadow.
35°



3. A ladder leaning against a building makes an angle of 78° with the ground. The foot of the ladder is 5 feet from the building. How long is the ladder?
24 ft



4. A person whose eyes are 5 feet above the ground is standing on the runway of an airport 100 feet from the control tower. That person observes an air traffic controller at the window of the 132-foot tower. What is the angle of elevation?
52°

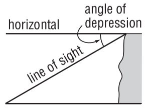


NAME ______________________ DATE __________ PERIOD _____

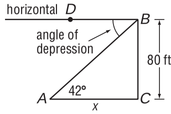
7-5 Study Guide and Intervention *(continued)*

Angles of Elevation and Depression

Angles of Depression When an observer is looking down, the **angle of depression** is the angle between the observer's line of sight and a horizontal line.



Example **The angle of depression from the top of an 80-foot building to point A on the ground is 42°. How far is the foot of the building from point A?**



Let x = the distance from point A to the foot of the building. Since the horizontal line is parallel to the ground, the angle of depression $\angle DBA$ is congruent to $\angle BAC$.

$\tan 42° = \frac{80}{x}$ — $\tan = \frac{\text{opposite}}{\text{adjacent}}$

$x(\tan 42°) = 80$ — Multiply each side by x.

$x = \frac{80}{\tan 42°}$ — Divide each side by tan 42°.

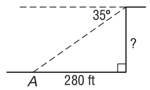
$x \approx 88.8$ — Use a calculator.

Point A is about 89 feet from the base of the building.

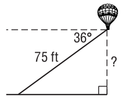
Exercises

Solve each problem. Round measures of segments to the nearest whole number and angles to the nearest degree.

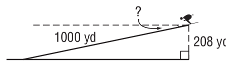
1. The angle of depression from the top of a sheer cliff to point A on the ground is 35°. If point A is 280 feet from the base of the cliff, how tall is the cliff?
196 ft



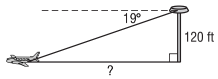
2. The angle of depression from a balloon on a 75-foot string to a person on the ground is 36°. How high is the balloon?
44 ft



3. A ski run is 1000 yards long with a vertical drop of 208 yards. Find the angle of depression from the top of the ski run to the bottom.
12°



4. From the top of a 120-foot-high tower, an air traffic controller observes an airplane on the runway at an angle of depression of 19°. How far from the base of the tower is the airplane?
349 ft



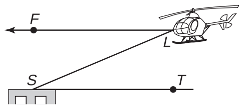
NAME ______________________ DATE __________ PERIOD _____

7-5 Skills Practice

Angles of Elevation and Depression

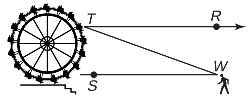
Name the angle of depression or angle of elevation in each figure.

1.



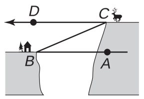
∠*FLS*; ∠*TSL*

2.



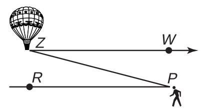
∠*RTW*; ∠*SWT*

3.



∠*DCB*; ∠*ABC*

4.

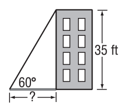


∠*WZP*; ∠*RPZ*

5. MOUNTAIN BIKING On a mountain bike trip along the Gemini Bridges Trail in Moab, Utah, Nabuko stopped on the canyon floor to get a good view of the twin sandstone bridges. Nabuko is standing about 60 meters from the base of the canyon cliff, and the natural arch bridges are about 100 meters up the canyon wall. If her line of sight is five feet above the ground, what is the angle of elevation to the top of the bridges? Round to the nearest tenth degree.

about 57.7°

6. SHADOWS Suppose the sun casts a shadow off a 35-foot building. If the angle of elevation to the sun is 60°, how long is the shadow to the nearest tenth of a foot?

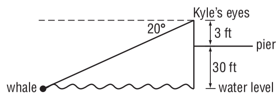


about 20.2 ft

7. BALLOONING From her position in a hot-air balloon, Angie can see her car parked in a field. If the angle of depression is 8° and Angie is 38 meters above the ground, what is the straight-line distance from Angie to her car? Round to the nearest whole meter.

about 273 m

8. INDIRECT MEASUREMENT Kyle is at the end of a pier 30 feet above the ocean. His eye level is 3 feet above the pier. He is using binoculars to watch a whale surface. If the angle of depression of the whale is 20°, how far is the whale from Kyle's binoculars? Round to the nearest tenth foot.



about 96.5 ft

Lesson 7-5

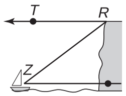
NAME ______________________ DATE __________ PERIOD _____

7-5 Practice (Average)

Angles of Elevation and Depression

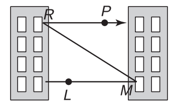
Name the angle of depression or angle of elevation in each figure.

1.



∠*TRZ*; ∠*YZR*

2.



∠*PRM*; ∠*LMR*

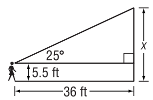
3. WATER TOWERS A student can see a water tower from the closest point of the soccer field at San Lobos High School. The edge of the soccer field is about 110 feet from the water tower and the water tower stands at a height of 32.5 feet. What is the angle of elevation if the eye level of the student viewing the tower from the edge of the soccer field is 6 feet above the ground? Round to the nearest tenth degree.

about 13.5°

4. CONSTRUCTION A roofer props a ladder against a wall so that the top of the ladder reaches a 30-foot roof that needs repair. If the angle of elevation from the bottom of the ladder to the roof is 55°, how far is the ladder from the base of the wall? Round your answer to the nearest foot.

about 21 ft

5. TOWN ORDINANCES The town of Belmont restricts the height of flagpoles to 25 feet on any property. Lindsay wants to determine whether her school is in compliance with the regulation. Her eye level is 5.5 feet from the ground and she stands 36 feet from the flagpole. If the angle of elevation is about 25°, what is the height of the flagpole to the nearest tenth foot?

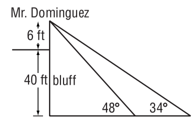


about 22.3 ft

6. GEOGRAPHY Stephan is standing on a mesa at the Painted Desert. The elevation of the mesa is about 1380 meters and Stephan's eye level is 1.8 meters above ground. If Stephan can see a band of multicolored shale at the bottom and the angle of depression is 29°, about how far is the band of shale from his eyes? Round to the nearest meter.

about 2850 m

7. INDIRECT MEASUREMENT Mr. Dominguez is standing on a 40-foot ocean bluff near his home. He can see his two dogs on the beach below. If his line of sight is 6 feet above the ground and the angles of depression to his dogs are 34° and 48°, how far apart are the dogs to the nearest foot?



about 27 ft

Answers

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7-5 Reading to Learn Mathematics

Angles of Elevation and Depression

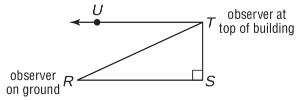
Pre-Activity **How do airline pilots use angles of elevation and depression?**

Read the introduction to Lesson 7-5 at the top of page 371 in your textbook.

What does the angle measure tell the pilot? **Sample answer: how steep her ascent must be to clear the peak**

Reading the Lesson

1. Refer to the figure. The two observers are looking at one another. Select the correct choice for each question.



a. What is the line of sight? **iii**
(i) line RS (ii) line ST (iii) line RT (iv) line TU

b. What is the angle of elevation? **ii**
(i) $\angle RST$ (ii) $\angle SRT$ (iii) $\angle RTS$ (iv) $\angle UTR$

c. What is the angle of depression? **iv**
(i) $\angle RST$ (ii) $\angle SRT$ (iii) $\angle RTS$ (iv) $\angle UTR$

d. How are the angle of elevation and the angle of depression related? **ii**
(i) They are complementary.
(ii) They are congruent.
(iii) They are supplementary.
(iv) The angle of elevation is larger than the angle of depression.

e. Which postulate or theorem that you learned in Chapter 3 supports your answer for part c? **iv**
(i) Corresponding Angles Postulate
(ii) Alternate Exterior Angles Theorem
(iii) Consecutive Interior Angles Theorem
(iv) Alternate Interior Angles Theorem

2. A student says that the angle of elevation from his eye to the top of a flagpole is 135°. What is wrong with the student's statement?
An angle of elevation cannot be obtuse.

Helping You Remember

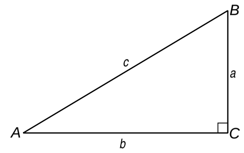
3. A good way to remember something is to explain it to someone else. Suppose a classmate finds it difficult to distinguish between angles of elevation and angles of depression. What are some hints you can give her to help her get it right every time? **Sample answers: (1) The angle of depression and the angle of elevation are both measured between the horizontal and the line of sight. (2) The angle of depression is always congruent to the angle of elevation in the same diagram. (3) Associate the word *elevation* with the word *up* and the word *depression* with the word *down*.**

NAME ______________________ DATE __________ PERIOD _____

7-5 Enrichment

Reading Mathematics

The three most common trigonometric ratios are **sine**, **cosine**, and **tangent**. Three other ratios are the **cosecant**, **secant**, and **cotangent**. The chart below shows abbreviations and definitions for all six ratios. Refer to the triangle at the right.



Abbreviation	Read as:	Ratio
sin A	the sine of $\angle A$	$\frac{\text{leg opposite } \angle A}{\text{hypotenuse}} = \frac{a}{c}$
cos A	the cosine of $\angle A$	$\frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}} = \frac{b}{c}$
tan A	the tangent of $\angle A$	$\frac{\text{leg opposite } \angle A}{\text{leg adjacent to } \angle A} = \frac{a}{b}$
csc A	the cosecant of $\angle A$	$\frac{\text{hypotenuse}}{\text{leg opposite } \angle A} = \frac{c}{a}$
sec A	the secant of $\angle A$	$\frac{\text{hypotenuse}}{\text{leg adjacent to } \angle A} = \frac{c}{b}$
cot A	the cotangent of $\angle A$	$\frac{\text{leg adjacent to } \angle A}{\text{leg opposite } \angle A} = \frac{b}{a}$

Use the abbreviations to rewrite each statement as an equation.

1. The secant of angle A is equal to 1 divided by the cosine of angle A. **$\sec A = \frac{1}{\cos A}$**

2. The cosecant of angle A is equal to 1 divided by the sine of angle A. **$\csc A = \frac{1}{\sin A}$**

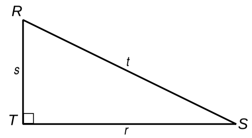
3. The cotangent of angle A is equal to 1 divided by the tangent of angle A. **$\cot A = \frac{1}{\tan A}$**

4. The cosecant of angle A multiplied by the sine of angle A is equal to 1. **$\csc A \sin A = 1$**

5. The secant of angle A multiplied by the cosine of angle A is equal to 1. **$\sec A \cos A = 1$**

6. The cotangent of angle A times the tangent of angle A is equal to 1. **$\cot A \tan A = 1$**

Use the triangle at right. Write each ratio.



7. sec R **$\frac{t}{s}$** **8.** csc R **$\frac{t}{r}$** **9.** cot R **$\frac{s}{r}$**

10. sec S **$\frac{t}{r}$** **11.** csc S **$\frac{t}{s}$** **12.** cot S **$\frac{r}{s}$**

13. If $\sin x° = 0.289$, find the value of $\csc x°$. **≈ 3.46**

14. If $\tan x° = 1.376$, find the value of $\cot x°$. **≈ 0.727**

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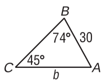
7-6 Study Guide and Intervention

The Law of Sines

The Law of Sines In any triangle, there is a special relationship between the angles of the triangle and the lengths of the sides opposite the angles.

Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example 1 **In $\triangle ABC$, find b.**



$\frac{\sin C}{c} = \frac{\sin B}{b}$ Law of Sines

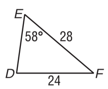
$\frac{\sin 45°}{30} = \frac{\sin 74°}{b}$ $m\angle C = 45$, $c = 30$, $m\angle B = 74$

$b \sin 45° = 30 \sin 74°$ Cross multiply.

$b = \frac{30 \sin 74°}{\sin 45°}$ Divide each side by sin 45°.

$b \approx 40.8$ Use a calculator.

Example 2 **In $\triangle DEF$, find $m\angle D$.**



$\frac{\sin D}{d} = \frac{\sin E}{e}$ Law of Sines

$\frac{\sin D}{28} = \frac{\sin 58°}{24}$ $d = 28$, $m\angle E = 58$, $e = 24$

$24 \sin D = 28 \sin 58°$ Cross multiply.

$\sin D = \frac{28 \sin 58°}{24}$ Divide each side by 24.

$D = \sin^{-1} \frac{28 \sin 58°}{24}$ Use the inverse sine.

$D \approx 81.6°$ Use a calculator.

Exercises

Find each measure using the given measures of $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $c = 12$, $m\angle A = 80$, and $m\angle C = 40$, find a.
18.4

2. If $b = 20$, $c = 26$, and $m\angle C = 52$, find $m\angle B$.
37

3. If $a = 18$, $c = 16$, and $m\angle A = 84$, find $m\angle C$.
62

4. If $a = 25$, $m\angle A = 72$, and $m\angle B = 17$, find b.
7.7

5. If $b = 12$, $m\angle A = 89$, and $m\angle B = 80$, find a.
12.2

6. If $a = 30$, $c = 20$, and $m\angle A = 60$, find $m\angle C$.
35

Lesson 7-6

NAME ______________________ DATE __________ PERIOD _____

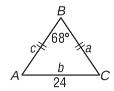
7-6 Study Guide and Intervention *(continued)*

The Law of Sines

Use the Law of Sines to Solve Problems You can use the **Law of Sines** to solve some problems that involve triangles.

Law of Sines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

Example **Isosceles $\triangle ABC$ has a base of 24 centimeters and a vertex angle of 68°. Find the perimeter of the triangle.**



The vertex angle is 68°, so the sum of the measures of the base angles is 112 and $m\angle A = m\angle C = 56$.

$\frac{\sin B}{b} = \frac{\sin A}{a}$ Law of Sines

$\frac{\sin 68°}{24} = \frac{\sin 56°}{a}$ $m\angle B = 68$, $b = 24$, $m\angle A = 56$

$a \sin 68° = 24 \sin 56°$ Cross multiply.

$a = \frac{24 \sin 56°}{\sin 68°}$ Divide each side by sin 68°.

≈ 21.5 Use a calculator.

The triangle is isosceles, so $c = 21.5$.
The perimeter is $24 + 21.5 + 21.5$ or about 67 centimeters.

Exercises

Draw a triangle to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. One side of a triangular garden is 42.0 feet. The angles on each end of this side measure 66° and 82°. Find the length of fence needed to enclose the garden.
192.9 ft

2. Two radar stations A and B are 32 miles apart. They locate an airplane X at the same time. The three points form $\angle XAB$, which measures 46°, and $\angle XBA$, which measures 52°. How far is the airplane from each station?
25.5 mi from *A*; 23.2 mi from *B*

3. A civil engineer wants to determine the distances from points A and B to an inaccessible point C in a river. $\angle BAC$ measures 67° and $\angle ABC$ measures 52°. If points A and B are 82.0 feet apart, find the distance from C to each point.
86.3 ft to point *B*; 73.9 ft to point *A*

4. A ranger tower at point A is 42 kilometers north of a ranger tower at point B. A fire at point C is observed from both towers. If $\angle BAC$ measures 43° and $\angle ABC$ measures 68°, which ranger tower is closer to the fire? How much closer?
Tower *B* is 11 km closer than Tower *A*.

Answers

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7-6 Skills Practice

The Law of Sines

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.

1. If $m\angle A = 35$, $m\angle B = 48$, and $b = 28$, find a. **21.6**

2. If $m\angle B = 17$, $m\angle C = 46$, and $c = 18$, find b. **7.3**

3. If $m\angle C = 86$, $m\angle A = 51$, and $a = 38$, find c. **48.8**

4. If $a = 17$, $b = 8$, and $m\angle A = 73$, find $m\angle B$. **26.7**

5. If $c = 38$, $b = 34$, and $m\angle B = 36$, find $m\angle C$. **41.1 or 138.9**

6. If $a = 12$, $c = 20$, and $m\angle C = 83$, find $m\angle A$. **36.6**

7. If $m\angle A = 22$, $a = 18$, and $m\angle B = 104$, find b. **46.6**

Solve each $\triangle PQR$ described below. Round measures to the nearest tenth.

8. $p = 27$, $q = 40$, $m\angle P = 33$ **$m\angle Q \approx 53.8$, $m\angle R \approx 93.2$, $r \approx 49.5$; or $m\angle Q \approx 126.2$, $m\angle R \approx 20.8$, $r \approx 17.6$**

9. $q = 12$, $r = 11$, $m\angle R = 16$ **$m\angle P \approx 146.5$, $m\angle Q \approx 17.5$, $p \approx 22.0$; or $m\angle P \approx 1.5$, $m\angle Q \approx 162.5$, $p \approx 1.0$**

10. $p = 29$, $q = 34$, $m\angle Q = 111$ **$m\angle P \approx 52.8$, $m\angle R \approx 16.2$, $r \approx 10.2$**

11. If $m\angle P = 89$, $p = 16$, $r = 12$ **$m\angle Q \approx 42.4$, $m\angle R \approx 48.6$, $q \approx 10.8$**

12. If $m\angle Q = 103$, $m\angle P = 63$, $p = 13$ **$m\angle R \approx 14$, $q \approx 14.2$, $r \approx 3.5$**

13. If $m\angle P = 96$, $m\angle R = 82$, $r = 35$ **$m\angle Q \approx 2$, $p \approx 35.2$, $q \approx 1.2$**

14. If $m\angle R = 49$, $m\angle Q = 76$, $r = 26$ **$m\angle P \approx 55$, $p \approx 28.2$, $q \approx 33.4$**

15. If $m\angle Q = 31$, $m\angle P = 52$, $p = 20$ **$m\angle R \approx 97$, $q \approx 13.1$, $r \approx 25.2$**

16. If $q = 8$, $m\angle Q = 28$, $m\angle R = 72$ **$m\angle P \approx 80$, $p \approx 16.8$, $r \approx 16.2$**

17. If $r = 15$, $p = 21$, $m\angle P = 128$ **$m\angle Q \approx 17.7$, $m\angle R \approx 34.3$, $q \approx 8.1$**

NAME ______________________ DATE ________ PERIOD _____

7-6 Practice (Average)

The Law of Sines

Find each measure using the given measures from $\triangle EFG$. Round angle measures to the nearest tenth degree and side measures to the nearest tenth.

1. If $m\angle G = 14$, $m\angle E = 67$, and $e = 14$, find g. **3.7**

2. If $e = 12.7$, $m\angle E = 42$, and $m\angle F = 61$, find f. **16.6**

3. If $g = 14$, $f = 5.8$, and $m\angle G = 83$, find $m\angle F$. **24.3**

4. If $e = 19.1$, $m\angle G = 34$, and $m\angle E = 56$, find g. **12.9**

5. If $f = 9.6$, $g = 27.4$, and $m\angle G = 43$, find $m\angle F$. **13.8**

Solve each $\triangle STU$ described below. Round measures to the nearest tenth.

6. $m\angle T = 85$, $s = 4.3$, $t = 8.2$ **$m\angle S \approx 31.5$, $m\angle U \approx 63.5$, $u \approx 7.4$**

7. $s = 40$, $u = 12$, $m\angle S = 37$ **$m\angle T \approx 132.6$, $m\angle U \approx 10.4$, $t \approx 48.9$**

8. $m\angle U = 37$, $t = 2.3$, $m\angle T = 17$ **$m\angle S \approx 126$, $s \approx 6.4$, $u \approx 4.7$**

9. $m\angle S = 62$, $m\angle U = 59$, $s = 17.8$ **$m\angle T \approx 59$, $t \approx 17.3$, $u \approx 17.3$**

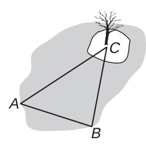
10. $t = 28.4$, $u = 21.7$, $m\angle T = 66$ **$m\angle S \approx 69.7$, $m\angle U \approx 44.3$, $s \approx 29.2$**

11. $m\angle S = 89$, $s = 15.3$, $t = 14$ **$m\angle T \approx 66.2$, $m\angle U \approx 24.8$, $u \approx 6.4$**

12. $m\angle T = 98$, $m\angle U = 74$, $u = 9.6$ **$m\angle S \approx 8$, $s \approx 1.4$, $t \approx 9.9$**

13. $t = 11.8$, $m\angle S = 84$, $m\angle T = 47$ **$m\angle U = 49$, $s \approx 16.0$, $u \approx 12.2$**

14. INDIRECT MEASUREMENT To find the distance from the edge of the lake to the tree on the island in the lake, Hannah set up a triangular configuration as shown in the diagram. The distance from location A to location B is 85 meters. The measures of the angles at A and B are 51° and 83°, respectively. What is the distance from the edge of the lake at B to the tree on the island at C?



about 91.8 m

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7-6 Reading to Learn Mathematics

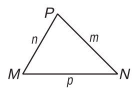
The Law of Sines

Pre-Activity **How are triangles used in radio astronomy?**

Read the introduction to Lesson 7-6 at the top of page 377 in your textbook.

Why might several antennas be better than one single antenna when studying distant objects? **Sample answer: Observing an object from only one position often does not provide enough information to calculate things such as the distance from the observer to the object.**

Reading the Lesson



1. Refer to the figure. According to the Law of Sines, which of the following are correct statements? **A, F**

A. $\frac{m}{\sin M} = \frac{n}{\sin N} = \frac{p}{\sin P}$

B. $\frac{\sin m}{M} = \frac{\sin n}{N} = \frac{\sin p}{P}$

C. $\frac{\cos M}{m} = \frac{\cos N}{n} = \frac{\cos P}{p}$

D. $\frac{\sin M}{m} + \frac{\sin N}{n} = \frac{\sin P}{p}$

E. $(\sin M)^2 + (\sin N)^2 = (\sin P)^2$

F. $\frac{\sin P}{p} = \frac{\sin M}{m} = \frac{\sin N}{n}$

2. State whether each of the following statements is *true* or *false*. If the statement is false, explain why.

a. The Law of Sines applies to all triangles. **true**

b. The Pythagorean Theorem applies to all triangles. **False; sample answer: It only applies to right triangles.**

c. If you are given the length of one side of a triangle and the measures of any two angles, you can use the Law of Sines to find the lengths of the other two sides. **true**

d. If you know the measures of two angles of a triangle, you should use the Law of Sines to find the measure of the third angle. **False; sample answer: You should use the Angle Sum Theorem.**

e. A friend tells you that in triangle RST, $m\angle R = 132$, $r = 24$ centimeters, and $s = 31$ centimeters. Can you use the Law of Sines to solve the triangle? Explain. **No; sample answer: In any triangle, the longest side is opposite the largest angle. Because a triangle can have only one obtuse angle, $\angle R$ must be the largest angle, but $s > r$, so it is impossible to have a triangle with the given measures.**

Helping You Remember

3. Many students remember mathematical equations and formulas better if they can state them in words. State the Law of Sines in your own words without using variables or mathematical symbols.

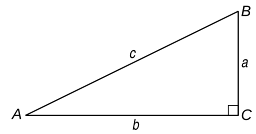
Sample answer: In any triangle, the ratio of the sine of an angle to the length of the opposite side is the same for all three angles.

NAME ______________________ DATE __________ PERIOD _____

7-6 Enrichment

Identities

An **identity** is an equation that is true for all values of the variable for which both sides are defined. One way to verify an identity is to use a right triangle and the definitions for trigonometric functions.



Example 1 **Verify that $(\sin A)^2 + (\cos A)^2 = 1$ is an identity.**

$$(\sin A)^2 + (\cos A)^2 = \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$
$$= \frac{a^2 + b^2}{c} = \frac{c^2}{c^2} = 1$$

To check whether an equation *may* be an identity, you can test several values. However, since you cannot test all values, you cannot be *certain* that the equation is an identity.

Example 2 **Test $\sin 2x = 2 \sin x \cos x$ to see if it could be an identity.**

Try $x = 20$. Use a calculator to evaluate each expression.

$\sin 2x = \sin 40 \approx 0.643$

$2 \sin x \cos x = 2(\sin 20)(\cos 20) \approx 2(0.342)(0.940) \approx 0.643$

Since the left and right sides seem equal, the equation may be an identity.

Use triangle *ABC* shown above. Verify that each equation is an identity.

1. $\frac{\cos A}{\sin A} = \frac{1}{\tan A}$

$\frac{\cos A}{\sin A} = \frac{b}{c} \div \frac{a}{c} = \frac{b}{a} = \frac{1}{\tan A}$

2. $\frac{\tan B}{\sin B} = \frac{1}{\cos B}$

$\frac{\tan B}{\sin B} = \frac{b}{a} \div \frac{b}{c} = \frac{c}{a} = \frac{1}{\cos B}$

3. $\tan B \cos B = \sin B$

$\tan B \cos B = \frac{b}{a} \cdot \frac{a}{c} = \frac{b}{c} = \sin B$

4. $1 - (\cos B)^2 = (\sin B)^2$

$1(\cos B)^2 = 1 - \left(\frac{a}{c}\right)^2$
$= \frac{c^2}{c^2} - \frac{a^2}{c^2}$
$= \frac{c^2 - a^2}{c^2} = \frac{b^2}{c^2}$ or $(\sin B)^2$

Try several values for x to test whether each equation could be an identity.

5. $\cos 2x = (\cos x)^2 - (\sin x)^2$

Yes; see students' work.

6. $\cos (90 - x) = \sin x$

Yes; see students' work.

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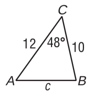
7-7 Study Guide and Intervention

The Law of Cosines

The Law of Cosines Another relationship between the sides and angles of any triangle is called the **Law of Cosines**. You can use the Law of Cosines if you know three sides of a triangle or if you know two sides and the included angle of a triangle.

Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Example 1 **In $\triangle ABC$, find c.**



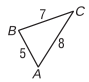
$c^2 = a^2 + b^2 - 2ab \cos C$ — Law of Cosines

$c^2 = 12^2 + 10^2 - 2(12)(10)\cos 48°$ — $a = 12$, $b = 10$, $m\angle C = 48$

$c = \sqrt{12^2 + 10^2 - 2(12)(10)\cos 48°}$ — Take the square root of each side.

$c \approx 9.1$ — Use a calculator.

Example 2 **In $\triangle ABC$, find $m\angle A$.**



$a^2 = b^2 + c^2 - 2bc \cos A$ — Law of Cosines

$7^2 = 5^2 + 8^2 - 2(5)(8) \cos A$ — $a = 7$, $b = 5$, $c = 8$

$49 = 25 + 64 - 80 \cos A$ — Multiply.

$-40 = -80 \cos A$ — Subtract 89 from each side.

$\frac{1}{2} = \cos A$ — Divide each side by -80.

$\cos^{-1}\frac{1}{2} = A$ — Use the inverse cosine.

$60° = A$ — Use a calculator.

Exercises

Find each measure using the given measures from $\triangle ABC$. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. If $b = 14$, $c = 12$, and $m\angle A = 62$, find a. **13.5**

2. If $a = 11$, $b = 10$, and $c = 12$, find $m\angle B$. **51**

3. If $a = 24$, $b = 18$, and $c = 16$, find $m\angle C$. **42**

4. If $a = 20$, $c = 25$, and $m\angle B = 82$, find b. **29.8**

5. If $b = 18$, $c = 28$, and $m\angle A = 59$, find a. **24.3**

6. If $a = 15$, $b = 19$, and $c = 15$, find $m\angle C$. **51**

Lesson 7-7

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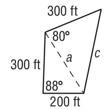
7-7 Study Guide and Intervention *(continued)*

The Law of Cosines

Use the Law of Cosines to Solve Problems You can use the **Law of Cosines** to solve some problems involving triangles.

Law of Cosines	Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then the following equations are true. $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Example **Ms. Jones wants to purchase a piece of land with the shape shown. Find the perimeter of the property.**



Use the Law of Cosines to find the value of a.

$a^2 = b^2 + c^2 - 2bc \cos A$ — Law of Cosines

$a^2 = 300^2 + 200^2 - 2(300)(200) \cos 88°$ — $b = 300$, $c = 200$, $m\angle A = 88$

$a = \sqrt{130{,}000 - 120{,}000 \cos 88°}$ — Take the square root of each side.

≈ 354.7 — Use a calculator.

Use the Law of Cosines again to find the value of c.

$c^2 = a^2 + b^2 - 2ab \cos C$ — Law of Cosines

$c^2 = 354.7^2 + 300^2 - 2(354.7)(300) \cos 80°$ — $a = 354.7$, $b = 300$, $m\angle C = 80$

$c = \sqrt{215{,}812.09 - 212{,}820 \cos 80°}$ — Take the square root of each side.

≈ 422.9 — Use a calculator.

The perimeter of the land is 300 + 200 + 422.9 + 200 or about 1223 feet.

Exercises

Draw a figure or diagram to go with each exercise and mark it with the given information. Then solve the problem. Round angle measures to the nearest degree and side measures to the nearest tenth.

1. A triangular garden has dimensions 54 feet, 48 feet, and 62 feet. Find the angles at each corner of the garden.
75°; 48°; 57°

2. A parallelogram has a 68° angle and sides 8 and 12. Find the lengths of the diagonals.
11.7; 16.7

3. An airplane is sighted from two locations, and its position forms an acute triangle with them. The distance to the airplane is 20 miles from one location with an angle of elevation 48.0°, and 40 miles from the other location with an angle of elevation of 21.8°. How far apart are the two locations?
50.5 mi

4. A ranger tower at point A is directly north of a ranger tower at point B. A fire at point C is observed from both towers. The distance from the fire to tower A is 60 miles, and the distance from the fire to tower B is 50 miles. If $m\angle ACB = 62$, find the distance between the towers.
57.3 mi

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7-7 Skills Practice

The Law of Cosines

In $\triangle RST$, given the following measures, find the measure of the missing side.

1. $r = 5, s = 8, m\angle T = 39$ **$t \approx 5.2$**

2. $r = 6, t = 11, m\angle S = 87$ **$s \approx 12.3$**

3. $r = 9, t = 15, m\angle S = 103$ **$s \approx 19.2$**

4. $s = 12, t = 10, m\angle R = 58$ **$r \approx 10.8$**

In $\triangle HIJ$, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.

5. $h = 12, i = 18, j = 7; m\angle H$ **24.7**

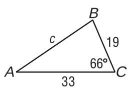
6. $h = 15, i = 16, j = 22; m\angle I$ **46.7**

7. $h = 23, i = 27, j = 29; m\angle J$ **70.4**

8. $h = 37, i = 21, j = 30; m\angle H$ **91.3**

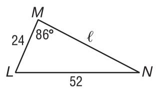
Determine whether the Law of *Sines* or the Law of *Cosines* should be used first to solve each triangle. Then solve each triangle. Round angle measures to the nearest degree and side measures to the nearest tenth.

9.



Cosines; $m\angle A \approx 34$; $m\angle B \approx 80$; $c \approx 30.7$

10.



Sines; $m\angle L \approx 67$; $m\angle N \approx 27$; $\ell \approx 47.8$

11. $a = 10, b = 14, c = 19$
Cosines; $m\angle A \approx 31$; $m\angle B \approx 46$; $m\angle C \approx 103$

12. $a = 12, b = 10, m\angle C = 27$
Cosines; $m\angle A \approx 97$; $m\angle B \approx 56$; $c \approx 5.5$

Solve each $\triangle RST$ described below. Round measures to the nearest tenth.

13. $r = 12, s = 32, t = 34$ **$m\angle R \approx 20.7$, $m\angle S \approx 70.2$, $m\angle T \approx 89.1$**

14. $r = 30, s = 25, m\angle T = 42$ **$m\angle R \approx 82.2$, $m\angle S \approx 55.7$, $t \approx 20.3$**

15. $r = 15, s = 11, m\angle R = 67$ **$m\angle S \approx 42.5$, $m\angle T \approx 70.5$, $t \approx 15.4$**

16. $r = 21, s = 28, t = 30$ **$m\angle R \approx 42.3$, $m\angle S \approx 63.8$, $m\angle T \approx 74.0$**

Lesson 7-7

NAME ________________________ DATE ____________ PERIOD _____

7-7 Practice (Average)

The Law of Cosines

In $\triangle JKL$, given the following measures, find the measure of the missing side.

1. $j = 1.3, k = 10, m\angle L = 77$ **$\ell \approx 9.8$**

2. $j = 9.6, \ell = 1.7, m\angle K = 43$ **$k \approx 8.4$**

3. $j = 11, k = 7, m\angle L = 63$ **$\ell \approx 10.0$**

4. $k = 4.7, \ell = 5.2, m\angle J = 112$ **$j \approx 8.2$**

In $\triangle MNQ$, given the lengths of the sides, find the measure of the stated angle to the nearest tenth.

5. $m = 17, n = 23, q = 25; m\angle Q$ **75.7**

6. $m = 24, n = 28, q = 34; m\angle M$ **44.2**

7. $m = 12.9, n = 18, q = 20.5; m\angle N$ **60.2**

8. $m = 23, n = 30.1, q = 42; m\angle Q$ **103.7**

Determine whether the Law of Sines or the Law of Cosines should be used first to solve $\triangle ABC$. Then sole each triangle. Round angle measures to the nearest degree and side measure to the nearest tenth.

9. $a = 13, b = 18, c = 19$
Cosines; $m\angle A \approx 41$; $m\angle B \approx 65$; $m\angle C \approx 74$

10. $a = 6, b = 19, m\angle C = 38$
Cosines; $m\angle A \approx 15$; $m\angle B \approx 127$; $c \approx 14.7$

11. $a = 17, b = 22, m\angle B = 49$
Sines; $m\angle A \approx 36$; $m\angle C \approx 95$; $c \approx 29.0$

12. $a = 15.5, b = 18, m\angle C = 72$
Cosines; $m\angle A \approx 48$; $m\angle B \approx 60$; $c \approx 19.8$

Solve each $\triangle FGH$ described below. Round measures to the nearest tenth.

13. $m\angle F = 54, f = 12.5, g = 11$ **$m\angle G \approx 45.4$, $m\angle H \approx 80.6$, $h \approx 15.2$**

14. $f = 20, g = 23, m\angle H = 47$ **$m\angle F \approx 57.4$, $m\angle G \approx 75.6$, $h \approx 17.4$**

15. $f = 15.8, g = 11, h = 14$ **$m\angle F \approx 77.4$, $m\angle G \approx 42.8$, $m\angle H \approx 59.8$**

16. $f = 36, h = 30, m\angle G = 54$ **$m\angle F \approx 73.1$, $m\angle H \approx 52.9$, $g \approx 30.4$**

17. REAL ESTATE The Esposito family purchased a triangular plot of land on which they plan to build a barn and corral. The lengths of the sides of the plot are 320 feet, 286 feet, and 305 feet. What are the measures of the angles formed on each side of the property?
65.5, 54.4, 60.1

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7-7 Reading to Learn Mathematics

The Law of Cosines

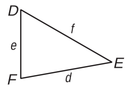
Pre-Activity **How are triangles used in building design?**

Read the introduction to Lesson 7-7 at the top of page 385 in your textbook.

What could be a disadvantage of a triangular room? **Sample answer: Furniture will not fit in the corners.**

Reading the Lesson

1. Refer to the figure. According to the Law of Cosines, which statements are correct for $\triangle DEF$? **B, E, H**



A. $d^2 = e^2 + f^2 - ef \cos D$

B. $e^2 = d^2 + f^2 - 2df \cos E$

C. $d^2 = e^2 + f^2 + 2ef \cos D$

D. $f^2 = d^2 + e^2 - 2ef \cos F$

E. $f^2 = d^2 + e^2 - 2de \cos F$

F. $d^2 = e^2 + f^2$

G. $\frac{\sin D}{d} = \frac{\sin E}{e} = \frac{\sin F}{f}$

H. $d = \sqrt{e^2 + f^2 - 2ef \cos D}$

2. Each of the following describes three given parts of a triangle. In each case, indicate whether you would use the Law of Sines or the Law of Cosines first in solving a triangle with those given parts. (In some cases, only one of the two laws would be used in solving the triangle.)

a. SSS **Law of Cosines**

b. ASA **Law of Sines**

c. AAS **Law of Sines**

d. SAS **Law of Cosines**

e. SSA **Law of Sines**

3. Indicate whether each statement is *true* or *false*. If the statement is false, explain why.

a. The Law of Cosines applies to right triangles. **true**

b. The Pythagorean Theorem applies to acute triangles. **False; sample answer: It only applies to right triangles.**

c. The Law of Cosines is used to find the third side of a triangle when you are given the measures of two sides and the nonincluded angle. **False; sample answer: It is used when you are given the measures of two sides and the included angle.**

d. The Law of Cosines can be used to solve a triangle in which the measures of the three sides are 5 centimeters, 8 centimeters, and 15 centimeters. **False; sample answer: 5 + 8 < 15, so, by the Triangle Inequality Theorem, no such triangle exists.**

Helping You Remember

4. A good way to remember a new mathematical formula is to relate it to one you already know. The Law of Cosines looks somewhat like the Pythagorean Theorem. Both formulas must be true for a right triangle. How can that be? **cos 90 = 0, so in a right triangle, where the included angle is the right angle, the Law of Cosines becomes the Pythagorean Theorem.**

Lesson 7-7

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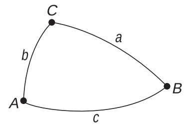
7-7 Enrichment

Spherical Triangles

Spherical trigonometry is an extension of plane trigonometry. Figures are drawn on the surface of a sphere. Arcs of great circles correspond to line segments in the plane. The arcs of three great circles intersecting on a sphere form a spherical triangle. Angles have the same measure as the tangent lines drawn to each great circle at the vertex. Since the sides are arcs, they too can be measured in degrees.



The sum of the sides of a spherical triangle is less than 360°.
The sum of the angles is greater than 180° and less than 540°.
The Law of Sines for spherical triangles is as follows.

$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C}$$

There is also a Law of Cosines for spherical triangles.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos b = \cos a \cos c + \sin a \sin c \cos B$$
$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

Example **Solve the spherical triangle given $a = 72°$, $b = 105°$, and $c = 61°$.**

Use the Law of Cosines.

$0.3090 = (-0.2588)(0.4848) + (0.9659)(0.8746) \cos A$
$\cos A = 0.5143$
$A = 59°$

$-0.2588 = (0.3090)(0.4848) + (0.9511)(0.8746) \cos B$
$\cos B = -0.4912$
$B = 119°$

$0.4848 = (0.3090)(-0.2588) + (0.9511)(0.9659) \cos C$
$\cos C = 0.6148$
$C = 52°$

Check by using the Law of Sines.

$$\frac{\sin 72°}{\sin 59°} = \frac{\sin 105°}{\sin 119°} = \frac{\sin 61°}{\sin 52°} = 1.1$$

Solve each spherical triangle.

1. $a = 56°, b = 53°, c = 94°$
$A = 41°, B = 39°, C = 128°$

2. $a = 110°, b = 33°, c = 97°$
$A = 116°, B = 31°, C = 71°$

3. $a = 76°, b = 110°, C = 49°$
$A = 59°, B = 124°, c = 59°$

4. $b = 94°, c = 55°, A = 48°$
$a = 60°, B = 121°, C = 45°$