Vectors: Motion and Forces in Two Dimensions

The Physics Classroom Tutorial

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Vectors: Motion and Forces in Two Dimensions - Chapter Outline

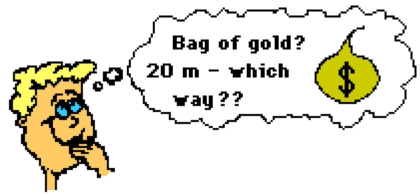
Vectors: Motion and Forces in Two Dimensions - Lesson 1

Vectors - Fundamentals and Operations

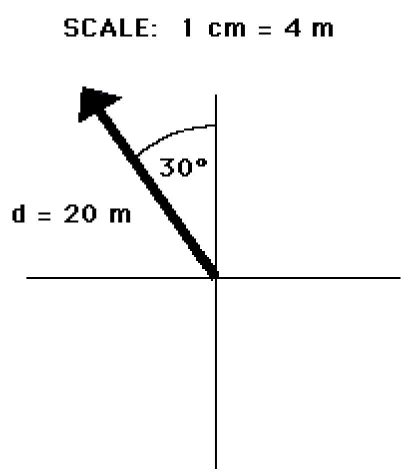
Vectors and Direction | Vector Addition | Resultants | Vector Components | Vector Resolution | Component Addition
Relative Velocity and Riverboat Problems | Independence of Perpendicular Components of Motion

Vectors and Direction

A study of motion will involve the introduction of a variety of quantities that are used to describe the physical world. Examples of such quantities include distance, displacement, speed, velocity, acceleration, force, mass, momentum, energy, work, power, etc. All these quantities can by divided into two categories - vectors and scalars. A vector quantity is a quantity that is fully described by both magnitude and direction. On the other hand, a scalar quantity is a quantity that is fully described by its magnitude. The emphasis of this unit is to understand some fundamentals about vectors and to apply the fundamentals in order to understand motion and forces that occur in two dimensions.

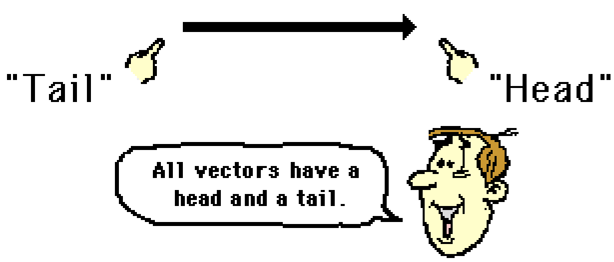


Examples of vector quantities that have been previously discussed include displacement, velocity, acceleration, and force. Each of these quantities are unique in that a full description of the quantity demands that both a magnitude and a direction are listed. For example, suppose your teacher tells you "A bag of gold is located outside the classroom. To find it, displace yourself 20 meters." This statement may provide yourself enough information to pique your interest; yet, there is not enough information included in the statement to find the bag of gold. The displacement required to find the bag of gold has not been fully described. On the other hand, suppose your teacher tells you "A bag of gold is located outside the classroom. To find it, displace yourself from the center of the classroom door 20 meters in a direction 30 degrees to the west of north." This statement now provides a complete description of the displacement vector - it lists both magnitude (20 meters) and direction (30 degrees to the west of north) relative to a reference or starting position (the center of the classroom door). Vector quantities are not fully described unless both magnitude and direction are listed.

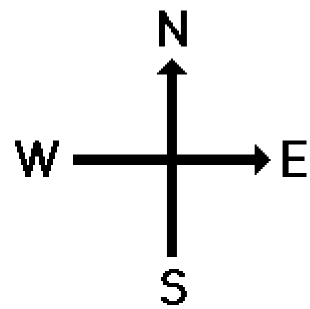


Vector quantities are often represented by scaled vector diagrams. Vector diagrams depict a vector by use of an arrow drawn to scale in a specific direction. Vector diagrams were introduced and used in earlier units to depict the forces acting upon an object. Such diagrams are commonly called as free-body diagrams. An example of a scaled vector diagram is shown in the diagram at the right. The vector diagram depicts a displacement vector. Observe that there are several characteristics of this diagram that make it an appropriately drawn vector diagram.

- a scale is clearly listed
- a vector arrow (with arrowhead) is drawn in a specified direction. The vector arrow has a *head* and a *tail*.
- the magnitude and direction of the vector is clearly labeled. In this case, the diagram shows the magnitude is 20 m and the direction is (30 degrees West of North).



Conventions for Describing Directions of Vectors



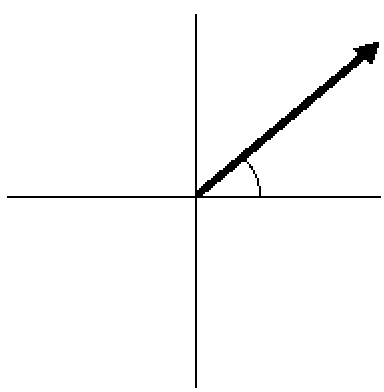
Vectors can be directed due East, due West, due South, and due North. But some vectors are directed *northeast* (at a 45 degree angle); and some vectors are even directed *northeast*, yet more north than east. Thus, there is a clear need for some form of a convention for identifying the direction of a vector that is not due East, due West, due South, or due North. There are a variety of conventions for describing the direction of any vector. The two conventions that will be discussed and used in this unit are described below:

a. The direction of a vector is often expressed as an angle of rotation of the vector about its "tail" from east, west, north, or south. For example, a vector can be said to have a direction of 40 degrees North of West (meaning a vector pointing West has been rotated 40 degrees towards the northerly direction) of 65 degrees East of South (meaning a vector pointing South has been rotated 65 degrees towards the easterly direction).

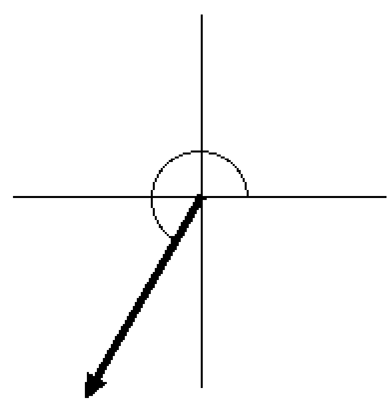
b. The direction of a vector is often expressed as a counterclockwise angle of rotation of the vector about its "tail" from due East. Using this convention, a vector with a direction of 30 degrees is a vector that has been rotated 30 degrees in a counterclockwise direction relative to due east. A vector with a direction of 160 degrees is a vector that has been rotated 160 degrees in a counterclockwise direction relative to due east. A vector with a direction of 270 degrees is a vector that has been rotated 270 degrees in a counterclockwise direction relative to due east. This is one of the most common conventions for the direction of a vector and will be utilized throughout this unit.

Two illustrations of the second convention (discussed above) for identifying the direction of a vector are shown below.

40° counter-clockwise rotation from East

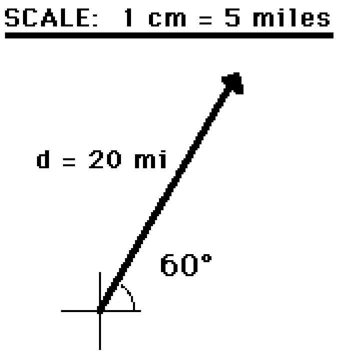


240° counter-clockwise rotation from East



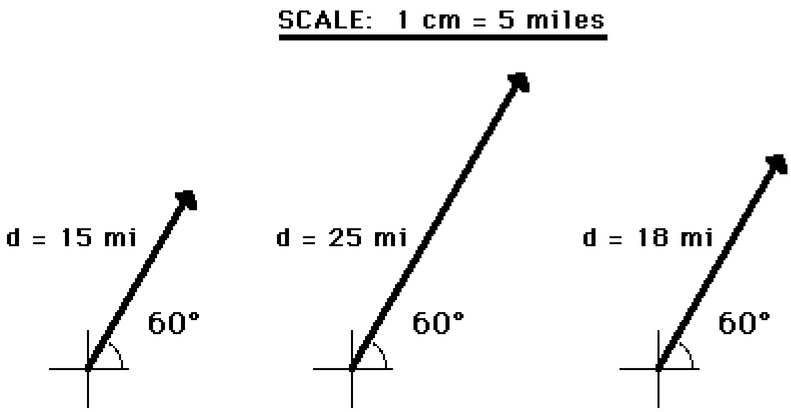
Observe in the first example that the vector is said to have a direction of 40 degrees. You can think of this direction as follows: suppose a vector pointing East had its tail pinned down and then the vector was rotated an angle of 40 degrees in the counterclockwise direction. Observe in the second example that the vector is said to have a direction of 240 degrees. This means that the tail of the vector was pinned down and the vector was rotated an angle of 240 degrees in the counterclockwise direction beginning from due east. A rotation of 240 degrees is equivalent to rotating the vector through two quadrants (180 degrees) and then an additional 60 degrees *into the* third quadrant.

Representing the Magnitude of a Vector



The magnitude of a vector in a scaled vector diagram is depicted by the length of the arrow. The arrow is drawn a precise length in accordance with a chosen scale. For example, the diagram at the right shows a vector with a magnitude of 20 miles. Since the scale used for constructing the diagram is 1 cm = 5 miles, the vector arrow is drawn with a length of 4 cm. That is, 4 cm x (5 miles/1 cm) = 20 miles.

Using the same scale (1 cm = 5 miles), a displacement vector that is 15 miles will be represented by a vector arrow that is 3 cm in length. Similarly, a 25-mile displacement vector is represented by a 5-cm long vector arrow. And finally, an 18-mile displacement vector is represented by a 3.6-cm long arrow. See the examples shown below.



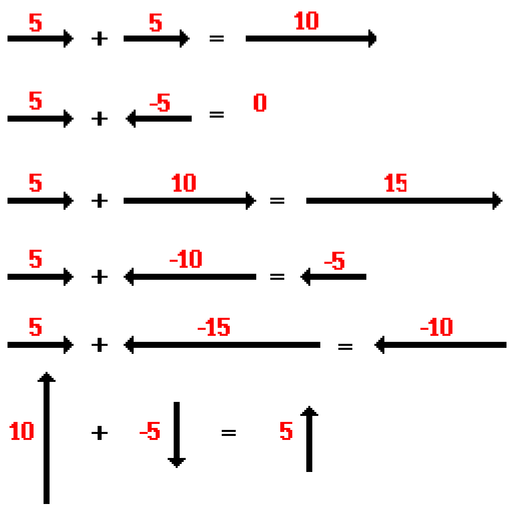
In conclusion, vectors can be represented by use of a scaled vector diagram. On such a diagram, a vector arrow is drawn to represent the vector. The arrow has an obvious tail and arrowhead. The magnitude of a vector is represented by the length of the arrow. A scale is indicated (such as, 1 cm = 5 miles) and the arrow is drawn the proper length according to the chosen scale. The arrow points in the precise direction. Directions are described by the use of some convention. The most common convention is that the direction of a vector is the counterclockwise angle of rotation which that vector makes with respect to due East.

In the remainder of this lesson, in the entire unit, and in future units, scaled vector diagrams and the above convention for the direction of a vector will be frequently used to describe motion and solve problems concerning motion. For this reason, it is critical that you have a comfortable understanding of the means of representing and describing vector quantities. Some practice problems are available on-line at the following WWW page:

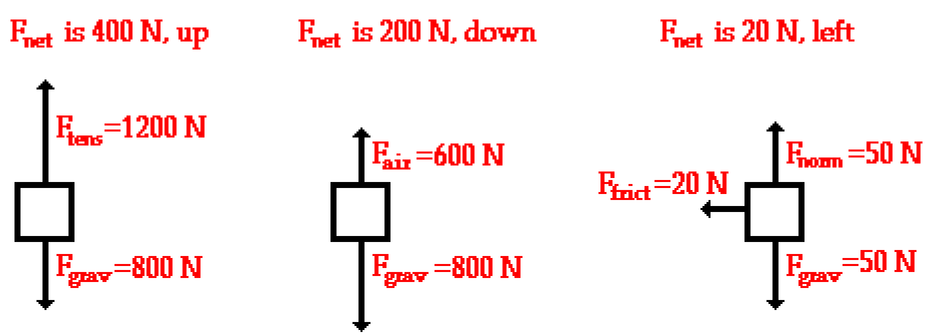
Visit the Vector Direction Practice Page

Vector Addition

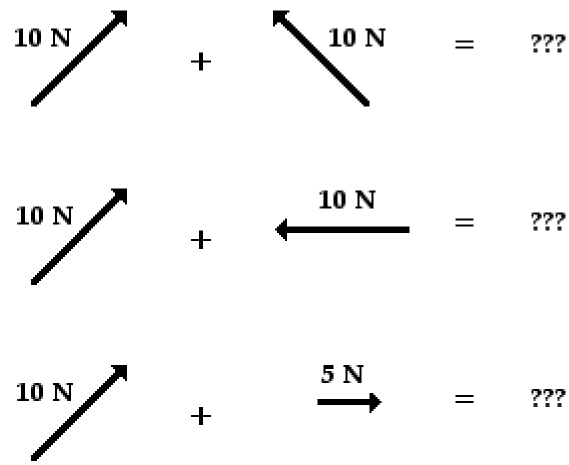
A variety of mathematical operations can be performed with and upon vectors. One such operation is the addition of vectors. Two vectors can be added together to determine the result (or resultant). This process of adding two or more vectors has already been discussed in an earlier unit. Recall in our discussion of Newton's laws of motion, that the *net force* experienced by an object was determined by computing the vector sum of all the individual forces acting upon that object. That is the net force was the result (or resultant) of adding up all the force vectors. During that unit, the rules for summing vectors (such as force vectors) were kept relatively simple. Observe the following summations of two force vectors:



These rules for summing vectors were applied to free-body diagrams in order to determine the net force (i.e., the vector sum of all the individual forces). Sample applications are shown in the diagram below.



In this unit, the task of summing vectors will be extended to more complicated cases in which the vectors are directed in directions other than purely vertical and horizontal directions. For example, a vector directed up and to the right will be added to a vector directed up and to the left. The *vector sum* will be determined for the more complicated cases shown in the diagrams below.

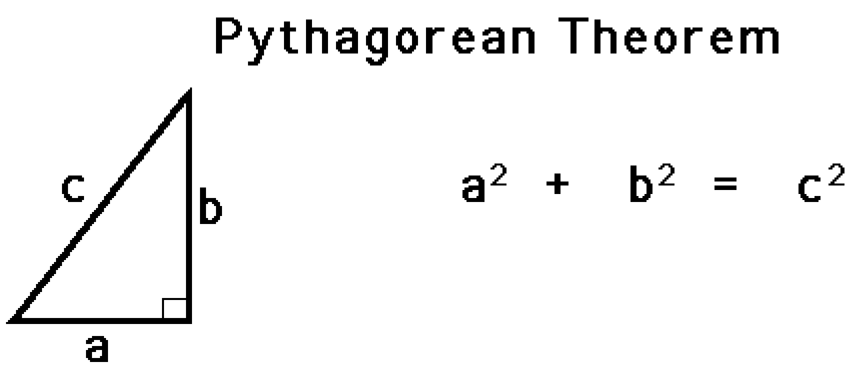


There are a variety of methods for determining the magnitude and direction of the result of adding two or more vectors. The two methods that will be discussed in this lesson and used throughout the entire unit are:

- the Pythagorean theorem and trigonometric methods
- the head-to-tail method using a scaled vector diagram

The Pythagorean Theorem

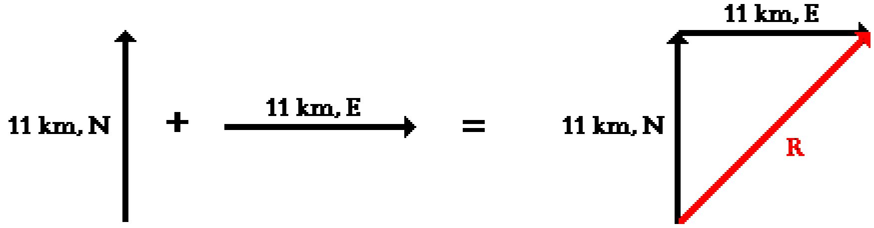
The Pythagorean theorem is a useful method for determining the result of adding two (and only two) vectors that make a right angle to each other. The method is not applicable for adding more than two vectors or for adding vectors that are not at 90-degrees to each other. The Pythagorean theorem is a mathematical equation that relates the length of the sides of a right triangle to the length of the hypotenuse of a right triangle.



To see how the method works, consider the following problem:

Eric leaves the base camp and hikes 11 km, north and then hikes 11 km east. Determine Eric's resulting displacement.

This problem asks to determine the result of adding two displacement vectors that are at right angles to each other. The result (or resultant) of walking 11 km north and 11 km east is a vector directed northeast as shown in the diagram to the right. Since the northward displacement and the eastward displacement are at right angles to each other, the Pythagorean theorem can be used to determine the resultant (i.e., the hypotenuse of the right triangle).



$$11^2 + 11^2 = R^2$$

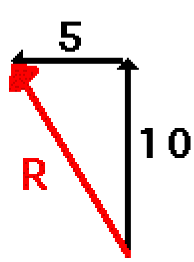
$$242 = R^2$$

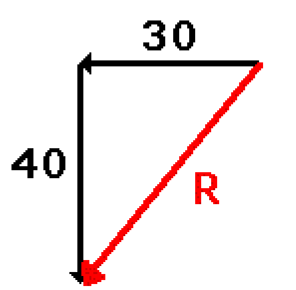
$$15.6 = R$$

The result of adding 11 km, north plus 11 km, east is a vector with a magnitude of 15.6 km. Later, the method of determining the direction of the vector will be discussed.

Quick Quiz Let's test your understanding with the following two practice problems. In each case, use the Pythagorean theorem to determine the magnitude of the *vector sum*. When finished, click the button to view the answer.

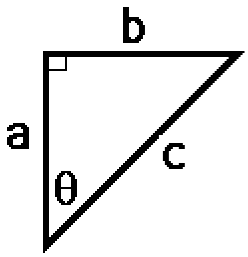
Practice A	Practice B
10 km, North	30 km, West
+	+
5 km, West	40 km, South





Using Trigonometry to Determine a Vector's Direction

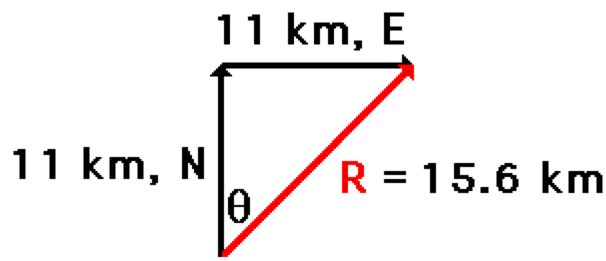
The direction of a *resultant* vector can often be determined by use of trigonometric functions. Most students recall the meaning of the useful mnemonic SOH CAH TOA from their course in trigonometry. SOH CAH TOA is a mnemonic that helps one remember the meaning of the three common trigonometric functions - sine, cosine, and tangent functions. These three functions relate an acute angle in a right triangle to the ratio of the lengths of two of the sides of the right triangle. The **sine function** relates the measure of an acute angle to the ratio of the length of the side opposite the angle to the length of the hypotenuse. The **cosine function** relates the measure of an acute angle to the ratio of the length of the side adjacent the angle to the length of the hypotenuse. The **tangent function** relates the measure of an angle to the ratio of the length of the side opposite the angle to the length of the side adjacent to the angle. The three equations below summarize these three functions in equation form.



a=side adjacent to angle Θ
b=side opposite to angle Θ
c=hypotenuse of triangle

$$\sin\Theta = \frac{b}{c} \qquad \cos\Theta = \frac{a}{c} \qquad \tan\Theta = \frac{b}{a}$$

These three trigonometric functions can be applied to the hiker problem in order to determine the direction of the hiker's overall displacement. The process begins by the selection of one of the two angles (other than the right angle) of the triangle. Once the angle is selected, any of the three functions can be used to find the measure of the angle. Write the function and proceed with the proper algebraic steps to solve for the measure of the angle. The work is shown below.



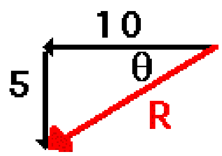
$$\sin\Theta = \frac{11\text{ km}}{15.6\text{ km}} = 0.7051$$

$$\Theta = \sin^{-1}(0.7051) = 45°$$

Once the measure of the angle is determined, the direction of the vector can be found. In this case the vector makes an angle of 45 degrees with due East. Thus, the direction of this vector is written as 45 degrees. (Recall from earlier in this lesson that the direction of a vector is the counterclockwise angle of rotation that the vector makes with due East.)

The measure of an angle as determined through use of SOH CAH TOA is <u>not</u> always the direction of the vector. The following vector addition diagram is an example of such a situation. Observe that the angle within the triangle is determined to be 26.6 degrees using SOH CAH TOA. This angle is the southward angle of rotation that the vector R makes with respect to West. Yet the direction of the vector as expressed with the CCW (counterclockwise from East) convention is 206.6 degrees.

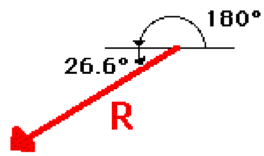
10 km, West + 5 km, South



$$\tan\Theta = \frac{5\text{ km}}{10\text{ km}} = 0.5$$

$$\Theta = \tan^{-1}(0.5) = 26.6°$$

Direction of R is 180° + 26.6°



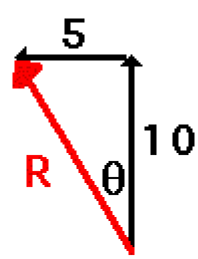
Quick Quiz Test your understanding of the use of SOH CAH TOA to determine the vector direction by trying the following two practice problems. In each case, use SOH CAH TOA to determine the direction of the resultant. When finished, click the button to view the answer.

Practice A

10 km, North

+

5 km, West

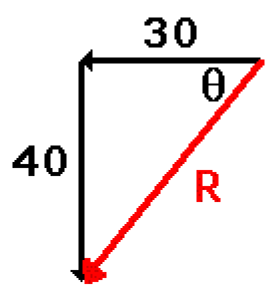


Practice B

30 km, West

+

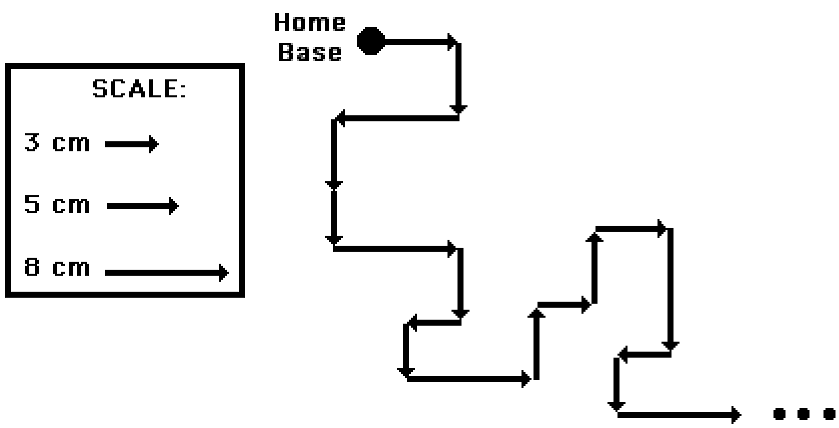
40 km, South



In the above problems, the magnitude and direction of the sum of two vectors is determined using the Pythagorean theorem and trigonometric methods (SOH CAH TOA). The procedure is restricted to the addition of <u>two vectors that make right angles to each other</u>. When the two vectors that are to be added do not make right angles to one another, or when there are more than two vectors to add together, we will employ a method known as the head-to-tail vector addition method. This method is described below.

Use of Scaled Vector Diagrams to Determine a Resultant

The magnitude and direction of the sum of two or more vectors can also be determined by use of an accurately drawn scaled vector diagram. Using a scaled diagram, the **head-to-tail method** is employed to determine the vector sum or resultant. A common Physics lab involves a *vector walk*. Either using centimeter-sized displacements upon a map or meter-sized displacements in a large open area, a student makes several consecutive displacements beginning from a designated starting position. Suppose that you were given a map of your local area and a set of 18 directions to follow. Starting at *home base*, these 18 displacement vectors could be *added together* in consecutive fashion to determine the result of adding the set of 18 directions. Perhaps the first vector is measured 5 cm, East. Where this measurement ended, the next measurement would begin. The process would be repeated for all 18 directions. Each time one measurement ended, the next measurement would begin. In essence, you would be using the head-to-tail method of vector addition.



The head-to-tail method involves drawing a vector to scale on a sheet of paper beginning at a designated starting position. Where the head of this first vector ends, the tail of the second vector begins (thus, *head-to-tail* method). The process is repeated for all vectors that are being added. Once all the vectors have been added head-to-tail, the resultant is then drawn from the tail of the first vector to the head of the last vector; i.e., from start to finish. Once the resultant is drawn, its length can be measured and converted to *real* units using the given scale. The direction of the resultant can be determined by using a protractor and measuring its counterclockwise angle of rotation from due East.

A step-by-step method for applying the head-to-tail method to determine the sum of two or more vectors is given below.

a. Choose a scale and indicate it on a sheet of paper. The best choice of scale is one that will result in a diagram that is as large as possible, yet fits on the sheet of paper.

b. Pick a starting location and draw the first vector *to scale* in the indicated direction. Label the magnitude and direction of the scale on the diagram (e.g., SCALE: 1 cm = 20 m).

c. Starting from where the head of the first vector ends, draw the second vector *to scale* in the indicated direction. Label the magnitude and direction of this vector on the diagram.

d. Repeat steps 2 and 3 for all vectors that are to be added

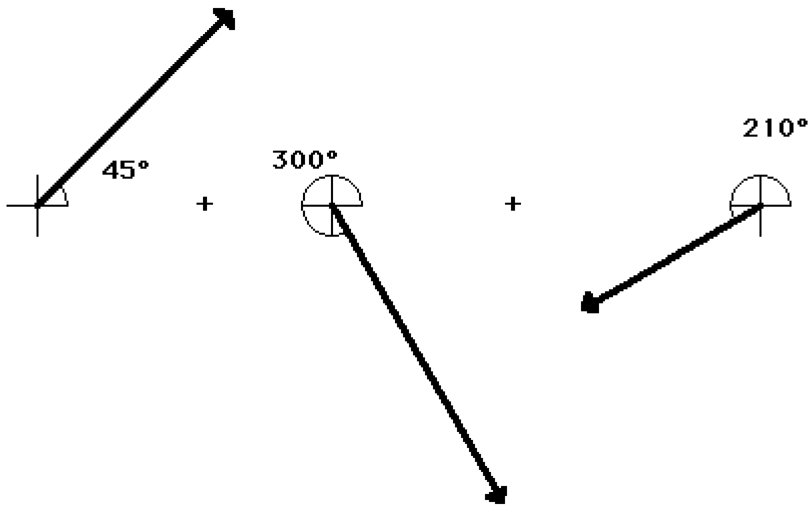
e. Draw the resultant from the tail of the first vector to the head of the last vector. Label this vector as **Resultant** or simply **R**.

f. Using a ruler, measure the length of the resultant and determine its magnitude by converting to real units using the scale (4.4 cm x 20 m/1 cm = 88 m).

g. Measure the direction of the resultant using the counterclockwise convention discussed earlier in this lesson.

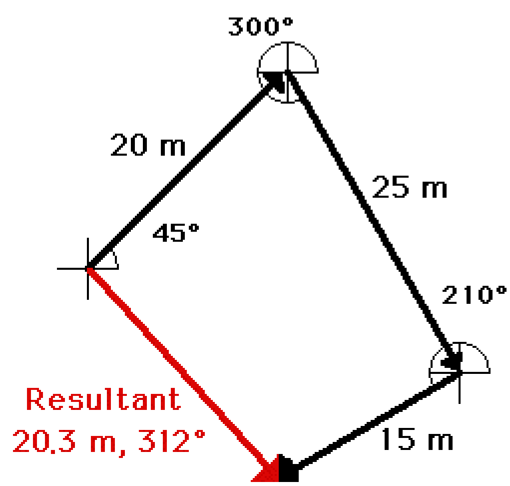
An example of the use of the head-to-tail method is illustrated below. The problem involves the addition of three vectors:

20 m, 45 deg. + 25 m, 300 deg. + 15 m, 210 deg.
SCALE: 1 cm = 5 m



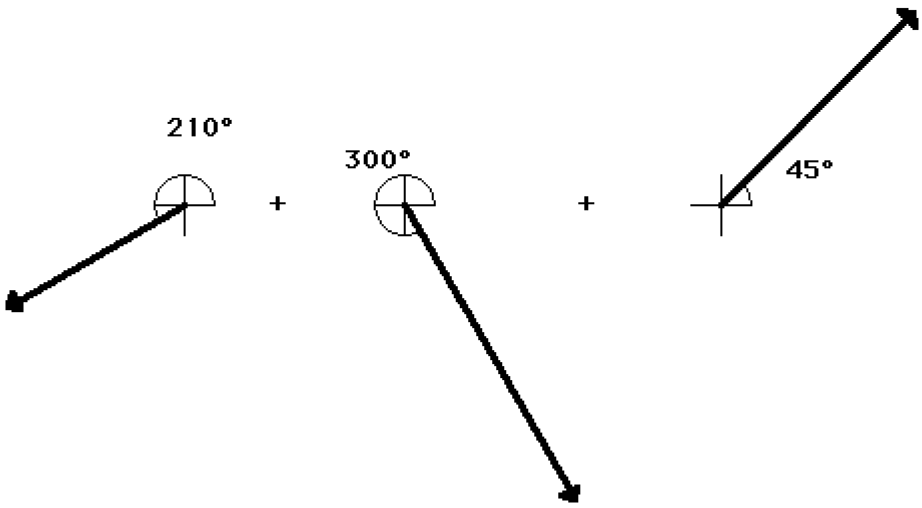
The head-to-tail method is employed as described above and the resultant is determined (drawn in red). Its magnitude and direction is labeled on the diagram.

SCALE: 1 cm = 5 m



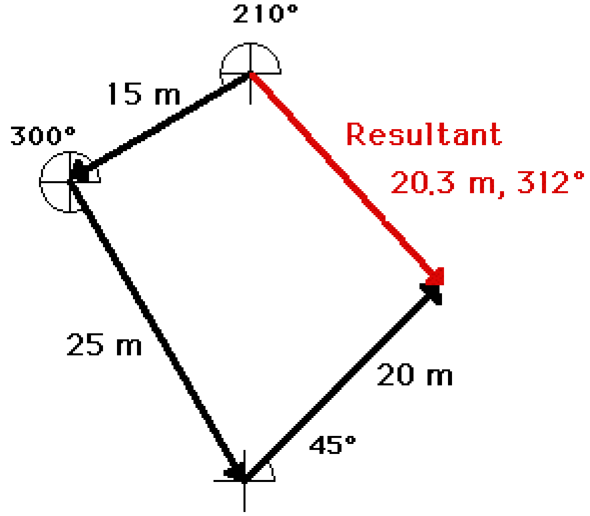
Interestingly enough, the order in which three vectors are added has no affect upon either the magnitude or the direction of the resultant. The resultant will still have the same magnitude and direction. For example, consider the addition of the same three vectors in a different order.

15 m, 210 deg. + 25 m, 300 deg. + 20 m, 45 deg.
SCALE: 1 cm = 5 m



When added together in this different order, these same three vectors still produce a resultant with the same magnitude and direction as before (20. m, 312 degrees). The order in which vectors are added using the head-to-tail method is insignificant.

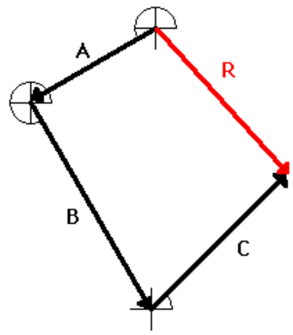
SCALE: 1 cm = 5 m



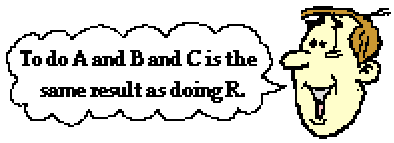
Additional examples of vector addition using the head-to-tail method are given on a separate web page.

Resultants

The **resultant** is the vector sum of two or more vectors. It is *the result* of adding two or more vectors together. If displacement vectors A, B, and C are added together, the result will be vector R. As shown in the diagram, vector R can be determined by the use of an accurately drawn, scaled, vector addition diagram.



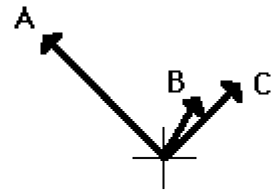
To say that vector R is the *resultant displacement* of displacement vectors A, B, and C is to say that a person who walked with displacements A, then B, and then C would be displaced by the same amount as a person who walked with displacement R. Displacement vector R gives the same *result* as displacement vectors A + B + C. That is why it can be said that

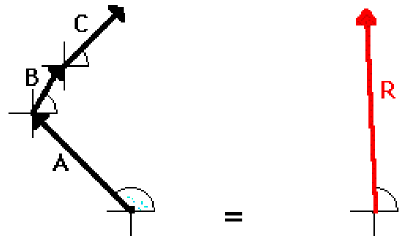


A + B + C = R

The above discussion pertains to the result of adding displacement vectors. When displacement vectors are added, the result is a *resultant displacement*. But any two vectors can be added as long as they are the same vector quantity. If two or more velocity vectors are added, then the result is a *resultant velocity*. If two or more force vectors are added, then the result is a *resultant force*. If two or more momentum vectors are added, then the result is ...

In all such cases, the resultant vector (whether a displacement vector, force vector, velocity vector, etc.) is the result of adding the individual vectors. It is the same thing as adding A + B + C + ... "To do A + B + C is the same as to do R." As an example, consider a football player who gets hit simultaneously by three players on the opposing team (players A, B, and C). The football player experiences three different applied forces. Each applied force contributes to a total or resulting force. If the three forces are added together using methods of vector addition (discussed earlier), then the resultant vector R can be determined. In this case, to experience the three forces A, B and C is the same as experiencing force R. To be hit by players A, B, and C would result in the same force as being hit by one player applying force R. "To do A + B + C is the same as to do R." Vector R is the same result as vectors A + B + C!!

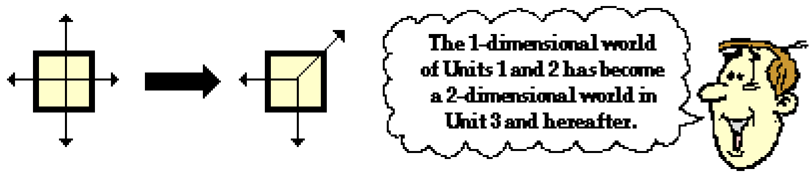




In summary, the resultant is the vector sum of all the individual vectors. The resultant is the result of combining the individual vectors together. The resultant can be determined by adding the individual forces together using vector addition methods.

Vector Components

A vector is a quantity that has both magnitude and direction. Displacement, velocity, acceleration, and force are the vector quantities that we have discussed thus far in the Physics Classroom Tutorial. In the first couple of units, all vectors that we discussed were simply directed up, down, left or right. When there was a free-body diagram depicting the forces acting upon an object, each individual force was directed in *one dimension* - either up or down or left or right. When an object had an acceleration and we described its direction, it was directed in *one dimension* - either up or down or left or right. Now in this unit, we begin to see examples of vectors that are directed in *two dimensions* - upward and rightward, northward and westward, eastward and southward, etc.

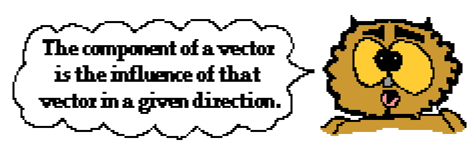


In situations in which vectors are directed at angles to the customary coordinate axes, a useful mathematical trick will be employed to *transform* the vector into two parts with each part being directed along the coordinate axes. For example, a vector that is directed northwest can be thought of as having two parts - a northward part and a westward part. A vector that is directed upward and rightward can be thought of as having two parts - an upward part and a rightward part.

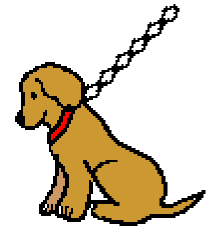
Northwest vectors have a northward and westward part.

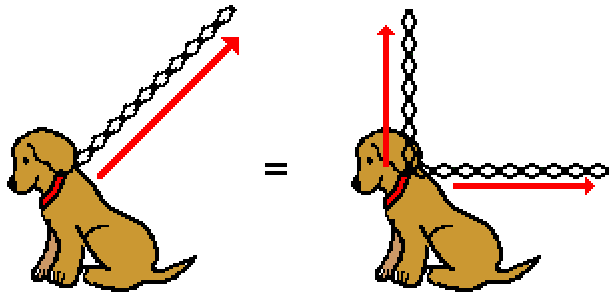
An upward and rightward vector has an upward and rightward part.

Any vector directed in two dimensions can be thought of as having an influence in two different directions. That is, it can be thought of as having two parts. Each part of a two-dimensional vector is known as a **component**. The components of a vector depict the influence of that vector in a given direction. The combined influence of the two components is equivalent to the influence of the single two-dimensional vector. The single two-dimensional vector could be replaced by the two components.

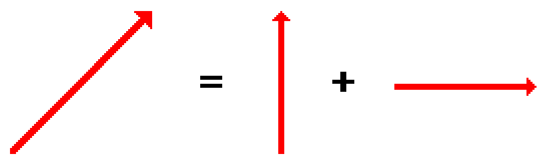


If Fido's dog chain is stretched upward and rightward and pulled tight by his master, then the tension force in the chain has two components - an upward component and a rightward component. To Fido, the influence of the chain on his body is equivalent to the influence of two chains on his body - one pulling upward and the other pulling rightward. If the single chain were replaced by two chains. with each chain having the magnitude and direction of the components, then Fido would not know the difference. This is not because Fido is *dumb* (a quick glance at his picture reveals that he is certainly not that), but rather because the combined influence of the two components is equivalent to the influence of the single two-dimensional vector.

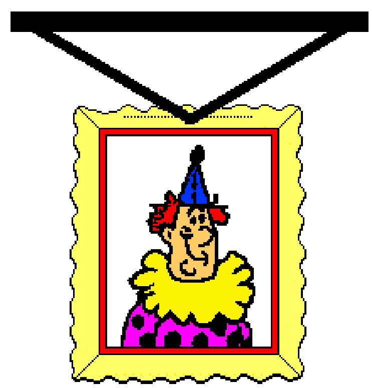


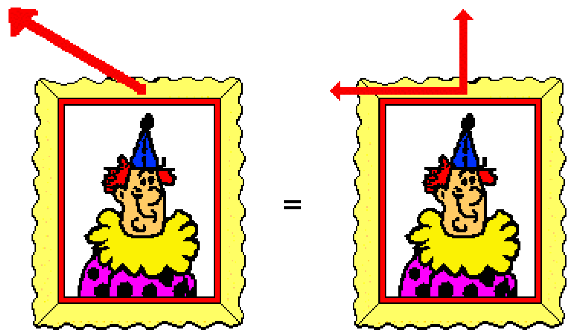


The upward and rightward force of the chain is equivalent to an upward force and a rightward force by two chains.

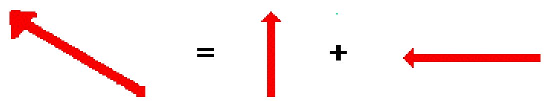


Consider a picture that is hung to a wall by means of two wires that are stretched vertically and horizontally. Each wire exerts a tension force upon the picture to support its weight. Since each wire is stretched in two dimensions (both vertically and horizontally), the tension force of each wire has two components - a vertical component and a horizontal component. Focusing on the wire on the left, we could say that the wire has a leftward and an upward component. This is to say that the wire on the left could be replaced by two wires, one pulling leftward and the other pulling upward. If the single wire were replaced by two wires (each one having the magnitude and direction of the components), then there would be no affect upon the stability of the picture. The combined influence of the two components is equivalent to the influence of the single two-dimensional vector.

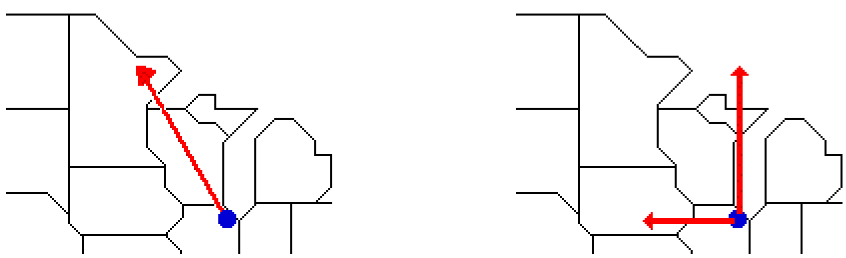




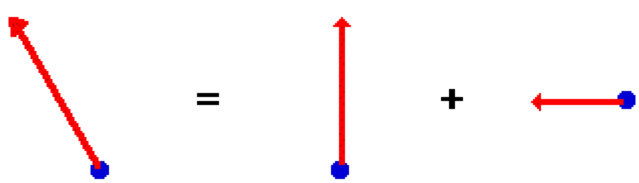
The wire's upward and rightward force is equivalent to a upward and rightward force exerted by two separate wires.



Consider an airplane that is flying from Chicago's O'Hare International Airport to a destination in Canada. Suppose that the plane is flying in such a manner that its resulting displacement vector is northwest. If this is the case, then the displacement of the plane has two components - a component in the northward direction and a component in the westward direction. This is to say that the plane would have the same displacement if it were to take the trip into Canada in two segments - one directed due North and the other directed due West. If the single displacement vector were replaced by these two individual displacement vectors, then the passengers in the plane would end up in the same final position. The combined influence of the two components is equivalent to the influence of the single two-dimensional displacement.



The plane's northwest displacement is equivalent to a northward plus a westward displacement.



Any vector directed in two dimensions can be thought of as having two different components. The component of a single vector describes the influence of that vector in a given direction. In the next part of this lesson, we will investigate two methods for determining the magnitude of the components. That is, we will investigate *how much* influence a vector exerts in a given direction.

Vector Resolution

As mentioned earlier in this lesson, any vector directed at an angle to the horizontal (or the vertical) can be thought of as having two parts (or components). That is, any vector directed in two dimensions can be thought of as having two components. For example, if a chain pulls upward at an angle on the collar of a dog, then there is a tension force directed in two dimensions. This tension force has two components: an upward component and a rightward component. As another example, consider an airplane that is displaced northwest from O'Hare International Airport (in Chicago) to a destination in Canada. The displacement vector of the plane is in two dimensions (northwest). Thus, this displacement vector has two components: a northward component and a westward component.

In this unit, we learn two basic methods for determining the magnitudes of the components of a vector directed in two dimensions. The process of determining the magnitude of a vector is known as **vector resolution**. The two methods of vector resolution that we will examine are

- the parallelogram method
- the trigonometric method

Parallelogram Method of Vector Resolution

The parallelogram method of vector resolution involves using an accurately drawn, scaled vector diagram to determine the components of the vector. Briefly put, the method involves drawing the vector to scale in the indicated direction, sketching a parallelogram around the vector such that the vector is the diagonal of the parallelogram, and determining the magnitude of the components (the sides of the parallelogram) using the scale. If one desires to determine the components as directed along the traditional x- and y-coordinate axes, then the parallelogram is a rectangle with sides that stretch vertically and horizontally. A step-by-step procedure for using the parallelogram method of vector resolution is:

a. Select a scale and accurately draw the vector to scale in the indicated direction.

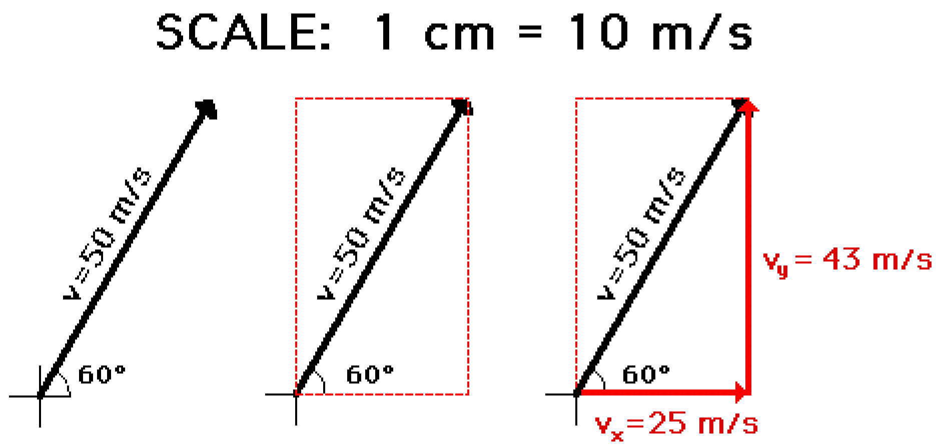
b. Sketch a parallelogram around the vector: beginning at the tail of the vector, sketch vertical and horizontal lines; then sketch horizontal and vertical lines at the head of the vector; the sketched lines will meet to form a rectangle (a special case of a parallelogram).

c. Draw the components of the vector. The components are the *sides* of the parallelogram. The tail of the components start at the tail of the vector and stretches along the axes to the nearest corner of the parallelogram. Be sure to place arrowheads on these components to indicate their direction (up, down, left, right).

d. Meaningfully label the components of the vectors with symbols to indicate which component represents which side. A northward force component might be labeled F_{north}. A rightward velocity component might be labeled v_x; etc.

e. Measure the length of the sides of the parallelogram and use the scale to determine the magnitude of the components in *real* units. Label the magnitude on the diagram.

The step-by-step procedure above is illustrated in the diagram below to show how a velocity vector with a magnitude of 50 m/s and a direction of 60 degrees above the horizontal may be resolved into two components. The diagram shows that the vector is first drawn to scale in the indicated direction; a parallelogram is sketched about the vector; the components are labeled on the diagram; and the result of measuring the length of the vector components and converting to m/s using the scale. (NOTE: because different computer monitors have different resolutions, the actual length of the vector on your monitor may not be 5 cm.)



Trigonometric Method of Vector Resolution

The trigonometric method of vector resolution involves using trigonometric functions to determine the components of the vector. Earlier in lesson 1, the use of trigonometric functions to determine the direction of a vector was described. Now in this part of lesson 1, trigonometric functions will be used to determine the components of a single vector. Recall from the earlier discussion that trigonometric functions relate the ratio of the lengths of the sides of a right triangle to the measure of an acute angle within the right triangle. As such, trigonometric functions can be used to determine the length of the sides of a right triangle if an angle measure and the length of one side are known.

The method of employing trigonometric functions to determine the components of a vector are as follows:

a. Construct a *rough* sketch (no scale needed) of the vector in the indicated direction. Label its magnitude and the angle that it makes with the horizontal.

b. Draw a rectangle about the vector such that the vector is the diagonal of the rectangle. Beginning at the tail of the vector, sketch vertical and horizontal lines. Then sketch horizontal and vertical lines at the head of the vector. The sketched lines will meet to form a rectangle.

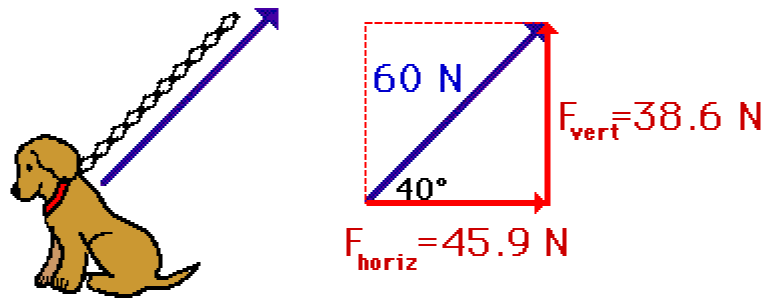
c. Draw the components of the vector. The components are the *sides* of the rectangle. The tail of each component begins at the tail of the vector and stretches along the axes to the nearest corner of the rectangle. Be sure to place arrowheads on these components to indicate their direction (up, down, left, right).

d. Meaningfully label the components of the vectors with symbols to indicate which component represents which side. A northward force component might be labeled F_{north}. A rightward force velocity component might be labeled v_x; etc.

e. To determine the length of the side opposite the indicated angle, use the sine function. Substitute the magnitude of the vector for the length of the hypotenuse. Use some algebra to solve the equation for the length of the side opposite the indicated angle.

f. Repeat the above step using the cosine function to determine the length of the side adjacent to the indicated angle.

The above method is illustrated below for determining the components of the force acting upon Fido. As the 60-Newton tension force acts upward and rightward on Fido at an angle of 40 degrees, the components of this force can be determined using trigonometric functions.



$$\sin 40° = \frac{F_{vert}}{60\ N} \qquad \cos 40° = \frac{F_{horiz}}{60\ N}$$

$$F_{vert} = 60\ N \times \sin 40° \qquad F_{horiz} = 60\ N \times \cos 40°$$

$$F_{vert} = 38.6\ N \qquad F_{horiz} = 45.9\ N$$

In conclusion, a vector directed in two dimensions has two components - that is, an influence in two separate directions. The amount of influence in a given direction can be determined using methods of vector resolution. Two methods of vector resolution have been described here - a graphical method (parallelogram method) and a trigonometric method.

More Practice

Use the **Components of a Vector** *widget* below to resolve a vector into its components. Simply enter the magnitude and direction of a vector. Then click the **Submit** button to view the horizontal and vertical components. Use the *widget* as a practice tool.

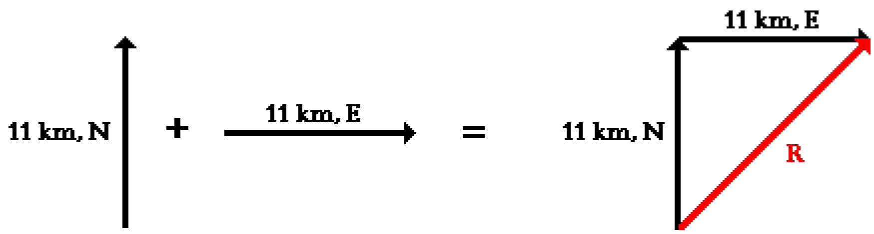
Components of a Vector

Magnitude	25	any unit
Direction (CCW)	30	degrees
	{21.6506, 12.5}	

See http://www.physicsclassroom.com/Class/vectors/u3l1d.cfm.

Component Method of Vector Addition

Earlier in this lesson, we learned that vectors oriented at right angles to one another can be added together using the Pythagorean theorem. For instance, two displacement vectors with magnitude and direction of 11 km, North and 11 km, East can be added together to produce a resultant vector that is directed both north and east. When the two vectors are added head-to-tail as shown below, the resultant is the hypotenuse of a right triangle. The sides of the right triangle have lengths of 11 km and 11 km. The resultant can be determined using the Pythagorean theorem; it has a magnitude of 15.6 km. The solution is shown below the diagram.



$$11^2 + 11^2 = R^2$$

$$242 = R^2$$

$$15.6 = R$$

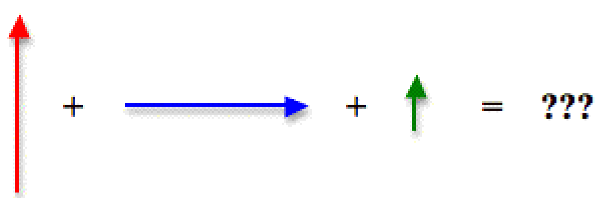
This *Pythagorean approach* is a useful approach for adding any two vectors that are directed at right angles to one another. A right triangle has two sides plus a hypotenuse; so the Pythagorean theorem is perfect for adding two right angle vectors. But there are limits to the usefulness of the Pythagorean theorem in solving vector addition problems. For instance, the addition of three or four vectors does not lead to the formation of a right triangle with two sides and a hypotenuse. So at first glance it may seem that it is impossible to use the Pythagorean theorem to determine the resultant for the addition of three or four vectors. Furthermore, the Pythagorean theorem works when the two added vectors are at right angles to one another - such as for adding a north vector and an east vector. But what can one do if the two vectors that are being added are not at right angles to one another? Is there a means of using mathematics to reliably determine the resultant for such vector addition situations? Or is the student of physics left to determining such resultants using a scaled vector diagram? Here on this page, we will learn how to approach more complex vector addition situations by combining the concept of vector components (discussed earlier) and the principles of vector resolution (discussed earlier) with the use of the Pythagorean theorem (discussed earlier).

Addition of Three or More Right Angle Vectors

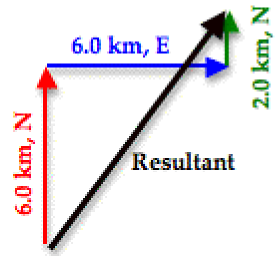
As our first example, consider the following vector addition problem:

Example 1:
A student drives his car 6.0 km, North before making a right hand turn and driving 6.0 km to the East. Finally, the student makes a left hand turn and travels another 2.0 km to the north. What is the magnitude of the overall displacement of the student?

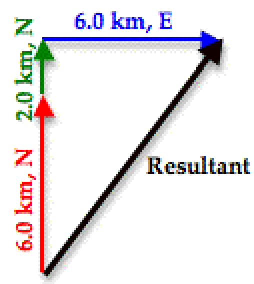
Like any problem in physics, a successful solution begins with the development of a mental picture of the situation. The construction of a diagram like that below often proves useful in the visualization process.



When these three vectors are added together in head-to-tail fashion, the resultant is a vector that extends from the tail of the first vector (6.0 km, North, shown in red) to the arrowhead of the third vector (2.0 km, North, shown in green). The head-to-tail vector addition diagram is shown below.



As can be seen in the diagram, the resultant vector (drawn in black) is not the hypotenuse of any right triangle - at least not of any immediately obvious right triangle. But would it be possible to force this resultant vector to be the hypotenuse of a right triangle? The answer is Yes! To do so, the order in which the three vectors are added must be changed. The vectors above were drawn in the order in which they were driven. The student drove north, then east, and then north again. But if the three vectors are added in the order 6.0 km, N + 2.0 km, N + 6.0 km, E, then the diagram will look like this:



After rearranging the order in which the three vectors are added, the resultant vector is now the hypotenuse of a right triangle. The lengths of the perpendicular sides of the right triangle are 8.0 m, North (6.0 km + 2.0 km) and 6.0 km, East. The magnitude of the resultant vector (R) can be determined using the Pythagorean theorem.

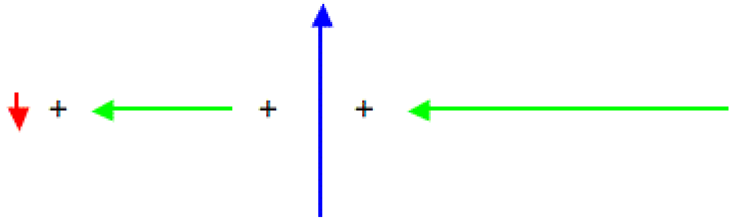
$$R^2 = (8.0 \text{ km})^2 + (6.0 \text{ km})^2$$
$$R^2 = 64.0 \text{ km}^2 + 36.0 \text{ km}^2$$
$$R^2 = 100.0 \text{ km}^2$$
$$R = \text{SQRT } (100.0 \text{ km2})$$
$$\mathbf{R = 10.0 \text{ km}}$$

(SQRT indicates square root)

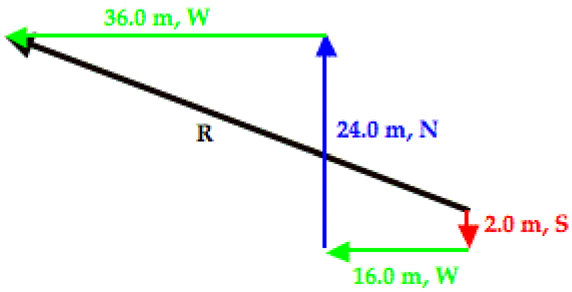
In the first vector addition diagram above, the three vectors were added in the order in which they are driven. In the second vector addition diagram (immediately above), the order in which the vectors were added was switched around. The size of the resultant was not affected by this change in order. This illustrates an important point about adding vectors: the resultant is independent by the order in which they are added. Adding vectors **A + B + C** gives the same resultant as adding vectors **B + A + C** or even **C + B + A**. As long as all three vectors are included with their specified magnitude and direction, the resultant will be the same. This property of vectors is the key to the strategy used in the determination of the answer to the above example problem. To further illustrate the strategy, let's consider the vector addition situation described in Example 2 below.

Example 2:
Mac and Tosh are doing the *Vector Walk Lab*. Starting at the door of their physics classroom, they walk 2.0 meters, south. They make a right hand turn and walk 16.0 meters, west. They turn right again and walk 24.0 meters, north. They then turn left and walk 36.0 meters, west. What is the magnitude of their overall displacement?

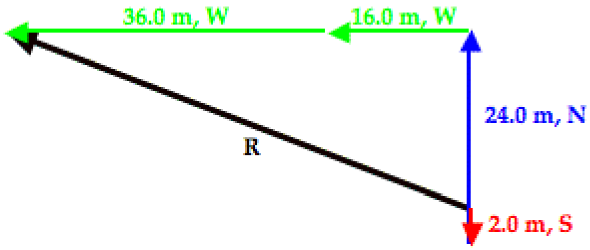
A graphical representation of the given problem will help visualize what is happening. The diagram below depicts such a representation.



When these four vectors are added together in head-to-tail fashion, the resultant is a vector that extends from the tail of the first vector (2.0 m, South, shown in red) to the arrowhead of the fourth vector (36.0 m, West, shown in green). The head-to-tail vector addition diagram is shown below.



The resultant vector (drawn in black and labeled **R**) in the vector addition diagram above is not the hypotenuse of any immediately obvious right trangle. But by changing the order of addition of these four vectors, one can force this resultant vector to be the hypotenuse of a right triangle. For instance, by adding the vectors in the order of 2.0 m, S + 24.0 m, N + 16.0 m, W + 36.0 m. W, the resultant becomes the hypotenuse of a right triangle. This is shown in the vector addition diagram below.



With the vectors rearranged, the resultant is now the hypotenuse of a right triangle that has two perpendicular sides with lengths of 22.0 m, North and 52.0 m, West. The 22.0 m, North side is the result of 2.0 m, South and 24.0 m, North added together. The 52.0 m, West side is the result of 16.0 m, West and 36.0 m, West added together. The magnitude of the resultant vector (R) can be determined using the Pythagorean theorem.

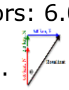
$$R^2 = (22.0\text{ m})^2 + (52.0\text{ m})^2$$
$$R^2 = 484.0\text{ m}^2 + 2704.0\text{ m}^2$$
$$R^2 = 3188.0\text{ m}^2$$
$$R = \text{SQRT}(3188.0\text{ m2}^2)$$
$$\mathbf{R = 56.5\text{ m}}$$

(SQRT indicates square root)

As can be seen in these two examples, the resultant of the addition of three or more right angle vectors can be easily determined using the Pythagorean theorem. Doing so involves the adding of the vectors in a different order.

SOH CAH TOA and the Direction of Vectors

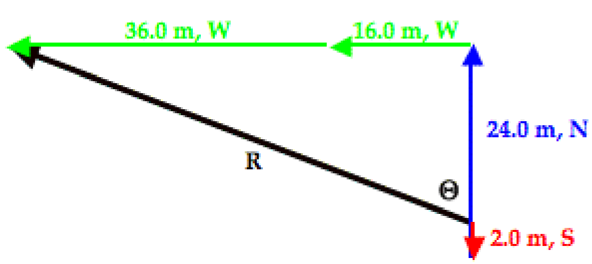
The above discussion explains the method for determining the magnitude of the resultant for three or more perpendicular vectors. The topic of direction has been ignored in the discussion. Now we will turn our attention to the method for determining the direction of the resultant vector. As a quick review, recall that earlier in this lesson a convention for expressing the direction of a vector was introduced. The convention is known as the counter-clockwise from east convention, often abbreviated as the **CCW** convention. Using this convention, the direction of a vector is often expressed as a counter-clockwise angle of rotation of the vector about its *tail* from due East.

To begin our discussion, let's return to **Example 1** above where we made an effort to add three vectors: 6.0 km, N + 6.0 km, E + 2.0 km, N. In the solution, the order of addition of the three vectors was rearranged so that a right triangle was formed with the resultant being the hypotenuse of the triangle.  The triangle is redrawn at the right. Observe that the angle in the lower left of the triangle has been labeled as theta (Θ). Theta (Θ) represents the angle that the vector makes with the north axis. Theta (Θ) can be calculated using one of the three trigonometric functions introduced earlier in this lesson - sine, cosine or tangent. The mnemonic **SOH CAH TOA** is a helpful way of remembering which function to use. In this problem, we wish to determine the angle measure of theta (Θ) and we know the length of the side opposite theta (Θ) - 6.0 km - and the length of the side adjacent the angle theta (Θ) - 8.0 km. The TOA of SOH CAH TOA indicates that the tangent of any angle is the ratio of the lengths of the side opposite to the side adjacent that angle. Thus, the tangent function will be used to calculate the angle measure of theta (Θ). The work is shown below.

$$\text{Tangent}(\Theta) = \text{Opposite/Adjacent}$$
$$\text{Tangent}(\Theta) = 6.0/8.0$$
$$\text{Tangent}(\Theta) = 0.75$$
$$\Theta = \tan^{-1}(0.75)$$
$$\Theta = 36.869\ldots°$$
$$\mathbf{\Theta = 37°}$$

The problem is not over once the value of theta (Θ) has been calculated. This angle measure must now be used to state the direction. One means of doing so is to simply state that the direction of the resultant is 37° east of north. Alternatively, the counter-clockwise convention could be used. Since the angle that the resultant makes with east is the complement of the angle that it makes with north, we could express the direction as 53° CCW.

We will now consider **Example 2** as a second example of how to use SOH CAH TOA to determine the direction of a resultant. In Example 2, we were trying to determine the magnitude of 2.0 m. S + 16.0 m, W + 24.0 m, N + 36.0 m, W. The solution involved re-ordering the addition so that the resultant was the hypotenuse of a right triangle with known sides. The right triangle is shown below. The resultant is drawn in black. Once more, observe that the angle in the lower right of the triangle has been labeled as theta (Θ). Theta (Θ) represents the angle that the vector makes with the north axis.



Theta (Θ) can be calculated using the tangent function. In this problem, we wish to determine the angle measure of theta (Θ) and we know the length of the side opposite theta (Θ) - 52.0 m - and the length of the side adjacent the angle theta (Θ) - 22.0 m. The TOA of SOH CAH TOA indicates that the tangent of any angle is the ratio of the lengths of the side opposite to the side adjacent that angle. Thus, the tangent function will be used to calculate the angle measure of theta (Θ). The work is shown below.

$$\text{Tangent}(\Theta) = \text{Opposite}/\text{Adjacent}$$
$$\text{Tangent}(\Theta) = 52.0/22.0$$
$$\text{Tangent}(\Theta) = 2.3636\ \ldots$$
$$\Theta = \tan^{-1}(2.3636\ \ldots)$$
$$\Theta = 67.067\ \ldots°$$
$$\mathbf{\Theta = 67.1°}$$

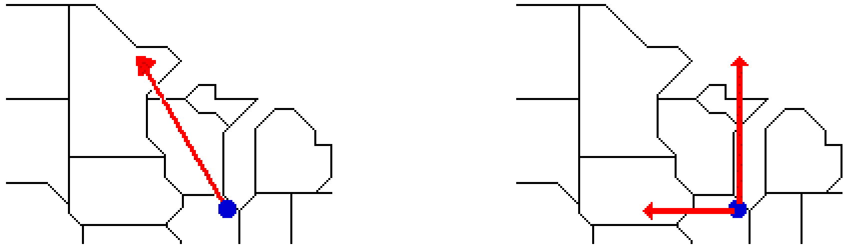
The problem is not over once the value of theta (Θ) has been calculated. This angle measure must now be used to state the direction. One means of doing so is to simply state that the direction of the resultant is 67.1° west of north. Alternatively, the counter-clockwise convention could be used. The north axis is rotated 90° counter-clockwise from east and this vector is an additional 67.1° counter-clockwise past north. Thus the CCW direction is 157.1° CCW.

In summary, the direction of a vector can be determined in the same way that it is always determined - by finding the angle of rotation counter-clockwise from due east. Since the resultant is the hypotenuse of a right triangle, this can be accomplished by first finding an angle that the resultant makes with one of the nearby axes of the triangle. Once done, a little thinking is required in order to relate the angle to a direction.

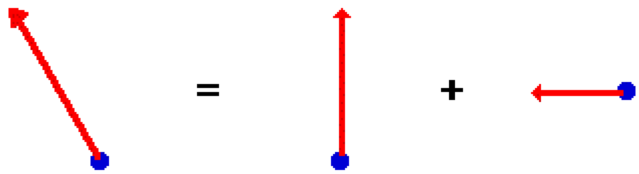
Addition of Non-Perpendicular Vectors

Now we will consider situations in which the two (or more) vectors that are being added are not at right angles to each other. The Pythagorean theorem is not applicable to such situations since it applies only to right triangles. Two non-perpendicular vectors will not form a right triangle. Yet it is possible to force two (or more) non-perpendicular vectors to be transformed into other vectors that do form a right triangle. The trick involves the concept of a vector component and the process of vector resolution.

A vector component describes the effect of a vector in a given direction. Any *angled* vector has two components; one is directed horizontally and the other is directed vertically. For instance, a northwest vector has a northward component and a westward component. Together, the effect these two components are equal to the overall effect of the *angled* vector. As an example, consider a plane that flies northwest from Chicago O'Hare airport towards the Canada border. The northwest displacement vector of the plane has two components - a northward component and a westward component. When added together, these two components are equal to the overall northwest displacement. This is shown in the diagram below.



The plane's northwest displacement is equivalent to
a northward plus a westward displacement.

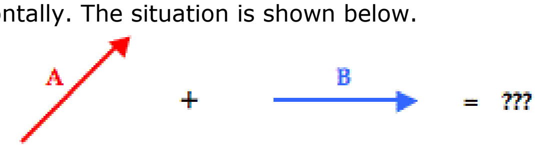


The northwest vector has north and west components that are represented as **A_x** and **A_y**. It can be said that

$A = A_x + A_y$

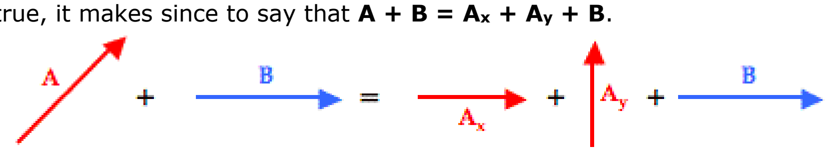
So whenever we think of a northwest vector, we can think instead of two vectors - a north and a west vector. The two components **$A_x + A_y$** can be substituted in for the single vector **A** in the problem.

Now suppose that your task involves adding two non-perpendicular vectors together. We will call the vectors **A** and **B**. Vector **A** is a *nasty angled vector* that is neither horizontal nor vertical. And vector **B** is a nice, polite vector directed horizontally. The situation is shown below.

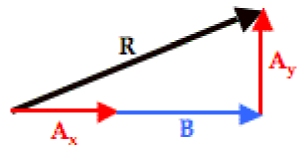


Of course nasty vector **A** has two components - **A_x** and **A_y**. These two components together are equal to vector **A**. That is, **$A = A_x + A_y$** .

And since this is true, it makes since to say that **$A + B = A_x + A_y + B$**.



And so the problem of A + B has been transformed into a problem in which all vectors are at right angles to each other. *Nasty* has been replaced by *nice* and that should make any physics student happy. With all vectors being at right angles to one another, their addition leads to a resultant that is at the hypotenuse of a right triangle. The Pythagorean theorem can then be used to determine the magnitude of the resultant.

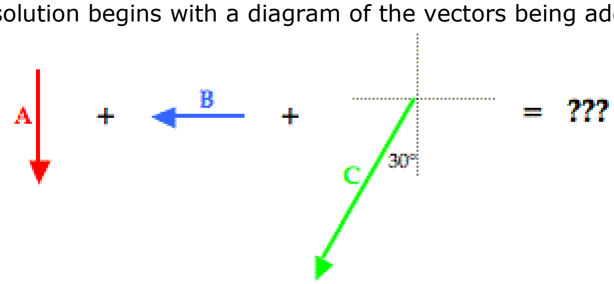


$$R^2 = (A_x + B)^2 + A_y^2$$

To see how this process works with an actual vector addition problem, consider Example 3 shown below.

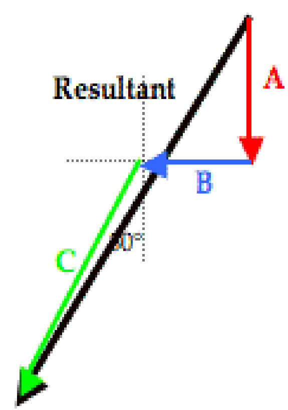
Example 3:
Max plays middle linebacker for South's football team. During one play in last Friday night's game against New Greer Academy, he made the following movements after the ball was snapped on third down. First, he back-pedaled in the southern direction for 2.6 meters. He then shuffled to his left (west) for a distance of 2.2 meters. Finally, he made a half-turn and ran downfield a distance of 4.8 meters in a direction of 240° counter-clockwise from east (30° W of S) before finally knocking the wind out of New Greer's wide receiver. Determine the magnitude and direction of Max's overall displacement.

As is the usual case, the solution begins with a diagram of the vectors being added.

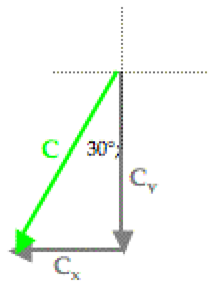


To assist in the discussion, the three vectors have been labeled as vectors **A**, **B**, and **C**. The resultant is the vector sum of these three vectors; a head-to-tail vector addition diagram reveals that the resultant is directed southwest. Of the three vectors being added, vector C is clearly the *nasty* vector. Its direction is neither due south nor due west. The solution involves resolving this vector into its components.

The process of vector resolution was discussed earlier in this lesson. The process involves using the magnitude and the sine and cosine functions to determine the x- and y-components of the vector. Vector C makes a 30° angle with the southern direction. By sketching a right triangle with horizontal and vertical legs and C as the hypotenuse, it becomes possible to determine the components of vector C. This is shown in the diagram below.



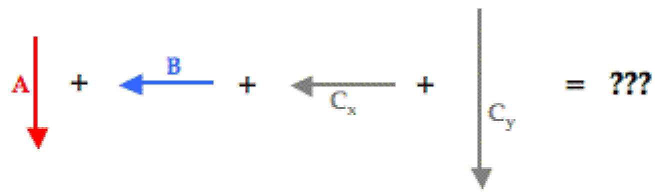
The side adjacent this 30° angle in the triangle is the vertical side; the vertical side represents the vertical (southward) component of C - C_y. So to determine C_y, the cosine function is used. The side opposite the 30° angle is the horizontal side; the horizontal side represents the horizontal (westward) component of C - C_x. The values of C_x and C_y can be determined by using SOH CAH TOA. The cosine function is used to determine the southward component since the southward component is adjacent to the 30° angle. The sine function is used to determine the westward component since the westward component is the side opposite to the 30° angle. The work is shown below.



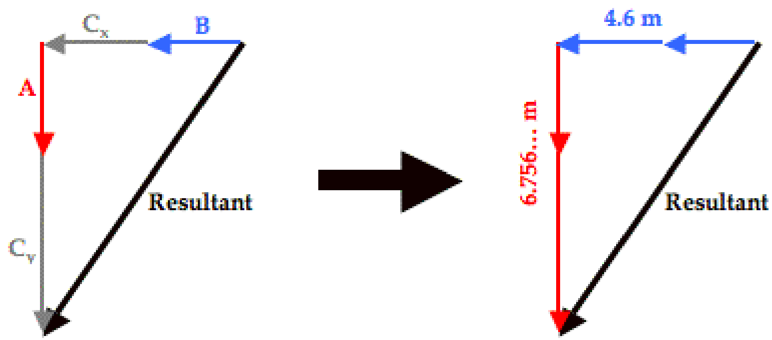
$C_x = C \bullet \text{sine}(30°)$
$C_x = 4.8 \bullet \text{sine}(30°) = 2.400 \ldots \text{ m}$

$C_y = C \bullet \text{cosine}(30°)$
$C_y = 4.8 \bullet \text{cosine}(30°) = 4.156 \ldots \text{ m}$

Now our vector addition problem has been transformed from the addition of two nice vectors and one nasty vector into the addition of four nice vectors.

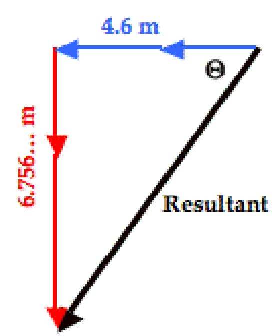


With all vectors oriented along are customary north-south and east-west axes, they can be added head-to-tail in any order to produce a right triangle whose the hypotenuse is the resultant. Such a diagram is shown below.



The triangle's perpendicular sides have lengths of 4.6 meters and 6.756 meters. The length of the horizontal side (4.6 m) was determined by adding the values of B (2.2 m) and C_x (2.4 m). The length of the vertical side (6.756… m) was determined by adding the values of A (2.6 m) and C_y (4.156… m). The resultant's magnitude (R) can now be determined using the Pythagorean theorem.

$R^2 = (6.756\ldots \text{ m})^2 + (4.6 \text{ m})^2$
$R^2 = 45.655\ldots \text{ m}^2 + 21.16 \text{ m}^2$
$R^2 = 66.815\ldots \text{ m}^2$
$R = \text{SQRT}(66.815\ldots \text{ m}^2)$
$R = 8.174 \ldots \text{ m}$
R = ~8.2 m



The direction of the resultant can be determined by finding the angle that the resultant makes with either the north-south or the east-west vector. The diagram at the right shows the angle theta (Θ) marked inside the vector addition triangle. This angle theta is the angle that the resultant makes with west. Its value can be determined using the tangent function. The tangent function (as in TOA) relates the angle value to the ratio of the lengths of the opposite side to the adjacent side. That is, tangent(Θ) = (6.756… m)/(4.6 m) = 1.46889…

Using the inverse tangent function, the angle theta (Θ) can be determined. On most calculators, this involves using the 2nd-Tangent buttons.

$$\Theta = \tan^{-1}(1.46889...) = 55.7536...\ °$$

$$\mathbf{\Theta = \sim 56°}$$

This 56° angle is the angle between the resultant vector (drawn in black above) and the westward direction. This makes the direction 56° south of west. The direction of the resultant based on the counter-clockwise from east convention (CCW) can be determined by adding 180° to the 56°. So the CCW direction is 236°.

Example 4 provides one final example of how to combine vector resolution with vector addition in order to add three or more non-perpendicular vectors. Because this example includes three particularly *nasty vectors*, a table will be used to organize the information about he magnitude and direction of the components. The use of a table is a wise idea when problems get complicated.

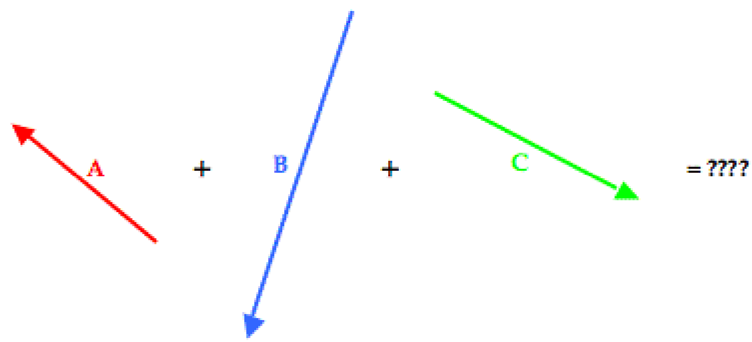
Example 4:

Cameron Per (his friends call him Cam) and Baxter Nature are on a hike. Starting from home base, they make the following movements.

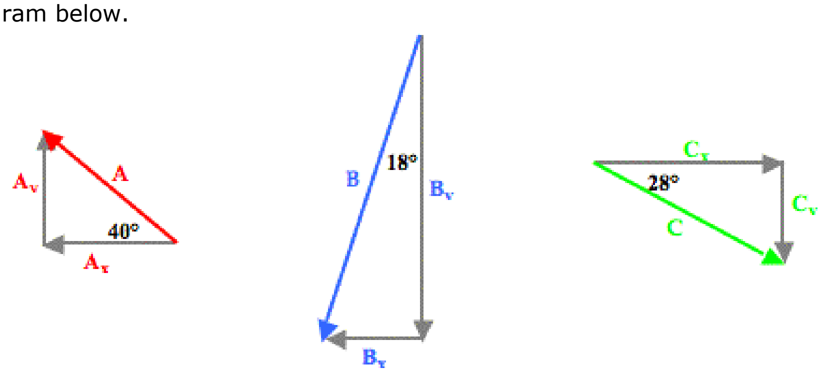
A: 2.65 km, 140° CCW
B: 4.77 km, 252° CCW
C: 3.18 km, 332° CCW

Determine the magnitude and direction of their overall displacement.

The visual representation of the situation is shown below.



To determine the resultant, the three individual vectors are resolved into horizontal and vertical components. The angle information about each vector is used to form a right triangle in which the vector is the hypotenuse and the perpendicular sides are oriented along the east-west and north-south axes. This is shown in the diagram below.



Trigonometric functions - sine, cosine and tangent - are then used to determine the magnitude of the horizontal and vertical component of each vector. The work is shown and organized in the table below.

Vector	East-West Component	North-South Component
A **2.65 km** **140° CCW**	(2.65 km)•cos(40°) = 2.030… km, West	(2.65 km)•sin(40°) = 1.703… km, North
B **4.77 km** **252° CCW**	(4.77 km)•sin(18°) = 1.474… km, West	(4.77 km)•cos(18°) = 4.536… km, South
C **3.18 km** **332° CCW**	(3.18 km)•cos(28°) = 2.808… km, East	(3.18 km)•sin(28°) = 1.493… km, South
Sum of **A + B + C**	0.696 km, West	4.326 km, South

The last row of the above table represents the sum of all the East-West components and the sum of all the North-South components. The resultant consists of these two components. The resultant is determined by adding together these two the components to form a right triangle that has a hypotenuse that is equal to the resultant. This typically involves adding all the horizontal components to determine the total length of the horizontal side of the right triangle … and adding all the vertical components to determine the total length of the vertical side of the right triangle. This is done in the table above by simple adding another row to the table for the vector sum of all the components. In adding the east-west components of all the individual vectors, one must consider that an eastward component and a westward component would add together as a *positive* and a *negative*. Some students prefer to think of this as subtraction as opposed to addition. In actuality, it really is addition - the addition of vectors with opposite direction. Similarly, a northward and a southward component would also add together as a *positive* and a *negative*. Once the bottom row is accurately determined, the magnitude of the resultant can be determined using Pythagorean theorem.

$$R^2 = (0.696\text{ km})^2 + (4.326\text{ km})^2$$
$$R^2 = 0.484\text{ km}^2 + 18.714\text{ km}^2$$
$$R^2 = 19.199\text{ km}^2$$
$$R = \text{SQRT}(19.199\text{ km}^2)$$
$$\mathbf{R = \sim 4.38\text{ km}}$$

The direction of the resulting displacement can be determined by constructing the final triangle from the components of the resultant. The components of the resultant are simply the sum the east-west and north-south components. Once done, SOH CAH TOA is used to determine the angle that the resultant makes with a nearby axis. The diagram is shown at the right. The angle labeled as theta (Θ) is the angle between the resultant vector and the west axis. This angle can be calculated as follows:

Tangent(Θ) = opposite/adjacent
Tangent(Θ) = (4.326 km)/(0.696 km)
Tangent(Θ) = 6.216
Θ = $\tan^{-1}(6.216)$
Θ = 80.9°

This angle measure represents the angle of rotation of the vector south of due west. It would be worded as 80.9° south of west. Since west is 180° counterclockwise from east, the direction could also be expressed in the counterclockwise (CCW) from east convention as 260.9°.

So the result of our analysis is that the overall displacement is 4.38 km with a direction of 260.9° (CCW).

The questions that have been addressed on this page are:

a. How can three or more perpendicular vectors be added together to determine the resultant?

b. How can two or more non-perpendicular vectors be added together to determine the resultant?

For both questions, we have found that any two or three or more vectors can be transformed or rearranged so that they add together to form a right triangle with the hypotenuse being the resultant. Once the right triangle is formed, Pythagorean theorem and SOH CAH TOA can be used to calculate the resultant.

Experiment with the widget below and then try the problems in the **Check Your Understanding** section to test your skill at adding vectors using components.

Practice!

The widget below computes the sum of three vectors if the x- and y-components are known. Use the widget to practice and check a problem.

Use Components to Add Three Vectors

Enter the magnitude of the components of the three vectors.

Use - signs for west and for south. When done, click Add 'Em Up.

Vector 1

x-component:	3
y-component:	2

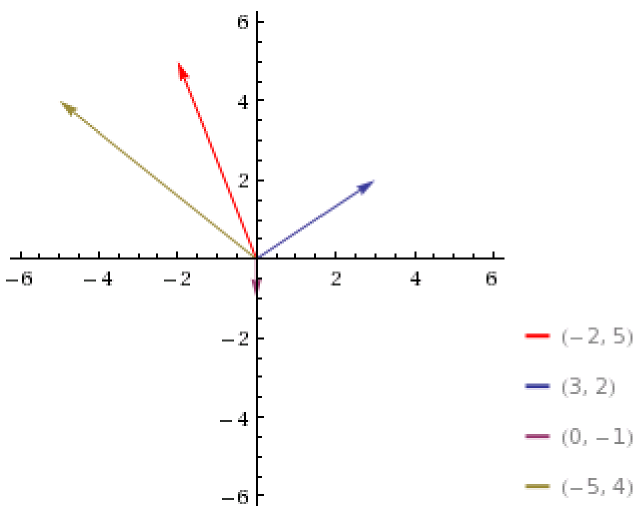
Vector 2

x-component	0
y-component:	-1

Vector 3

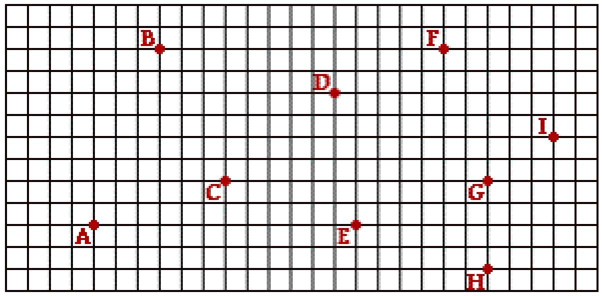
x-component:	-5
y-component:	4

Add 'Em Up



Check Your Understanding

Consider the diagram below. Nine unique, labeled locations are provided on a grid. Each square on the grid represents a 20-meter x 20-meter area. Rightward on the grid is in the eastward direction and upward on the grid in the northward direction. Use the grid in answering the next few questions.



1. Suppose that a person starts at position A and walks to position E and then to position G. Fill in the table below to indicate the east-west and the north-south components of the individual legs of the walk and the components of the resulting displacement. Make the measurements off the grid. Finally, use the Pythagorean theorem and SOH CAH TOA to determine the magnitude and the direction of the resulting displacement.

Vector	East-West Component	North-South Component
A to E		
E to G		
Resultant A to G		

Magnitude of Resultant: ___________________________

Direction of Resultant: ___________________________

2. Using the same grid, repeat the measurements for a walk from location C to location B to location F. Make the measurements off the grid and use the Pythagorean theorem and SOH CAH TOA to determine the magnitude and the direction of the resulting displacement.

Vector	East-West Component	North-South Component
C to B		
B to F		
Resultant C to F		

Magnitude of Resultant: ___________________________

Direction of Resultant: ___________________________

3. Finally, use the same grid to repeat the measurements for a walk from location I to location B to location G to location H. Make the measurements off the grid and use the Pythagorean theorem and SOH CAH TOA to determine the magnitude and the direction of the resulting displacement.

Vector	**East-West Component**	**North-South Component**
I to B		
B to G		
G to H		
Resultant I to H		

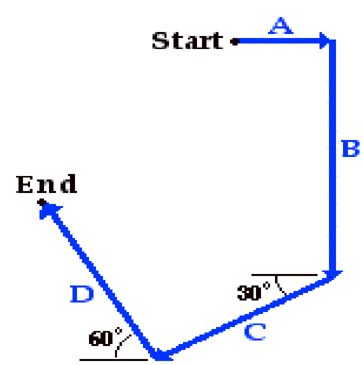
Magnitude of Resultant: ___________________________

Direction of Resultant: ___________________________

4. During her recent trip to the grocery store, Claire de Iles walked 28 m to the end of an aisle. She then made a right hand turn and walked 12 m down the end aisle. Finally, she made another right hand turn and walked 12 m in the opposite direction as her original direction. Determine the magnitude of Claire's resultant displacement. (The actual direction - east, west, north, south are not the focus.)

5. In the final game of last year's regular season, South was playing New Greer Academy for the Conference Championship. In the last play of the game, star quarterback Avery took a snap from scrimmage and scooted backwards (northwards) 8.0 yards. He then ran sideways (westward) *out of the pocket* for 12.0 yards before finally throwing a 34.0 yard pass directly downfield (southward) to Kendall for the game-winning touchdown. Determine the magnitude and direction of the ball's displacement.

6. Mia Ander exits the front door of her home and walks along the path shown in the diagram at the right (not to scale). The walk consists of four legs with the following magnitudes:



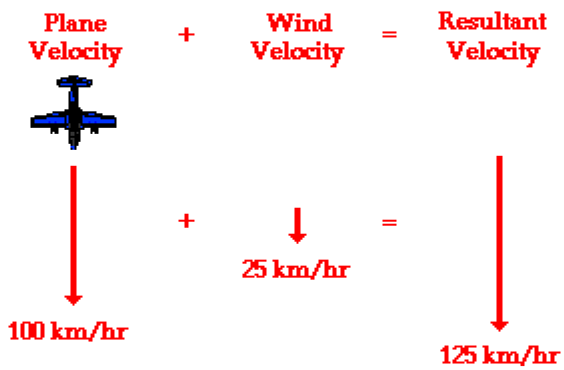
A = 46 m
B = 142 m
C = 78 m
D = 89 m

Determine the magnitude and direction of Mia's resultant displacement. Consider using a table to organize your calculations.

Relative Velocity and Riverboat Problems

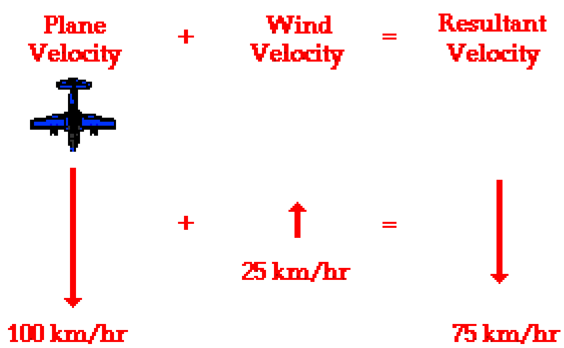
On occasion objects move within a medium that is moving with respect to an observer. For example, an airplane usually encounters a wind - air that is moving with respect to an observer on the ground below. As another example, a motorboat in a river is moving amidst a river current - water that is moving with respect to an observer on dry land. In such instances as this, the magnitude of the velocity of the moving object (whether it be a plane or a motorboat) with respect to the observer on land will not be the same as the speedometer reading of the vehicle. That is to say, the speedometer on the motorboat might read 20 mi/hr; yet the motorboat might be moving relative to the observer on shore at a speed of 25 mi/hr. Motion is relative to the observer. The observer on land, often named (or misnamed) the "stationary observer" would measure the speed to be different than that of the person on the boat. The observed speed of the boat must always be described relative to who the observer is.

To illustrate this principle, consider a plane flying amidst a **tailwind**. A tailwind is merely a wind that approaches the plane from behind, thus increasing its resulting velocity. If the plane is traveling at a velocity of 100 km/hr with respect to the air, and if the wind velocity is 25 km/hr, then what is the velocity of the plane relative to an observer on the ground below? The resultant velocity of the plane (that is, the result of the wind velocity contributing to the velocity due to the plane's motor) is the vector sum of the velocity of the plane and the velocity of the wind. This resultant velocity is quite easily determined if the wind approaches the plane directly from behind. As shown in the diagram below, the plane travels with a resulting velocity of 125 km/hr relative to the ground.

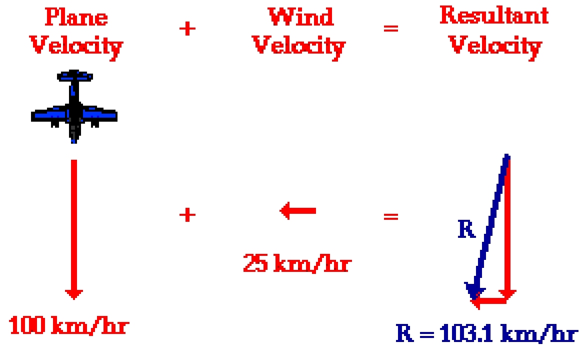


The plane travels with a velocity relative to the ground which is the vector sum of the plane's velocity (relative to the air) plus the wind velocity.

If the plane encounters a headwind, the resulting velocity will be less than 100 km/hr. Since a headwind is a wind that approaches the plane from the front, such a wind would decrease the plane's resulting velocity. Suppose a plane traveling with a velocity of 100 km/hr with respect to the air meets a headwind with a velocity of 25 km/hr. In this case, the resultant velocity would be 75 km/hr; this is the velocity of the plane relative to an observer on the ground. This is depicted in the diagram below.



Now consider a plane traveling with a velocity of 100 km/hr, South that encounters a **side wind** of 25 km/hr, West. Now what would the resulting velocity of the plane be? This question can be answered in the same manner as the previous questions. The resulting velocity of the plane is the vector sum of the two individual velocities. To determine the resultant velocity, the plane velocity (relative to the air) must be added to the wind velocity. This is the same procedure that was used above for the headwind and the tailwind situations; only now, the resultant is not as easily computed. Since the two vectors to be added - the southward plane velocity and the westward wind velocity - are at right angles to each other, the Pythagorean theorem can be used. This is illustrated in the diagram below.



In this situation of a side wind, the southward vector can be added to the westward vector using the usual methods of vector addition. The magnitude of the resultant velocity is determined using Pythagorean theorem. The algebraic steps are as follows:

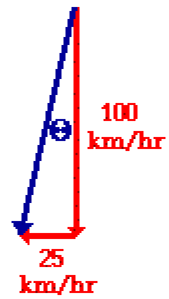
$$(100 \text{ km/hr})^2 + (25 \text{ km/hr})^2 = R^2$$
$$10\,000 \text{ km}^2/\text{hr}^2 + 625 \text{ km}^2/\text{hr}^2 = R^2$$

$$10\,625 \text{ km}^2/\text{hr}^2 = R^2$$

$$\text{SQRT}(10\,625 \text{ km}^2/\text{hr}^2) = R$$

$$\mathbf{103.1 \text{ km/hr} = R}$$

The direction of the resulting velocity can be determined using a trigonometric function. Since the plane velocity and the wind velocity form a right triangle when added together in head-to-tail fashion, the angle between the resultant vector and the southward vector can be determined using the sine, cosine, or tangent functions. The tangent function can be used; this is shown below:



$$\tan(\text{theta}) = (\text{opposite}/\text{adjacent})$$
$$\tan(\text{theta}) = (25/100)$$

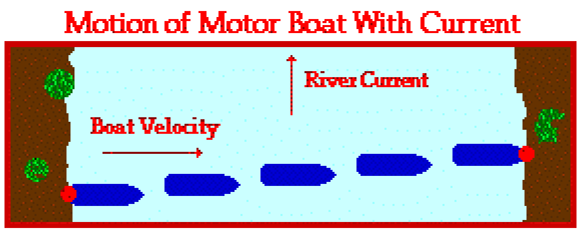
$$\text{theta} = \text{invtan}(25/100)$$

$$\textbf{theta = 14.0 degrees}$$

If the resultant velocity of the plane makes a 14.0 degree angle with the southward direction (theta in the above diagram), then the direction of the resultant is 256 degrees. Like any vector, the resultant's direction is measured as a counterclockwise angle of rotation from due East.

Analysis of a Riverboat's Motion

The affect of the wind upon the plane is similar to the affect of the river current upon the motorboat. If a motorboat were to head straight across a river (that is, if the boat were to point its bow straight towards the other side), it would not reach the shore directly across from its starting point. The river current influences the motion of the boat and carries it downstream. The motorboat may be moving with a velocity of 4 m/s directly across the river, yet the resultant velocity of the boat will be greater than 4 m/s and at an angle in the downstream direction. While the speedometer of the boat may read 4 m/s, its speed with respect to an observer on the shore will be greater than 4 m/s.



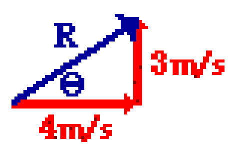
The resultant velocity of the motorboat can be determined in the same manner as was done for the plane. The resultant velocity of the boat is the vector sum of the boat velocity and the river velocity. Since the boat heads straight across the river and since the current is always directed straight downstream, the two vectors are at right angles to each other. Thus, the Pythagorean theorem can be used to determine the resultant velocity. Suppose that the river was moving with a velocity of 3 m/s, North and the motorboat was moving with a velocity of 4 m/s, East. What would be the resultant velocity of the motorboat (i.e., the velocity relative to an observer on the shore)? The magnitude of the resultant can be found as follows:

$$(4.0\ \text{m/s})^2 + (3.0\ \text{m/s})^2 = R^2$$
$$16\ \text{m}^2/\text{s}^2 + 9\ \text{m}^2/\text{s}^2 = R^2$$

$$25\ \text{m}^2/\text{s}^2 = R^2$$

$$\text{SQRT}\ (25\ \text{m}^2/\text{s}^2) = R$$

$$\mathbf{5.0\ m/s = R}$$



The direction of the resultant is the counterclockwise angle of rotation that the resultant vector makes with due East. This angle can be determined using a trigonometric function as shown below.

$$\tan(\text{theta}) = (\text{opposite}/\text{adjacent})$$
$$\tan(\text{theta}) = (3/4)$$

$$\text{theta} = \text{invtan}\ (3/4)$$

$$\mathbf{theta = 36.9\ degrees}$$

Given a boat velocity of 4 m/s, East and a river velocity of 3 m/s, North, the resultant velocity of the boat will be 5 m/s at 36.9 degrees.

Motorboat problems such as these are typically accompanied by three separate questions:

a. What is the resultant velocity (both magnitude and direction) of the boat?
b. If the width of the river is *X* meters wide, then how much time does it take the boat to travel shore to shore?
c. What distance downstream does the boat reach the opposite shore?

The first of these three questions was answered above; the resultant velocity of the boat can be determined using the Pythagorean theorem (magnitude) and a trigonometric function (direction). The second and third of these questions can be answered using the average speed equation (and a lot of logic).

ave. speed = distance/time

Consider the following example.

Example 1:

A motorboat traveling 4 m/s, East encounters a current traveling 3.0 m/s, North.

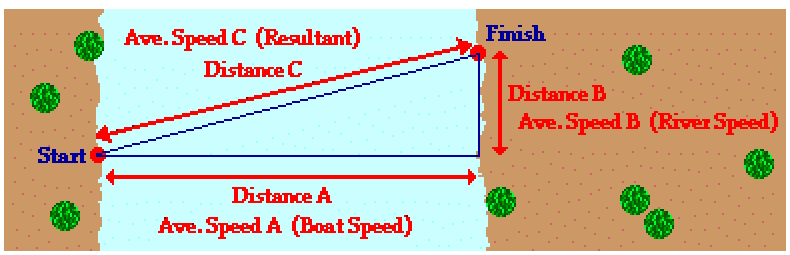
a. What is the resultant velocity of the motorboat?
b. If the width of the river is 80 meters wide, then how much time does it take the boat to travel shore to shore?
c. What distance downstream does the boat reach the opposite shore?

The solution to the first question has already been shown in the above discussion. The resultant velocity of the boat is 5 m/s at 36.9 degrees. We will start in on the second question.

The river is 80-meters wide. That is, the distance from shore to shore as measured straight across the river is 80 meters. The time to cross this 80-meter wide river can be determined by rearranging and substituting into the average speed equation.

time = distance /(ave. speed)

The distance of 80 m can be substituted into the numerator. But what about the denominator? What value should be used for average speed? Should 3 m/s (the current velocity), 4 m/s (the boat velocity), or 5 m/s (the resultant velocity) be used as the average speed value for covering the 80 meters? With what average speed is the boat traversing the 80 meter wide river? Most students want to use the resultant velocity in the equation since that is the actual velocity of the boat with respect to the shore. Yet the value of 5 m/s is the speed at which the boat covers the diagonal dimension of the river. And the diagonal distance across the river is not known in this case. If one knew the **distance C** in the diagram below, then the **average speed C** could be used to calculate the time to reach the opposite shore. Similarly, if one knew the **distance B** in the diagram below, then the **average speed B** could be used to calculate the time to reach the opposite shore. And finally, if one knew the **distance A** in the diagram below, then the **average speed A** could be used to calculate the time to reach the opposite shore.



In our problem, the 80 m corresponds to the distance A, and so the average speed of 4 m/s (average speed in the direction straight across the river) should be substituted into the equation to determine the time.

time = (80 m)/(4 m/s) = 20 s

It requires 20 s for the boat to travel across the river. During this 20 s of crossing the river, the boat also drifts downstream. Part c of the problem asks "What distance downstream does the boat reach the opposite shore?" The same equation must be used to calculate this *downstream distance*. And once more, the question arises, which one of the three average speed values must be used in the equation to calculate the distance downstream? The distance downstream corresponds to **Distance B** on the above diagram. The speed at which the boat covers this distance corresponds to **Average Speed B** on the diagram above (i.e., the speed at which the current moves - 3 m/s). And so the average speed of 3 m/s (average speed in the downstream direction) should be substituted into the equation to determine the distance.

distance = ave. speed * time = (3 m/s) * (20 s)
distance = 60 m

The boat is carried 60 meters downstream during the 20 seconds it takes to cross the river.

The mathematics of the above problem is no more difficult than dividing or multiplying two numerical quantities by each other. The mathematics is easy! The difficulty of the problem is conceptual in nature; the difficulty lies in deciding which numbers to use in the equations. That decision emerges from one's conceptual understanding (or unfortunately, one's misunderstanding) of the complex motion that is occurring. The motion of the riverboat can be divided into two simultaneous parts - a motion in the direction straight across the river and a motion in the downstream direction. These two parts (or components) of the motion occur simultaneously for the same time duration (which was 20 seconds in the above problem). The decision as to which velocity value or distance value to use in the equation must be consistent with the diagram above. The boat's motor is what carries the boat across the river the **Distance A**; and so any calculation involving the **Distance A** must involve the speed value labeled as **Speed A** (the boat speed relative to the water). Similarly, it is the current of the river that carries the boat downstream for the **Distance B**; and so any calculation involving the **Distance B** must involve the speed value labeled as **Speed B** (the river speed). Together, these two parts (or components) add up to give the resulting motion of the boat. That is, the across-the-river component of displacement adds to the downstream displacement to equal the resulting displacement. And likewise, the boat velocity (across the river) adds to the river velocity (down the river) to equal the resulting velocity. And so any calculation of the Distance C or the Average Speed C ("Resultant Velocity") can be performed using the Pythagorean theorem.

Now to illustrate an important point, let's try a second example problem that is similar to the first example problem. Make an attempt to answer the three questions and then click the button to check your answer.

Example 2:

A motorboat traveling 4 m/s, East encounters a current traveling 7.0 m/s, North.

a. What is the resultant velocity of the motorboat?
b. If the width of the river is 80 meters wide, then how much time does it take the boat to travel shore to shore?
c. What distance downstream does the boat reach the opposite shore?

An important concept emerges from the analysis of the two example problems above. In Example 1, the time to cross the 80-meter wide river (when moving 4 m/s) was 20 seconds. This was in the presence of a 3 m/s current velocity. In Example 2, the current velocity was much greater - 7 m/s - yet the time to cross the river remained unchanged. In fact, the current velocity itself has no affect upon the time required for a boat to cross the river. The river moves downstream parallel to the banks of the river. As such, there is no way that the current is capable of assisting a boat in crossing a river. While the increased current may affect the resultant velocity - making the boat travel with a greater speed with respect to an observer on the ground - it does not increase the speed in the direction across the river. The component of the resultant velocity that is increased is the component that is in a direction pointing down the river. It is often said that "perpendicular components of motion are independent of each other." As applied to riverboat problems, this would mean that an across-the-river variable would be independent of (i.e., not be affected by) a downstream variable. The time to cross the river is dependent upon the velocity at which the boat crosses the river. It is only the component of motion directed across the river (i.e., the boat velocity) that affects the time to travel the distance directly across the river (80 m in this case). The component of motion perpendicular to this direction - the current velocity - only affects the distance that the boat travels down the river. This concept of perpendicular components of motion will be investigated in more detail in the next part of Lesson 1.

Animation

Check Your Understanding

1. A plane can travel with a speed of 80 mi/hr with respect to the air. Determine the resultant velocity of the plane (magnitude only) if it encounters a

a. 10 mi/hr headwind.

b. 10 mi/hr tailwind.

c. 10 mi/hr crosswind.

d. 60 mi/hr crosswind.

2. A motorboat traveling 5 m/s, East encounters a current traveling 2.5 m/s, North.

a. What is the resultant velocity of the motor boat?

b. If the width of the river is 80 meters wide, then how much time does it take the boat to travel shore to shore?

c. What distance downstream does the boat reach the opposite shore?

3. A motorboat traveling 5 m/s, East encounters a current traveling 2.5 m/s, South.

a. What is the resultant velocity of the motor boat?

b. If the width of the river is 80 meters wide, then how much time does it take the boat to travel shore to shore?

c. What distance downstream does the boat reach the opposite shore?

4. A motorboat traveling 6 m/s, East encounters a current traveling 3.8 m/s, South.

a. What is the resultant velocity of the motor boat?

b. If the width of the river is 120 meters wide, then how much time does it take the boat to travel shore to shore?

c. What distance downstream does the boat reach the opposite shore?

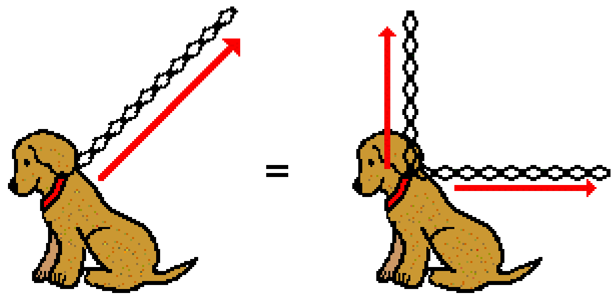
5. If the current velocity in question #4 were increased to 5 m/s, then:

a. how much time would be required to cross the same 120-m wide river?

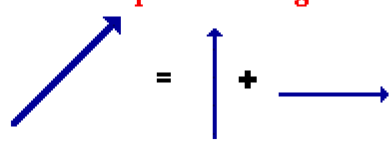
b. what distance downstream would the boat travel during this time?

Independence of Perpendicular Components of Motion

A force vector that is directed upward and rightward has two parts - an upward part and a rightward part. That is to say, if you pull upon an object in an upward and rightward direction, then you are exerting an influence upon the object in two separate directions - an upward direction and a rightward direction. These two parts of the two-dimensional vector are referred to as components. A **component** describes the affect of a single vector in a given direction. Any force vector that is exerted at an angle to the horizontal can be considered as having two parts or components. The vector sum of these two components is always equal to the force at the given angle. This is depicted in the diagram below.

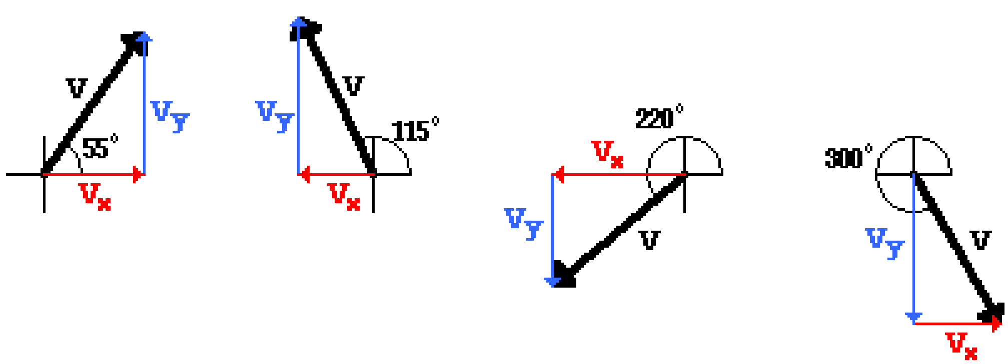


A pull upon Fido's chain in an upward and a rightward direction exerts two separate influences upon Fido - an upward and a rightward influence.



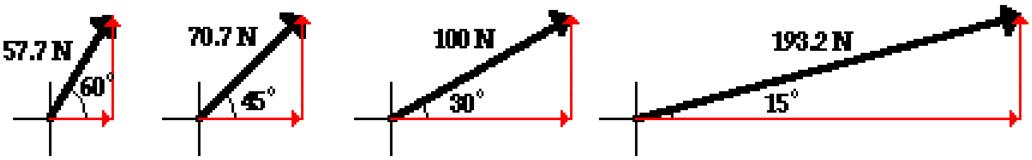
The force exerted at the angle is equal to the vector sum of the two individual forces.

Any vector - whether it is a force vector, displacement vector, velocity vector, etc. - directed at an angle can be thought of as being composed of two perpendicular components. These two components can be represented as legs of a right triangle formed by projecting the vector onto the x- and y-axis.



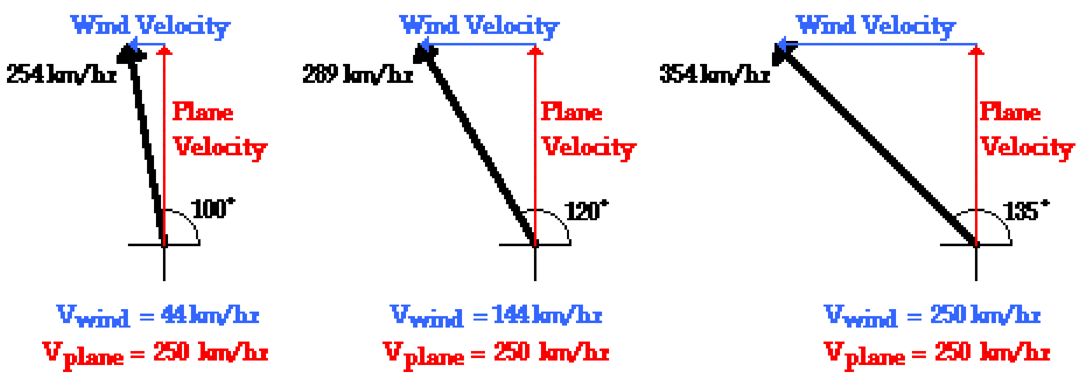
Four velocity vectors - labeled V - with varying directions are shown. The horizontal and vertical components of these vectors are drawn and labeled. Note that a northwest vector has a north and a west component and a southeast vector has a south and an east component.

The two perpendicular parts or components of a vector are independent of each other. Consider the pull upon Fido as an example. If the horizontal pull upon Fido increases, then Fido would be accelerated at a greater rate to the right; yet this greater horizontal pull would not exert any vertical influence upon Fido. Pulling horizontally with more force does not lift Fido vertically off the ground. A change in the horizontal component does not affect the vertical component. This is what is meant by the phrase "perpendicular components of vectors are independent of each other." A change in one component does not affect the other component. Changing a component will affect the motion in that specific direction. While the change in one of the components will alter the magnitude of the resulting force, it does not alter the magnitude of the other component.



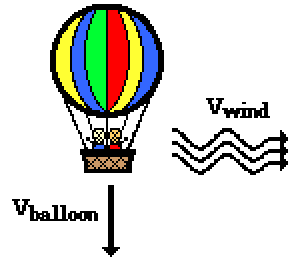
Each of the four vectors above has the same vertical component of force - 50 N. The four vectors have different horizontal components of force. Altering the horizontal component will affect the horizontal moton of the object to which this force is applied.

The resulting motion of a plane flying in the presence of a crosswind is the combination (or sum) of two simultaneous velocity vectors that are perpendicular to each other. Suppose that a plane is attempting to fly northward from Chicago to the Canada border by simply directing the plane due northward. If the plane encounters a crosswind directed towards the west, then the resulting velocity of the plane would be northwest. The northwest velocity vector consists of two components - a north component resulting from the plane's motor (the *plane velocity*) and a westward component resulting from the crosswind (the *wind velocity*). These two components are independent of each other. An alteration in one of the components will not affect the other component. For instance, if the wind velocity increased, then the plane would still be covering ground in the northerly direction at the same rate. It is true that the alteration of the wind velocity would cause the plane to travel more westward; however, the plane still flies northward at the same speed. Perpendicular components of motion do not affect each other.



The resulting motion of a plane in the presence of a wind is dependent upon the velocity of the crosswind. An alteration of the wind velocity affects the resulting motion but does NOT affect the velocity at which the plane flies northward. Perpendicular components of motion are independent of each other.

Now consider an air balloon descending through the air toward the ground in the presence of a wind that blows eastward. Suppose that the downward velocity of the balloon is 3 m/s and that the wind is blowing east with a velocity of 4 m/s. The resulting velocity of the air balloon would be the combination (i.e., the vector sum) of these two simultaneous and independent velocity vectors. The air balloon would be moving downward and eastward.

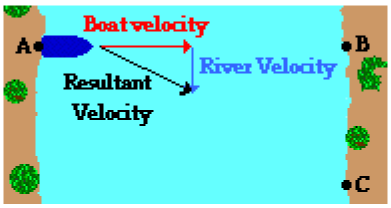


If the wind velocity increased, the air balloon would begin moving faster in the eastward direction, but its downward velocity would not be altered. If the balloon were located 60 meters above the ground and was moving downward at 3 m/s, then it would take a time of 20 seconds to travel this vertical distance.

d = v • t So **t = d / v** = (60 m) / (3 m/s) = **20 seconds**

During the 20 seconds taken by the air balloon to fall the 60 meters to the ground, the wind would be carrying the balloon in the eastward direction. With a wind speed of 4 m/s, the distance traveled eastward in 20 seconds would be 80 meters. If the wind speed increased from the value of 4 m/s to a value of 6 m/s, then it would still take 20 seconds for the balloon to fall the 60 meters of downward distance. A motion in the downward direction is affected only by downward components of motion. An alteration in a horizontal component of motion (such as the eastward wind velocity) will have no affect upon vertical motion. Perpendicular components of motion are independent of each other. A variation of the eastward wind speed from a value of 4 m/s to a value of 6 m/s would only cause the balloon to be blown eastward a distance of 120 meters instead of the original 80 meters.

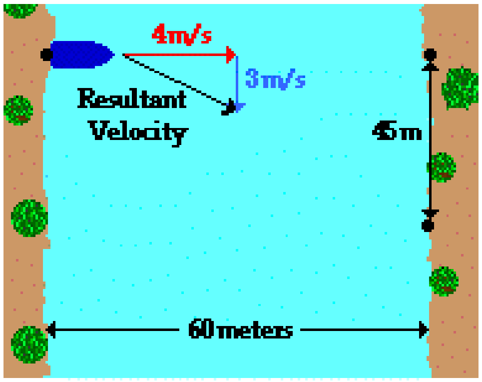
In the most recent section of Lesson 1, the topic of relative velocity and riverboat motion was discussed. A boat on a river often heads straight across the river, perpendicular to its banks. Yet because of the flow of water (i.e., the current) moving parallel to the river banks, the boat does not land on the bank directly across from the starting location. The resulting motion of the boat is the combination (i.e., the vector sum) of these two simultaneous and independent velocity vectors - the boat velocity plus the river velocity. In the diagram at the right, the boat is depicted as moving eastward across the river while the river flows southward. The boat starts at Point A and heads itself towards Point B. But because of the flow of the river southward, the boat reaches the opposite bank of the river at Point C. The time required for the boat to cross the river from one side to the other side is dependent upon the boat velocity and the width of the river. Only an eastward component of motion could affect the time to move eastward across a river.

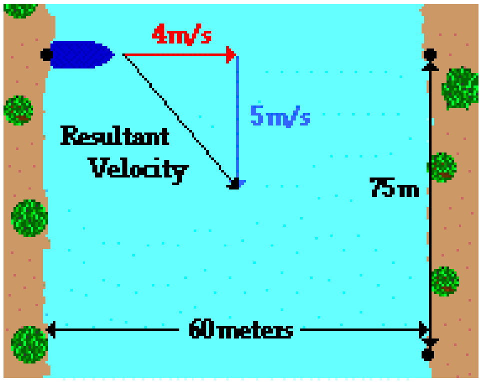


Suppose that the boat velocity is 4 m/s and the river velocity is 3 m/s. The magnitude of the resultant velocity could be determined to be 5 m/s using the Pythagorean Theorem. The time required for the boat to cross a 60-meter wide river would be dependent upon the boat velocity of 4 m/s. It would require 15 seconds to cross the 60-meter wide river.

d = v • t So **t = d / v** = (60 m) / (4 m/s) = **15 seconds**

The southward river velocity will not affect the time required for the boat to travel in the eastward direction. If the current increased such that the river velocity became 5 m/s, then it would still require 15 seconds to cross the river. Perpendicular components of motion are independent of each other. An increase in the river velocity would simply cause the boat to travel further in the southward direction during these 15 seconds of motion. An alteration in a southward component of motion only affects the southward motion.



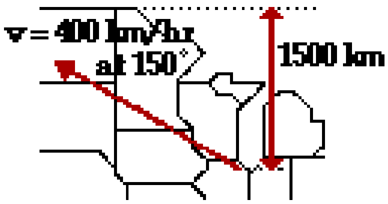


Altering the river velocity does not affect the time to cross a river. A change of the current from 3 m/s to 5 m/s does not change the time to cross the river. It simply changes the distance traveled down the river from a value of 45 meters to a value of 75 meters.

All vectors can be thought of as having perpendicular components. In fact, any motion that is at an angle to the horizontal or the vertical can be thought of as having two perpendicular motions occurring simultaneously. These perpendicular components of motion occur independently of each other. Any component of motion occurring strictly in the horizontal direction will have no affect upon the motion in the vertical direction. Any alteration in one set of these components will have no affect on the other set. In Lesson 2, we will apply this principle to the motion of projectiles that typically encounter both horizontal and vertical motion.

Check Your Understanding

1. A plane flies northwest out of O'Hare Airport in Chicago at a speed of 400 km/hr in a direction of 150 degrees (i.e., 30 degrees north of west). The Canadian border is located a distance of 1500 km due north of Chicago. The plane will cross into Canada after approximately _____ hours.



a. 0.13 b. 0.23 c. 0.27

d. 3.75 e. 4.33 f. 6.49

g. 7.50

h. None of these are even close.

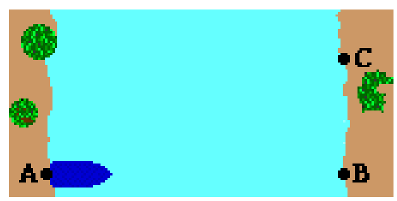
2. Suppose that the plane in question 1 was flying with a velocity of 358 km/hr in a direction of 146 degrees (i.e., 34 degrees north of west). If the Canadian border is still located a distance of 1500 km north of Chicago, then how much time would it take to cross the border?

3. **TRUE** or **FALSE**:

A boat heads straight across a river. The river flows north at a speed of 3 m/s. If the river current were greater, then the time required for the boat to reach the opposite shore would not change.

a. True
b. False

4. A boat begins at point A and heads straight across a 60-meter wide river with a speed of 4 m/s (relative to the water). The river water flows north at a speed of 3 m/s (relative to the shore). The boat reaches the opposite shore at point C. Which of the following would cause the boat to reach the opposite shore at a location SOUTH of C?



a. The boat heads across the river at 5 m/s.

b. The boat heads across the river at 3 m/s.

c. The river flows north at 4 m/s.

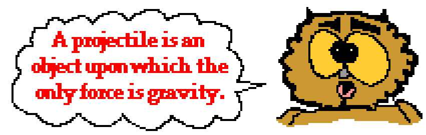
d. The river flows north at 2 m/s.

e. Nonsense! None of these affect the location where the boat lands.

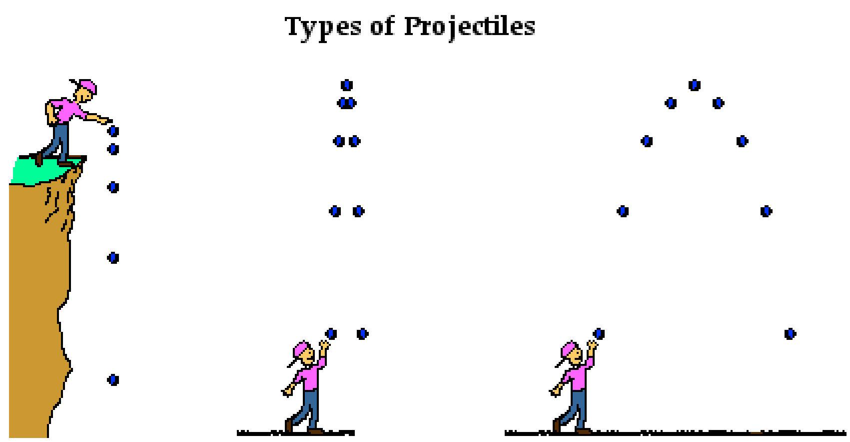
Projectile Motion

What is a Projectile?

In Unit 1 of the Physics Classroom Tutorial, we learned a variety of means to describe the 1-dimensional motion of objects. In Unit 2 of the Physics Classroom Tutorial, we learned how Newton's laws help to explain the motion (and specifically, the changes in the state of motion) of objects that are either at rest or moving in 1-dimension. Now in this unit we will apply both kinematic principles and Newton's laws of motion to understand and explain the motion of objects moving in two dimensions. The most common example of an object that is moving in *two dimensions* is a projectile. Thus, Lesson 2 of this unit is devoted to understanding the motion of projectiles.



A projectile is an object upon which the only force acting is gravity. There are a variety of examples of projectiles. An object dropped from rest is a projectile (provided that the influence of air resistance is negligible). An object that is thrown vertically upward is also a projectile (provided that the influence of air resistance is negligible). And an object which is thrown upward at an angle to the horizontal is also a projectile (provided that the influence of air resistance is negligible). A projectile is any object that once *projected* or dropped continues in motion by its own inertia and is influenced only by the downward force of gravity.

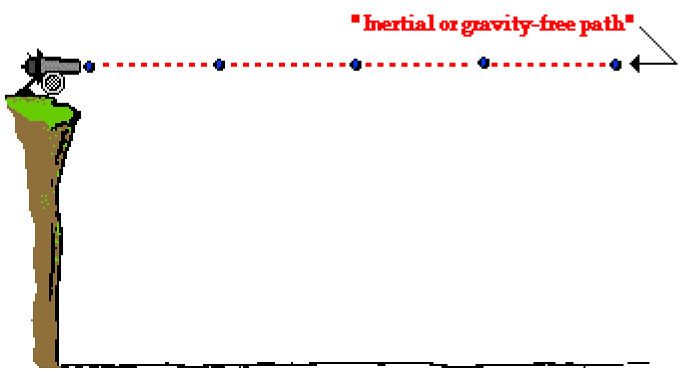


By definition, a projectile has a single force that acts upon it - the force of gravity. If there were any other force acting upon an object, then that object would not be a projectile. Thus, the free-body diagram of a projectile would show a single force acting downwards and labeled force of gravity (or simply F_{grav}). Regardless of whether a projectile is moving downwards, upwards, upwards and rightwards, or downwards and leftwards, the free-body diagram of the projectile is still as depicted in the diagram at the right. By definition, a projectile is any object upon which the only force is gravity.

Projectile Motion and Inertia

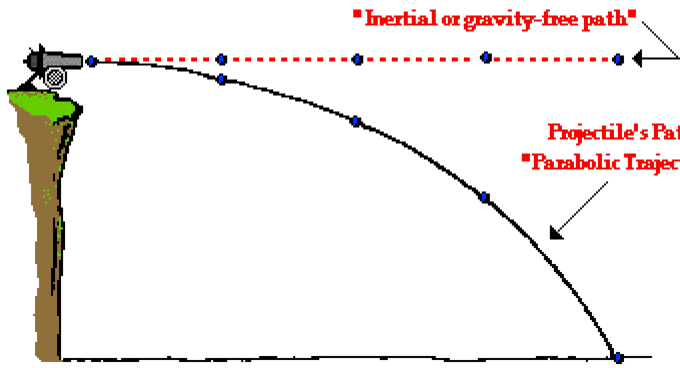
Many students have difficulty with the concept that the only force acting upon an upward moving projectile is gravity. Their conception of motion prompts them to think that if an object is moving upward, then there *must* be an upward force. And if an object is moving upward and rightward, there *must* be both an upward and rightward force. Their belief is that forces cause motion; and if there is an upward motion then there must be an upward force. They reason, "How in the world can an object be moving upward if the only force acting upon it is gravity?" Such students do not *believe* in Newtonian physics (or at least do not believe strongly in Newtonian physics). Newton's laws suggest that forces are only required to cause an acceleration (not a motion). Recall from the Unit 2 that Newton's laws stood in direct opposition to the common misconception that a force is required to keep an object in motion. This idea is simply not true! A force is not required to keep an object in motion. A force is only required to maintain an acceleration. And in the case of a projectile that is moving upward, there is a downward force and a downward acceleration. That is, the object is moving upward and slowing down.

To further ponder this concept of the downward force and a downward acceleration for a projectile, consider a cannonball shot horizontally from a very high cliff at a high speed. And suppose for a moment that the *gravity switch* could be *turned off* such that the cannonball would travel in the absence of gravity? What would the motion of such a cannonball be like? How could its motion be described? According to Newton's first law of motion, such a cannonball would continue in motion in a straight line at constant speed. If not acted upon by an unbalanced force, "an object in motion will ...". This is Newton's law of inertia.



In the absence of gravity, an object in motion will continue in motion with the same speed and in the same direction.

Now suppose that the *gravity switch* is turned on and that the cannonball is projected horizontally from the top of the same cliff. What effect will gravity have upon the motion of the cannonball? Will gravity affect the cannonball's horizontal motion? Will the cannonball travel a greater (or shorter) horizontal distance due to the influence of gravity? The answer to both of these questions is "No!" Gravity will act downwards upon the cannonball to affect its vertical motion. Gravity causes a vertical acceleration. The ball will drop vertically below its otherwise straight-line, inertial path. Gravity is the downward force upon a projectile that influences its vertical motion and causes the parabolic trajectory that is characteristic of projectiles.



With gravity, a "projectile" will fall below its inertial path. Gravity acts downward to cause a downward acceleration. There are no horizontal forces needed to maintain the horizontal motion - consistent with the concept of inertia.

Animation

A projectile is an object upon which the only force is gravity. Gravity acts to influence the vertical motion of the projectile, thus causing a vertical acceleration. The horizontal motion of the projectile is the result of the tendency of any object in motion to remain in motion at constant velocity. Due to the absence of horizontal forces, a projectile remains in motion with a constant horizontal velocity. Horizontal forces are not required to keep a projectile moving horizontally. The only force acting upon a projectile is gravity!

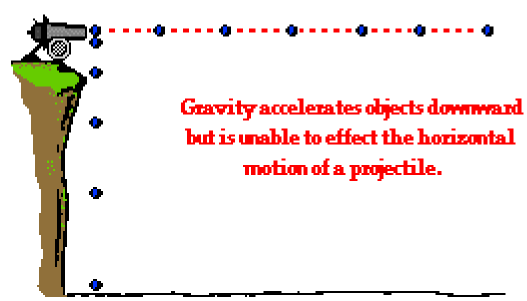
Characteristics of a Projectile's Trajectory

As discussed earlier in this lesson, a projectile is an object upon which the only force acting is gravity. Many projectiles not only undergo a vertical motion, but also undergo a horizontal motion. That is, as they move upward or downward they are also moving horizontally. There are the two components of the projectile's motion - horizontal and vertical motion. And since perpendicular components of motion are independent of each other, these two components of motion can (and must) be discussed separately. The goal of this part of the lesson is to discuss the horizontal and vertical components of a projectile's motion; specific attention will be given to the presence/absence of forces, accelerations, and velocity.

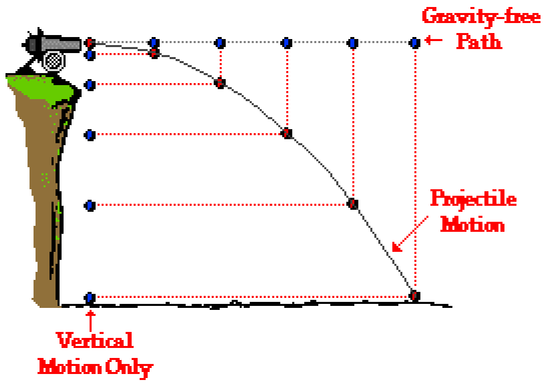

A projectile often moves horizontally as it moves upward and/or downward.

Horizontally Launched Projectiles

Let's return to our *thought experiment* from earlier in this lesson. Consider a cannonball projected horizontally by a cannon from the top of a very high cliff. In the absence of gravity, the cannonball would continue its horizontal motion at a constant velocity. This is consistent with the law of inertia. And furthermore, if merely dropped from rest in the presence of gravity, the cannonball would accelerate downward, gaining speed at a rate of 9.8 m/s every second. This is consistent with our conception of free-falling objects accelerating at a rate known as the **acceleration of gravity**.



If our thought experiment continues and we project the cannonball horizontally in the presence of gravity, then the cannonball would maintain the same horizontal motion as before - a constant horizontal velocity. Furthermore, the force of gravity will act upon the cannonball to cause the same vertical motion as before - a downward acceleration. The cannonball falls the same amount of distance as it did when it was merely dropped from rest (refer to diagram below). However, the presence of gravity does not affect the horizontal motion of the projectile. The force of gravity acts downward and is unable to alter the horizontal motion. There must be a horizontal force to cause a horizontal acceleration. (And we know that there is only a vertical force acting upon projectiles.) The vertical force acts perpendicular to the horizontal motion and will not affect it since perpendicular components of motion are independent of each other. Thus, the projectile travels with a constant horizontal velocity and a downward vertical acceleration.

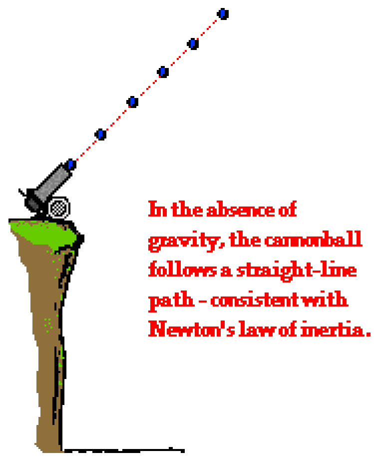


The above information can be summarized by the following table.

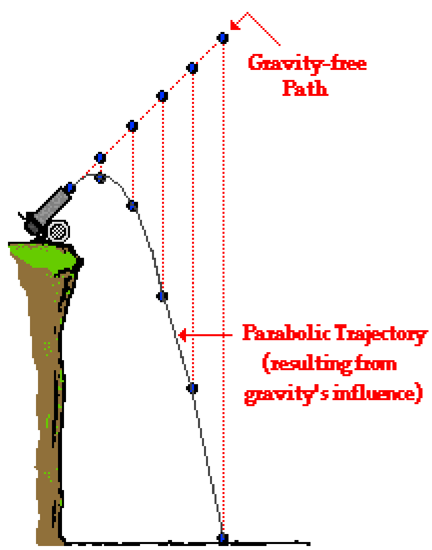
	Horizontal Motion	Vertical Motion
Forces (Present? - Yes or No) (If present, what dir'n?)	No	Yes The force of gravity acts downward
Acceleration (Present? - Yes or No) (If present, what dir'n?)	No	Yes "g" is downward at 9.8 m/s/s
Velocity (Constant or Changing?)	Constant	Changing (by 9.8 m/s each second)

Animation

Non-Horizontally Launched Projectiles



Now suppose that our cannon is aimed upward and shot at an angle to the horizontal from the same cliff. In the absence of gravity (i.e., supposing that the *gravity switch* could be *turned off*) the projectile would again travel along a straight-line, inertial path. An object in motion would continue in motion at a constant speed in the same direction if there is no unbalanced force. This is the case for an object moving through space in the absence of gravity. However, if the *gravity switch* could be *turned on* such that the cannonball is truly a projectile, then the object would once more *free-fall* below this straight-line, inertial path. In fact, the projectile would travel with a *parabolic trajectory*. The downward force of gravity would act upon the cannonball to cause the same vertical motion as before - a downward acceleration. The cannonball falls the same amount of distance in every second as it did when it was merely dropped from rest (refer to diagram below). Once more, the presence of gravity does not affect the horizontal motion of the projectile. The projectile still moves the same horizontal distance in each second of travel as it did when the *gravity switch* was turned off. The force of gravity is a vertical force and does not affect horizontal motion; perpendicular components of motion are independent of each other.

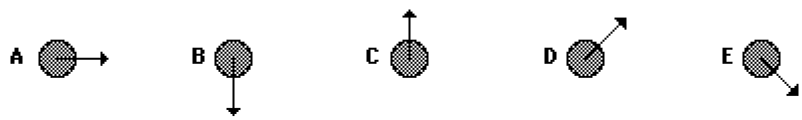


In conclusion, projectiles travel with a parabolic trajectory due to the fact that the downward force of gravity accelerates them downward from their otherwise straight-line, gravity-free trajectory. This downward force and acceleration results in a downward displacement from the position that the object would be if there were no gravity. The force of gravity does not affect the horizontal component of motion; a projectile maintains a constant horizontal velocity since there are no horizontal forces acting upon it.

Check Your Understanding

Use your understanding of projectiles to answer the following questions. When finished, click the button to view your answers.

1. Consider these diagrams in answering the following questions.



Which diagram (if any) might represent ...

a. ... the initial horizontal velocity?

b. ... the initial vertical velocity?

c. ... the horizontal acceleration?

d. ... the vertical acceleration?

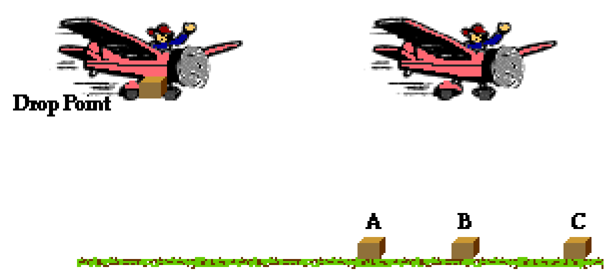
e. ... the net force?

2. Supposing a snowmobile is equipped with a flare launcher that is capable of launching a sphere vertically (relative to the snowmobile). If the snowmobile is in motion and launches the flare and maintains a constant horizontal velocity after the launch, then where will the flare land (neglect air resistance)?

a. in front of the snowmobile

b. behind the snowmobile

c. in the snowmobile



3. Suppose a rescue airplane drops a relief package while it is moving with a constant horizontal speed at an elevated height. Assuming that air resistance is negligible, where will the relief package land relative to the plane?

a. below the plane and behind it.

b. directly below the plane

c. below the plane and ahead of it

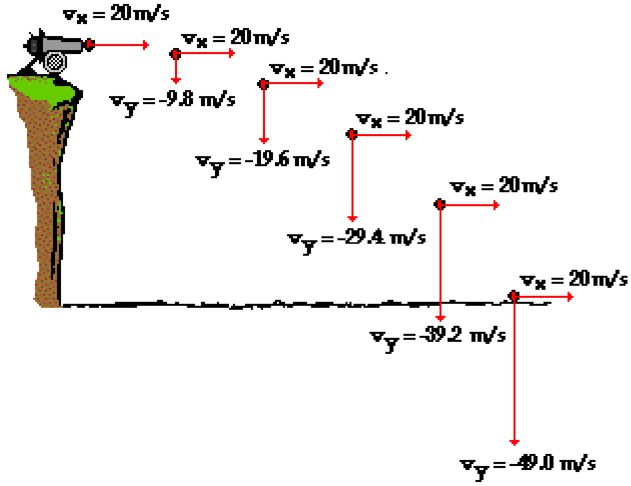
Describing Projectiles With Numbers: (Horizontal and Vertical Velocity)

So far in Lesson 2 you have learned the following conceptual notions about projectiles.

- A projectile is any object upon which the only force is gravity,
- Projectiles travel with a parabolic trajectory due to the influence of gravity,
- There are no horizontal forces acting upon projectiles and thus no horizontal acceleration,
- The horizontal velocity of a projectile is constant (a never changing in value),
- There is a vertical acceleration caused by gravity; its value is 9.8 m/s/s, down,
- The vertical velocity of a projectile changes by 9.8 m/s each second,
- The horizontal motion of a projectile is independent of its vertical motion.

In this portion of Lesson 2 you will learn how to describe the motion of projectiles numerically. You will learn how the numerical values of the x- and y-components of the velocity and displacement change with time (or remain constant). As you proceed through this part of Lesson 2, pay careful attention to how a conceptual understanding of projectiles translates into a numerical understanding.

Consider again the cannonball launched by a cannon from the top of a very high cliff. Suppose that the cannonball is launched horizontally with no upward angle whatsoever and with an initial speed of 20 m/s. If there were no gravity, the cannonball would continue in motion at 20 m/s in the horizontal direction. Yet in actuality, gravity causes the cannonball to accelerate downwards at a rate of 9.8 m/s/s. This means that the vertical velocity is changing by 9.8 m/s every second. If a vector diagram (showing the velocity of the cannonball at 1-second intervals of time) is used to represent how the x- and y-components of the velocity of the cannonball is changing with time, then x- and y- velocity vectors could be drawn and their magnitudes labeled. The lengths of the vector arrows are representative of the magnitudes of that quantity. Such a diagram is shown below.



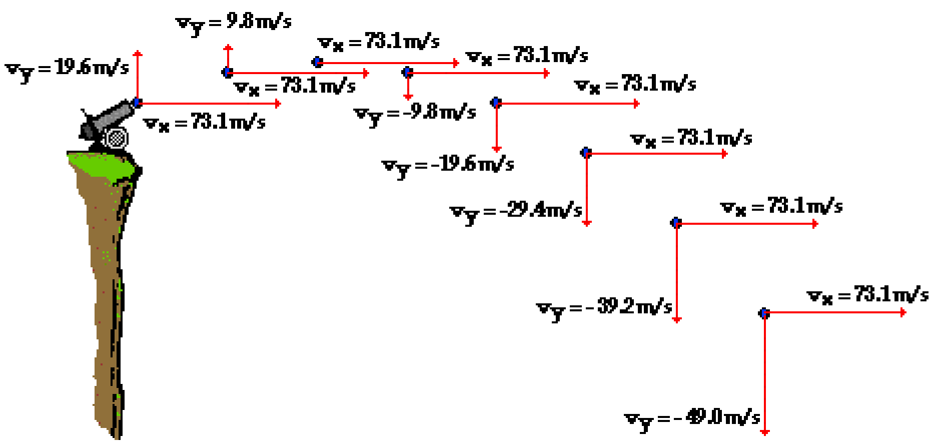
The important concept depicted in the above vector diagram is that the horizontal velocity remains constant during the course of the trajectory and the vertical velocity changes by 9.8 m/s every second. These same two concepts could be depicted by a table illustrating how the x- and y-component of the velocity vary with time.

Time	Horizontal Velocity	Vertical Velocity
0 s	20 m/s, right	0
1 s	20 m/s, right	9.8 m/s, down
2 s	20 m/s, right	19.6 m/s, down
3 s	20 m/s, right	29.4 m/s, down
4 s	20 m/s, right	39.2 m/s, down
5 s	20 m/s, right	49.0 m/s, down

The numerical information in both the diagram and the table above illustrate identical points - a projectile has a vertical acceleration of 9.8 m/s/s, downward and no horizontal acceleration. This is to say that the vertical velocity changes by 9.8 m/s each second and the horizontal velocity never changes. This is indeed consistent with the fact that there is a vertical force acting upon a projectile but no horizontal force. A vertical force causes a vertical acceleration - in this case, an acceleration of 9.8 m/s/s.

Animation

But what if the projectile is launched upward at an angle to the horizontal? How would the horizontal and vertical velocity values change with time? How would the numerical values differ from the previously shown diagram for a horizontally launched projectile? The diagram below reveals the answers to these questions. The diagram depicts an object launched upward with a velocity of 75.7 m/s at an angle of 15 degrees above the horizontal. For such an initial velocity, the object would initially be moving 19.6 m/s, upward and 73.1 m/s, rightward. These values are x- and y-components of the initial velocity and will be discussed in more detail in the next part of this lesson.



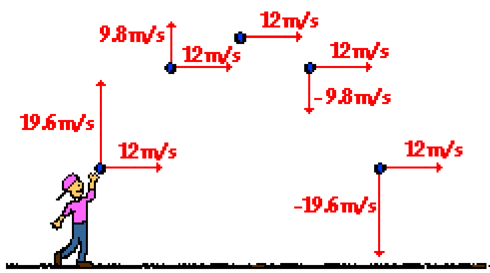
Again, the important concept depicted in the above diagram is that the horizontal velocity remains constant during the course of the trajectory and the vertical velocity changes by 9.8 m/s every second. These same two concepts could be depicted by a table illustrating how the x- and y-component of the velocity vary with time.

Time	Horizontal Velocity	Vertical Velocity
0 s	73.1 m/s, right	19.6 m/s, up
1 s	73.1 m/s, right	9.8 m/s, up
2 s	73.1 m/s, right	0 m/s
3 s	73.1 m/s, right	9.8 m/s, down

4 s	73.1 m/s, right	19.6 m/s, down
5 s	73.1 m/s, right	29.4 m/s, down
6 s	73.1 m/s, right	39.2 m/s, down
7 s	73.1 m/s, right	49.0 m/s, down

Animation

The numerical information in both the diagram and the table above further illustrate the two key principles of projectile motion - there is a horizontal velocity that is constant and a vertical velocity that changes by 9.8 m/s each second. As the projectile rises towards its peak, it is slowing down (19.6 m/s to 9.8 m/s to 0 m/s); and as it falls from its peak, it is speeding up (0 m/s to 9.8 m/s to 19.6 m/s to ...). Finally, the *symmetrical* nature of the projectile's motion can be seen in the diagram above: the vertical speed one second before reaching its peak is the same as the vertical speed one second after falling from its peak. The vertical speed two seconds before reaching its peak is the same as the vertical speed two seconds after falling from its peak. For non-horizontally launched projectiles, the direction of the velocity vector is sometimes considered + on the way up and - on the way down; yet the magnitude of the vertical velocity (i.e., vertical speed) is the same an equal interval of time on either side of its peak. At the peak itself, the vertical velocity is 0 m/s; the velocity vector is entirely horizontal at this point in the trajectory. These concepts are further illustrated by the diagram below for a non-horizontally launched projectile that lands at the same height as which it is launched.



The above diagrams, tables, and discussion pertain to how the horizontal and vertical components of the velocity vector change with time during the course of projectile's trajectory. Another vector quantity that can be discussed is the displacement. The numerical description of the displacement of a projectile is discussed in the next section of Lesson 2.

Describing Projectiles With Numbers: (Horizontal and Vertical Displacement)

The previous diagrams, tables, and discussion pertain to how the horizontal and vertical components of the velocity vector change with time during the course of projectile's trajectory. Now we will investigate the manner in which the horizontal and vertical components of a projectile's displacement vary with time. As has already been discussed, the vertical displacement (denoted by the symbol ***y*** in the discussion below) of a projectile is dependent only upon the acceleration of gravity and not dependent upon the horizontal velocity. Thus, the vertical displacement (***y***) of a projectile can be predicted using the same equation used to find the displacement of a free-falling object undergoing one-dimensional motion. This equation was discussed in Unit 1 of The Physics Classroom. The equation can be written as follows.

$$y = 0.5 \bullet g \bullet t^2$$

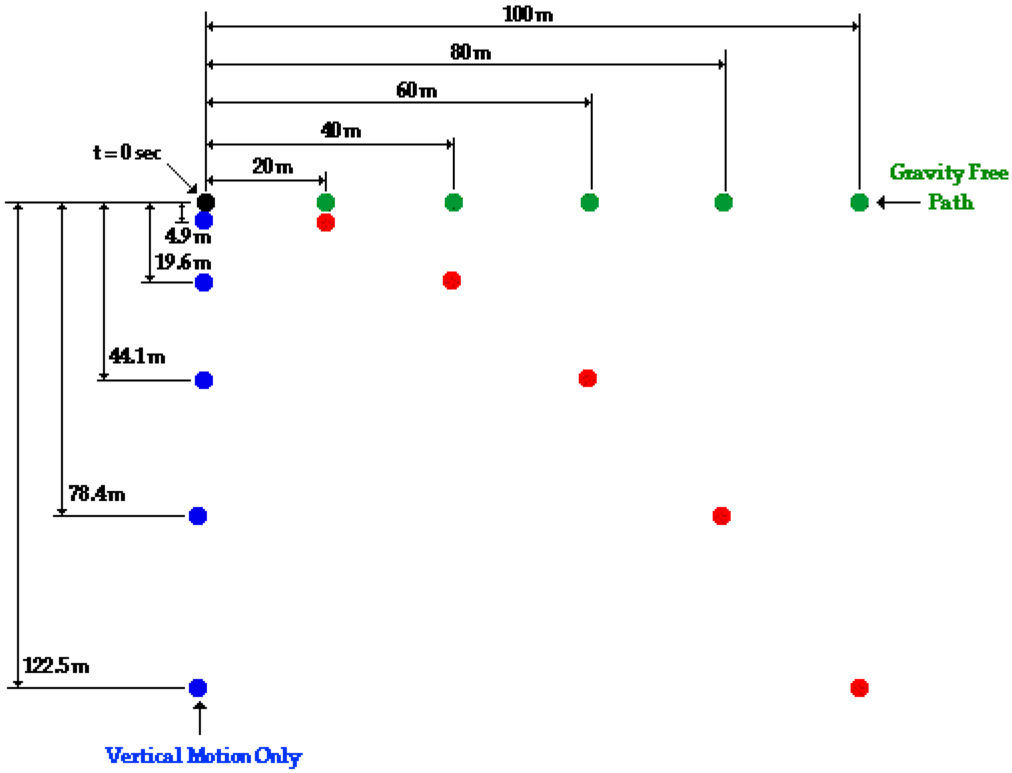
(equation for vertical displacement for a horizontally launched projectile)

where **g** is -9.8 m/s/s and **t** is the time in seconds. The above equation pertains to a projectile with no initial vertical velocity and as such predicts the vertical distance that a projectile falls if dropped from rest. It was also discussed earlier, that the force of gravity does not influence the horizontal motion of a projectile.

The horizontal displacement of a projectile is only influenced by the speed at which it moves horizontally (**v_{ix}**) and the amount of time (**t**) that it has been moving horizontally. Thus, if the horizontal displacement (**x**) of a projectile were represented by an equation, then that equation would be written as

$$x = v_{ix} \bullet t$$

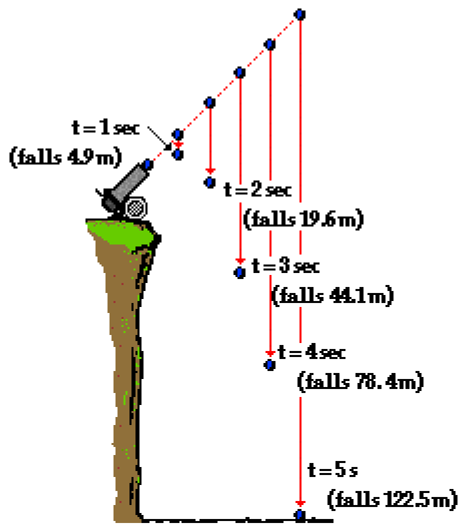
The diagram below shows the trajectory of a projectile (in red), the path of a projectile released from rest with no horizontal velocity (in blue) and the path of the same object when gravity is turned off (in green). The position of the object at 1-second intervals is shown. In this example, the initial horizontal velocity is 20 m/s and there is no initial vertical velocity (i.e., a case of a horizontally launched projectile).



As can be seen in the diagram above, the vertical distance fallen from rest during each consecutive second is increasing (i.e., there is a vertical acceleration). It can also be seen that the vertical displacement follows the equation above ($y = 0.5 \bullet g \bullet t^2$). Furthermore, since there is no horizontal acceleration, the horizontal distance traveled by the projectile each second is a constant value - the projectile travels a horizontal distance of 20 meters each second. This is consistent with the initial horizontal velocity of 20 m/s. Thus, the horizontal displacement is 20 m at 1 second, 40 meters at 2 seconds, 60 meters at 3 seconds, etc. This information is summarized in the table below.

Time	Horizontal Displacement	Vertical Displacement
0 s	0 m	0 m
1 s	20 m	-4.9 m
2 s	40 m	-19.6 m
3 s	60 m	-44.1 m
4 s	80m	-78.4 m
5 s	100 m	-122.5 m

Now consider displacement values for a projectile launched at an angle to the horizontal (i.e., a non-horizontally launched projectile). How will the presence of an initial vertical component of velocity affect the values for the displacement? The diagram below depicts the position of a projectile launched at an angle to the horizontal. The projectile still falls 4.9 m, 19.6 m, 44.1 m, and 78.4 m below the straight-line, gravity-free path. These distances are indicated on the diagram below.



The projectile still falls below its gravity-free path by a vertical distance of 0.5*g*t^2. However, the gravity-free path is no longer a horizontal line since the projectile is not launched horizontally. In the absence of gravity, a projectile would rise a vertical distance equivalent to the time multiplied by the vertical component of the initial velocity (v_{iy}• t). In the presence of gravity, it will fall a distance of 0.5 • g • t^2. Combining these two influences upon the vertical displacement yields the following equation.

$$\mathbf{y = v_{iy} \bullet t + 0.5 \bullet g \bullet t^2}$$

(equation for vertical displacement for an angled-launched projectile)

where $\mathbf{v_{iy}}$ is the initial vertical velocity in m/s, **t** is the time in seconds, and **g** = -9.8 m/s/s (an approximate value of the acceleration of gravity). If a projectile is launched with an initial vertical velocity of 19.6 m/s and an initial horizontal velocity of 33.9 m/s, then the x- and y- displacements of the projectile can be calculated using the equations above. A sample calculation is shown below.

Calculations for t = 1 second

$y = v_{iy} * t + 0.5*g*t^2$ where v_{iy} = 19.6 m/s	$x = v_{ix} * t$ where v_{ix} = 33.9 m/s
y = (19.6 m/s) * (1 s) + 0.5*(-9.8 m/s/s)*$(1\ s)^2$	x = (33.9 m/s) * (1 s)
y = 19.6 m + (-4.9 m)	x = 33.9 m
y = 14.7 m (approximately)	

The following table lists the results of such calculations for the first four seconds of the projectile's motion.

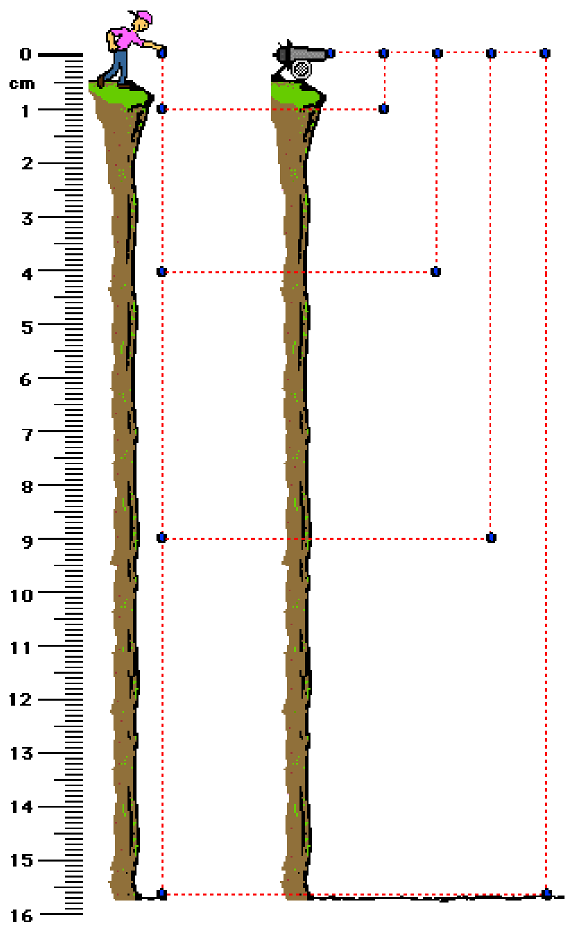
Time	Horizontal Displacement	Vertical Displacement
0 s	0 m	0 m
1 s	33.9 m	14.7 m
2 s	67.8 m	19.6 m
3 s	101.7 m	14.7 m
4 s	135.6 m	0 m

The data in the table above show the symmetrical nature of a projectile's trajectory. The vertical displacement of a projectile *t* seconds before reaching the peak is the same as the vertical displacement of a projectile *t* seconds after reaching the peak. For example, the projectile reaches its peak at a time of 2 seconds; the vertical displacement is the same at 1 second (1 s before reaching the peak) is the same as it is at 3 seconds (1 s after reaching its peak). Furthermore, the time to reach the peak (2 seconds) is the same as the time to fall from its peak (2 seconds).

Check Your Understanding

Use your understanding of projectiles to answer the following questions. Then click the button to view the answers.

The following diagram pertains to questions #1 and #2 below. A scale is used where 1 cm = 5 meters. (Note that 1-cm may be a different distance for different computer monitors; thus, a cm-ruler is given in the diagram.)



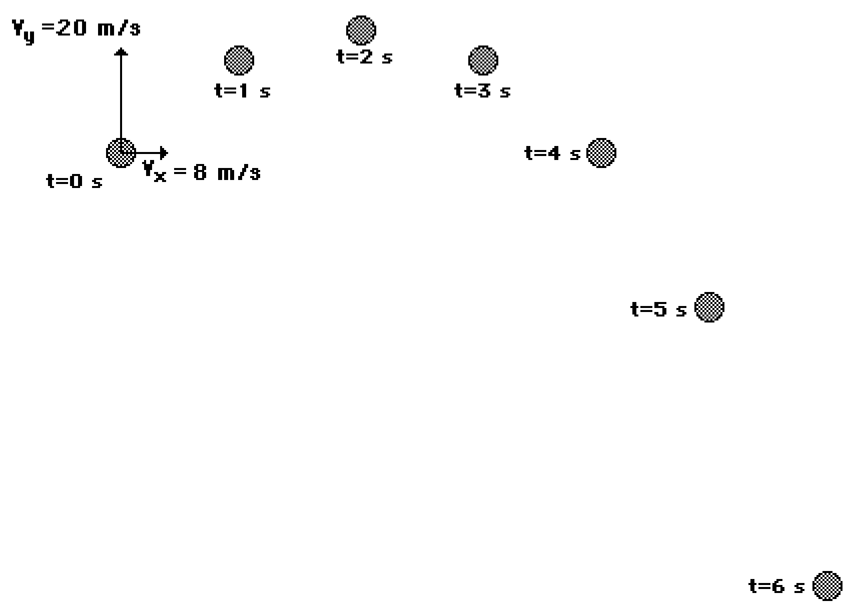
1. Anna Litical drops a ball from rest from the top of 78.4-meter high cliff. How much time will it take for the ball to reach the ground and at what height will the ball be after each second of motion?

2. A cannonball is launched horizontally from the top of an 78.4-meter high cliff. How much time will it take for the ball to reach the ground and at what height will the ball be after each second of travel?

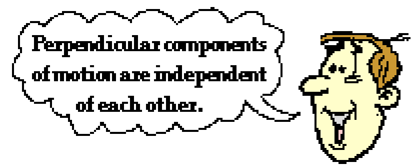
3. Fill in the table below indicating the value of the horizontal and vertical components of velocity and acceleration for a projectile.

Time (s)	v_x (m/s)	v_y (m/s)	a_x (m/s2)	a_y (m/s2)
0	15.0	29.4		
1				
2				
3				
4				
5				
6				

4. The diagram below shows the trajectory for a projectile launched non-horizontally from an elevated position on top of a cliff. The initial horizontal and vertical components of the velocity are 8 m/s and 19.6 m/s respectively. Positions of the object at 1-second intervals are shown. Determine the horizontal and vertical velocities at each instant shown in the diagram.



Initial Velocity Components



It has already been stated and thoroughly discussed that the horizontal and vertical motions of a projectile are independent of each other. The horizontal velocity of a projectile does not affect how far (or how fast) a projectile falls vertically. Perpendicular components of motion are independent of each other. Thus, an analysis of the motion of a projectile demands that the two components of motion are analyzed independent of each other, being careful not to *mix* horizontal motion information with vertical motion information. That is, if analyzing the motion to determine the vertical displacement, one would use kinematic equations with vertical motion parameters (initial *vertical* velocity, final *vertical* velocity, *vertical* acceleration) and not horizontal motion parameters (initial horizontal velocity, final horizontal velocity, horizontal acceleration). It is for this reason that one of the initial steps of a projectile motion problem is to determine the components of the initial velocity.

Earlier in this unit, the method of vector resolution was discussed. Vector resolution is the method of taking a single vector at an angle and separating it into two perpendicular parts. The two parts of a vector are known as components and describe the influence of that vector in a single direction. If a projectile is launched at an angle to the horizontal, then the initial velocity of the projectile has both a horizontal and a vertical component. The horizontal velocity component (**v_x**) describes the influence of the velocity in displacing the projectile horizontally. The vertical velocity component (**v_y**) describes the influence of the velocity in displacing the projectile vertically. Thus, the analysis of projectile motion problems begins by using the trigonometric methods discussed earlier to determine the horizontal and vertical components of the initial velocity.

Consider a projectile launched with an initial velocity of 50 m/s at an angle of 60 degrees above the horizontal. Such a projectile begins its motion with a horizontal velocity of 25 m/s and a vertical velocity of 43 m/s. These are known as the horizontal and vertical components of the initial velocity. These numerical values were determined by constructing a sketch of the velocity vector with the given direction and then using trigonometric functions to determine the sides of the *velocity* triangle. The sketch is shown at the right and the use of trigonometric functions to determine the magnitudes is shown below. (If necessary, review this method on an earlier page in this unit.)

$$\cos 60° = \frac{v_x}{50\text{ m/s}}$$

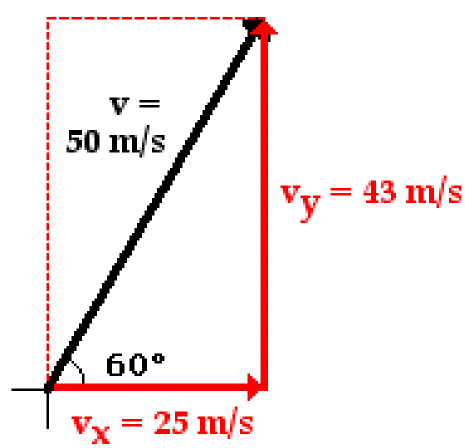
$$v_x = 50\text{ m/s} * \cos 60°$$

$$v_x = 25\text{ m/s}$$

$$\sin 60° = \frac{v_y}{50\text{ m/s}}$$

$$v_y = 50\text{ m/s} * \sin 60°$$

$$v_y = 43\text{ m/s}$$



All vector resolution problems can be solved in a similar manner. As a test of your understanding, utilize trigonometric functions to determine the horizontal and vertical components of the following initial velocity values. When finished, click the button to check your answers.

Practice A: A water balloon is launched with a speed of 40 m/s at an angle of 60 degrees to the horizontal.

Practice B: A motorcycle stunt person traveling 70 mi/hr jumps off a ramp at an angle of 35 degrees to the horizontal.

Practice C: A springboard diver jumps with a velocity of 10 m/s at an angle of 80 degrees to the horizontal.

Try Some More!

Need more practice? Use the **Velocity Components for a Projectile** *widget* below to try some additional problems. Enter any velocity magnitude and angle with the horizontal. Use your calculator to determine the values of v_x and v_y. Then click the **Submit** button to check your answers.

Velocity Components for a Projectile

Velocity magnitude: 25 m/s

Angle w/horizontal: 30 deg

Input:

$\{25\cos(30°), 25\sin(30°)\}$

Decimal approximation:

$\{21.6506, 12.5\}$

As mentioned above, the point of resolving an initial velocity vector into its two components is to use the values of these two components to analyze a projectile's motion and determine such parameters as the horizontal displacement, the vertical displacement, the final vertical velocity, the time to reach the peak of the trajectory, the time to fall to the ground, etc.

Determination of the Time of Flight

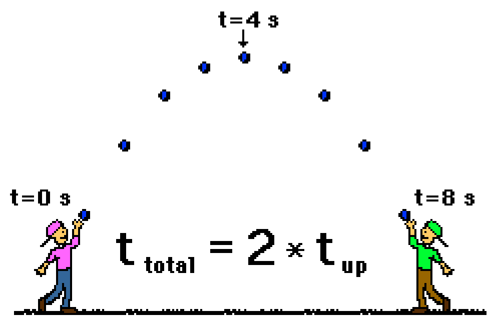
The time for a projectile to rise vertically to its peak (as well as the time to fall from the peak) is dependent upon vertical motion parameters. The process of rising vertically to the peak of a trajectory is a vertical motion and is thus dependent upon the initial vertical velocity and the vertical acceleration (g = 9.8 m/s/s, down). The process of determining the time to rise to the peak is an easy process - provided that you have a solid grasp of the concept of acceleration. When first introduced, it was said that acceleration is the rate at which the velocity of an object changes. An acceleration value indicates the amount of velocity change in a given interval of time. To say that a projectile has a vertical acceleration of -9.8 m/s/s is to say that the vertical velocity changes by 9.8 m/s (in the - or downward direction) each second. For example, if a projectile is moving upwards with a velocity of 39.2 m/s at 0 seconds, then its velocity will be 29.4 m/s after 1 second, 19.6 m/s after 2 seconds, 9.8 m/s after 3 seconds, and 0 m/s after 4 seconds. For such a projectile with an initial vertical velocity of 39.2 m/s, it would take 4 seconds for it to reach the peak where its vertical velocity is 0 m/s. With this notion in mind, it is evident that the time for a projectile to rise to its peak is a matter of dividing the vertical component of the initial velocity (v_{iy}) by the acceleration of gravity.

Time	Vertical Velocity
0 s	39.2 m/s, up
1 s	29.4 m/s, up
2 s	19.6 m/s, up
3 s	9.8 m/s, up
4 s	0 m/s

A projectile with an initial vertical velocity of 39.2 m/s requires 4 seconds to rise to the peak of its trajectory.

$$t_{up} = \frac{v_{iy}}{g}$$

Once the time to rise to the peak of the trajectory is known, the total time of flight can be determined. For a projectile that lands at the same height which it started, the total time of flight is twice the time to rise to the peak. Recall from the last section of Lesson 2 that the trajectory of a projectile is symmetrical about the peak. That is, if it takes 4 seconds to rise to the peak, then it will take 4 seconds to fall from the peak; the total time of flight is 8 seconds. The time of flight of a projectile is twice the time to rise to the peak.



If it takes a projectile 4 seconds to rise to its peak, then it will take a total of 8 seconds to move through the air from start to finish.

Determination of Horizontal Displacement

The horizontal displacement of a projectile is dependent upon the horizontal component of the initial velocity. As discussed in the previous part of this lesson, the horizontal displacement of a projectile can be determined using the equation

$$x = v_{ix} \bullet t$$

If a projectile has a time of flight of 8 seconds and a horizontal velocity of 20 m/s, then the horizontal displacement is 160 meters (20 m/s • 8 s). If a projectile has a time of flight of 8 seconds and a horizontal velocity of 34 m/s, then the projectile has a horizontal displacement of 272 meters (34 m/s • 8 s). The horizontal displacement is dependent upon the only horizontal parameter that exists for projectiles - the horizontal velocity (**v_{ix}**).

Determination of the Peak Height

A non-horizontally launched projectile with an initial vertical velocity of 39.2 m/s will reach its peak in 4 seconds. The process of rising to the peak is a vertical motion and is again dependent upon vertical motion parameters (the initial vertical velocity and the vertical acceleration). The height of the projectile at this peak position can be determined using the equation

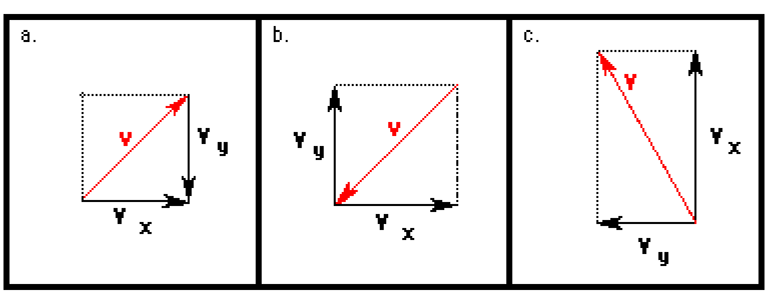
$$y = v_{iy} \bullet t + 0.5 \bullet g \bullet t^2$$

where **v_{iy}** is the initial vertical velocity in m/s, **g** is the acceleration of gravity (-9.8 m/s/s) and **t** is the time in seconds it takes to reach the peak. This equation can be successfully used to determine the vertical displacement of the projectile through the first half of its trajectory (i.e., peak height) provided that the algebra is properly performed and the proper values are substituted for the given variables. Special attention should be given to the facts that the **t** in the equation is the time up to the peak and the g has a negative value of -9.8 m/s/s.

Check Your Understanding

Answer the following questions and click the button to see the answers.

1. Aaron Agin is resolving velocity vectors into horizontal and vertical components. For each case, evaluate whether Aaron's diagrams are correct or incorrect. If incorrect, explain the problem or make the correction.



2. Use trigonometric functions to resolve the following velocity vectors into horizontal and vertical components. Then utilize kinematic equations to calculate the other motion parameters. Be careful with the equations; be guided by the principle that "perpendicular components of motion are independent of each other."

Bob Beamon's record breaking long jump (8.9 m) in the 1968 Olympics resulted from an initial velocity of 9.5 m/s at an angle of 40° to the horizontal.	Megan Progress, GBS golf standout, hits a nine-iron with a velocity of 25 m/s at an angle of 60° to the horizontal.	Kevin Butler, Chicago Bear place kicker, lauches a kickoff at an angle of 30° to the horizontal and a velocity of 30 m/s.
v_{ix} = ____A____ m/s	v_{ix} = ____G____ m/s	v_{ix} = ____M____ m/s
v_{iy} = ____B____ m/s	v_{iy} = ____H____ m/s	v_{iy} = ____N____ m/s
t_{up} = ____C____ s	t_{up} = ____I____ s	t_{up} = ____O____ s
t_{total} = ____D____ s	t_{total} = ____J____ s	t_{total} = ____P____ s
x = ____E____ m	x = ____K____ m	x = ____Q____ m
y @ peak = ____F____ m	y @ peak = ____L____ m	y @ peak = ____R____ m

3. Utilize kinematic equations and projectile motion concepts to fill in the blanks in the following tables.

vi (m/s)	angle (°)	vix (m/s)	viy (m/s)	t_{up} (s)	t_{total} (s)	peak ht. (m)	x (m)
50	20	47.0	17.1	1.75	3.49	A	B
50	35	41.0	28.7	C	D	E	F
25	40	G	H	I	J	13.2	62.8
20	K	3.5	L	M	N	19.8	14.1
20	10	O	P	Q	R	0.615	14.0

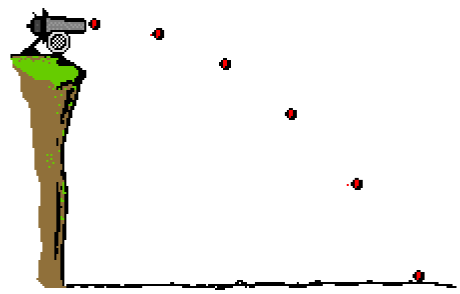
Horizontally Launched Projectile Problems

One of the powers of physics is its ability to use physics principles to make predictions about the final outcome of a moving object. Such predictions are made through the application of physical principles and mathematical formulas to a given set of initial conditions. In the case of projectiles, a student of physics can use information about the initial velocity and position of a projectile to predict such things as how much time the projectile is in the air and how far the projectile will go. The physical principles that must be applied are those discussed previously in Lesson 2. The mathematical formulas that are used are commonly referred to as kinematic equations. Combining the two allows one to make predictions concerning the motion of a projectile. In a typical physics class, the predictive ability of the principles and formulas are most often demonstrated in word story problems known as projectile problems.

There are two basic types of projectile problems that we will discuss in this course. While the general principles are the same for each type of problem, the approach will vary due to the fact the problems differ in terms of their initial conditions. The two types of problems are:

Problem Type 1:

A projectile is launched with an initial horizontal velocity from an elevated position and follows a parabolic path to the ground. Predictable unknowns include the initial speed of the projectile, the initial height of the projectile, the time of flight, and the horizontal distance of the projectile.



This projectile is launched with an initial horizontal velocity from an elevated position.

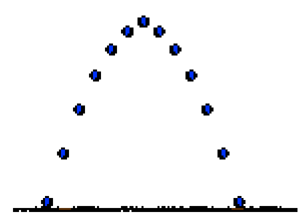
Examples of this type of problem are

a. A pool ball leaves a 0.60-meter high table with an initial horizontal velocity of 2.4 m/s. Predict the time required for the pool ball to fall to the ground and the horizontal distance between the table's edge and the ball's landing location.

b. A soccer ball is kicked horizontally off a 22.0-meter high hill and lands a distance of 35.0 meters from the edge of the hill. Determine the initial horizontal velocity of the soccer ball.

Problem Type 2:

A projectile is launched at an angle to the horizontal and rises upwards to a peak while moving horizontally. Upon reaching the peak, the projectile falls with a motion that is symmetrical to its path upwards to the peak. Predictable unknowns include the time of flight, the horizontal range, and the height of the projectile when it is at its peak.



This projectile is launched at an angle to the horizontal; it rises upward to a peak before falling back down.

Examples of this type of problem are

a. A football is kicked with an initial velocity of 25 m/s at an angle of 45-degrees with the horizontal. Determine the time of flight, the horizontal distance, and the peak height of the football.

b. A long jumper leaves the ground with an initial velocity of 12 m/s at an angle of 28-degrees above the horizontal. Determine the time of flight, the horizontal distance, and the peak height of the long-jumper.

The second problem type will be the subject of the next part of Lesson 2. In this part of Lesson 2, we will focus on the first type of problem - sometimes referred to as horizontally launched projectile problems. Three common kinematic equations that will be used for both type of problems include the following:

$$d = v_i * t + \frac{1}{2} * a * t^2$$

$$v_f = v_i + a * t$$

$$v_f^2 = v_i^2 + 2 * a * d$$

d = displacement a = acceleration t = time

v_f = final velocity v_i = initial velocity

Equations for the Horizontal Motion of a Projectile

The above equations work well for motion in one-dimension, but a projectile is usually moving in two dimensions - both horizontally and vertically. Since these two components of motion are independent of each other, two distinctly separate sets of equations are needed - one for the projectile's horizontal motion and one for its vertical motion. Thus, the three equations above are transformed into two sets of three equations. For the horizontal components of motion, the equations are

$$x = v_{ix} * t + \frac{1}{2} * a_x * t^2$$

$$v_{fx} = v_{ix} + a_x * t$$

$$v_{fx}^2 = v_{ix}^2 + 2 * a_x * x$$

x = horiz. displacement a_x = horiz. acceler'n t = time

v_{ix} = init. horiz. velocity v_{fx} = final horiz. velocity

Of these three equations, the top equation is the most commonly used. An application of projectile concepts to each of these equations would also lead one to conclude that any term with a_x in it would cancel out of the equation since $a_x = 0$ m/s/s.

Equations for the Vertical Motion of a Projectile

For the vertical components of motion, the three equations are

$$y = v_{iy} * t + \frac{1}{2} * a_y * t^2$$

$$v_{fy} = v_{iy} + a_y * t$$

$$v_{fy}^2 = v_{iy}^2 + 2 * a_y * y$$

y = vert. displacement $\quad a_y$ = vert. acceler'n $\quad t$ = time

v_{iy} = init. vert. velocity $\quad v_{fy}$ = final vert. velocity

In each of the above equations, the vertical acceleration of a projectile is known to be -9.8 m/s/s (the acceleration of gravity). Furthermore, for the special case of the first type of problem (horizontally launched projectile problems), v_{iy} = 0 m/s. Thus, any term with v_{iy} in it will cancel out of the equation.

The two sets of three equations above are the kinematic equations that will be used to solve projectile motion problems.

Solving Projectile Problems

To illustrate the usefulness of the above equations in making predictions about the motion of a projectile, consider the solution to the following problem.

Example

A pool ball leaves a 0.60-meter high table with an initial horizontal velocity of 2.4 m/s. Predict the time required for the pool ball to fall to the ground and the horizontal distance between the table's edge and the ball's landing location.

The solution of this problem begins by equating the known or given values with the symbols of the kinematic equations - x, y, v_{ix}, v_{iy}, a_x, a_y, and t. Because horizontal and vertical information is used separately, it is a wise idea to organized the given information in two columns - one column for horizontal information and one column for vertical information. In this case, the following information is either given or implied in the problem statement:

Horizontal Information	Vertical Information
x = ???	y = -0.60 m
v_{ix} = 2.4 m/s	v_{iy} = 0 m/s
a_x = 0 m/s/s	a_y = -9.8 m/s/s

As indicated in the table, the unknown quantity is the horizontal displacement (and the time of flight) of the pool ball. The solution of the problem now requires the selection of an appropriate strategy for using the kinematic equations and the known information to solve for the unknown quantities. It will almost always be the case that such a strategy demands that one of the vertical equations be used to determine the time of flight of the projectile and then one of the horizontal equations be used to find the other unknown quantities (or vice versa - first use the horizontal and then the vertical equation). An organized listing of known quantities (as in the table above) provides cues for the selection of the strategy. For example, the table above reveals that there are three quantities known about the vertical motion of the pool ball. Since each equation has four variables in it, knowledge of three of the variables allows one to calculate a fourth variable. Thus, it would be reasonable that a vertical equation is used with the vertical values to determine time and then the horizontal equations be used to determine the horizontal displacement (x). The first vertical equation ($y = v_{iy} \bullet t + 0.5 \bullet a_y \bullet t^2$) will allow for the determination of the time. Once the appropriate equation has been selected, the physics problem becomes transformed into an algebra problem. By substitution of known values, the equation takes the form of

$$-0.60 \text{ m} = (0 \text{ m/s}) \bullet t + 0.5 \bullet (-9.8 \text{ m/s/s}) \bullet t^2$$

Since the first term on the right side of the equation reduces to 0, the equation can be simplified to

$$-0.60 \text{ m} = (-4.9 \text{ m/s/s}) \bullet t^2$$

If both sides of the equation are divided by -5.0 m/s/s, the equation becomes

$$0.122 \text{ s}^2 = t^2$$

By taking the square root of both sides of the equation, the time of flight can then be determined.

$$t = 0.350 \text{ s (rounded from 0.3499 s)}$$

Once the time has been determined, a horizontal equation can be used to determine the horizontal displacement of the pool ball. Recall from the given information, v_{ix} = 2.4 m/s and a_x = 0 m/s/s. The first horizontal equation ($x = v_{ix} \bullet t + 0.5 \bullet a_x \bullet t^2$) can then be used to solve for "x." With the equation selected, the physics problem once more becomes transformed into an algebra problem. By substitution of known values, the equation takes the form of

$$x = (2.4 \text{ m/s}) \bullet (0.3499 \text{ s}) + 0.5 \bullet (0 \text{ m/s/s}) \bullet (0.3499 \text{ s})^2$$

Since the second term on the right side of the equation reduces to 0, the equation can then be simplified to

$$x = (2.4 \text{ m/s}) \bullet (0.3499 \text{ s})$$

Thus,

$$x = 0.84 \text{ m (rounded from 0.8398 m)}$$

The answer to the stated problem is that the pool ball is in the air for 0.35 seconds and lands a horizontal distance of 0.84 m from the edge of the pool table.

The following procedure summarizes the above problem-solving approach.

a. Carefully read the problem and list known and unknown information in terms of the symbols of the kinematic equations. For convenience sake, make a table with horizontal information on one side and vertical information on the other side.

b. Identify the unknown quantity that the problem requests you to solve for.

c. Select either a horizontal or vertical equation to solve for the time of flight of the projectile.

d. With the time determined, use one of the other equations to solve for the unknown. (Usually, if a horizontal equation is used to solve for time, then a vertical equation can be used to solve for the final unknown quantity.)

One caution is in order. The sole reliance upon 4- and 5-step procedures to solve physics problems is always a dangerous approach. Physics problems are usually just that - problems! While problems can often be simplified by the use of short procedures as the one above, not all problems can be solved with the above procedure. While steps 1 and 2 above are critical to your success in solving horizontally launched projectile problems, there will always be a problem that doesn't *fit the mold*. Problem solving is not like cooking; it is not a mere matter of following a recipe. Rather, problem solving requires careful reading, a firm grasp of conceptual physics, critical thought and analysis, and lots of disciplined practice. Never divorce conceptual understanding and critical thinking from your approach to solving problems.

Check Your Understanding

A soccer ball is kicked horizontally off a 22.0-meter high hill and lands a distance of 35.0 meters from the edge of the hill. Determine the initial horizontal velocity of the soccer ball.

Non-Horizontally Launched Projectile Problems

In the previous part of Lesson 2, the use of kinematic equations to solve projectile problems was introduced and demonstrated. These equations were used to solve problems involving the launching of projectiles in a horizontal direction from an elevated position. In this section of Lesson 2, the use of kinematic equations to solve non-horizontally launched projectiles will be demonstrated. A non-horizontally launched projectile is a projectile that begins its motion with an initial velocity that is both horizontal and vertical. To treat such problems, the same principles that were discussed earlier in Lesson 2 will have to be combined with the kinematic equations for projectile motion. You may recall from earlier that there are two sets of kinematic equations - a set of equations for the horizontal components of motion and a similar set for the vertical components of motion. For the horizontal components of motion, the equations are

$$x = v_{ix} * t + \frac{1}{2} * a_x * t^2$$

$$v_{fx} = v_{ix} + a_x * t$$

$$v_{fx}^2 = v_{ix}^2 + 2 * a_x * x$$

x = horiz. displacement $\quad a_x$ = horiz. acceler'n $\quad t$ = time

v_{ix} = init. horiz. velocity $\quad v_{fx}$ = final horiz. velocity

Of these three equations, the top equation is the most commonly used. The other two equations are seldom (if ever) used. An application of projectile concepts to each of these equations would also lead one to conclude that any term with a_x in it would cancel out of the equation since $a_x = 0$ m/s/s.

For the vertical components of motion, the three equations are

$$y = v_{iy} * t + \frac{1}{2} * a_y * t^2$$

$$v_{fy} = v_{iy} + a_y * t$$

$$v_{fy}^2 = v_{iy}^2 + 2 * a_y * y$$

y = vert. displacement $\quad a_y$ = vert. acceler'n $\quad t$ = time

v_{iy} = init. vert. velocity $\quad v_{fy}$ = final vert. velocity

In each of the above equations, the vertical acceleration of a projectile is known to be -9.8 m/s/s (the acceleration of gravity).

As discussed earlier in Lesson 2, the v_{ix} and v_{iy} values in each of the above sets of kinematic equations can be determined by the use of trigonometric functions. The initial x-velocity (v_{ix}) can be found using the equation $v_{ix} = v_i \bullet \text{cosine(Theta)}$ where Theta is the angle that the velocity vector makes with the horizontal. The initial y-velocity (v_{iy}) can be found using the equation $v_{iy} = v_i \bullet \text{sine(Theta)}$ where Theta is the angle that the velocity vector makes with the horizontal. The topic of components of the velocity vector was discussed earlier in Lesson 2.

To illustrate the usefulness of the above equations in making predictions about the motion of a projectile, consider their use in the solution of the following problem.

Example

A football is kicked with an initial velocity of 25 m/s at an angle of 45-degrees with the horizontal. Determine the time of flight, the horizontal displacement, and the peak height of the football.

The solution of any non-horizontally launched projectile problem (in which v_i and Theta are given) should begin by first resolving the initial velocity into horizontal and vertical components using the trigonometric functions discussed above. Thus,

Horizontal Component	Vertical Component
$v_{ix} = v_i \bullet \cos(\text{Theta})$	$v_{iy} = v_i \bullet \sin(\text{Theta})$
v_{ix} = 25 m/s•cos(45 deg)	v_{iy} = 25 m/s•sin(45 deg)
v_{ix} = 17.7 m/s	v_{iy} = 17.7 m/s

In this case, it happens that the v_{ix} and the v_{iy} values are the same as will always be the case when the angle is 45-degrees.

The solution continues by declaring the values of the known information in terms of the symbols of the kinematic equations - x, y, v_{ix}, v_{iy}, a_x, a_y, and t. In this case, the following information is either explicitly given or implied in the problem statement:

Horizontal Information	Vertical Information
x = ???	**y** = ???
v_{ix} = 17.7 m/s	**v_{iy}** = 17.7 m/s
v_{fx} = 17.7 m/s	**v_{fy}** = -17.7 m/s
a_x = 0 m/s/s	**a_y** = -9.8 m/s/s

As indicated in the table, the final x-velocity (v_{fx}) is the same as the initial x-velocity (v_{ix}). This is due to the fact that the horizontal velocity of a projectile is constant; there is no horizontal acceleration. The table also indicates that the final y-velocity (v_{fy}) has the same magnitude and the opposite direction as the initial y-velocity (v_{iy}). This is due to the symmetrical nature of a projectile's trajectory.

The unknown quantities are the horizontal displacement, the time of flight, and the height of the football at its peak. The solution of the problem now requires the selection of an appropriate strategy for using the kinematic equations and the known information to solve for the unknown quantities. There are a variety of possible strategies for solving the problem. An organized listing of known quantities in two columns of a table provides clues for the selection of a useful strategy.

From the vertical information in the table above and the second equation listed among the vertical kinematic equations ($v_{fy} = v_{iy} + a_y{*}t$), it becomes obvious that the time of flight of the projectile can be determined. By substitution of known values, the equation takes the form of

$$-17.7 \text{ m/s} = 17.7 \text{ m/s} + (-9.8 \text{ m/s/s}) \bullet t$$

The physics problem now takes the form of an algebra problem. By subtracting 17.7 m/s from each side of the equation, the equation becomes

$$-35.4 \text{ m/s} = (-9.8 \text{ m/s/s}) \bullet t$$

If both sides of the equation are divided by -9.8 m/s/s, the equation becomes

$$3.61\ \text{s} = t$$

(rounded from 3.6077 s)

The total time of flight of the football is 3.61 seconds.

With the time determined, information in the table and the horizontal kinematic equations can be used to determine the horizontal displacement (x) of the projectile. The first equation ($x = v_{ix} \bullet t + 0.5 \bullet a_x \bullet t^2$) listed among the horizontal kinematic equations is suitable for determining x. With the equation selected, the physics problem once more becomes transformed into an algebra problem. By substitution of known values, the equation takes the form of

$$x = (17.7\ \text{m/s}) \bullet (3.6077\ \text{s}) + 0.5 \bullet (0\ \text{m/s/s}) \bullet (3.6077\ \text{s})^2$$

Since the second term on the right side of the equation reduces to 0, the equation can then be simplified to

$$x = (17.7\ \text{m/s}) \bullet (3.6077\ \text{s})$$

Thus,

$$x = 63.8\ \text{m}$$

The horizontal displacement of the projectile is 63.8 m.

Finally, the problem statement asks for the height of the projectile at is peak. This is the same as asking, "what is the vertical displacement (y) of the projectile when it is halfway through its trajectory?" In other words, find y when t = 1.80 seconds (one-half of the total time). To determine the peak height of the projectile (y with t = 1.80 sec), the first equation ($y = v_{iy} \bullet t + 0.5 \bullet a_y \bullet t^2$) listed among the vertical kinematic equations can be used. By substitution of known values into this equation, it takes the form of

$$y = (17.7\ \text{m/s}) \bullet (1.80\ \text{s}) + 0.5 * (-10\ \text{m/s/s}) \bullet (1.80\ \text{s})^2$$

Using a calculator, this equation can be simplified to

$$y = 31.9\ \text{m} + (-15.9\ \text{m})$$

And thus,

$$y = 15.9\ \text{m}$$

The solution to the problem statement yields the following answers: the time of flight of the football is 3.61 s, the horizontal displacement of the football is 63.8 m, and the peak height of the football 15.9 m.

(Note that in all calculations performed above, unrounded numbers were used. The numbers reported in the preliminary steps and in the final answer were the rounded form of the actual unrounded values.)

The following procedure summarizes the above problem-solving approach.

a. Use the given values of the initial velocity (the magnitude and the angle) to determine the horizontal and vertical components of the velocity (v_{ix} and v_{iy}).

b. Carefully read the problem and list known and unknown information in terms of the symbols of the kinematic equations. For convenience sake, make a table with horizontal information on one side and vertical information on the other side.

c. Identify the unknown quantity that the problem requests you to solve for.

d. Select either a horizontal or vertical equation to solve for the time of flight of the projectile. For non-horizontally launched projectiles, the second equation listed among the vertical equations ($v_{fy} = v_{iy} + a_y*t$) is usually the most useful equation.

e. With the time determined, use a horizontal equation (usually $x = v_{ix}*t + 0.5*a_x*t^2$) to determine the horizontal displacement of the projectile.

f. Finally, the peak height of the projectile can be found using a time value that is one-half the total time of flight. The most useful equation for this is usually $y = v_{iy}*t +0.5*a_y*t^2$.

One caution is in order: the sole reliance upon 4- and 5-step procedures to solve physics problems is always a dangerous approach. Physics problems are usually just that - problems! And problems can often be simplified by the use of short procedures as the one above. However, not all problems can be solved with the above procedure. While steps 1, 2 and 3 above are critical to your success in solving non-horizontally launched projectile problems, there will always be a problem that doesn't "fit the mold." Problem solving is not like cooking; it is not a mere matter of following a recipe. Rather, problem solving requires careful reading, a firm grasp of conceptual physics, critical thought and analysis, and lots of disciplined practice. Never divorce conceptual understanding and critical thinking from your approach to solving problems.

Your Turn to Try It!

Use the **Range of an Angle-Launched Projectile** *widget* to practice a projectile problem (or two) (or three). Using the given launch velocity and launch angle, determine the expected horizontal displacement (d_x). After completing your calculation, use the **Submit** button to check your answer.

Range of an Angle-Launched Projectile

Launch Velocity:	25	m/s
Launch Angle:	30	°

$$y = 55.2312$$

See http://www.physicsclassroom.com/Class/vectors/U3L2a.cfm.

Check Your Understanding

A long jumper leaves the ground with an initial velocity of 12 m/s at an angle of 28-degrees above the horizontal. Determine the time of flight, the horizontal distance, and the peak height of the long-jumper.

Vectors: Motion and Forces in Two Dimensions - Lesson 3

Forces in Two Dimensions

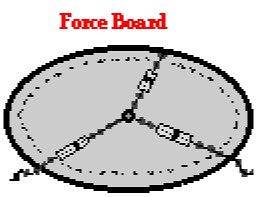
Addition of Forces | Resolution of Forces | Equilibrium and Statics | Net Force Problems Revisited
Inclined Planes | Double Trouble in 2 Dimensions

Addition of Forces

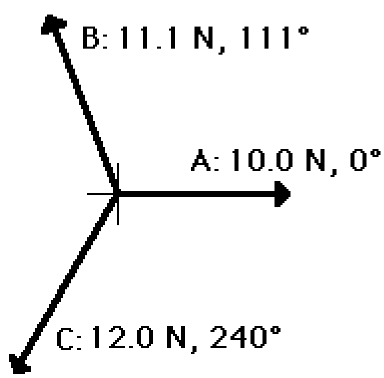
In Unit 2 we studied the use of Newton's second law and free-body diagrams to determine the net force and acceleration of objects. In that unit, the forces acting upon objects were always directed in one dimension. There may have been both horizontal and vertical forces acting upon objects; yet there were never individual forces that were directed both horizontally and vertically. Furthermore, when a free-body diagram analysis was performed, the net force was either horizontal or vertical; the net force (and corresponding acceleration) was never both horizontal and vertical. Now times have changed and you are ready for situations involving forces in two dimensions. In this unit, we will examine the affect of forces acting at angles to the horizontal, such that the force has an influence in two dimensions - horizontally and vertically. For such situations, Newton's second law applies as it always did for situations involving one-dimensional net forces. However, to use Newton's laws, common vector operations such as vector addition and vector resolution will have to be applied. In this part of Lesson 3, the rules for adding vectors will be reviewed and applied to the addition of force vectors.

Methods of adding vectors were discussed earlier in Lesson 1 of this unit. During that discussion, the head to tail method of vector addition was introduced as a useful method of adding vectors that are not at right angles to each other. Now we will see how that method applies to situations involving the addition of force vectors.

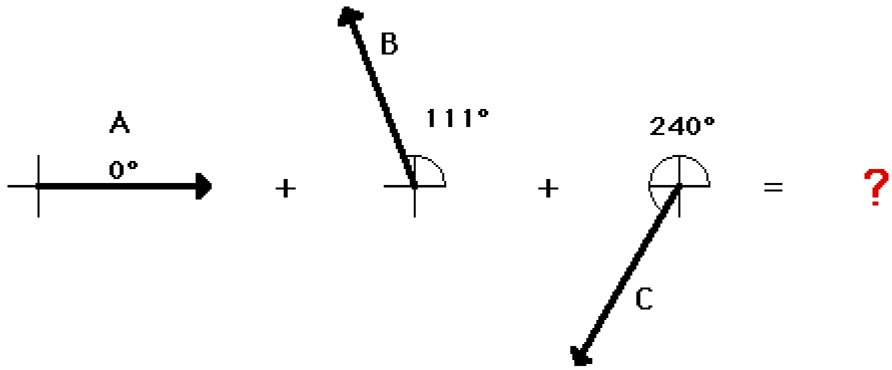
A force board (or force table) is a common physics lab apparatus that has three (or more) chains or cables attached to a center ring. The chains or cables exert forces upon the center ring in three different directions. Typically the experimenter adjusts the direction of the three forces, makes measurements of the amount of force in each direction, and determines the vector sum of three forces. Forces perpendicular to the plane of the force board are typically ignored in the analysis.



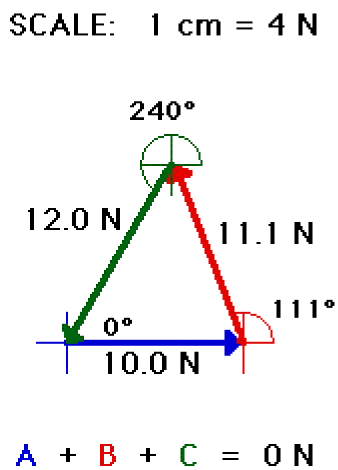
Suppose that a force board or a force table is used such that there are three forces acting upon an object. (The object is the ring in the center of the force board or force table.) In this situation, two of the forces are acting in two-dimensions. A *top view* of these three forces could be represented by the following diagram.



The goal of a force analysis is to determine the net force and the corresponding acceleration. The net force is the vector sum of all the forces. That is, the net force is the resultant of all the forces; it is the result of adding all the forces together as vectors. For the situation of the three forces on the force board, the net force is the sum of force vectors A + B + C.



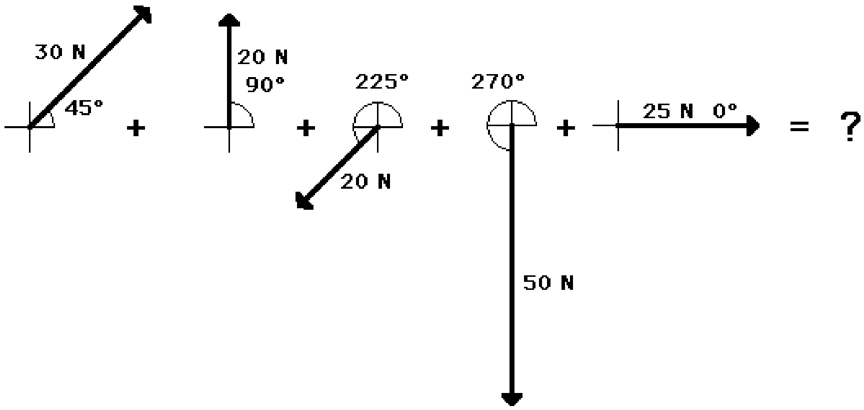
One method of determining the vector sum of these three forces (i.e., the net force") is to employ the method of head-to-tail addition. In this method, an accurately drawn scaled diagram is used and each individual vector is drawn to scale. Where the head of one vector ends, the tail of the next vector begins. Once all vectors are added, the resultant (i.e., the vector sum) can be determined by drawing a vector from the tail of the first vector to the head of the last vector. This procedure is shown below. The three vectors are added using the head-to-tail method. Incidentally, the vector sum of the three vectors is 0 Newton - the three vectors add up to 0 Newton. The last vector ends where the first vector began such that there is no resultant vector.

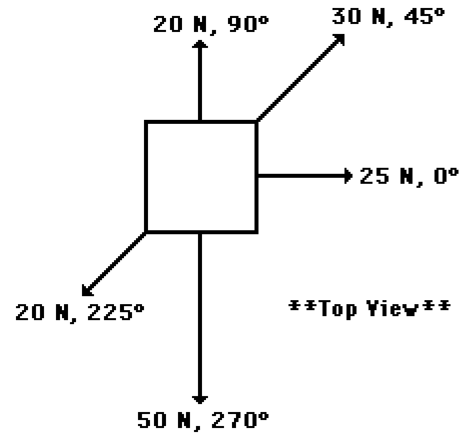


The purpose of adding force vectors is to determine the net force acting upon an object. In the above case, the net force (vector sum of all the forces) is 0 Newton. This would be expected for the situation since the object (the ring in the center of the force table) is at rest and staying at rest. We would say that the object is at *equilibrium*. Any object upon which all the forces are balanced (F_{net} = 0 N) is said to be at *equilibrium*.

Quite obviously, the net force is <u>not always</u> 0 Newton. In fact, whenever objects are accelerating, the forces will <u>not</u> balance and the net force will be <u>nonzero</u>. This is consistent with Newton's first law of motion. For example consider the situation described below.

An Example to Test Your Understanding

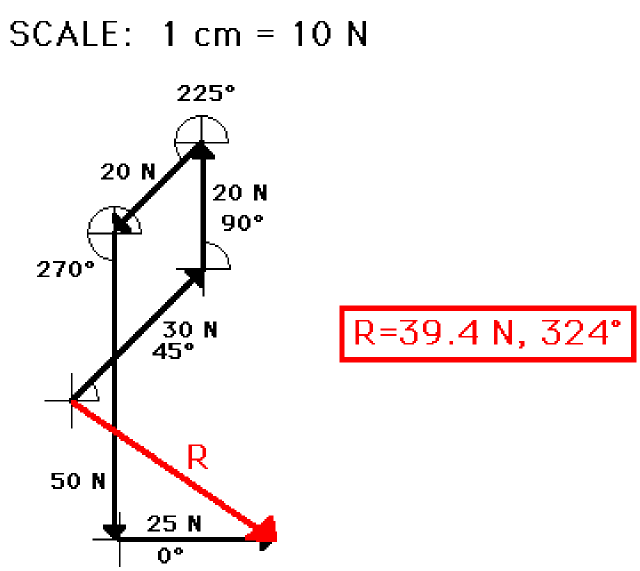




A pack of five Artic wolves are exerting five different forces upon the carcass of a 500-kg dead polar bear. A *top view* showing the magnitude and direction of each of the five individual forces is shown in the diagram at the right. The counterclockwise convention is used to indicate the direction of each force vector. Remember that this is a top view of the situation and as such does not depict the gravitational and normal forces (since they would be perpendicular to the *plane of your computer monitor*); it can be assumed that the gravitational and normal forces balance each other. Use a scaled vector diagram to determine the net force acting upon the polar bear. Then compute the acceleration of the polar bear (both magnitude and direction). When finished, check your answer by clicking the button and then view the solution to the problem by analyzing the diagrams shown below.

The task of determining the vector sum of all the forces for the polar bear problem involves constructing an accurately drawn scaled vector diagram in which all five forces are added head-to-tail. The following five forces must be added.

The scaled vector diagram for this problem would look like the following:

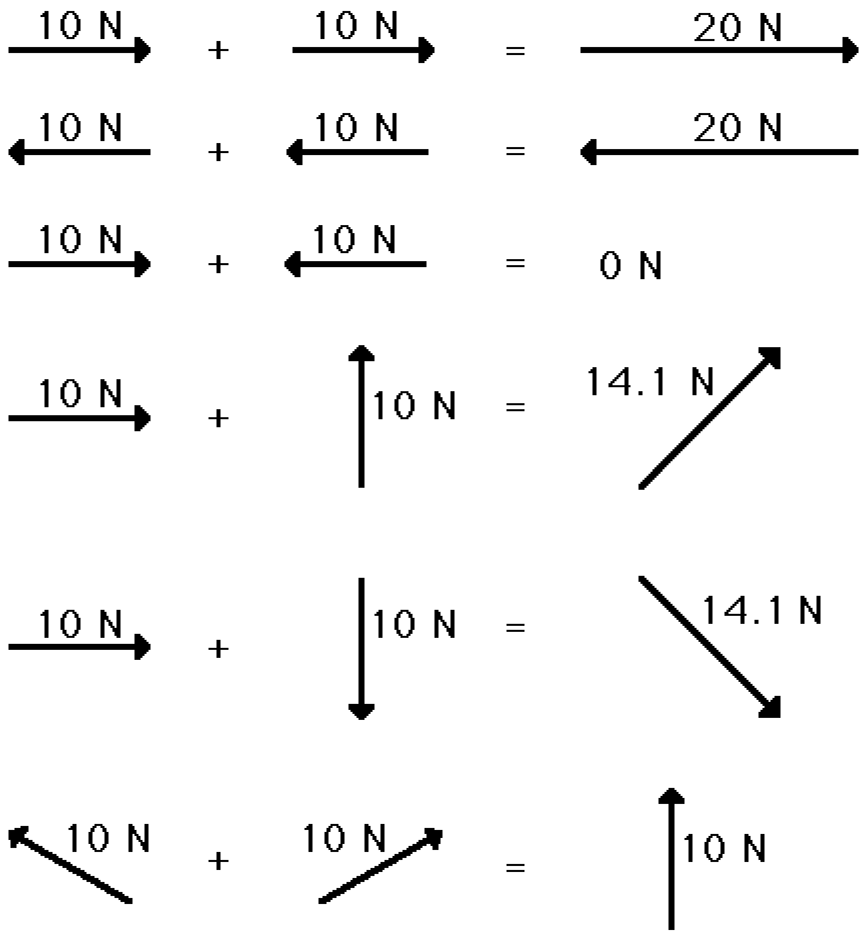


The above two problems (the force table problem and the polar bear problem) illustrate the use of the head-to-tail method for determining the vector sum of all the forces. The resultants in each of the above diagrams represent the net force acting upon the object. This net force is related to the acceleration of the object. Thus, to put the contents of this page in perspective with other material studied in this course, vector addition methods can be utilized to determine the sum of all the forces acting upon an object and subsequently the acceleration of that object. And the acceleration of an object can be combined with kinematic equations to determine motion information (i.e., the final velocity, the distance traveled, etc.) for a given object.

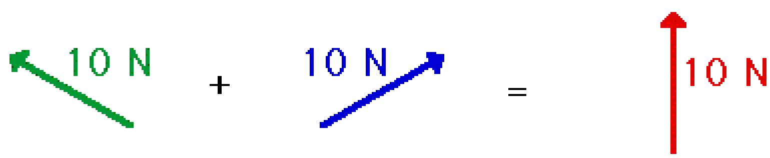
In addition to knowing graphical methods of adding the forces acting upon an object, it is also important to have a conceptual grasp of the principles of adding forces. Let's begin by considering the addition of two forces, both having a magnitude of 10 Newton. Suppose the question is posed:

10 Newton + 10 Newton = ???

How would you answer such a question? Would you quickly conclude 20 Newton, thinking that two force vectors can be added like any two numerical quantities? Would you pause for a moment and think that the quantities to be added are vectors (force vectors) and the addition of vectors follow a different set of rules than the addition of scalars? Would you pause for a moment, pondering the possible ways of adding 10 Newton and 10 Newton and conclude, "it depends upon their direction?" In fact, 10 Newton + 10 Newton could give almost any resultant, provided that it has a magnitude between 0 Newton and 20 Newton. Study the diagram below in which 10 Newton and 10 Newton are added to give a variety of answers; each answer is dependent upon the direction of the two vectors that are to be added. For this example, the minimum magnitude for the resultant is 0 Newton (occurring when 10 N and 10 N are in the opposite direction); and the maximum magnitude for the resultant is 20 N (occurring when 10 N and 10 N are in the same direction).

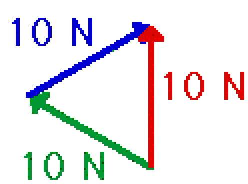


The above diagram shows what is occasionally a difficult concept to believe. Many students find it difficult to see how 10 N + 10 N could ever be equal to 10 N. For reasons to be discussed in the next section of this lesson, 10 N + 10 N would equal 10 N whenever the two forces to be added are at 30 degrees to the horizontal. For now, it ought to be sufficient to merely show a simple vector addition diagram for the addition of the two forces (see diagram below).



Scaled Vector Addition Diagram

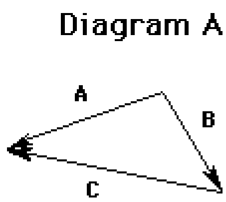
1 cm = 5 N

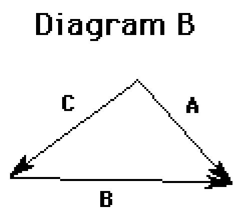


Check Your Understanding

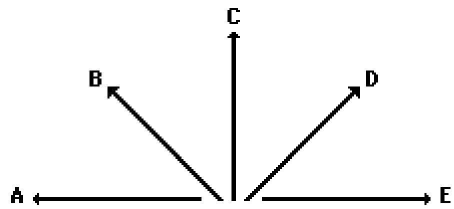
Answer the following questions and then view the answers by clicking on the button.

1. Barb Dwyer recently submitted her vector addition homework assignment. As seen below, Barb added two vectors and drew the resultant. However, Barb Dwyer failed to label the resultant on the diagram. For each case, that is the resultant (A, B, or C)? Explain.





2. Consider the following five force vectors.



Sketch the following and draw the resultant (R). Do not draw a scaled vector diagram; merely make a sketch. Label each vector. Clearly label the resultant (R).

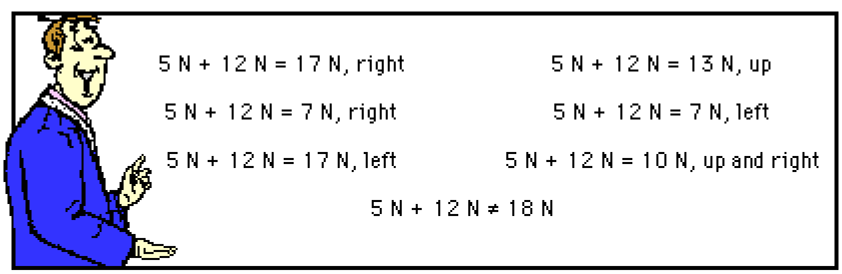
A + C + D

B + E + D

3. On two different occasions during a high school soccer game, the ball was kicked simultaneously by players on opposing teams. In which case (Case 1 or Case 2) does the ball undergo the greatest acceleration? Explain your answer.

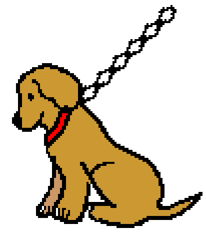
4. Billie Budten and Mia Neezhirt are having an intense argument at the lunch table. They are adding two force vectors together to determine the resultant force. The magnitude of the two forces are 3 N and 4 N. Billie is arguing that the sum of the two forces is 7 N. Mia argues that the two forces add together to equal 5 N. Who is right? Explain.

5. Matt Erznott entered the classroom for his physics class. He quickly became amazed by the remains of some of teacher's whiteboard scribblings. Evidently, the teacher had taught his class on that day that

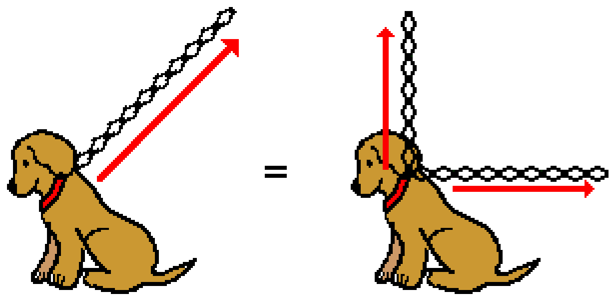


Explain why the equalities are indeed equalities and the inequality must definitely be an inequality.

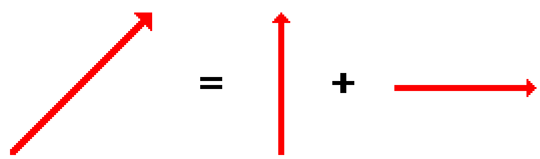
Resolution of Forces



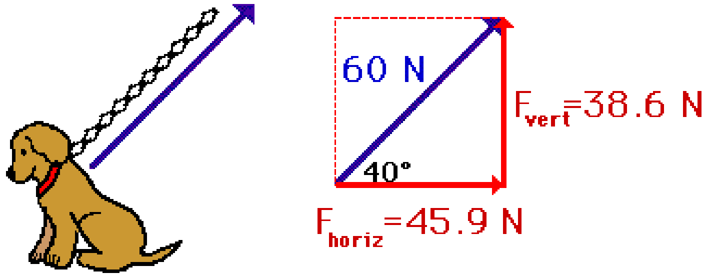
Earlier in Lesson 1, the method of resolving a vector into its components was thoroughly discussed. During that lesson, it was said that any vector that is directed at an angle to the customary coordinate axis can be considered to have two parts - each part being directed along one of the axes - either horizontally or vertically. The parts of the single vector are called *components* and describe the influence of that single vector in that given direction. One example that was given during Lesson 1 was the example of Fido being pulled upon by a dog chain. If the chain is pulled upwards and to the right, then there is a tensional force acting upwards and rightwards upon Fido. That single force can be *resolved* into two components - one directed upwards and the other directed rightwards. Each component describes the influence of that chain in the given direction. The vertical component describes the upward influence of the force upon Fido and the horizontal component describes the rightward influence of the force upon Fido.



The upward and rightward force of the chain is equivalent to an upward force and a rightward force by two chains.



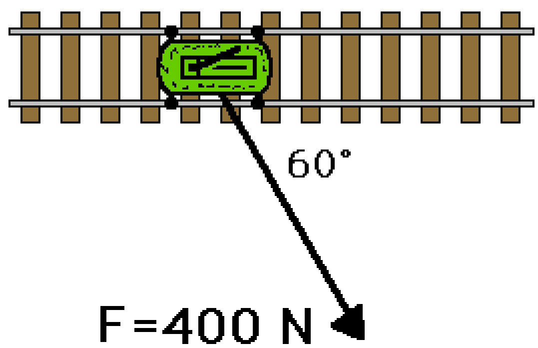
The task of determining the amount of influence of a single vector in a given direction involves the use of trigonometric functions. The use of these functions to determine the components of a single vector was also discussed in Lesson 1 of this unit. As a quick review, let's consider the use of *SOH CAH TOA* to determine the components of force acting upon Fido. Assume that the chain is exerting a 60 N force upon Fido at an angle of 40 degrees above the horizontal. A quick sketch of the situation reveals that to determine the vertical component of force, the sine function can be used and to determine the horizontal component of force, the cosine function can be used. The solution to this problem is shown below.



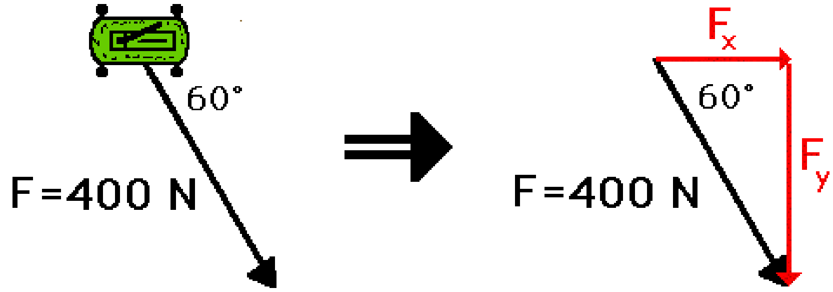
$$\sin 40° = \frac{F_{vert}}{60\ N} \qquad \cos 40° = \frac{F_{horiz}}{60\ N}$$

$$F_{vert} = 60\ N \times \sin 40° \qquad F_{horiz} = 60\ N \times \cos 40°$$

$$F_{vert} = 38.6\ N \qquad F_{horiz} = 45.9\ N$$



As another example of the use of SOH CAH TOA to resolve a single vector into its two components, consider the diagram at the right. A 400-N force is exerted at a 60-degree angle (a direction of 300 degrees) to move a railroad car eastward along a railroad track. A *top view* of the situation is depicted in the diagram. The force applied to the car has both a vertical (southward) and a horizontal component (eastward). To determine the magnitudes of these two components, the sine and cosine function will have to be used. The task is made clearer by beginning with a diagram of the situation with a labeled angle and a labeled hypotenuse. Once a triangle is constructed, it becomes obvious that the sine function will have to be used to determine the *vertical* (southward) component and the cosine function will have to be used to determine the *horizontal* (eastward) component. The triangle and accompanying work is shown below.



$$\sin 60° = \frac{F_y}{400\ \text{N}} \qquad \cos 60° = \frac{F_x}{400\ \text{N}}$$

$$F_y = 400\ \text{N} * \sin 60° \qquad F_x = 400\ \text{N} * \cos 60°$$

$$F_y = 346\ \text{N} \qquad F_x = 200\ \text{N}$$

Quick Quiz Anytime a force vector is directed at an angle to the horizontal, the trigonometric functions can be used to determine the components of that force vector. To assure that you understand the use of SOH CAH TOA to determine the components of a vector, try the following three practice problems.

Diagram A

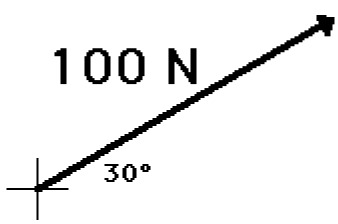


Diagram B

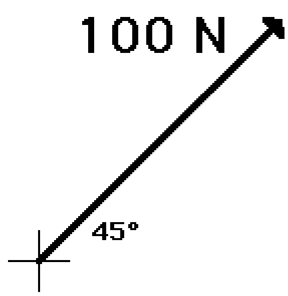
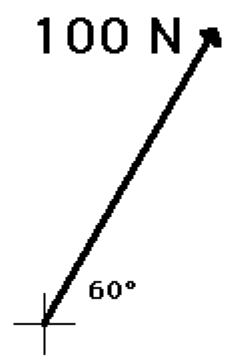
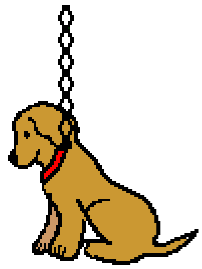


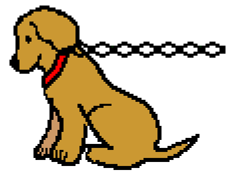
Diagram C



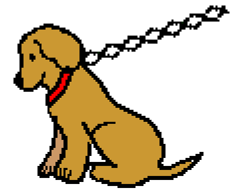
An important concept is revealed by the above three diagrams. Observe that the force is the same magnitude in each diagram; only the angle with the horizontal is changing. As the angle that a force makes with the horizontal increases, the component of force in the horizontal direction (F_x) decreases. The principle makes some sense; the more that a force is directed upwards (the angle with the horizontal increases), the less that the force is able to exert an influence in the horizontal direction. If you wish to drag Fido horizontally, then you would make an effort to pull in as close to a horizontal direction as possible; you would not pull vertically on Fido's chain if you wish to pull him horizontally.



This force exerts no horizontal influence.

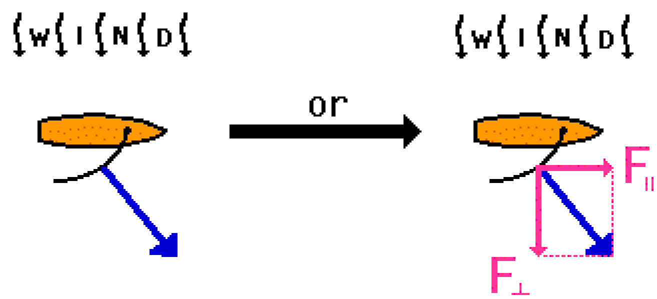


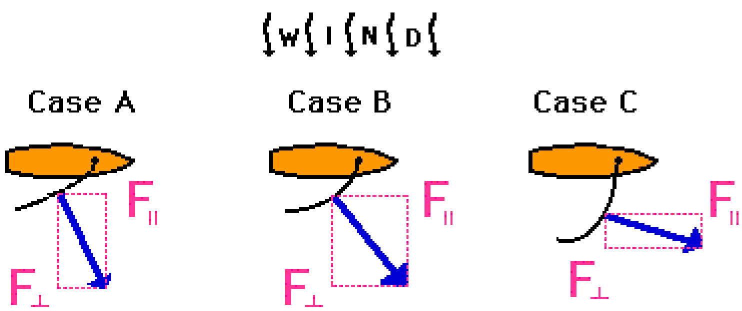
This force exerts no vertical influence.



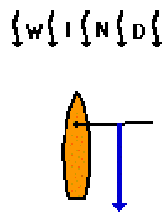
This force exerts both a horizontal and vertical influence, but mostly a horizontal one.

One important application of this principle is in the recreational sport of sail boating. Sailboats encounter a force of wind resistance due to the impact of the moving air molecules against the sail. This force of wind resistance is directed perpendicular to the face of the sail, and as such is often directed at an angle to the direction of the sailboat's motion. The actual direction of this force is dependent upon the orientation of the sail. To determine the influence of the wind resistance force in the direction of motion, that force will have to be resolved into two components - one in the direction that the sailboat is moving and the other in a direction perpendicular to the sailboat's motion. See diagram at right. In the diagram below, three different sail orientations are shown. Assuming that the wind resistance force is the same in each case, which case would produce the greatest influence in the direction of the sailboat's motion? That is, which case has the greatest component of force in the direction parallel to the boats' heading?

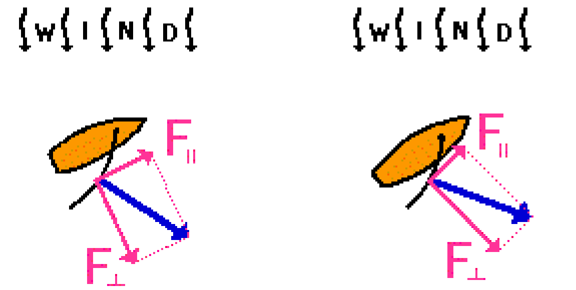




Many people believe that a sailboat cannot travel "upwind." It is their perception that if the wind blows from north to south, then there is no possible way for a sailboat to travel from south to north. This is simply not true. Sailboats can travel "upwind" and commonly do so by a method known as *tacking into the wind*. It is true to say that a sailboat can never travel upwind by heading its boat <u>directly</u> into the wind. As seen in the diagram below, if the boat heads directly into the wind, then the wind force is directed due opposite its heading. In such a case, there is no component of force in the direction that the sailboat is heading. That is, there is no "propelling force." On the other hand, if the boat heads at an angle into the wind, then the wind force can be resolved into two components. In the two orientations of the sailboat shown below, the component of force in the direction parallel to the sailboat's heading will propel the boat at an angle into the wind. When tacking into the wind, a sailboat will typically travel at 45-degree angles, tacking back and forth into the wind.



This sailboat cannot move "upwind" since there is no component of force parallel to the boat's heading.

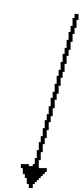


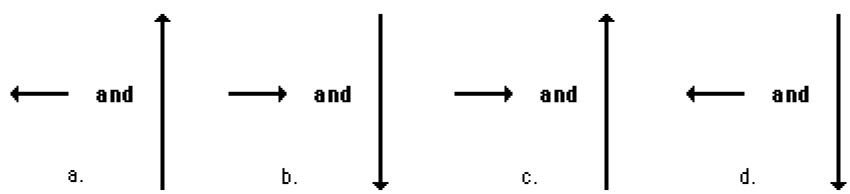
These sailboats can move "upwind" since there is a component of force parallel to the boat's heading.

Check Your Understanding

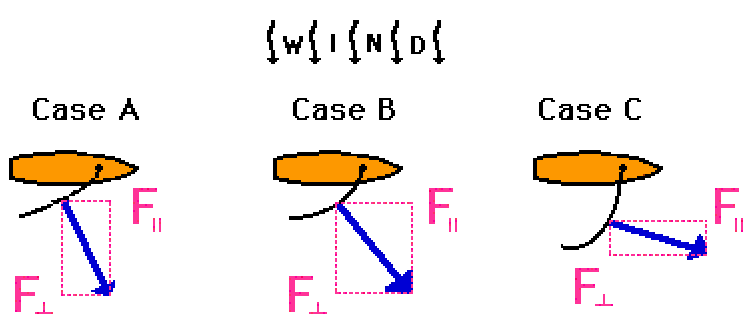
The following problems focus on concepts discussed in this lesson. Answer each question and then click the button to view the answer.

1. The diagram at the right depicts a force that makes an angle to the horizontal. This force will have horizontal and vertical components. Which one of the choices below best depicts the direction of the horizontal and vertical components of this force?



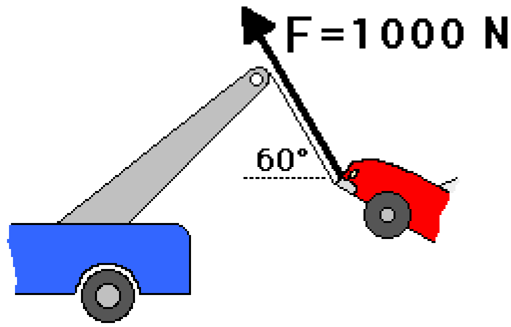


2. Three sailboats are shown below. Each sailboat experiences the same amount of force, yet has different sail orientations.



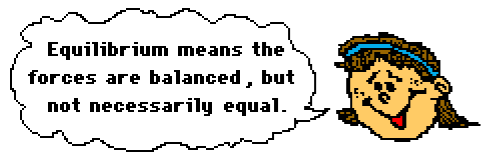
In which case (A, B or C) is the sailboat most likely to *tip over* sideways? Explain.

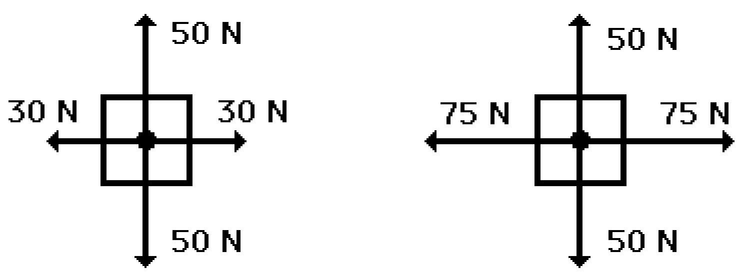
3. Consider the tow truck at the right. If the tensional force in the cable is 1000 N and if the cable makes a 60-degree angle with the horizontal, then what is the vertical component of force that lifts the car off the ground?



Equilibrium and Statics

When all the forces that act upon an object are balanced, then the object is said to be in a state of **equilibrium**. The forces are considered to be balanced if the rightward forces are balanced by the leftward forces and the upward forces are balanced by the downward forces. This however does not necessarily mean that all the forces are *equal* to each other. Consider the two objects pictured in the force diagram shown below. Note that the two objects are at equilibrium because the forces that act upon them are balanced; however, the individual forces are not equal to each other. The 50 N force is not equal to the 30 N force.



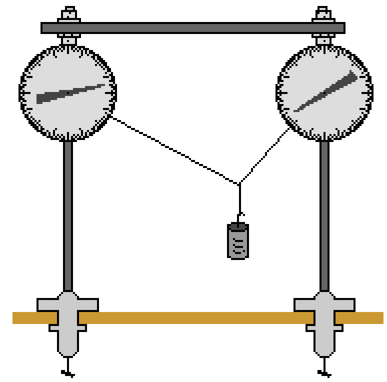


These two objects are at equilibrium since the forces are balanced. However, the forces are not equal.

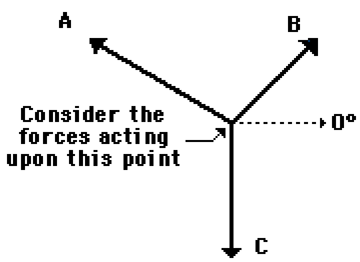
If an object is at equilibrium, then the forces are balanced. *Balanced* is the key word that is used to describe equilibrium situations. Thus, the net force is zero and the acceleration is 0 m/s/s. Objects at equilibrium must have an acceleration of 0 m/s/s. This extends from Newton's first law of motion. But having an acceleration of 0 m/s/s does not mean the object is at rest. An object at equilibrium is either ...

- at rest and staying at rest, or
- in motion and continuing in motion with the same speed and direction.

This too extends from Newton's first law of motion.

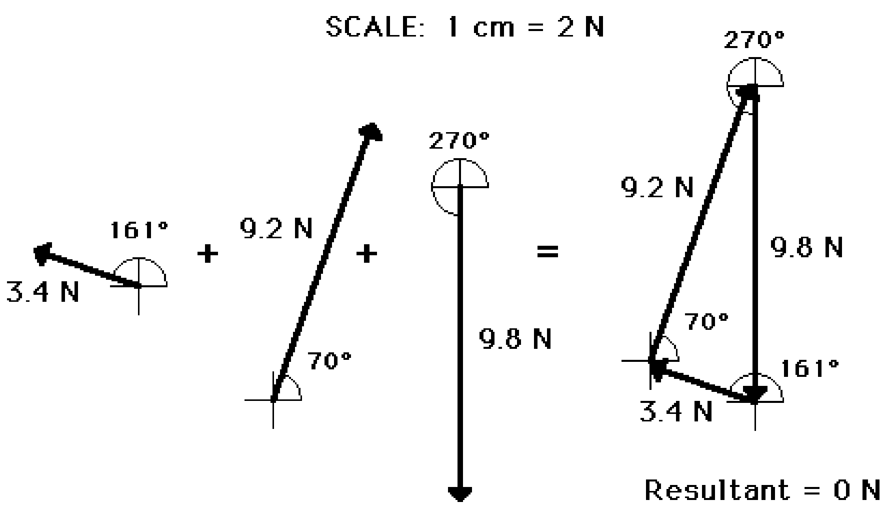


If an object is at rest and is in a state of equilibrium, then we would say that the object is at "static equilibrium." "Static" means *stationary* or *at rest*. A common physics lab is to hand an object by two or more strings and to measure the forces that are exerted at angles upon the object to support its weight. The state of the object is analyzed in terms of the forces acting upon the object. The object is a *point* on a string upon which three forces were acting. See diagram at right. If the object is at equilibrium, then the net force acting upon the object should be 0 Newton. Thus, if all the forces are added together as vectors, then the resultant force (the vector sum) should be 0 Newton. (Recall that the net force is "the vector sum of all the forces" or the resultant of adding all the individual forces head-to-tail.) Thus, an accurately drawn vector addition diagram can be constructed to determine the resultant. Sample data for such a lab are shown below.

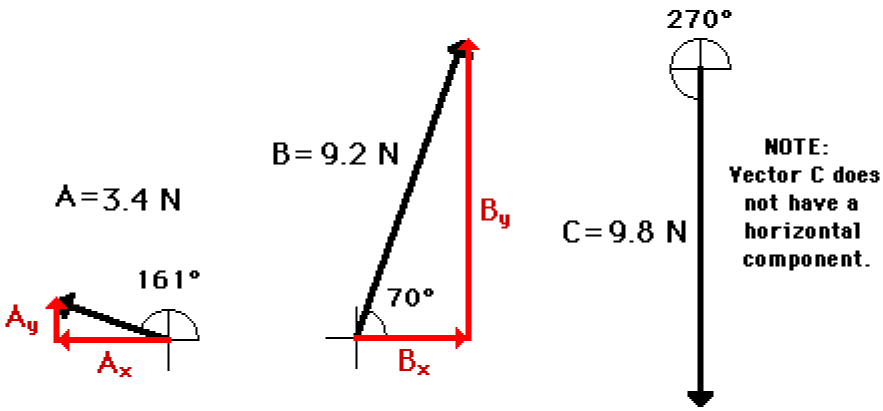


	Force A	Force B	Force C
Magnitude	3.4 N	9.2 N	9.8 N
Direction	161 deg.	70 deg.	270 deg

For most students, the resultant was 0 Newton (or at least very close to 0 N). This is what we expected - since the object was at equilibrium, the net force (vector sum of all the forces) should be 0 N.



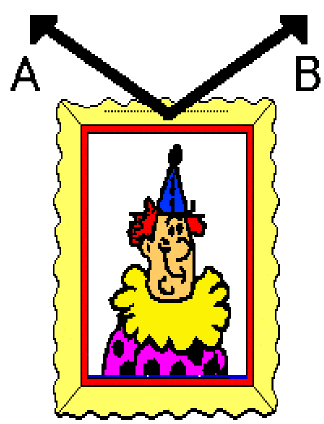
Another way of determining the net force (vector sum of all the forces) involves using the trigonometric functions to resolve each force into its horizontal and vertical components. Once the components are known, they can be compared to see if the vertical forces are balanced and if the horizontal forces are balanced. The diagram below shows vectors A, B, and C and their respective components. For vectors A and B, the vertical components can be determined using the sine of the angle and the horizontal components can be analyzed using the cosine of the angle. The magnitude and direction of each component for the sample data are shown in the table below the diagram.



Force	Horizontal Component (PSYW)	Vertical Component (PSYW)
A	$A_x = 3.4\ N*\cos(161°)$ $A_x = 3.2$ N, left	$A_y = 3.4\ N*\sin(161°)$ $A_y = 1.1$ N, up
B	$B_x = 9.2\ N*\cos(70°)$ $B_x = 3.1$ N, right	$B_y = 9.2\ N*\sin(70°)$ $B_y = 8.6$ N, up
C	$C_x = 0$ N	$C_y = 9.8$ N, down

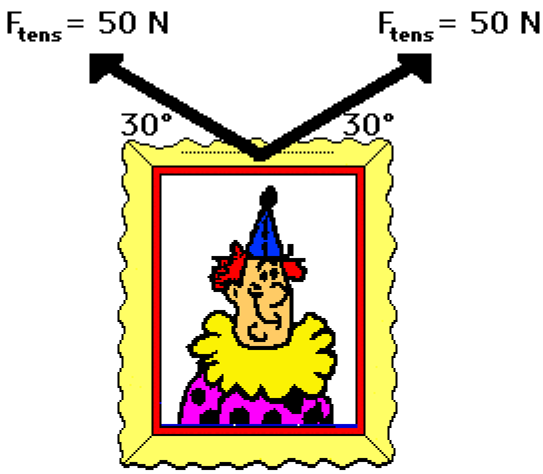
The data in the table above show that the forces *nearly* balance. An analysis of the horizontal components shows that the leftward component of A *nearly* balances the rightward component of B. An analysis of the vertical components show that the sum of the upward components of A + B *nearly* balance the downward component of C. The vector sum of all the forces is (*nearly*) equal to 0 Newton. But what about the 0.1 N difference between rightward and leftward forces and the 0.2 N difference between the upward and downward forces? Why do the components of force only *nearly* balance? The sample data used in this analysis are the result of measured data from an actual experimental setup. The difference between the actual results and the expected results is due to the error incurred when measuring force A and force B. We would have to conclude that this low margin of experimental error reflects an experiment with excellent results. We could say it's "close enough for government work."

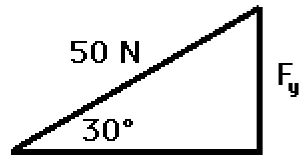
The above analysis of the forces acting upon an object in equilibrium is commonly used to analyze situations involving objects at static equilibrium. The most common application involves the analysis of the forces acting upon a sign that is at rest. For example, consider the picture at the right that hangs on a wall. The picture is in a state of equilibrium, and thus all the forces acting upon the picture must be balanced. That is, all horizontal components must add to 0 Newton and all vertical components must add to 0 Newton. The leftward pull of cable A must balance the rightward pull of cable B and the sum of the upward pull of cable A and cable B must balance the weight of the sign.



Suppose the tension in both of the cables is measured to be 50 N and that the angle that each cable makes with the horizontal is known to be 30 degrees. What is the weight of the sign? This question can be answered by conducting a force analysis using trigonometric functions. The weight of the sign is equal to the sum of the upward components of the tension in the two cables.

Thus, a trigonometric function can be used to determine this vertical component. A diagram and accompanying work is shown below.





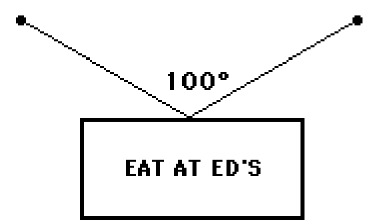
$$\sin 30^\circ = \frac{F_y}{50\text{ N}}$$

$$F_y = 50\text{ N}*\sin 30^\circ$$

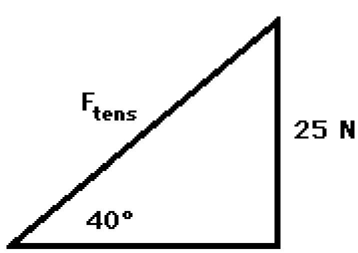
$$F_y = 25\text{ N}$$

Since each cable pulls upwards with a force of 25 N, the total upward pull of the sign is 50 N. Therefore, the force of gravity (also known as weight) is 50 N, down. The sign weighs 50 N.

In the above problem, the tension in the cable and the angle that the cable makes with the horizontal are used to determine the weight of the sign. The idea is that the tension, the angle, and the weight are related. If the any two of these three are known, then the third quantity can be determined using trigonometric functions.



As another example that illustrates this idea, consider the symmetrical hanging of a sign as shown at the right. If the sign is known to have a mass of 5 kg and if the angle between the two cables is 100 degrees, then the tension in the cable can be determined. Assuming that the sign is at equilibrium (a good assumption if it is remaining at rest), the two cables must supply enough upward force to balance the downward force of gravity. The force of gravity (also known as weight) is 49 N (Fgrav = m*g), so each of the two cables must pull upwards with 24.5 N of force. Since the angle between the cables is 100 degrees, then each cable must make a 50-degree angle with the vertical and a 40-degree angle with the horizontal. A sketch of this situation (see diagram below) reveals that the tension in the cable can be found using the sine function. The triangle below illustrates these relationships.

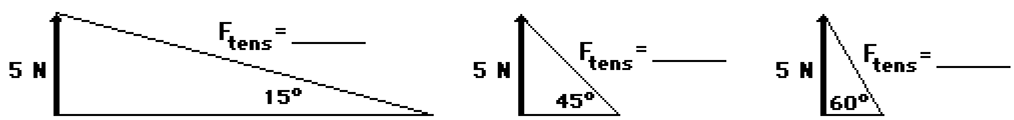


$$\sin 40^\circ = \frac{24.5\text{ N}}{F_{tens}}$$

$$\sin 40^\circ * F_{tens} = 24.5\text{ N}$$

$$F_{tens} = \frac{24.5\text{ N}}{\sin 40^\circ} = 38.1\text{ N}$$

There is an important principle that emanates from some of the trigonometric calculations performed above. The principle is that as the angle with the horizontal increases, the amount of tensional force required to hold the sign at equilibrium decreases. To illustrate this, consider a 10-Newton picture held by three different wire orientations as shown in the diagrams below. In each case, two wires are used to support the picture; each wire must support one-half of the sign's weight (5 N). The angle that the wires make with the horizontal is varied from 60 degrees to 15 degrees. Use this information and the diagram below to determine the tension in the wire for each orientation. When finished, click the button to view the answers.

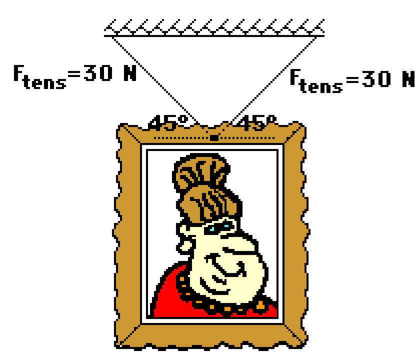


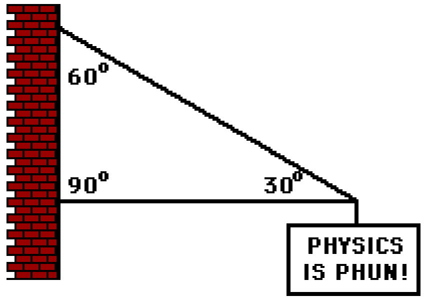
In conclusion, equilibrium is the state of an object in which all the forces acting upon it are balanced. In such cases, the net force is 0 Newton. Knowing the forces acting upon an object, trigonometric functions can be utilized to determine the horizontal and vertical components of each force. If at equilibrium, then all the vertical components must balance and all the horizontal components must balance.

Check Your Understanding

The following questions are meant to test your understanding of equilibrium situations. Click the button to view the answers to these questions.

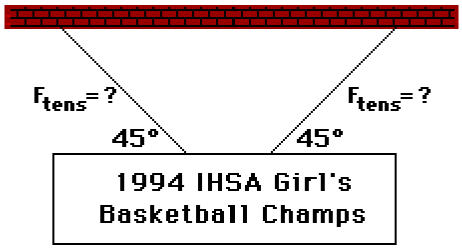
1. The following picture is hanging on a wall. Use trigonometric functions to determine the weight of the picture.



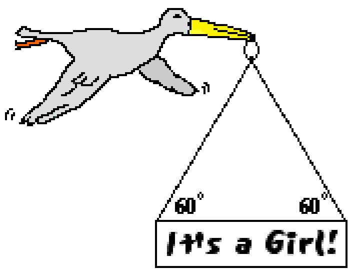


2. The sign below hangs outside the physics classroom, advertising the most important truth to be found inside. The sign is supported by a diagonal cable and a rigid horizontal bar. If the sign has a mass of 50 kg, then determine the tension in the diagonal cable that supports its weight.

3. The following sign can be found in Glenview. The sign has a mass of 50 kg. Determine the tension in the cables.



4. After its most recent delivery, the infamous stork announces the good news. If the sign has a mass of 10 kg, then what is the tensional force in each cable? Use trigonometric functions and a sketch to assist in the solution.

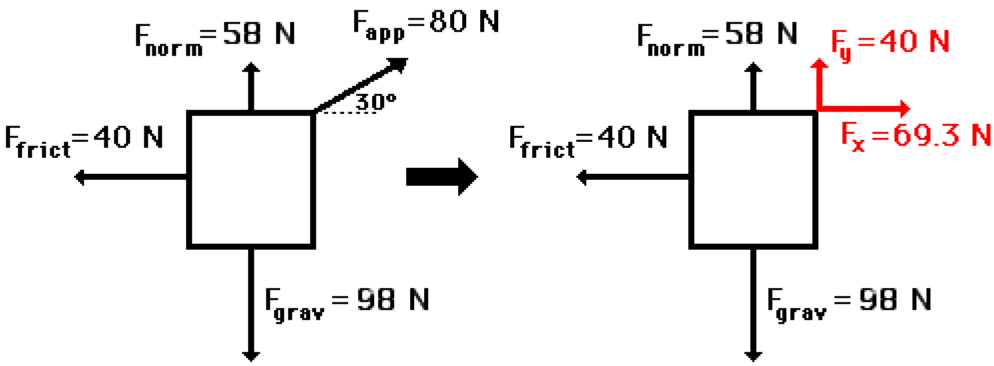


5. Suppose that a student pulls with two large forces (F_1 and F_2) in order to lift a 1-kg book by two cables. If the cables make a 1-degree angle with the horizontal, then what is the tension in the cable?

Net Force Problems Revisited

This part of Lesson 3 focuses on net force-acceleration problems in which an applied force is directed at an angle to the horizontal. We have already discussed earlier in Lesson 3 how a force directed an angle can be resolved into two components - a horizontal and a vertical component. We have also discussed in an earlier unit that the acceleration of an object is related to the net force acting upon the object and the mass of the object (Newton's second law). We had used this principle to solve net force-acceleration problems in an earlier unit. Therefore, it is a natural extension of this unit to combine our understanding of Newton's second law with our understanding of force vectors directed at angles.

Consider the situation below in which a force is directed at an angle to the horizontal. In such a situation, the applied force could be resolved into two components. These two components can be considered to replace the applied force at an angle. By doing so, the situation simplifies into a familiar situation in which all the forces are directed horizontally and vertically.

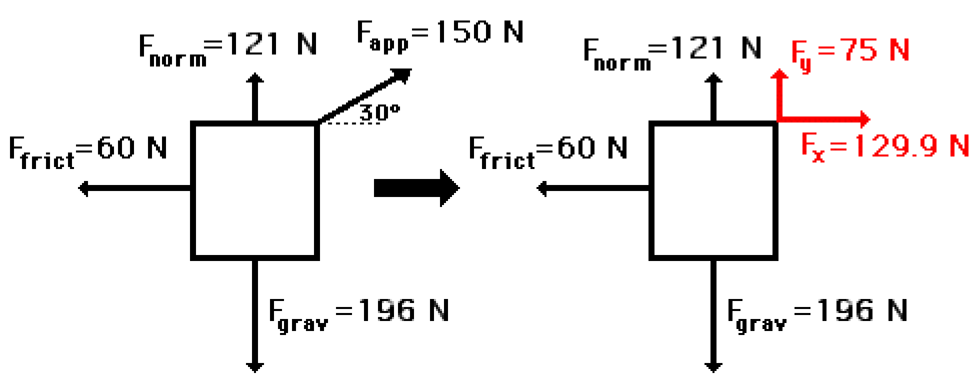


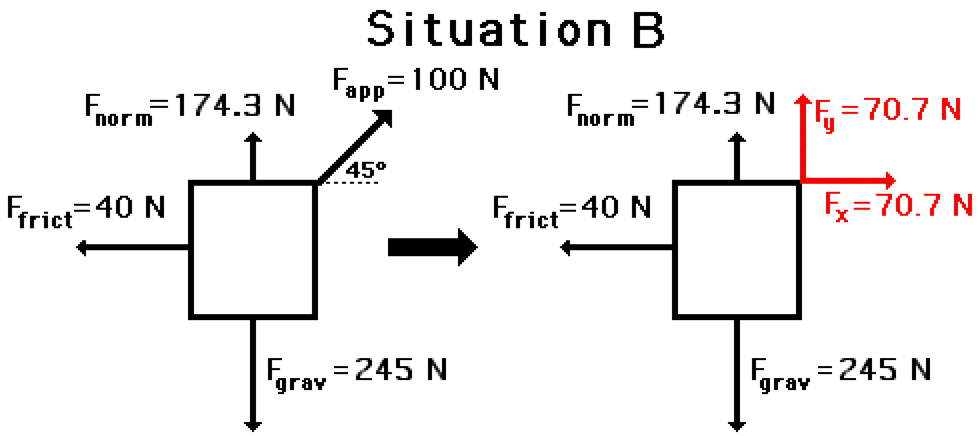
A force directed at an angle to the horizontal can be resolved into two components. Together, these two components are a replacement for the single force.

Once the situation has been simplified, the problem can be solved like any other problem. The task of determining the acceleration involves first determining the net force by adding up all the forces as vectors and then dividing the net force by the mass to determine the acceleration. In the above situation, the vertical forces are balanced (i.e., F_{grav}, F_y, and F_{norm} add up to 0 N), and the horizontal forces add up to 29.3 N, right (i.e., 69.3 N, right + 40 N, left = 29.3 N, right). The net force is 29.3 N, right and the mass is 10 kg ($m = F_{grav}/g$); therefore, the acceleration is 2.93 m/s/s, right.

To test your understanding, analyze the two situations below to determine the net force and the acceleration. When finished, click the button to view the answers.

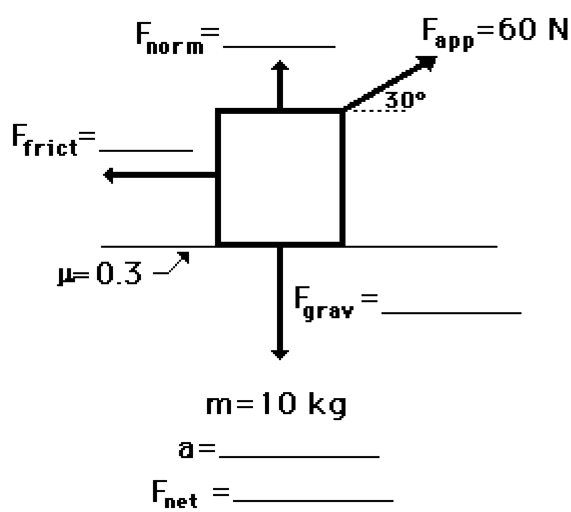
Situation A





There is one peculiarity about these types of problems that you need to be aware of. The normal force (F_{norm}) is not necessarily equal to the gravitational force (F_{grav}) as it has been in problems that we have previously seen. The principle is that the vertical forces must balance if there is no vertical acceleration. If an object is being dragged across a horizontal surface, then there is no vertical acceleration. For this reason, the normal force (F_{norm}) plus the vertical component (F_y) of the applied force must balance the gravitational force (F_{grav}). A quick review of these problems shows that this is the case. If there is an acceleration for an object being pulled across a floor, then it is a horizontal acceleration; and thus the only imbalance of force would be in the horizontal direction.

Now consider the following situation in which a force analysis must be conducted to fill in all the blanks and to determine the net force and acceleration. In a case such as this, a thorough understanding of the relationships between the various quantities must be fully understood. Make an effort to solve this problem. When finished, click the button to view the answers. (When you run into difficulties, consult the help from a previous unit.)

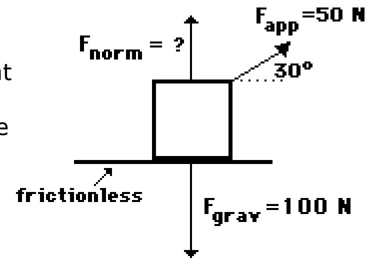


In conclusion, a situation involving a force at an angle can be simplified by using trigonometric relations to resolve that force into two components. Such a situation can be analyzed like any other situation involving individual forces. The net force can be determined by adding all the forces as vectors and the acceleration can be determined as the ratio of Fnet/mass.

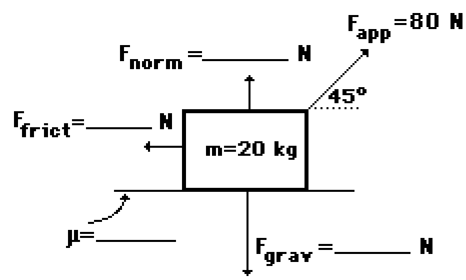
Check Your Understanding

The following problems provide plenty of practice with F_{net}= m • a problems involving forces at angles. Try each problem and then click the button to view the answers.

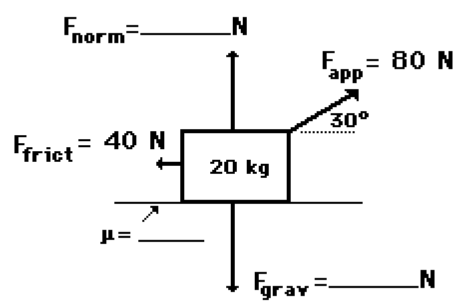
1. A 50-N applied force (30 degrees to the horizontal) accelerates a box across a horizontal sheet of ice (see diagram). Glen Brook, Olive N. Glenveau, and Warren Peace are discussing the problem. Glen suggests that the normal force is 50 N; Olive suggests that the normal force in the diagram is 75 N; and Warren suggests that the normal force is 100 N. While all three answers may seem reasonable, only one is correct. Indicate which two answers are wrong and explain why they are wrong.



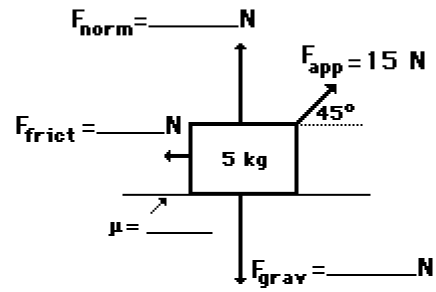
2. A box is pulled at a constant speed of 0.40 m/s across a frictional surface. Perform an extensive analysis of the diagram below to determine the values for the blanks.



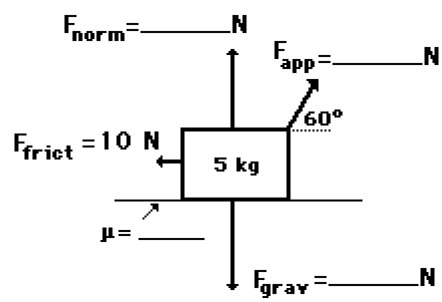
3. Use your understanding of force relationships and vector components to fill in the blanks in the following diagram and to determine the net force and acceleration of the object. (F_{net} = m•a; F_{frict} = μ•F_{norm}; F_{grav} = m•g)



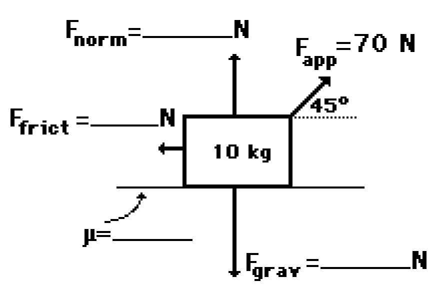
4. The 5-kg mass below is moving with a constant speed of 4 m/s to the right. Use your understanding of force relationships and vector components to fill in the blanks in the following diagram and to determine the net force and acceleration of the object. (F_{net} = m•a; F_{frict} = μ•F_{norm}; F_{grav} = m•g)

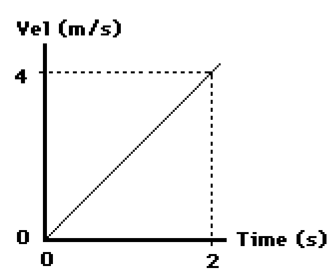


5. The following object is being pulled at a constant speed of 2.5 m/s. Use your understanding of force relationships and vector components to fill in the blanks in the following diagram and to determine the net force and acceleration of the object. (F_{net} = m•a; F_{frict} = μ•F_{norm}; F_{grav} = m•g)

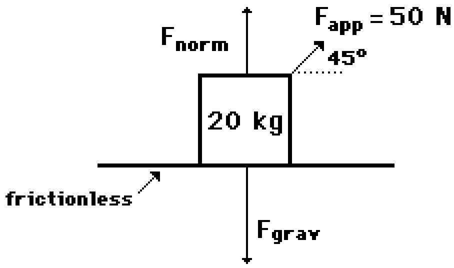


6. Use your understanding of force relationships and vector components to fill in the blanks in the following diagram and to determine the net force and acceleration of the object. (F_{net} = m•a; F_{frict} = μ•F_{norm}; F_{grav} = m•g)

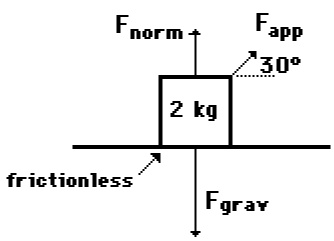


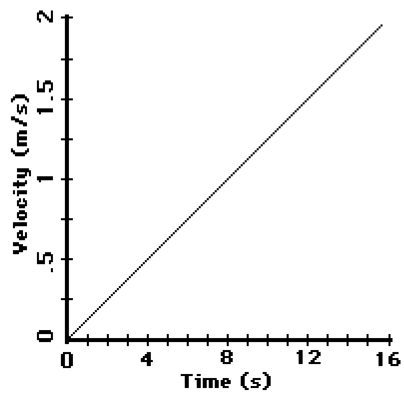


7. Study the diagram below and determine the acceleration of the box and its velocity after being pulled by the applied force for 2.0 seconds.

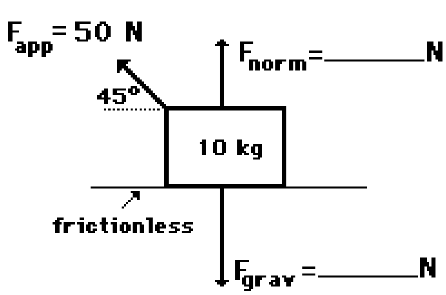


8. A student pulls a 2-kg backpack across the ice (assume friction-free) by pulling at a 30-degree angle to the horizontal. The velocity-time graph for the motion is shown. Perform a careful analysis of the situation and determine the applied force.



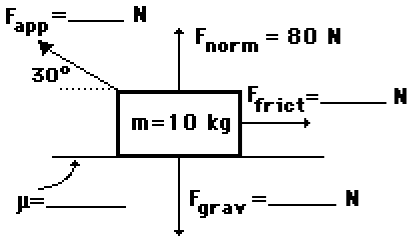


9. The following object is moving to the right and encountering the following forces. Use your understanding of force relationships and vector components to fill in the blanks in the following diagram <u>and</u> to determine the net force and acceleration of the object. (F_{net} = m•a; F_{frict} = μ•F_{norm}; F_{grav} = m•g)

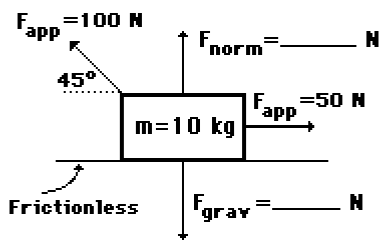


Time (s)	Vel (m/s)
0.0	21.0
1.0	________
2.0	________
3.0	________
4.0	________
5.0	________
6.0	________

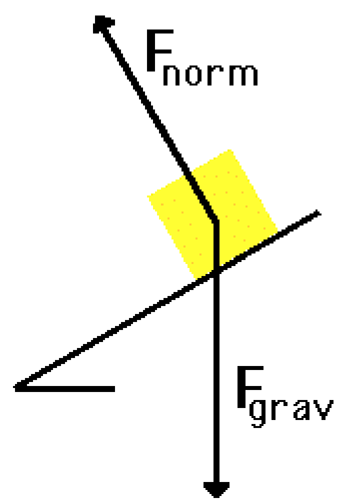
10. The 10-kg object is being pulled to the left at a constant speed of 2.5 m/s. Use your understanding of force relationships and vector components to fill in the blanks in the following diagram. (F_{net} = m•a; F_{frict} = μ•F_{norm}; F_{grav} = m•g)



11. Use your understanding of force relationships and vector components to fill in the blanks in the following diagram <u>and</u> to determine the net force and acceleration of the object. (F_{net} = m•a; F_{frict} = μ•F_{norm}; F_{grav} = m•g)

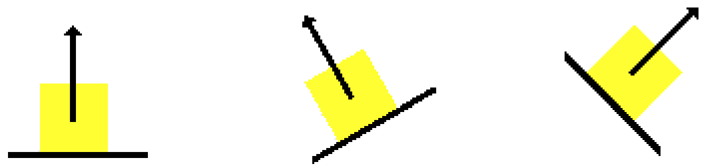


Inclined Planes



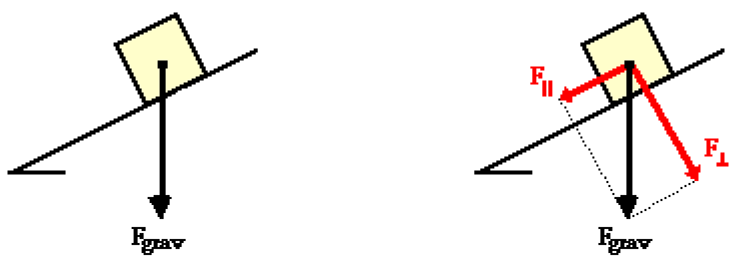
An object placed on a *tilted surface* will often slide down the surface. The rate at which the object slides down the surface is dependent upon how *tilted* the surface is; the greater the *tilt* of the surface, the faster the rate at which the object will slide down it. In physics, a *tilted surface* is called an inclined plane. Objects are known to accelerate down inclined planes because of an unbalanced force. To understand this type of motion, it is important to analyze the forces acting upon an object on an inclined plane. The diagram at the right depicts the two forces acting upon a crate that is positioned on an inclined plane (assumed to be friction-free). As shown in the diagram, there are always <u>at least two forces</u> acting upon any object that is positioned on an inclined plane - the force of gravity and the normal force. The force of gravity (also known as weight) acts in a downward direction; yet the normal force acts in a direction perpendicular to the surface (in fact, *normal* means "perpendicular").

The first peculiarity of inclined plane problems is that the normal force is <u>not</u> directed in the direction that we are accustomed to. Up to this point in the course, we have always seen normal forces acting in an upward direction, opposite the direction of the force of gravity. But this is only because the objects were always on horizontal surfaces and never upon inclined planes. The truth about normal forces is not that they are always upwards, but rather that they are always directed perpendicular to the surface that the object is on.



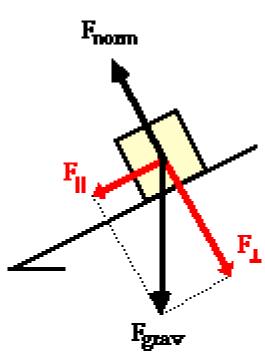
Normal forces are always directed perpendicular to the surface.

The task of determining the net force acting upon an object on an inclined plane is a difficult manner since the two (or more) forces are not directed in opposite directions. Thus, one (or more) of the forces will have to be resolved into perpendicular components so that they can be easily added to the other forces acting upon the object. Usually, any force directed at an angle to the horizontal is resolved into horizontal and vertical components. However, this is not the process that we will pursue with inclined planes. Instead, the process of analyzing the forces acting upon objects on inclined planes will involve resolving the weight vector (F_{grav}) into two perpendicular components. This is the second peculiarity of inclined plane problems. The force of gravity will be resolved into two components of force - one directed parallel to the inclined surface and the other directed perpendicular to the inclined surface. The diagram below shows how the force of gravity has been replaced by two components - a parallel and a perpendicular component of force.



The force of gravity can be resolved into two components. Together, these two components replace the affect of the force of gravity.

The perpendicular component of the force of gravity is directed opposite the normal force and as such balances the normal force. The parallel component of the force of gravity is not balanced by any other force. This object will subsequently accelerate down the inclined plane due to the presence of an unbalanced force. It is the parallel component of the force of gravity that causes this acceleration. The parallel component of the force of gravity is the net force.



For objects on inclined planes (with no friction)

$F_{\perp} = F_{norm}$

$F_{||}$ is the net force

The task of determining the magnitude of the two components of the force of gravity is a mere manner of using the equations. The equations for the parallel and perpendicular components are:

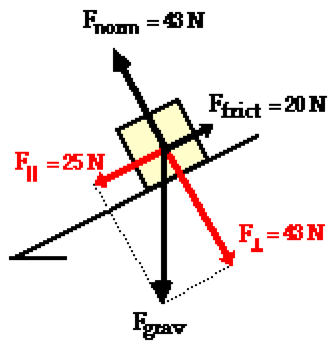
$$F_{\parallel} = m \cdot g \cdot \sin\theta \qquad F_{\perp} = m \cdot g \cdot \cos\theta$$

In the absence of friction and other forces (tension, applied, etc.), the acceleration of an object on an incline is the value of the parallel component (m*g*sine of angle) divided by the mass (m). This yields the equation

$$a = g \cdot \sin\theta$$

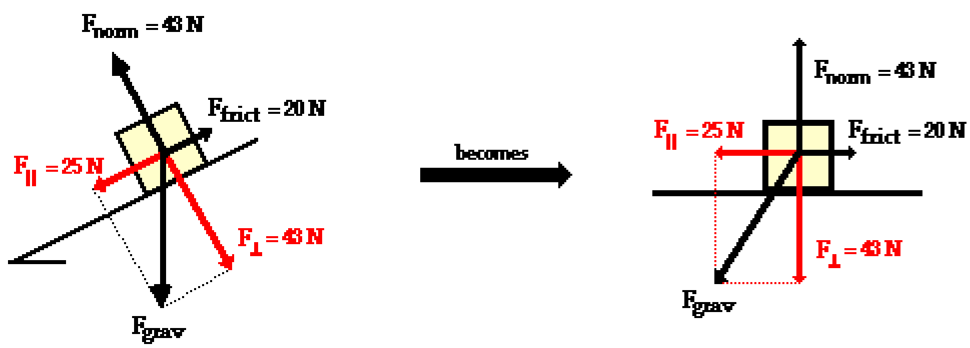
(in the absence of friction and other forces)

In the presence of friction or other forces (applied force, tensional forces, etc.), the situation is slightly more complicated. Consider the diagram shown at the right. The perpendicular component of force still balances the normal force since objects do not accelerate perpendicular to the incline. Yet the frictional force must also be considered when determining the net force. As in all net force problems, the net force is the vector sum of all the forces. That is, all the individual forces are added together as *vectors*. The perpendicular component and the normal force add to 0 N. The parallel component and the friction force add together to yield 5 N. The net force is 5 N, directed along the incline towards the floor.



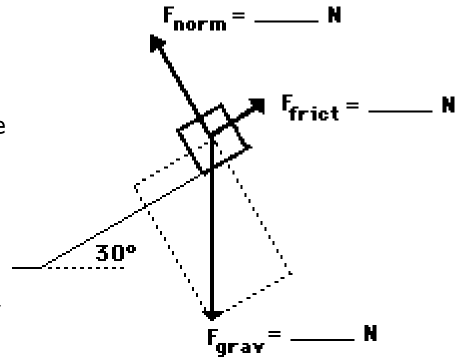
The net force is 5 N.

The above problem (and all inclined plane problems) can be simplified through a useful trick known as "tilting the head." An inclined plane problem is in every way like any other net force problem with the sole exception that the surface has been *tilted*. Thus, to transform the problem back into the form with which you are more comfortable, merely *tilt* your head in the same direction that the incline was *tilted*. Or better yet, merely *tilt* the page of paper (a sure remedy for TNS - "tilted neck syndrome" or "taco neck syndrome") so that the surface no longer appears level. This is illustrated below.



Once the force of gravity has been resolved into its two components and the inclined plane has been tilted, the problem should look very familiar. Merely ignore the force of gravity (since it has been replaced by its two components) and solve for the net force and acceleration.

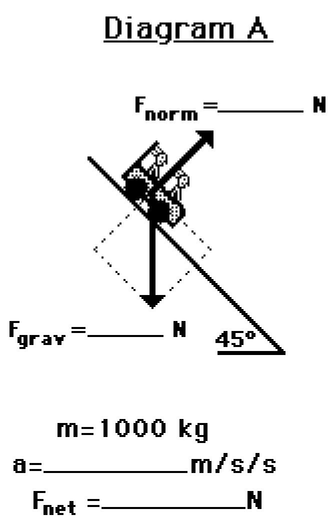
As an example consider the situation depicted in the diagram at the right. The free-body diagram shows the forces acting upon a 100-kg crate that is sliding down an inclined plane. The plane is inclined at an angle of 30 degrees. The coefficient of friction between the crate and the incline is 0.3. Determine the net force and acceleration of the crate.

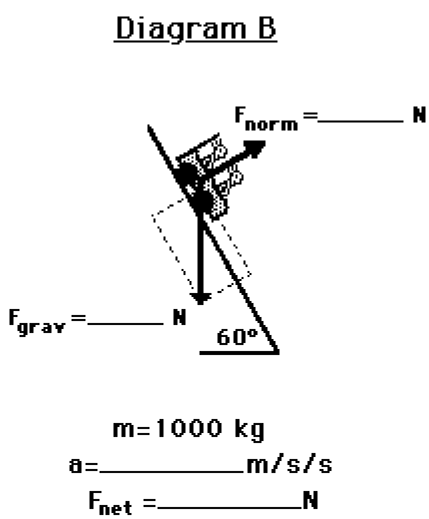


Begin the above problem by finding the force of gravity acting upon the crate and the components of this force parallel and perpendicular to the incline. The force of gravity is 980 N and the components of this force are $F_{parallel}$ = 490 N (980 N • sin 30 degrees) and $F_{perpendicular}$ = 849 N (980 N • cos30 degrees). Now the normal force can be determined to be 849 N (it must balance the perpendicular component of the weight vector). The force of friction can be determined from the value of the normal force and the coefficient of friction; F_{frict} is 255 N (F_{frict} = "mu"*F_{norm}= 0.3 • 849 N). The net force is the vector sum of all the forces. The forces directed perpendicular to the incline balance; the forces directed parallel to the incline do not balance. The net force is 235 N (490 N - 255 N). The acceleration is 2.35 m/s/s (F_{net}/m = 235 N/100 kg).

Practice

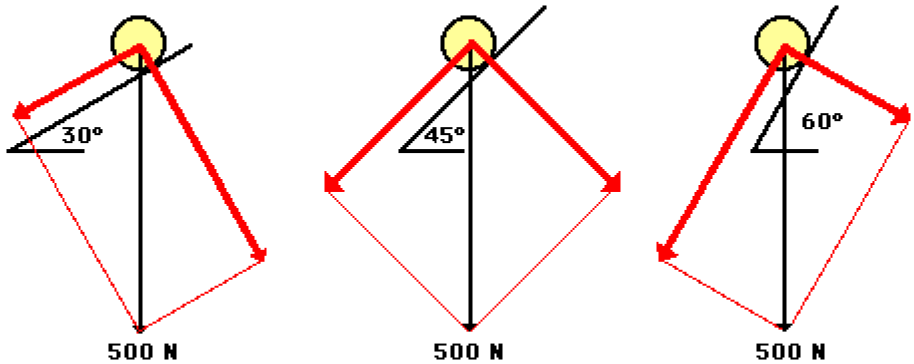
The two diagrams below depict the free-body diagram for a 1000-kg roller coaster on the first drop of two different roller coaster rides. Use the above principles of vector resolution to determine the net force and acceleration of the roller coaster cars. Assume a negligible affect of friction and air resistance. When done, click the button to view the answers.

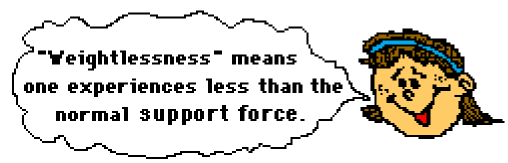




The affects of the incline angle on the acceleration of a roller coaster (or any object on an incline) can be observed in the two practice problems above. As the angle is increased, the acceleration of the object is increased. The explanation of this relates to the components that we have been drawing. As the angle increases, the component of force parallel to the incline increases and the component of force perpendicular to the incline decreases. It is the parallel component of the weight vector that causes the acceleration.

Thus, accelerations are greater at greater angles of incline. The diagram below depicts this relationship for three different angles of increasing magnitude.





Roller coasters produce two thrills associated with the initial drop down a steep incline. The thrill of acceleration is produced by using large angles of incline on the first drop; such large angles increase the value of the parallel component of the weight vector (the component that causes acceleration). The thrill of *weightlessness* is produced by reducing the magnitude of the normal force to values less than their usual values. It is important to recognize that the thrill of weightlessness is a feeling associated with a lower than usual normal force. Typically, a person weighing 700 N will experience a 700 N normal force when sitting in a chair. However, if the chair is accelerating down a 60-degrees incline, then the person will experience a 350 Newton normal force. This value is less than normal and contributes to the feeling of weighing less than one's normal weight - i.e., **weightlessness**.

More Practice

Use the *widget* below to investigate other inclined plane situations. Simply enter the mass, the incline angle and the coefficient of friction (use 0 for frictionless situations). Then click the **Submit** button to view the acceleration.

Calculate the Acceleration on an Inclined Plane with Friction

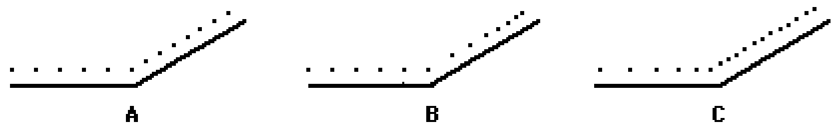
Mass:	2.0	kg
Incline Angle:	30	°
Coefficient of Friction (mu):	0.400	
	$a = 1.50518$	

More info at: www.physicsclassroom.com/Class/vectors/u3l3e.cfm

Check Your Understanding

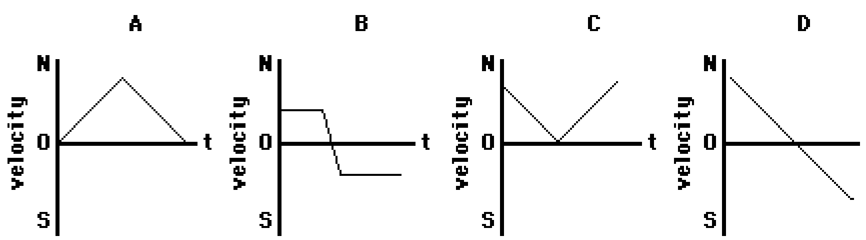
The following questions are intended to test your understanding of the mathematics and concepts of inclined planes. Once you have answered the question, click the button to see the answers.

1. Two boys are playing ice hockey on a neighborhood street. A stray puck travels across the friction-free ice and then up the friction-free incline of a driveway. Which one of the following ticker tapes (A, B, or C) accurately portrays the motion of the puck as it travels across the level street and then up the driveway?



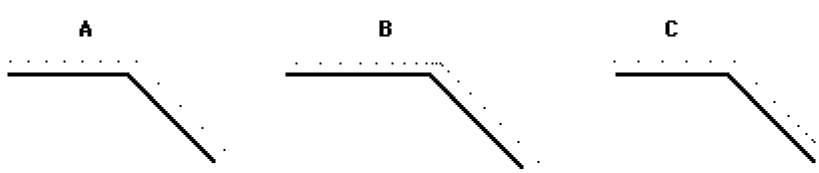
Explain your answer.

2. Little Johnny stands at the bottom of the driveway and kicks a soccer ball. The ball rolls northward up the driveway and then rolls back to Johnny. Which one of the following velocity-time graphs (A, B, C, or D) most accurately portrays the motion of the ball as it rolls up the driveway and back down?



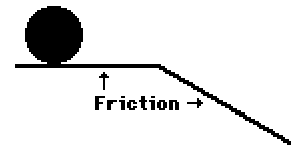
Explain your answer.

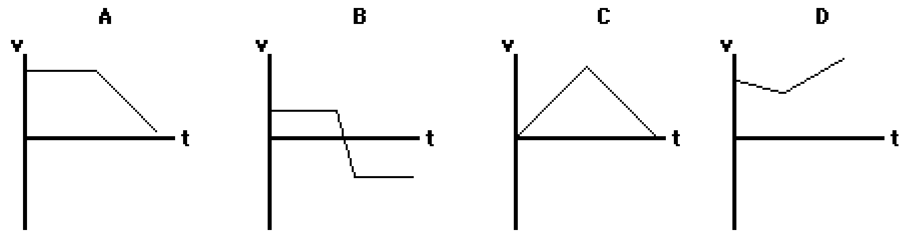
3. A golf ball is rolling across a horizontal section of the green on the 18th hole. It then encounters a steep downward incline (see diagram). Friction is involved. Which of the following ticker tape patterns (A, B, or C) might be an appropriate representation of the ball's motion?



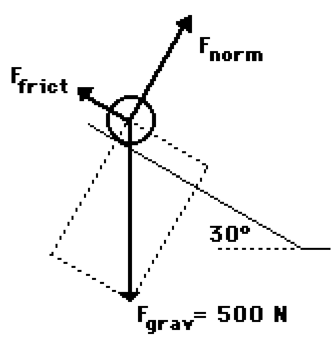
Explain why the inappropriate patterns are inappropriate.

4. Missy dePenn's eighth frame in the Wednesday night bowling league was a disaster. The ball rolled off the lane, passed through the freight door in the building's rear, and then down the driveway. Millie Meater (Missy's teammate), who was spending every free moment studying for her physics test, began visualizing the velocity-time graph for the ball's motion. Which one of the velocity-time graphs (A, B, C, or D) would be an appropriate representation of the ball's motion as it rolls across the horizontal surface and then down the incline? Consider frictional forces.

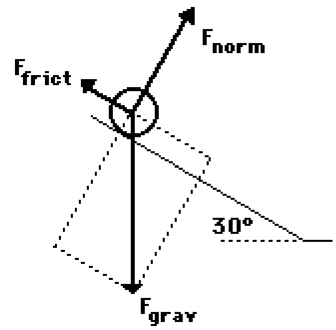




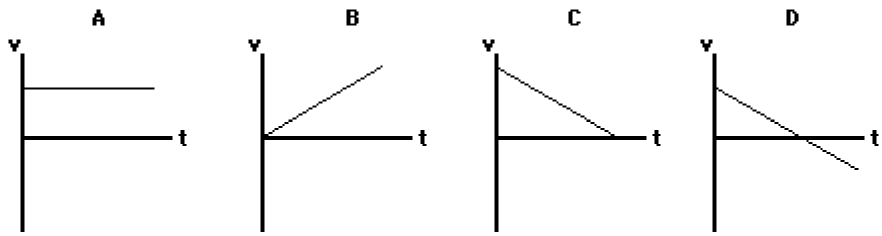
5. Three lab partners - Olive N. Glenveau, Glen Brook, and Warren Peace - are discussing an incline problem (see diagram). They are debating the value of the normal force. Olive claims that the normal force is 250 N; Glen claims that the normal force is 433 N; and Warren claims that the normal force is 500 N. While all three answers seem reasonable, only one is correct. Indicate which two answers are wrong and explain why they are wrong.



6a. Lon Scaper is doing some lawn work when a 2-kg tire escapes from his wheelbarrow and begins rolling down a steep hill (a 30° incline) in San Francisco. Sketch the parallel and perpendicular components of this weight vector. Determine the magnitude of the components using trigonometric functions. Then determine the acceleration of the tire. Ignore resistance force.



6b. Determine which one of the velocity-time graph would represent the motion of the tire as it rolls down the incline.



Explain your answer.

7. In each of the following diagrams, a 100-kg box is sliding down a frictional surface at a constant speed of 0.2 m/s. The incline angle is different in each situation. Analyze each diagram and fill in the blanks.

Diagram A

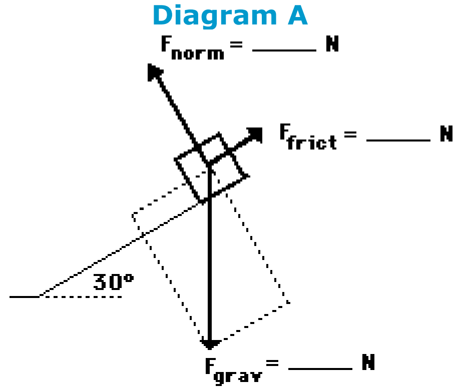
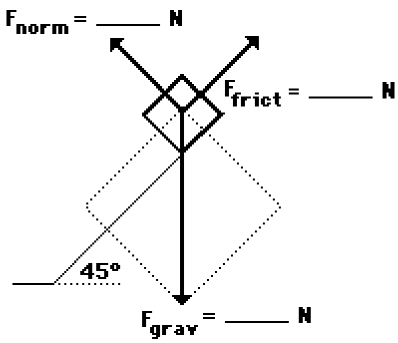
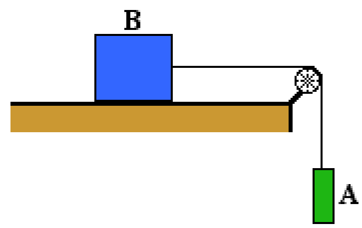


Diagram B



Double Trouble in 2 Dimensions (a.k.a., Two Body Problems)

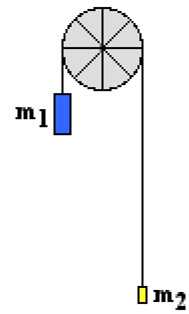
In the Newton's laws unit, the topic of two-body problems was introduced. A pair of problem-solving strategies were discussed and applied to solve three example problems. Such two-body problems typically involve solving for the acceleration of the objects and the force that is acting between the objects. One strategy for solving two-body problems involves the use of a system analysis to determine the acceleration combined with an individual object analysis to determine the force transmitted between the objects. The second strategy involved the use of two individual object analyses in order to develop a system of two equations for solving for the two unknown quantities. If necessary, take the time to review the page on solving two-body problems. This page will build upon the lessons learned earlier in the Newton's Laws unit.



In this lesson we will analyze two-body problems in which the objects are moving in different directions. In these problems, the two objects are connected by a string that transmits the force of one object to the other object. The string is wrapped around a pulley that changes the direction that the force is exerted without changing the magnitude. As an illustration of how a pulley works, consider the diagram at the right. Object A is connected to object B by a string. The string is wrapped around a pulley at the end of a table. Object A is suspended in mid-air while object B is on the table. In this situation, Object A will fall downward under the influence of gravity, pulling downward on one end of the string that it is connected to. According to Newton's law of action-reaction, this lower end of the string will pull upward on object A. The opposite end of the string is connected to object B. This end of the string pulls rightward upon object B. As such, the string connecting the two objects is pulling on both objects with the same amount of force, but in different directions. The string pulls upward on object A and rightward on object B. The pulley has changed the direction that the force is exerted.

Problems involving two objects, connecting strings and pulleys are characterized by objects that are moving (or even accelerating) in different directions. They move or accelerate at the same rate but in different directions. As such, it becomes important in approaching such problems to select a different reference frame and axes system for each object. Attention should be given to selecting an axes system such that both objects are accelerating along an axis in the positive direction. With the axes properly defined for each individual object, a free-body diagram can be constructed. Then Newton's laws can be applied to each diagram to develop a system of two equations for solving for the two unknowns. This problem-solving process will be demonstrated for three different example problems.

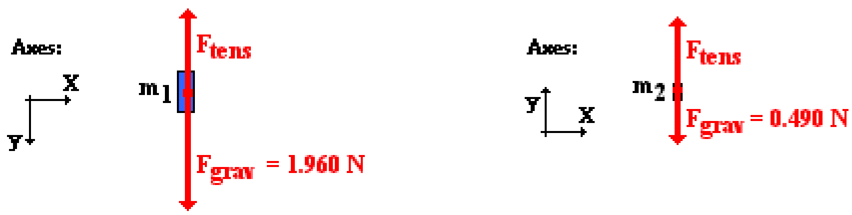
Example Problem 1



A 200.0-gram mass (**m_1**) and 50.0-gram mass (**m_2**) are connected by a string. The string is stretched over a pulley. Determine the acceleration of the masses and the tension in the string.

As is frequently the case, this example problem requests information about two unknowns - the acceleration of the objects and the force acting between the objects. In a situation such as this one with two objects suspended over a pulley, the more massive object will accelerate downward and the least massive object will accelerate upward. The magnitude of the acceleration will be the same for each object. The coordinate system chosen for m_1 has the positive y-axis directed downwards; the coordinate system chosen for m_2 has the positive y-axis directed upwards. With this selection of axes, the direction of acceleration will be positive for each object. The free-body diagrams for each individual mass are shown below. Each object is experiencing a downward force of gravity - calculated as $m_1 \bullet g$ and $m_2 \bullet g$ respectively. Each object is also experiencing an upward tension force that pulls the two objects towards each other.

Free-Body Diagrams for m_1 and m_2



Newton's second law equation ($F_{net} = m \bullet a$) can be applied to both diagrams in order to write two equations for the two unknowns. The F_{net} will be expressed as the force in the direction of the acceleration minus the one that opposes it. So for the 200.0-gram mass, F_{net} is written as 1.960 N - F_{tens}. For the 50.0-gram mass, F_{net} is written as F_{tens} - 0.490 N. Equations 1 and 2 are the result of applying the Newton's second law equation to the 200.0-gram and 50.0-gram masses. (Note that the mass values are converted to the standard kilogram unit before use in the equations. Also note that the units have been dropped in order for the equations to read more cleanly.)

$$1.960 - F_{tens} = 0.2000 \bullet a \longleftarrow \textbf{Equation 1}$$

$$F_{tens} - 0.490 = 0.0500 \bullet a \longleftarrow \textbf{Equation 2}$$

From this point, a few steps of algebra lead to the answers to the problem. Equation 2 can be rearranged to develop an expression for F_{tens} written in terms of the acceleration.

$$F_{tens} = 0.0500 \bullet a + 0.490 \longleftarrow \textbf{Equation 3}$$

This expression for F_{tens} can now be substituted into Equation 1 in order to change it into a single-unknown equation. That equation and the subsequent steps of algebra leading to the value of acceleration are shown below.

$$1.96 - (0.0500 \bullet a + 0.490) = 0.2000 \bullet a$$
$$1.96 - 0.0500 \bullet a - 0.490 = 0.2000 \bullet a$$
$$1.47 = 0.2500 \bullet a$$
$$a = 1.47/.2500 = \mathbf{5.88\ m/s^2}$$

Now that the acceleration has been found from Equation 1, its value can be substituted into Equation 3 in order to determine the tension.

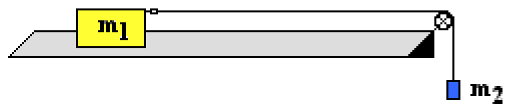
$$F_{tens} = 0.0500 \bullet (5.88) + 0.490$$

$$\mathbf{F_{tens} = 0.784\ N}$$

The pulley system analyzed here is sometimes referred to as an **Atwood's machine**. The problem-solving approach is the standard approach that will be used throughout this page in order to solve for the two unknowns. It will be repeated in Example Problem 2 in order to solve what is commonly referred to as a modified Atwood's machine problem.

Example Problem 2

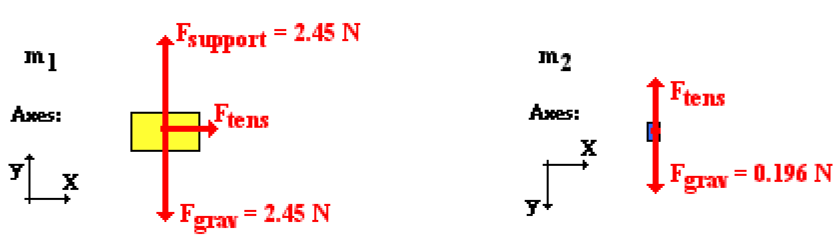
Consider the two-body situation at the right. A 20.0-gram hanging mass (m_2) is attached to a 250.0-gram air track glider (m_1). Determine the acceleration of the system and the tension in the string.



As in Example Problem 1, this system must first be analyzed conceptually in order to determine the direction of acceleration of the two objects. This will allow for the assignment of a coordinate axes for each object. Since there is nothing pushing m_1 to the left, we would reason that it would accelerate to the right due to the pull of the string. The *hanging mass* (m_2) will clearly accelerate downward under the influence of gravity. Thus, the coordinate system is chosen for m_2 has the positive y-axis directed downward; the coordinate system chosen for m_1 has the positive x-axis directed rightward. With this selection of axes, the direction of acceleration will be positive for each object.

The free-body diagram for each individual mass is shown below. Each object is experiencing a downward force of gravity (F_{grav}) - calculated as $m_1 \bullet g$ and $m_2 \bullet g$ respectively. The glider (m_1) is experiencing an upward support force (air pushing up on it) to balance the force of gravity. The glider is also experiencing a horizontal force - the tension force (F_{tens}) to the right. The hanging mass (m_2) is experiencing an upward tension force (F_{tens}) that offers some resistance to the downward pull of gravity.

Free-Body Diagrams for m_1 and m_2



Newton's second law equation ($F_{net} = m \bullet a$) can be applied to both free-body diagrams in order to write two equations for the two unknowns. The F_{net} will be expressed as the force in the direction of the acceleration minus any that oppose it. For the 250.0-gram (0.250 kg) glider, F_{net} is simply the unbalanced tension force (F_{tens}). For the 20.0-gram (0.020 kg) hanging mass, F_{net} is written as 0.196 N - F_{tens}. Equations 4 and 5 are the result of applying the Newton's second law equation to the 250.0-gram glider and 20.0-gram hanging mass. (Note that the mass values are converted to the standard kilogram unit before use in the equations. Also note that the units have been dropped in order for the equations to read more cleanly.)

$$F_{tens} = 0.2500 \bullet a \longleftarrow \textbf{Equation 4}$$

$$0.196 - F_{tens} = 0.0200 \bullet a \longleftarrow \textbf{Equation 5}$$

From this point, a few steps of algebra lead to the answers to the problem. Equation 4 expresses the F_{tens} value in terms of the acceleration. This expression for F_{tens} can be substituted into Equation 5 in order to convert it to a single-unknown equation. That equation and the subsequent steps of algebra leading to the value of acceleration are shown below.

$$0.196 - 0.2500 \bullet a = 0.0200 \bullet a$$
$$0.196 = 0.2700 \bullet a$$
$$a = .196/.2700 = 0.72593 \text{ m/s}^2$$
$$a = \mathbf{\sim 0.726 \text{ m/s}^2}$$

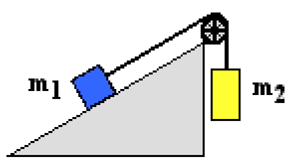
Now that the acceleration has been found from Equation 5, its value can be substituted into Equation 4 in order to determine the tension.

$$F_{tens} = 0.2500 \bullet (0.72593) = 0.18148$$
$$\mathbf{F_{tens} = \sim 0.181 \text{ N}}$$

The pulley system analyzed in Example Problem 2 is sometimes referred to as a modified Atwood's machine. The analysis is slightly more complicated than the Atwood's machine of Example Problem 1. The final example problem will be a case of a modified Atwood's machine with the surface inclined as shown below. The problem-solving approach will be the same.

Example Problem 3

Consider the two-body situation at the right. A 2.50x103-kg crate (m_1) rests on an inclined plane and is connected by a cable to a 4.00x103-kg mass (m_2). This second mass (m_2) is suspended over a pulley. The incline angle is 30.0° and the surface is frictionless. Determine the acceleration of the system and the tension in the cable.



Like the previous problem, the first task involves analyzing the situation to determine which direction the objects will accelerate. Such an analysis will allow a coordinate axes system to be assigned to each object. In this case, the hanging mass (m_2) could be accelerated upward or downward. The direction it accelerates depends upon a comparison of its weight (the force of gravity) to the opposing force acting on the other mass (m_1). The mass on the inclined plane encounters three forces - the gravity force, the normal force and the tension force. The gravity force is directed downward (as is usual) and calculated as $m_1 \bullet g$. The normal force is directed perpendicular to the surface (as is usual). The tension force is directed upwards and rightwards - parallel to the inclined plane and along the same orientation as the string that provides this force. As discussed on the previous page, objects placed on inclined planes are analyzed by resolving the force of gravity into two components. One component is directed parallel to the plane (and downwards at this angle) and the other component is directed perpendicular to the plane (and upwards at this angle). It is the parallel component of gravity that attempts to pull m_1 down the inclined plane. As mentioned previously, this component can be calculated by multiplying the weight of the object ($m_1 \bullet g$) by the sine of the incline angle (30°). The value for $F_{parallel}$ is

$$F_{parallel} = m_1 \bullet g \bullet \text{sine}(\theta) = (2500 \text{ kg}) \bullet (9.8 \text{ N/kg}) \bullet \text{sine}(30°)$$
$$F_{parallel} = 12250 \text{ N}$$

This parallel component of gravity is attempting to pull m_1 down the inclined plane. Since m_1 is attached by the cable to m_2, the hanging mass would be pulled with it. However, there is the opposing action of gravity pulling m_2 downward; this opposing action, if dominant, would drag the object m_1 up the inclined plane. The force of gravity on m_2 is

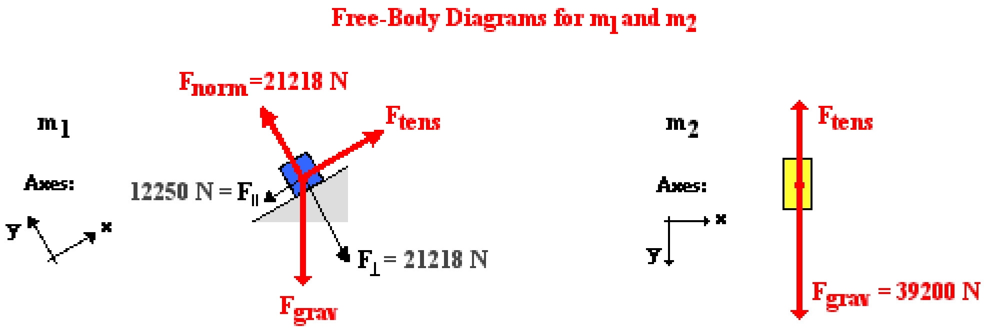
$$F_{grav\text{-}2} = m_2 \bullet g = (4000 \text{ kg}) \bullet (9.8 \text{ N/kg}) = 39200 \text{ N}$$

This gravitational force on m_2 is the dominant force. And so m_1 will accelerate up the inclined plane and m_2 will accelerate downward. The coordinate axes are assigned accordingly so that each object has a positive acceleration.

The diagrams below show these coordinate axes and the forces acting upon the two objects. The three forces on m_1 have already been discussed. The diagram shows the two components of F_{grav}. As mentioned on the previous page, the perpendicular component of gravity is calculated as

$$F_{perpendicular} = m_1 \bullet g \bullet \cos\theta = (2500 \text{ kg}) \bullet (9.8 \text{ N/kg}) \bullet \cos(30°)$$
$$F_{perpendicular} = 21218 \text{ N}$$

The normal force (F_{norm}) acting upon m_1 balances the $F_{perpendicular}$ so that there is no acceleration perpendicular to the inclined plane. The F_{norm} value is also 21218 N. The hanging mass (m_2) experiences only two forces - the downward pull of gravity and the upward tension force.



Now the Newton's second law equation ($F_{net} = m \bullet a$) can be applied to both free-body diagrams in order to write two equations for the two unknowns. The F_{net} is expressed as the force in the direction of the acceleration minus any that oppose it. For the 2500-kg mass on the incline (m_1), F_{net} is simply the tension force (F_{tens}) minus the parallel component of gravity. For the 4000-kg hanging mass (m_2), F_{net} is the force of gravity (39200 N) minus the tension force (F_{tens}). Equations 6 and 7 are the result of applying the Newton's second law equation to m_1 and m_2. (Note that the units have been dropped in order for the equations to read more cleanly.)

$$F_{tens} - 12250 = 2500 \bullet a \longleftarrow \textbf{Equation 6}$$

$$39200 - F_{tens} = 4000 \bullet a \longleftarrow \textbf{Equation 7}$$

From this point, a few steps of algebra lead to the answers to the problem. Equation 6 can be rearranged to create an expression for F_{tens}, expressed in terms of the acceleration.

$$F_{tens} = 2500 \bullet a + 12250 \longleftarrow \textbf{Equation 8}$$

This expression for F_{tens} can be substituted into Equation 7 in order to convert it to a single-unknown equation. That equation and the subsequent steps of algebra leading to the value of acceleration are shown below.

$$39200 - (2500 \bullet a + 12250) = 4000 \bullet a$$
$$39200 - 2500 \bullet a - 12250 = 4000 \bullet a$$
$$26950 = 6500 \bullet a$$
$$a = 26950/6500 = 4.1462 \text{ m/s}^2$$
$$a = \mathbf{\sim 4.15 \text{ m/s}^2}$$

Now that the acceleration has been found from Equation 7, its value can be substituted into Equation 8 in order to determine the tension force (F_{tens}).

$$F_{tens} = 2500•a + 12250 = 2500•(4.1462) + 12250 = 22615 \text{ N}$$

F_{tens} = ~2.26 x 103 N

Two-body problems like these three example problems can be quite a challenge. Having a systematic approach that is applied to every problem simplifies the analysis. Good conceptual understanding, a commitment to the use of free-body diagrams, and a solid grasp of Newton's second law are the essential ingredients of success.

Check Your Understanding

1. Consider the two-body situation at the right. A 100.0-gram hanging mass (m2) is attached to a 325.0-gram mass (m1) at rest on the table. The coefficient of friction between the 325.0-gram mass and the table is 0.215. Determine the acceleration of the system and the tension in the string.

2. Consider the two-body situation at the right. A 3.50x103-kg crate (m1) rests on an inclined plane and is connected by a cable to a 1.00x103-kg mass (m2). This second mass (m2) is suspended over a pulley. The incline angle is 30.0° and the surface has a coefficient of friction of 0.210. Determine the acceleration of the system and the tension in the cable.