Introduction to Econometrics (3rd Updated Edition)

by

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Solutions to Odd-Numbered End-of-Chapter Exercises: Chapter 5

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5.1 (a) The 95% confidence interval for β_1 is $\{-5.82 \pm 1.96 \times 2.21\}$, that is $-10.152 \le \beta_1 \le -1.4884.$

(b) Calculate the t-statistic:

$$t^{act} = \frac{\hat{\beta}_1 - 0}{\mathrm{SE}(\hat{\beta}_1)} = \frac{-5.82}{2.21} = -2.6335.$$

The p-value for the test $H_0 : \beta_1 = 0$ vs. $H_1 : \beta_1 \ne 0$ is

$$p\text{-value} = 2\Phi(-|t^{act}|) = 2\Phi(-2.6335) = 2 \times 0.0042 = 0.0084.$$

The p-value is less than 0.01, so we can reject the null hypothesis at the 5% significance level, and also at the 1% significance level.

(c) The t-statistic is

$$t^{act} = \frac{\hat{\beta}_1 - (-5.6)}{\mathrm{SE}(\hat{\beta}_1)} = \frac{0.22}{2.21} = 0.10$$

The p-value for the test $H_0 : \beta_1 = -5.6$ vs. $H_1 : \beta_1 \ne -5.6$ is

$$p\text{-value} = 2\Phi(-|t^{act}|) = 2\Phi(-0.10) = 0.92$$

The p-value is larger than 0.10, so we cannot reject the null hypothesis at the 10%, 5% or 1% significance level. Because $\beta_1 = -5.6$ is not rejected at the 5% level, this value is contained in the 95% confidence interval.

(d) The 99% confidence interval for □$_0$ is $\{520.4 \pm 2.58 \times 20.4\}$, that is, $467.7 \le \beta_0 \le 573.0.$

5.3. The 99% confidence interval is $1.5 \times \{3.94 \pm 2.58 \times 0.31)$ or

$$4.71 \text{ lbs} \leq \text{WeightGain} \leq 7.11 \text{ lbs.}$$

5. 5 (a) The estimated gain from being in a small class is 13.9 points. This is equal to approximately 1/5 of the standard deviation in test scores, a moderate increase.

(b) The *t*-statistic is $t^{act} = \frac{13.9}{2.5} = 5.56,$ which has a *p*-value of 0.00. Thus the null hypothesis is rejected at the 5% (and 1%) level.

(c) $13.9 \pm 2.58 \times 2.5 = 13.9 \pm 6.45$.

5.7. (a) The t-statistic is $\frac{3.2}{1.5} = 2.13$ with a p-value of 0.03; since the p-value is less than 0.05, the null hypothesis is rejected at the 5% level.

(b) $3.2 \pm 1.96 \times 1.5 = 3.2 \pm 2.94$

(c) Yes. If Y and X are independent, then $\beta_1 = 0$; but this null hypothesis was rejected at the 5% level in part (a).

(d) β_1 would be rejected at the 5% level in 5% of the samples; 95% of the confidence intervals would contain the value $\beta_1 = 0$.

5.9. (a) $\bar{\beta} = \frac{\frac{1}{n}(Y_1 + Y_2 + \cdots + Y_n)}{\bar{X}}$ so that it is linear function of Y_1, Y_2, …, Y_n.

(b) $E(Y_i|X_1, \ldots, X_n) = \beta_1 X_i$, thus

$$E(\bar{\beta}|X_1, \ldots, X_n) = E\frac{1}{\bar{X}}\frac{1}{n}(Y_1 + Y_2 + \cdots + Y_n)|X_1, \ldots, X_n)$$
$$= \frac{1}{\bar{X}}\frac{1}{n}\beta_1(X_1 + \cdots + X_n) = \beta_1$$

5.11. Using the results from 5.10, $\hat{\beta}_0 = \overline{Y}_m$ and $\hat{\beta}_1 = \overline{Y}_w - \overline{Y}_m$. From Chapter 3,

$\mathrm{SE}(\overline{Y}_m) = \frac{S_m}{\sqrt{n_m}}$ and $\mathrm{SE}(\overline{Y}_w - \overline{Y}_m) = \sqrt{\frac{s_m^2}{n_m} + \frac{s_w^2}{n_w}}$. Plugging in the numbers $\hat{\beta}_0 = 523.1$ and $\mathrm{SE}(\hat{\beta}_0) = 6.22$; $\hat{\beta}_1 = -38.0$ and $\mathrm{SE}(\hat{\beta}_1) = 7.65$.

5.13. (a) Yes, this follows from the assumptions in KC 4.3.

(b) Yes, this follows from the assumptions in KC 4.3 and conditional homoskedasticity

(c) They would be unchanged for the reasons specified in the answers to those questions.

(d) (a) is unchanged; (b) is no longer true as the errors are not conditionally homosckesdastic.

5.15. Because the samples are independent, $\hat{\beta}_{m,1}$ and $\hat{\beta}_{w,1}$ are independent. Thus $\text{var}(\hat{\beta}_{m,1} - \hat{\beta}_{w,1}) = \text{var}(\hat{\beta}_{m,1}) + \text{var}(\hat{\beta}_{w,1})$. $\text{Var}(\hat{\beta}_{m,1})$ is consistently estimated as $[SE(\hat{\beta}_{m,1})]^2$ and $\text{Var}(\hat{\beta}_{w,1})$ is consistently estimated as $[SE(\hat{\beta}_{w,1})]^2$, so that $\text{var}(\hat{\beta}_{m,1} - \hat{\beta}_{w,1})$ is consistently estimated by $[SE(\hat{\beta}_{m,1})]^2 + [SE(\hat{\beta}_{w,1})]^2$, and the result follows by noting the SE is the square root of the estimated variance.