

Compound Interest

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Some Vocabulary

When you invest or borrow money, the amount invested or borrowed is called the **principal**.

When you invest money, you are paid **interest** for your investment. When you borrow money, you pay **interest** to the bank.

The amount of interest you pay is usually stated as a percentage, called the **interest rate**.

There are two types of interest: **simple interest** and **compound interest**.

Simple Interest

Simple interest is interest earned when the interest rate is **only** on the principal.

For a formula:

$$A = P + Prt$$

where A is the account balance at the end of year t , P is the principal and r is the interest rate (written as a decimal).

For example, if you invest \$1000 at 5 % simple interest, then

$$A = 1000 + 50t.$$

What kind of function is this?

Compound Interest

Compound interest is interest earned when the interest rate is applied to the account balance **including previous interest earned**.

For a formula:

$$A = P(1 + r)^t$$

where A is the account balance at the end of year t , P is the principal and r is the interest rate (written as a decimal).

For example, if you invest \$1000 at 5 % interest compounded annually, then

$$A = 1000(1.05)^t.$$

What kind of function is this?

Compounding n Times per Year

Interest does not have to be compounded only at the end of each year. It can be compounded as many times per year as one wants.

For a formula:

$$A = P \left(1 + \frac{r}{n} \right)^{nt}$$

where A is the account balance at time t , P is the principal and r is the interest rate (written as a decimal) compounded n times per year.

The number $\left(1 + \frac{r}{n} \right)^n - 1$ is called the **annual percentage rate** (or APR).

Typical Values for n

What is n when interest is compounded:

- 1 annually?
- 2 quarterly?
- 3 monthly?
- 4 daily?

Example

Suppose you deposit \$2500 in a savings account earning 6 % interest compounded quarterly. What would your account balance be after 3 years?

CAUTION: Enter everything into your calculator in **one** step (to avoid unnecessary round-off error). Be careful with parentheses. Always ask whether your answer makes sense.

Example

Suppose you deposit \$2500 in a savings account earning 6 % interest compounded quarterly. What would your account balance be after 3 years?

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Continuous Compounding

Continuous compounding occurs when n is allowed to be infinite. When n becomes infinite:

$$\left(1 + \frac{1}{n}\right)^n \rightarrow e$$

where the number $e \approx 2.718$.

When an interest rate r (written as a decimal) is compounded continuously, the account balance A at time t is

$$A = Pe^{rt}.$$

Example

You have just started law school. As a reward for your hard work, your Aunt Frida has given you \$5000. This money must be used for living expenses while you are studying for the bar three years later.

You have two account options:

- 1 5 % interest, compounded monthly.
- 2 4 % interest, compounded continuously

Which option do you choose?

Example: How much to invest?

Determine the amount of money P that should be invested at 9 % interest compounded continuously in order to produce a balance of \$ 100,000 in t years.