Properties of Logarithms Worksheet

I. Model Problems.
II. Practice Expanding Logarithms
III. Rewrite expression as 1 Term
IV. Extension Problems
V. Answer Key

Relevant urls:
Log Rules: www.mathwarehouse.com/logs/

Online Scientific/Graphing Calculator
   http://www.meta-calculator.com/online/

(yes, it can graph logarithms!)

META
CALC

I) Model Problems

For any positive numbers *X, Y* and *N* and any positive base *b*, the following formulas are true:

$\log_b X^N = N \cdot \log_b X$	Power Rule for Logarithms
$\log_b\left(\dfrac{X}{Y}\right) = \log_b X - \log_b Y$	Quotient Rule for Logarithms
$\log_b(XY) = \log_b X + \log_b Y$	Product Rule for Logarithms

The following examples show how to expand logarithmic expressions using each of the rules above.

Example 1

Expand $\log_2 49^3$

$\log_2 49^3 = 3 \cdot \log_2 49$ Use the Power Rule for Logarithms.

The answer is $3 \cdot \log_2 49$

Example 2

Expand $\log_3(7a)$

$\log_3(7a) = \log_3(7 \cdot a)$ Since 7*a* is the product of 7 and *a*, you can write 7*a* as $7 \cdot a$.

$= \log_3 7 + \log_3 a$ Use the Product Rule for Logarithms.

The answer is $\log_3 7 + \log_3 a$

Example 3

Expand $\log_5\left(\dfrac{11}{3}\right)$

$\log_5\left(\dfrac{11}{3}\right) = \log_5 11 - \log_5 3$ Use the Quotient Rule for Logarithms.

The answer is $\log_5 11 - \log_5 3$

The following examples use more than one of the rules at a time.

Example 4

Expand $\log_2\left(\dfrac{a^2 b}{c}\right)$.

$\log_2\left(\dfrac{a^2 b}{c}\right) = \log_2 a^2 b - \log_2 c$ Use the Quotient Rule for Logarithms.

$= \log_2 a^2 + \log_2 b - \log_2 c$ Use the Product Rule for Logarithms.
$= 2 \cdot \log_2 a + \log_2 b - \log_2 c$ Use the Power Rule for Logarithms

The answer is $2 \cdot \log_2 a + \log_2 b - \log_2 c$.

Example 5

Expand $\log_5 \sqrt{8a^7}$.

$\log_5 \sqrt{8a^7} = \log_5 (8a^7)^{1/2}$ Rewrite the radical with a fractional exponent.
Use the Power Rule for Logarithms.

$= \dfrac{1}{2}\log_5 (8a^7)$

$= \dfrac{1}{2}(\log_5 8 + \log a^7)$ Use the Product Rule for Logarithms.

$= \dfrac{1}{2}(\log_5 8 + 7\log a)$ Use the Power Rule for Logarithms.

The answer is $\dfrac{1}{2}(\log_5 8 + 7\log a)$

II) Exercises
Expand the following logarithms.
Use either the power rule, product rule or quotient rule.

1. $\log_2(9^5) =$ _____

2. $\log_2(21) =$ _____

3. $\log_5\left(\dfrac{19}{2}\right) =$ _____

4. $\log_2(6a) =$ _____

5. $\log_3(xy) =$ _____

6. $\log_5\left(\dfrac{a}{3}\right) =$ _____

7. $\log_3(5y) =$ _____

8. $\log_3(a^{10}) =$ _____

Expand the following logarithms using one or more of the logarithm rules.

9. $\log_5\left(\dfrac{12a}{2}\right)$ = _____

10. $\log_2\left(\dfrac{a}{b}\right)^5$ = _____

11. $\log_5\sqrt{x^5 y}$ = _____

12. $\log_5\left(\dfrac{xy}{z}\right)^8$ = _____

13. $\log_2\left(\dfrac{1-x}{y}\right)^3$ = _____

14. $\log_3\sqrt[5]{9x^3}$ = _____

15. $\log_3\sqrt[3]{2x^5}$ = _____

16. $\log_2\left(\dfrac{9x^{10}}{y^2}\right)$ = _____

17. $\log_2\left(\dfrac{4a}{5}\right)$ = _____

18. $\log_2\sqrt[3]{x^2 a}$ = _____

Sometimes you need to write an expression as a single logarithm.
Use the rules to work backwards.

Example 6

Write 2 log₃x + log₃y as a single logarithm

$\log_3 x^2 + \log_3 y$ Use the Power Rule for Logarithms to move the 2 in 2 log₃x to the exponent of x

$= \log_3 x^2 y$ Use the Product Rule for Logarithms.

The answer is log₃$x^2 y$

Example 7

Simplify $\dfrac{1}{2}\log_5 100 - \log_5 2$

$\log_5 100^{1/2} - \log_5 2$ Use the Power Rule for Logarithms.

$= \log_5 10 - \log_5 2$ Simplify.

$= \log_5(10 \div 2) = \log_5 5$ Use the Quotient Rule for Logarithms.

$= 1$ Simplify.

The answer is 1

III) Rewrite as Single Expression
Write as a single logarithm.

19. $2 \log_3 10 - \log_3 4 \ = $ _____

20. $\dfrac{2}{3} \log_2 x \ + \log_2 y = $ _____

21. $\dfrac{1}{2} \log_5 x \ + \log_5 y = $ _____

22. $3 \log_3 x + 4 \log_3 y = $ _____

23. $6 \log_3 x + 2 \log_3 11 = $ _____

24. $4 \log_5 x - \log_5 y + \log_5 z \ = $ _____

25. $\dfrac{1}{2} \log_3 144 - \log_3 4 = $ _____

26. $\log_3 a + \log_3 b - 2 \log_3 c \ = $ _____

IV) Extension Problems

27. Let $\log_b 2 = x$, $\log_b 3 = y$ and $\log_b 5 = z$.

(a) What is the value of $\log_b 50$ in terms of x, y and z?

(b) What is the value of $\log_b 3000$ in terms of x, y and z?

28. Are $\log_2 16$ and $\log_4 64$ equal? Why or why not?

29. Correct the error
There is an error in the student work shown below.

Directions: Simplify $\log_2 (6x)^5$.

$$\log_2 (6x)^5 = 5 \bullet \log_2(6 \bullet x)$$
$$= 5 \bullet \log_2 6 + \log_2 x$$
$$= 5 \log_2 6 + \log_2 x$$

What is the error in the work above?

_____

_____

Answer Key
1. $5 \log_2 9 = 10 \log_2 3$
2. $\log_2 3 + \log_2 7$
3. $\log_5 19 - \log_5 2$
4. $\log_2 6 + \log_2 a$
5. $\log_3 x + \log_3 y$
6. $\log_5 a - \log_5 3$
7. $\log_3 5 + \log_3 y$
8. $10 \log_3 a$
9. $\log_5 6 + \log_5 a$
10. $5 (\log_2 a - \log_2 b)$
11. $\dfrac{1}{2}(5 \log_5 x + \log_5 y)$
12. $8 (\log_5 x + \log_5 y - \log_5 z)$
13. $3 (\log_2(1 - x) - \log_2 y)$
14. $\dfrac{1}{5}(2 - 3 \log_3 x)$
15. $\dfrac{1}{3}(\log_3 2 - 5 \log_3 x)$
16. $2 \log_2 3 + 10 \log_2 x - 2 \log_2 y$
17. $2 + \log_2 a - \log_2 5$
18. $\dfrac{1}{3}(2 \log_2 x + \log_2 a)$
19. $\log_3 25$
20. $\log_2(x^{2/3} y)$
21. $\log_5(x^{1/2} y)$
22. $\log_3(x^3 y^4)$
23. $\log_3(121 x^6)$
24. $\log_5\left(\dfrac{x^4 z}{y}\right)$
25. 1
26. $\log_5\left(\dfrac{ab}{c^2}\right)$
27. (a) $x + y + z$; (b) $3(x + z) + y$
28. Yes; they are both equal to 4.
29. The student did not distribute the 5 to $\log_2 6$ and $\log_2 x$; the correct answer is $5(\log_2 6 + \log_2 x)$, or $5 \log_2 6 + 5 \log_2 x$.