# Fractions — basic ideas

mc-TY-fracbasic-2009-1

In this unit we shall look at the basic concept of fractions — what they are, what they look like, why we have them and how we use them. We shall also look at different ways of writing down the same fraction.

In order to master the techniques explained here it is vital that you undertake plenty of practice exercises so that they become second nature.

After reading this text, and/or viewing the video tutorial on this topic, you should be able to:

- recognize when two fractions are equivalent;
- convert a fraction into its lowest form;
- convert an improper fraction into a mixed fraction, and vice versa.

## Contents



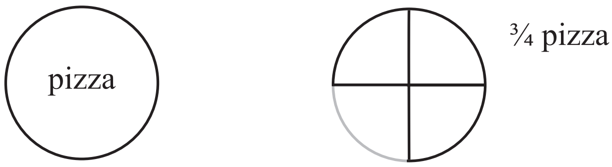
# 1. Introduction

What are fractions? Fractions are ways of writing parts of whole numbers. For example if we take a pizza, and divide it up equally between 4 people, each person will have $\frac{1}{4}$ or, written in words, one quarter of the pizza.


If one person were to take 2 quarters of the pizza, they would have $\frac{2}{4}$, which is the same as $\frac{1}{2}$ or half the pizza. So

$$\frac{2}{4} = \frac{1}{2}.$$


If three pieces of the pizza have been eaten, then $\frac{3}{4}$ or three quarters has gone, and $\frac{1}{4}$ or one quarter remains.




Finally, the whole pizza is $\frac{4}{4}$, or four quarters.

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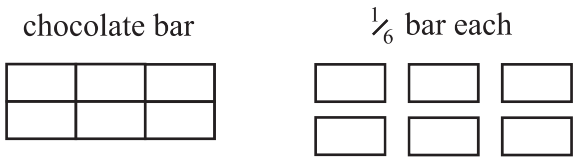
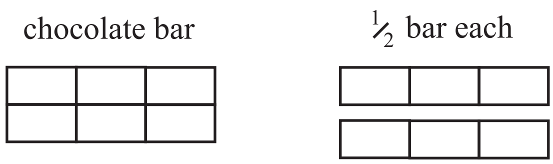
Some chocolate bars are conveniently marked to make them easier to break into pieces to eat. For instance, we might have a bar marked into 6 equal pieces, so each piece is $\frac{1}{6}$, or one sixth of the whole bar. So if we share this bar between 6 people, we would get 1 piece each.




If we share it between just 2 people, we could have half the bar each, which would be 3 pieces each. So

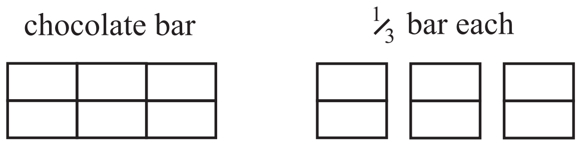
$$\frac{3}{6} = \frac{1}{2}.$$



Similarly, if it were to be shared between 3 people, they would get $\frac{1}{3}$ of the bar each, which is 2 pieces. So

$$\frac{1}{3} \quad = \quad \frac{2}{6}\,.$$




We are looking at exactly the same result each time, but in different ways. We can also think of its meaning in more than one way.

$$\frac{2}{6} \quad = \quad \frac{\text{number of pieces being used}}{\text{number of pieces that make up the whole}} \quad = \quad \frac{1}{3}\,,$$

or as

$$\frac{1}{3} \quad = \quad 1 \div 3 \quad = \quad 1 \text{ whole bar of chocolate divided into 3 pieces}\,.$$

If we take all 6 pieces we have $\frac{6}{6}$ which is the whole bar, so

$$\frac{6}{6} = 1$$

just as $6 \div 6 = 1$.

We can divide a whole number into any number of pieces of equal size, and then we can take any number of those pieces, for example $\frac{3}{8}$ is a whole divided into 8 pieces, and we have taken 3 of them. Similarly

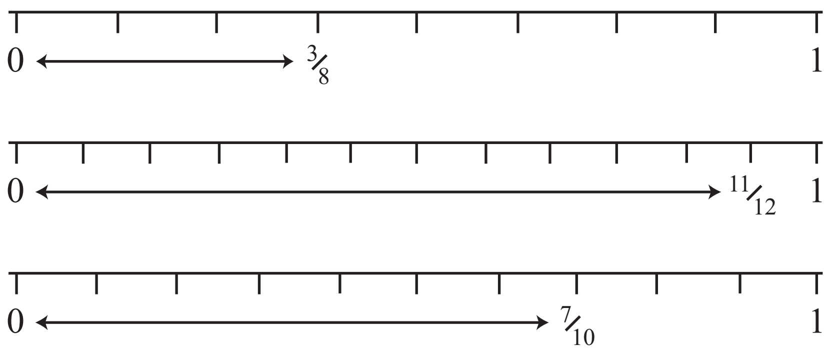
$$\frac{11}{12} \quad \text{means} \quad \text{11 pieces out of 12,}$$

$$\frac{7}{10} \quad \text{means} \quad \text{7 pieces out of 10,}$$

$$\frac{100}{500} \quad \text{means} \quad \text{100 pieces out of 500,}$$

$$\frac{3}{167} \quad \text{means} \quad \text{3 pieces out of 167.}$$

We can also represent fractions on a section of a number line. We take the section from 0 to 1, and divide it up into the total number of pieces. Then we count off the number of pieces we have taken.



**Key Point**

Fractions are formed by splitting a whole into any number of pieces of equal size.

## 2. Equivalent fractions

Let us examine more closely what fractions look like.

Take $\frac{1}{2}$ and you can see that the bottom number is twice the size of the top number, so any fraction where the bottom number is twice the top number is equivalent (the same as) a half. So

$$\frac{2}{4}, \quad \frac{3}{6}, \quad \frac{4}{8}, \quad \frac{5}{10}, \quad \frac{20}{40}, \quad \frac{99}{198} \ldots$$

are all equivalent fractions that mean $\frac{1}{2}$.

When a half is written as 1 over 2 rather than 2 over 4, or 5 over 10, or any other version, it is said to be in its *lowest* form. This is because no number, except 1, will divide into both the top number and the bottom number. So to put a fraction in its lowest form, you divide by any factors common to both the top number and the bottom number.

Equivalent fractions can be found for any fraction by multiplying the top number and the bottom number by the same number. For example, if we have $\frac{3}{4}$, then multiplying by 2 gives

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8},$$

or by 3 gives

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}.$$

Multiplying by 10 gives

$$\frac{3 \times 10}{4 \times 10} = \frac{30}{40},$$

and all of these fractions are exactly the same as $\frac{3}{4}$.

When dealing with fractions, we often use some special mathematical language. Instead of using the words 'top number' and 'bottom number' we use the words *numerator* and *denominator*. So in $\frac{3}{4}$, 3 is the numerator and 4 is the denominator:

$$\frac{\text{top number}}{\text{bottom number}} = \frac{\text{numerator}}{\text{denominator}}.$$

**Example**

Write $\frac{8}{100}$ in its lowest form.

**Solution**

Here, we are going backwards. From a fraction in its lowest form, we must have multiplied both the numerator and the denominator by the same number to obtain this equivalent fraction $\frac{8}{100}$. So now we must divide both the numerator and the denominator by the same number. We know that 2 goes into both 8 and 100, so let us divide both numbers by 2, giving $\frac{4}{50}$. Again both 4 and 50 will divide by 2, giving $\frac{2}{25}$. But now only 1 goes into both 2 and 25, so $\frac{2}{25}$ is the fraction in its lowest form.

Since we have divided by 2 twice here, we could have just divided by 4 originally. But we can't always spot the highest common factor of the two numbers straight away.

# 3. Different types of fraction

It doesn't matter how many equal pieces a whole is split into, if all the pieces are then taken, we have the whole again. For example,

$$\frac{6}{6} = \frac{3}{3} = \frac{8}{8} = 1\,,$$

just as $6 \div 6 = 1$, $3 \div 3 = 1$, $8 \div 8 = 1$, and so on.

We have some more mathematical names to describe some fractions. If the numerator is smaller than the denominator, the value of the fraction is less than 1 and it is called a *proper fraction*. For example

$$\frac{1}{2}\,,\quad \frac{3}{4}\,,\quad \frac{1}{6}\,,\quad \frac{7}{8}\,,\quad \frac{5}{10}\,,\quad \frac{11}{12}\,,\quad \frac{100}{150}\,.$$

If the numerator is larger than the denominator and hence the value of the fraction is greater than 1, then it is called an *improper fraction*. For example

$$\frac{3}{2}\,,\quad \frac{7}{5}\,,\quad \frac{8}{4}\,,\quad \frac{12}{8}\,,\quad \frac{200}{100}\,.$$

Here, $\frac{3}{2}$ means 3 lots of a half, $\frac{7}{5}$ means 7 lots of one fifth and so on.

Improper fractions arise where more than one whole has been split up, and they can also be written as a mixture of whole numbers and fractions. For example, if we have $\frac{3}{2}$ then we can think of this as $\frac{2}{2}$ plus another $\frac{1}{2}$, and the $\frac{2}{2}$ form a whole. So

$$\frac{3}{2} \text{ can be written as } 1\tfrac{1}{2}\,.$$

Similarly with, say, $\frac{8}{3}$. Every 3 lots of $\frac{1}{3}$ makes a whole one, so we have 2 whole ones and 2 left over. In other words, we calculate $8 \div 3$: 3 goes in to 8 twice remainder 2, so $\frac{8}{3} = 2\frac{2}{3}$.

Here are some more examples:

$$\frac{7}{4} = 1\tfrac{3}{4}\,,\quad \frac{37}{10} = 3\tfrac{7}{10}\,.$$

These are referred to as *mixed fractions*.

Now let us look at turning mixed fractions into improper fractions. Suppose we start with $3\frac{1}{4}$. We want it written in quarters. Now 3 wholes, divided into quarters, give us 12 quarters. And we also have another quarter. In total we have 13 quarters, so as an improper fraction $3\frac{1}{4} = \frac{13}{4}$. In effect we have multiplied each whole number by 4, then added on the one quarter.

So, to convert from mixed fractions to improper fractions you multiply the whole number by the denominator then add the numerator before writing it all over the denominator.

**Example**

Write $5\frac{2}{9}$ as an improper fraction.

**Solution**

$$5\tfrac{2}{9} \quad = \quad \frac{5 \times 9 + 2}{9} \quad = \quad \frac{45 + 2}{9} \quad = \quad \frac{47}{9}\,.$$

**Example**

We can even write any whole number as a fraction, in many different ways. For instance,

$$2 \quad = \quad \frac{2}{1} \quad = \quad \frac{4}{2} \quad = \quad \frac{30}{15}\,.$$

**Key Point**

Fractions may appear as proper fractions, improper fractions or mixed fractions. They may also appear in many equivalent forms.

**Exercises**

1. Write down five fractions equivalent to $\frac{2}{3}$.
2. Write down $1\frac{4}{9}$ in five different ways, including at least one improper fraction.
3. Share a chocolate bar with 32 pieces, equally between four friends. Write down the fraction they each receive in five different ways.
4. Write 7 as a fraction in five different ways.
5. How many thirds make 5 whole ones?
6. Convert these improper fractions into mixed fractions:

$$\frac{10}{3}\,, \quad \frac{7}{2}\,, \quad \frac{16}{5}\,, \quad \frac{29}{10}\,, \quad \frac{15}{4}\,.$$

7. Convert these mixed fractions into improper fractions:

$$2\tfrac{1}{2}\,, \quad 6\tfrac{1}{3}\,, \quad 7\tfrac{2}{5}\,, \quad 11\tfrac{1}{4}\,, \quad 7\tfrac{2}{9}\,.$$

## Answers

1. Any equivalent fraction where both numerator and denominator have been multiplied by the same number.
2. $1\frac{8}{18}$, $1\frac{20}{45}$, $1\frac{40}{90}$, $\frac{13}{9}$, $\frac{26}{18}$, or any other equivalent.
3. $\frac{8}{32}$, $\frac{1}{4}$, $\frac{2}{8}$, $\frac{16}{64}$, $\frac{5}{20}$, or any other equivalent where the numerator and denominator have been multiplied by the same number.
4. $\frac{7}{1}$, $\frac{14}{2}$, $\frac{70}{10}$, $\frac{700}{100}$, $\frac{21}{3}$, or any other equivalent where the numerator and denominator have been multiplied by the same number.
5. 15
6. $3\frac{1}{3}$, $3\frac{1}{2}$, $3\frac{1}{5}$, $2\frac{9}{10}$, $3\frac{3}{4}$ .
7. $\frac{5}{2}$, $\frac{19}{3}$, $\frac{37}{5}$, $\frac{45}{4}$, $\frac{65}{9}$ .