Secondary One Mathematics: An Integrated Approach
Module 2
Arithmetic and Geometric Sequences

By

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In partnership with the
Utah State Office of Education

Module 2 – Arithmetic and Geometric Sequences

Classroom Task: Growing Dots- A Develop Understanding Task
Representing arithmetic sequences with equations, tables, graphs, and story context **(F.BF.1, F.LE.1, F.LE.2, F.LE.5)**
Ready, Set, Go Homework: Sequences 1

Classroom Task: Growing, Growing Dots – A Develop Understanding Task
Representing geometric sequences with equations, tables, graphs, and story context **(F.BF.1, F.LE.1, F.LE.2, F.LE.5)**
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Classroom Task: Scott's Workout – A Solidify Understanding Task
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Classroom Task: Don't Break the Chain – A Solidify Understanding Task
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Homework Help for Students and Parents

Core standards addressed in this unit:

F-BF: Build a function that models a relationship between to quantities.
1: Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-LE: Linear, Quadratic, and Exponential Models* (Secondary I focus is linear and exponential only)
Construct and compare linear, quadratic and exponential models and solve problems.

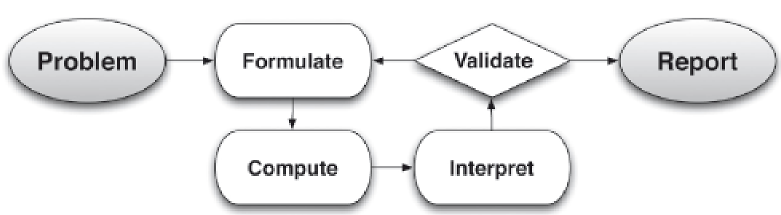
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which one quantity grows or decays by a constant percent rate per unit interval relative to another.

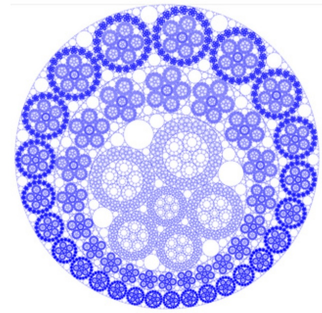
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Interpret expression for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

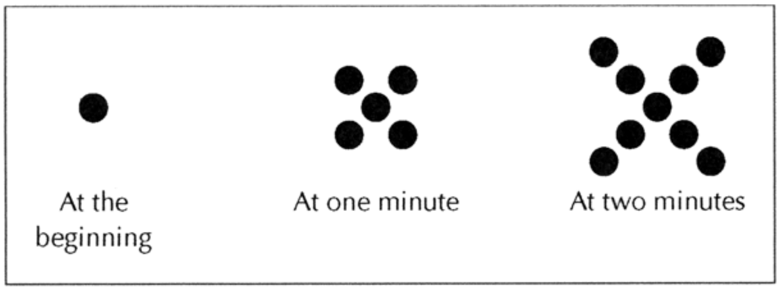
Tasks in this unit also follow the structure suggested in the Modeling standard:





Growing Dots*

A Develop Understanding Task



1. Describe the pattern that you see in the sequence of figures above.

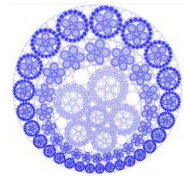
2. Assuming the sequence continues in the same way, how many dots are there at 3 minutes?

3. How many dots are there at 100 minutes?

4. How many dots are there at t minutes?

Solve the problems by your preferred method. Your solution should indicate how many dots will be in the pattern at 3 minutes, 100 minutes, and t minutes. Be sure to show how your solution relates to the picture and how you arrived at your solution.

*Adapted from: "Learning and Teaching Linear Functions", Nanette Seago, Judy Mumme, Nicholas Branca, Heinemann, 2004.



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Ready, Set, Go!

Ready

Topic: Exponents

Find each value.

1. 3^1
2. 3^2
3. 3^3
4. 3^4

Topic: Substitution and function notation

5. $f(x) = 2^x;\ find\ f(1), f(2), and\ f(3)$

6. $f(x) = 3^x;\ find\ f(1), f(2), and\ f(3)$

7. $f(x) = 2(x-1)+3;\ \ find\ f(1)\ and\ f(2)$

8. Complete each table.

Term	1st	2nd	3^{rd}	4th	5th	6th	7th	8th
Value	2	4	8	16	32			

Term	1st	2nd	3^{rd}	4th	5th	6th	7th	8th
Value	66	50	34	18				

Term	1st	2nd	3^{rd}	4th	5th	6th	7th	8th
Value	-3	9	-27	81				

Term	1st	2nd	3^{rd}	4th	5th	6th	7th	8th
Value	160	80	40	20				

Term	1st	2nd	3^{rd}	4th	5th	6th	7th	8th
Value	-9	-2	5	12				

Set

Topic: Completing a table

Fill in the table. Then write a sentence explaining how you figured out the values to put in each cell. Explain how to figure out what will be in cell #8.

9. You run a business making birdhouses. You spend $600 to start your business, and it costs you $5.00 to make each birdhouse.

# of birdhouses	1	2	3	4	5	6	7
Total cost to build							

Explanation:

10. You borrow $500 from a relative, and you agree to pay back the debt at a rate of $15 per month.

# of months	1	2	3	4	5	6	7
Amount of money owed							

Explanation:

11. You earn $10 per week.

# of weeks	1	2	3	4	5	6	7
Amount of money earned							

Explanation:

12. You are saving for a bike and can save $10 per week. You have $25 already saved.

# of weeks	1	2	3	4	5	6	7
Amount of money saved							

Explanation:

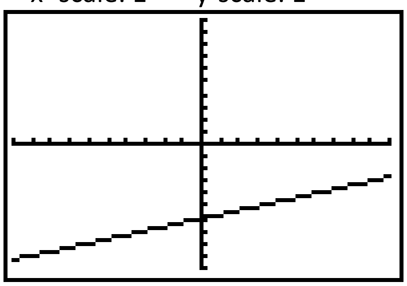
Go

Topic: Good viewing window

When sketching a graph of a function, it is important that we see important features of the graph. For linear functions, sometimes we want a window that shows important information related to a situation. Sometimes, this means including both the x- and y- intercepts. For the following equations, practice finding a 'good view' by graphing the problems below and including both intercepts within the window. Also include your scale for both axes.

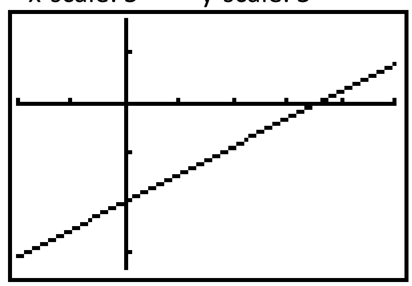
Example: $g(x) = \frac{1}{3}x - 6$

Window: [-10, 10] by [-10,10]
x- scale: 1 y-scale: 1



NOT a good window

Window: [-10, 25] by [-10, 5]
x-scale: 5 y-scale: 5



Good window

1. $f(x) = -\frac{1}{10}x + 1$

[ ] by [ ]
x-scale: y-scale:

2. $7x - 3y = 14$

[ ] by [ ]
x-scale: y-scale:

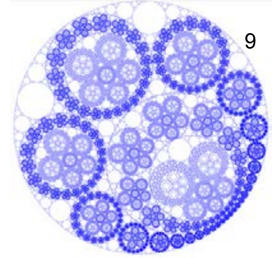
3. $y = 3(x - 5) + 12$

[ ] by [ ]
x-scale: y-scale:

4. $f(x) = -15\ (x + 10) - 45$

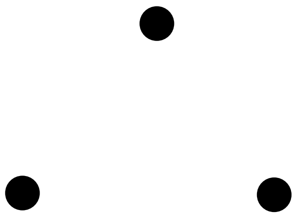
[ ] by [ ]
x-scale: y-scale:

5. Explain the pros and cons for this type of viewing window. Describe how some viewing windows are not good for showing how steep the slope may be in a linear equation. Use examples from above to discuss how the viewing window may be deceiving.

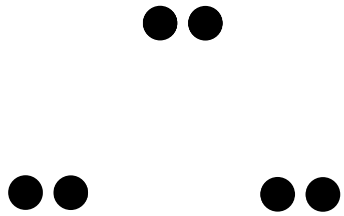


Growing, Growing Dots

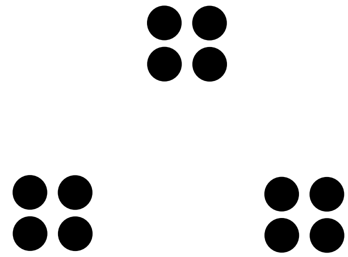
A Develop Understanding Task



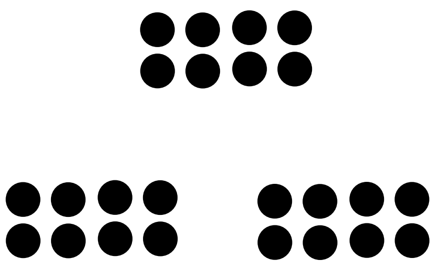
At the beginning



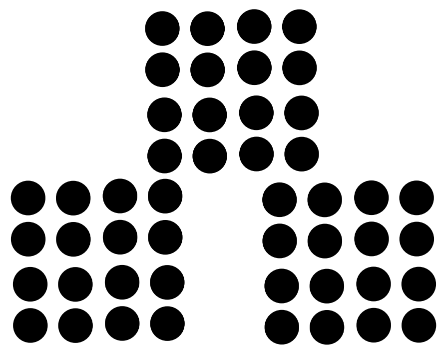
At one minute



At two minutes

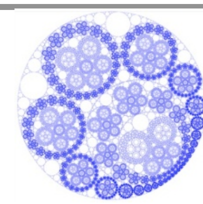


At three minutes



At four minutes

1. Describe and annotate the pattern of change you see in the above sequence of figures.
2. Assuming the sequence continues in the same way, how many dots are there at 5 minutes?
3. Write a recursive formula to describe how many dots there will be after t minutes?
4. Write an explicit formula to describe how many dots there will be after t minutes?



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Ready, Set, Go!

Ready

Topic: Finding values for a pattern

1. Bob Cooper was born in 1900. By 1930 he had 3 sons, all with the Cooper last name. By 1960 each of Bob's 3 boys had exactly 3 sons of their own. By the end of each 30 year time period, the pattern of each Cooper boy having exactly 3 sons of their own continued. How many Cooper sons were born in the 30 year period between 1960 and 1990?

2. Create a diagram that would show this pattern.

3. Predict how many Cooper sons will be born between 1990 and 2020, if the pattern continues.

4. Try to write an equation that would help you predict the number of Cooper sons that would be born between 2020 and 2050. If you can't find the equation, explain it in words.

Set

Topic: Evaluate the following equations when x = { 1, 2, 3, 4, 5 }. Organize your inputs and outputs into a table of values for each equation. Let x be the input and y be the output.

5. $y = 4^x$ 6. $y = (-3)^x$ 7. $y = -3^x$ 8. $y = 10^x$

Go

Topic: Solve equations

9. Solve the following equations for the unknown variable.

a. $3(x-1)=2(x+3)$

b. $7(x+20)=x+5$

c. $9(x-2)=3x+3$

d. $2\left(a-\frac{1}{3}\right)=\frac{2}{5}\left(a+\frac{2}{3}\right)$

e. $3(x+3)-2(x-1)=0$

Need help? Check out these related videos.

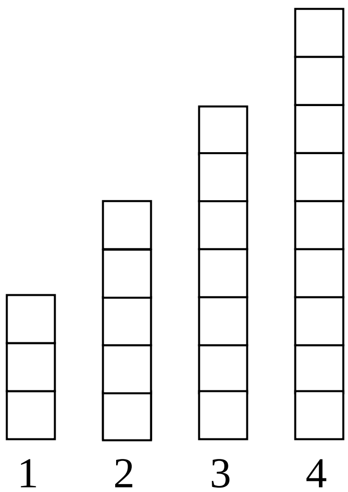
Evaluating with exponents http://www.khanacademy.org/math/algebra/exponents-radicals/v/level-1-exponents

Solving equations http://www.khanacademy.org/math/algebra/solving-linear-equations/v/solving-equations-with-the-distributive-property

Scott's Workout

A Solidify Understanding Task

Scott has decided to add push-ups to his daily exercise routine. He is keeping track of the number of push-ups he completes each day in the bar graph below, with day one showing he completed three push-ups. After four days, Scott is certain he can continue this pattern of increasing the number of push-ups he completes each day.



1. How many push-ups will Scott do on day 10?
2. How many push-ups will Scott do on day *n*?
3. Model the number of push-ups Scott will complete on any given day. Include both explicit and recursive equations.
4. Aly is also including push-ups in her workout and says she does more push-ups than Scott because she does fifteen push-ups every day. Is she correct? Explain.

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Ready, Set, Go!

Ready

Topic: Find the slope of the line that goes through each pair of points.

1. (3,7) and (5, 10)
2. (-1, 4) and (3,3)
3. (0,0) and (-2, 5)
4. (-1, -5) and (-4, -5)

Set

Topic: Finding terms for a given sequence.

Find the next 3 terms in each sequence. Identify the constant difference. Write recursive equations for the following arithmetic sequences, and then write the explicit equation. Identify where you see the constant difference in both equations.

4. Constant difference? ________________________________

3	8	13	18	23			

Recursive equation: Explicit equation:

5. Constant difference? ________________________________

11	9	7	5	3			

Recursive equation: Explicit equation:

6. Constant difference? ________________________________

3	1.5	0	-1.5	-3			

Recursive equation: Explicit equation:

Go

Topic: Write the equations in slope intercept form.

7. y = 12 + (x – 1)(-4)

8. $\frac{2}{3}(6y+9)=\frac{3}{5}(15x-20)$

9. $\frac{5}{7}(21y+7)=\frac{2}{9}(18x+27)$

Need Help? Check out these related videos:

Finding slope http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/slope-and-rate-of-change

Writing the explicit equation http://www.khanacademy.org/math/algebra/solving-linear-equations/v/equations-of-sequence-patterns

Writing equations in slope-intercept form http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/converting-to-slope-intercept-form

Don't Break the Chain

A Solidify Understanding Task

Maybe you've received an email like this before:

> Hi! My name is Bill Weights, founder of Super Scooper Ice Cream. I am offering you a gift certificate for our signature "Super Bowl" (a $4.95 value) if you forward this letter to 10 people.
>
> When you have finished sending this letter to 10 people, a screen will come up. It will be your Super Bowl gift certificate. Print that screen out and bring it to your local Super Scooper Ice Cream store. The server will bring you the most wonderful ice cream creation in the world—a Super Bowl with three yummy ice cream flavors and three toppings!
>
> This is a sales promotion to get our name out to young people around the country. We believe this project can be a success, but only with your help. Thank you for your support.
>
> Sincerely,
>
> Bill Weights
> Founder of Super Scooper Ice Cream

These chain emails rely on each person that receives the email to forward it on. Have you ever wondered how many people might receive the email if the chain remains unbroken? To figure this out, assume that it takes a day for the email to be opened, forwarded, and then received by the next person. On day 1, Bill Weights starts by sending the email out to his 8 closest friends. They each forward it to 10 people so that on day 2, it is received by 80 people. The chain continues unbroken.

1. How many people will receive the email on day 7?
2. How many people with receive the email on day *n*? Explain your answer with as many representations as possible.
3. If Bill gives away a Super Bowl that costs $4.95 to every person that receives the email during the first week, how much will he have spent?

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Ready, Set, Go!

Ready

Topic: Write the equation of a line given two points.

Find the equation of the line that goes through each pair of points. Graph the equation and explain the values for *x* that would work for each line.

1. (5,2) and (7,0)
2. (-4,2) and (6,6)
3. (3,0) and (0, 4)
4. (2,-4) and (2, 6)
5. (2,2) and (8,8)

Set

Topic: Find the recursive and explicit equations for each geometric sequence.

6. 2, 4, 8, 16…
7.

Time (days)	Number of cells
1	3
2	6
3	12
4	24

8. Claire has $300 in an account. She decides she is going to take out half of the money remaining in the account at the end of each month.

9. Tania creates a chain letter and sends it to four friends. Each friend is then instructed to send it to four of their friends, and so forth.

10.

Day 1	Day 2	Day 3
+ +	+ + + +	+ + + + + + + +
+ +	+ + + +	+ + + + + + + +
+ +	+ + + +	+ + + + + + + +

Go

Topic: Graph sequences

Graph each problem. Focus on creating a 'good viewing window'. Label axes and scale. Explain the values of *x* that make sense for each situation.

1. 2, 4, 8, 16…
2.

Time (days)	Number of cells
1	3
2	6
3	12
4	24

8. Claire has $300 in an account. She decides she is going to take out half of the money remaining in the account at the end of each month.

9. Tania creates a chain letter and sends it to four friends. Each friend is then instructed to send it to four of their friends, and so forth.

10. Day 1 Day 2 Day 3

Day 1	Day 2	Day 3
+ +	+ + + +	+ + + + + + + +
+ +	+ + + +	+ + + + + + + +
+ +	+ + + +	+ + + + + + + +

Need Help? Check out these related videos:

Find equation of line http://patrickjmt.com/find-the-equation-of-a-line-using-point-slope-form/

Something to Chew On

A Solidify Understanding Task

The Food-Mart grocery store has a candy machine like the one pictured here. Each time a child inserts a quarter, 7 candies come out of the machine. The machine holds 15 pounds of candy. Each pound of candy contains about 180 individual candies.

1. Represent the number of candies in the machine for any given number of customers. About how many customers will there be before the machine is empty?
2. Represent the amount of money in the machine for any given number of customers.
3. To avoid theft, the store owners don't want to let too much money collect in the machine, so they take all the money out when they think the machine has about $25 in it. The tricky part is that the store owners can't tell how much money is actually in the machine without opening it up, so they choose when to remove the money by judging how many candies are left in the machine. About how full should the machine look when they take the money out? How do you know?

Ready, Set, Go!

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Ready

Topic: Find the missing terms for each arithmetic sequence and state the common difference, *d*.

1. 5, 11, ___, 23, 29, ___... common difference:

2. 7, 3, -1, ___, ___, -13… common difference:

3. 8, ___, ___, 47, 60… common difference:

4. 0, ___, ___, 2, $\frac{8}{3}$ … common difference:

5. 5, ___, ___, ___, 25… common difference:

Set

Topic: Determine recursive equations

Two consecutive terms in an arithmetic sequence are given. Find the constant difference and the recursive equation.

6. If $f(3) = 5 \; and \; f(4) = 8$.

Find $f(5) and \; f(6)$. Then find $f(next) = d + \; f(now)$

7. If $f(2) = 20 \; and \; f(3) = 12$.

Find $f(4) and \; f(5)$. Then find $f(next) = d + f(now) + d$

8. If $f(5) = 3.7 \; and \; f(6) = 8.7$.

Find $f(7) and \; f(8)$. Then find $f(now) = d + f(previous)$

Go

Topic: Evaluate using function notation

Find each value.

9. Find $f(3); f(n) = 2^n$

10. Find $f(2); f(n) = 5^n$

11. Find $f(3); f(n) = (-2)^n$

12. Find $f(5) and\ f(6); f(n) = 3 + 4(n-1)$

13. Find $f(1) and\ f(2); f(n) = 2(n-1) + 6$

Need Help? Check out these videos:

Arithmetic sequences http://www.khanacademy.org/math/algebra/solving-linear-equations/v/patterns-in-sequences-1

Function notation http://www.youtube.com/watch?v=Kj3Aqov52TY

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Chew on This

A Solidify Understanding Task

Mr. and Mrs. Gloop want their son, Augustus, to do his homework every day. Augustus loves to eat candy, so his parents have decided to motivate him to do his homework by giving him candies for each day that the homework is complete. Mr. Gloop says that on the first day that Augustus turns in his homework, he will give him 10 candies. On the second day he promises to give 20 candies, on the third day he will give 30 candies, and so on.

1. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with his father's plan.
2. Use a formula to find how many candies Augustus will have on day 30 in this plan.

Augustus looks in the mirror and decides that he is gaining weight. He is afraid that all that candy will just make it worse, so he tells his parents that it would be ok if they just give him 1 candy on the first day, 2 on the second day, continuing to double the amount each day as he completes his homework. Mr. and Mrs. Gloop like Augustus' plan and agree to it.

3. Model the amount of candy that Augustus would get each day he reaches his goals with the new plan.
4. Use your model to predict the number of candies that Augustus would earn on the 30th day with this plan.
5. Write both a recursive and an explicit formula that shows the number of candies that Augustus earns on any given day with this plan.

Augustus is generally selfish and somewhat unpopular at school. He decides that he could improve his image by sharing his candy with everyone at school. When he has a pile of 100,000 candies, he generously plans to give away 60% of the candies that are in the pile each day. Although Augustus may be earning more candies for doing his homework, he is only giving away candies from the pile that started with 100,000. (He's not that generous.)

6. Model the amount of candy that would be left in the pile each day.
7. How many pieces of candy will be left on day 8?
8. When would the candy be gone?

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Ready, Set, Go!

Ready

Topic: Arithmetic and geometric sequences

Find the missing values for each arithmetic or geometric sequence. Then

1. 5, 10, 15, ___, 25, 30…
   Does this sequence have a constant difference or a constant rate?________________________
   what is the value? ______

2. 20, 10, ___, 2.5, ___...
   Does this sequence have a constant difference or a constant rate?________________________
   what is the value? ______

3. 2, 5, 8, ___, 14, ___...
   Does this sequence have a constant difference or a constant rate?________________________
   what is the value? ______

4. 30, 24, ___, 12, 6…
   Does this sequence have a constant difference or a constant rate?________________________
   what is the value? ______

Set

Topic: Recursive and explicit equations

Determine whether each situation represents an arithmetic or geometric sequence and then find the recursive and explicit equation for each.

5. 2, 4, 6, 8 …
6. 2, 4, 8, 16…
7.

Time (days)	Number of Dots
1	3
2	7
3	11
4	15

8.

Time (days)	Number of cells
1	5
2	8
3	12.8
4	20.48

9. Michelle likes chocolate but ths it causes acne. She chooses to limit herself to three pieces of chocolate every five days.
10. Scott decides to add running to his exercise routine and runs a total of one mile his first week. He plans to double the number of miles he runs each week.
11. Vanessa has $60 to spend on rides at the State Fair. Each ride cost $4.
12. Adella bought a car for $10,000. One year later, the car was worth $8,000. A year after that, the car was worth $6,400. The pattern continued and the next year the car was worth $5,120.
13. Cami invested $6,000 dollars into an account that earns 10% interest each year.
14. How are arithmetic and geometric sequences similar?
15. How are arithmetic and geometric sequences different?

Go

Topic: Solving systems of linear equations.

Solve the system of equations.

15. $y = 2x - 10$ and $x - 4y = 5$

16. $x - 7y = 6$ *and* $-3x + 21y = -18$

17. $5x - 4y = 3$ and $6x + 4y = 30$

Need help? Check out these related videos

Arithmetic and geometric sequences http://www.youtube.com/watch?v=THV2Wsf8hro

What Comes Next? What Comes Later?

A Practice Understanding Task

For each of the following tables,

- describe how to find the next term in the sequence,
- write a recursive rule for the function,
- describe how the features identified in the recursive rule can be used to write an explicit rule for the function, and
- write an explicit rule for the function.
- identify if the function is arithmetic, geometric or neither

Example:

x	**y**
0	5
1	8
2	11
3	14
4	?
…	…
n	?

- To find the next term: add 3 to the previous term
- Recursive rule: $f(0) = 5, f(n) = f(n-1) + 3$
- To find the n^{th} term: start with 5 and add 3 n times
- Explicit rule: $f(n) = 5 + 3n$
- Arithmetic, geometric, or neither? Arithmetic

Function A

x	**y**
1	5
2	10
3	20
4	40
5	?
…	…
n	?

1. To find the next term: ________________________________
2. Recursive rule: ____
3. To find the n^{th} term: ____________________________________
4. Explicit rule: ____
5. Arithmetic, geometric, or neither? _________________________

Function B

x	**y**
0	3
1	4
2	7
3	12
4	19
5	?
…	…
n	?

6. To find the next term: ________________________________
7. Recursive rule: ____
8. To find the n^{th} term: ____________________________________
9. Explicit rule: ____
10. Arithmetic, geometric, or neither? _________________________

Function C

x	y
1	3
2	5
3	9
4	17
5	33
6	?
…	…
n	?

11. To find the next term: ______________________________
12. Recursive rule: ______________________________
13. To find the n^{th} term: ______________________________
14. Explicit rule: ______________________________
15. Arithmetic, geometric, or neither? ______________________________

Function D

x	y
1	-8
2	-17
3	-26
4	-35
5	-44
6	-53
…	…
n	

16. To find the next term: ______________________________
17. Recursive rule: ______________________________
18. To find the n^{th} term: ______________________________
19. Explicit rule: ______________________________
20. Arithmetic, geometric, or neither? ______________________________

Function E

x	y
1	2
2	-6
3	18
4	-54
5	162
6	-486
…	…
n	

21. To find the next term: ______________________________
22. Recursive rule: ______________________________
23. To find the n^{th} term: ______________________________
24. Explicit rule: ______________________________
25. Arithmetic, geometric, or neither? ______________________________

Function F

x	y
0	1
1	$1\frac{3}{5}$
2	$2\frac{1}{5}$
3	$2\frac{4}{5}$
4	$3\frac{2}{5}$
5	4
…	…
n	

26. To find the next term: ______________________________
27. Recursive rule: ______________________________
28. To find the n^{th} term: ______________________________
29. Explicit rule: ______________________________
30. Arithmetic, geometric, or neither? ______________________________

x	y
1	10
2	2
3	$\frac{2}{5}$
4	$\frac{2}{25}$
5	$\frac{2}{125}$
6	$\frac{2}{625}$
…	…
n	

Function G

31. To find the next term: ______________________________

32. Recursive rule: ______________________________

33. To find the n^{th} term: ______________________________

34. Explicit rule: ______________________________

35. Arithmetic, geometric, or neither? ______________________________

x	y
1	-1
2	0.2
3	-0.04
4	0.008
5	-0.0016
6	0.00032
…	…
n	

Function H

36. To find the next term: ______________________________

37. Recursive rule: ______________________________

38. To find the n^{th} term: ______________________________

39. Explicit rule: ______________________________

40. Arithmetic, geometric, or neither? ______________________________


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Ready, Set, Go!

Ready

Topic: Find the constant ratio for each geometric sequence.

1. 1, 4, 8, 16…

2. $\frac{1}{2}$, 1, 2, 4, 8…

3. -5, 10, -20, 40…

4. 10, 5, 2.5, 1.25…

Set

Topic: Recursive and explicit equations

Fill in the blanks for each table, then write the recursive and explicit equation for each sequence.

5. Table 1

x	1	2	3	4	5
y	5	7	9		

Recursive:

Explicit:

6. Table 2

x	*y*
1	-2
2	-4
3	-6
4	
5	

Recursive:

Explicit:

7. Table 3

x	*y*
1	3
2	9
3	27
4	
5	

Recursive:

Explicit

8. Table 4

x	*y*
1	27
2	9
3	3
4	
5	

Recursive:

Explicit:

Go

Topic: Graphing linear equations and labeling windows

Graph the following linear equations. Label your window

13. $y = 4x + 7$

14. $y = \frac{-3}{4}x + 5$

15. $2x + 7y = 10$

16. $x - 3y = 7$

Need Help? Check out these related videos:

Graphing equations http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/graphs-using-slope-intercept-form

What Does It *Mean*?

A Solidify Understanding Task

Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

x	1	2	3
y	5		11

x	1	2	3	4	5
y	18				-10

x	1	2	3	4	5	6	7
y	12						-6

Describe your method for finding the missing terms. Will the method always work? How do you know?

Here are a few more arithmetic sequences with missing terms. Complete each table, either using the method you developed previously or by finding a new method.

x	1	2	3	4
y	50			86

x	1	2	3	4	5	6
y	40					10

x	1	2	3	4	5	6	7	8
y	-23							5

The missing terms in an arithmetic sequence are called “arithmetic means”. For example, in the problem above, you might say, “Find the 6 arithmetic means between -23 and 5”. Describe a method that will work to find arithmetic means and explain why this method works.

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Ready, Set, Go!

Ready

Topic: Comparing arithmetic and geometric sequences

1. How are arithmetic and geometric sequences similar?

2. How are they different?

Set

Topic: arithmetic sequences

Each of the tables below represents an arithmetic sequence. Find the missing terms in the sequence, showing your method.

3. Table 1

x	1	2	3
y	3		12

4. Table 2

x	y
1	2
2	
3	
4	26

5. Table 3

x	y
1	24
2	
3	6
4	

6. Table 4

x	y
1	16
2	
3	
4	2
5	

Go

Topic: Determine if the sequence is arithmetic, geometric, or neither. Then determine the recursive and explicit equations for each (if the sequence is not arithmetic or geometric, try your best).

7. 5, 9, 13, 17,…

8. 60, 30, 0, -30,…

9. 60, 30, 15, $\frac{15}{2}$, …

10. 

11. 4, 7, 12, 19, …

Geometric *Meanies*

A Solidify and Practice Task

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Each of the tables below represents a geometric sequence. Find the missing terms in the sequence, showing your method.

Table 1

x	1	2	3
y	3		12

Is the missing term that you identified the only answer? Why or why not?

Table 2

x	1	2	3	4
y	7			875

Are the missing terms that you identified the only answers? Why or why not?

Table 3

x	1	2	3	4	5
y	6				96

Are the missing terms that you identified the only answers? Why or why not?

Table 4

x	1	2	3	4	5	6
y	4					972

Are the missing terms that you identified the only answers? Why or why not?

A. Describe your method for finding the geometric means.

B. How can you tell if there will be more than one solution for the geometric means?

Ready, Set, Go!

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Ready

Topic: Arithmetic and geometric sequences

For each set of sequences, find the first five terms. Compare arithmetic sequences and geometric sequences. Use the given information to help explain how they are similar and how they are different. Which grows faster? When?

1. Arithmetic sequence: $f(1) = 2, common\ difference, d = 3$
Geometric sequence: $f(1) = 2, common\ ratio, r = 3$

Arithmetic:	Geometric:
$f(1) =$	$f(1) =$
$f(2) =$	$f(2) =$
$f(3) =$	$f(3) =$
$f(4) =$	$f(4) =$
$f(5) =$	$f(5) =$

Whose value do you think will be more for $f(100)$? Why?

2. Arithmetic sequence: $f(1) = 2, common\ difference, d = 10$
Geometric sequence: $f(1) = 2, common\ ratio, r = 3$

Arithmetic:	Geometric:
$f(1) =$	$f(1) =$
$f(2) =$	$f(2) =$
$f(3) =$	$f(3) =$
$f(4) =$	$f(4) =$
$f(5) =$	$f(5) =$

Whose value do you think will be more for $f(100)$? Why?

3. Arithmetic sequence: $f(1) = 20, d = 10$
Geometric sequence: $f(1) = 2, r = 2$

Arithmetic:	Geometric:
$f(1) =$	$f(1) =$
$f(2) =$	$f(2) =$
$f(3) =$	$f(3) =$
$f(4) =$	$f(4) =$
$f(5) =$	$f(5) =$

Whose value do you think will be more for $f(100)$? Why?

4. Arithmetic sequence: $f(1) = 50, common\ difference, d = 10$
Geometric sequence: $f(1) = 1, common\ ratio, r = 2$

Arithmetic:	Geometric:
$f(1) =$	$f(1) =$
$f(2) =$	$f(2) =$
$f(3) =$	$f(3) =$
$f(4) =$	$f(4) =$
$f(5) =$	$f(5) =$

Whose value do you think will be more for $f(100)$? Why?

5. Compare arithmetic sequences and geometric sequences growth rates. Which grows faster? When?

Set

Topic: Geometric sequences

Each of the tables below represents a geometric sequence. Find the missing terms in the sequence, showing your method.

6. Table 1

x	1	2	3
y	3		12

7. Table 2

x	y
1	2
2	
3	
4	54

8. Table 3

x	y
1	5
2	
3	20
4	

9. Table 4

x	y
1	4
2	
3	
4	
5	324

Go

Topic: Given the following information, determine the explicit equation for each geometric sequence.

10. $f(1) = 8, common\ ratio,\ r = 2$

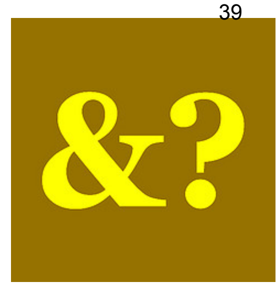
11. $f(1) = 4, f(n) = 3f(n-1)$

12. $f(n) = 4f(n-1);\ f(1) = \frac{5}{3}$

Which geometric sequence above has the greatest value at $f(100)$?

Need Help? Check out these videos:

Geometric sequence http://www.khanacademy.org/math/algebra/ck12-algebra-1/v/geometric-sequences--introduction



I Know . . . What Do You Know?

A Practice Task

In each of the problems below I share some of the information that I know about a sequence. Your job is to add all the things that you know about the sequence from the information that I have given. Depending on the sequence, some of things you may be able to figure out for the sequence are: a table, a graph, an explicit equation, a recursive equation, the constant ratio or constant difference between consecutive terms, any terms that are missing, the type of sequence, or a story context. Try to find as many as you can for each sequence, but you must have at least 4 things for each.

1. I know that: the recursive formula for the sequence is $f(1) = -12,\ f(n) = f(n-1) + 4$
   What do you know?

2. I know that: the first 5 terms of the sequence are 0, -6, -12, -18, -24 . . .
   What do you know?

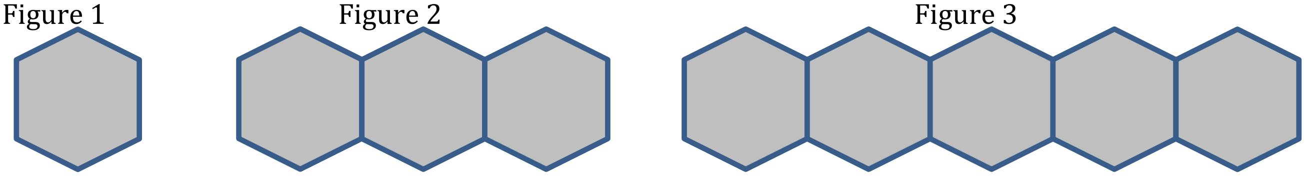
3. I know that: the explicit formula for the sequence is $f(n) = -10(3)^n$
   What do you know?

4. I know that: The first 4 terms of the sequence are 2, 3, 4.5, 6.75 . . .
   What do you know?

5. I know that: the sequence is arithmetic and $f(3) = 10$ and $f(7) = 26$
   What do you know?

6. I know that: the sequence is a model for the perimeter of the following figures:

Figure 1 Figure 2 Figure 3



Length of each side = 1

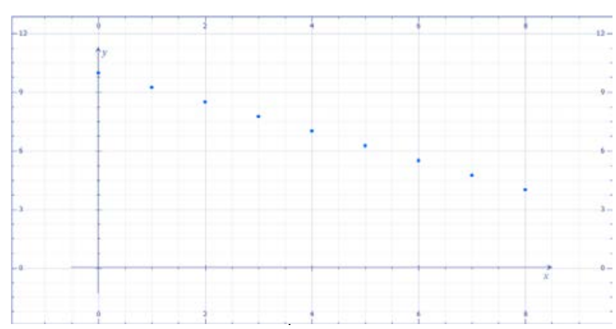
What do you know?

7. I know that: it is a sequence where a_1 = 5 and the constant ratio between terms is -2.
What do you know?

8. I know that: the sequence models the value of a car that originally cost $26,500, but loses 10% of its value each year.
What do you know?

9. I know that: the first term of the sequence is -2, and the fifth term is $-\frac{1}{8}$.
What do you know?

10. I know that: a graph of the sequence is:
What do you know?



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Ready, Set, Go!

Ready

Topic: Comparing linear equations and arithmetic sequences

1. Describe similarities and differences between linear equations and arithmetic sequences.

Similarities	Differences

Set

Topic: representations of arithmetic sequences

Use the given information to complete the other representations for each arithmetic sequence.

2.

<table>
<tr><td>Table

Days	1	2	3	4
Cost	8	16	24	32

</td><td>Graph</td></tr>
<tr><td>Recursive Equation</td><td>Explicit Equation</td></tr>
<tr><td colspan="2">Create a context:</td></tr>
</table>

3.

Table	Graph
Recursive Equation $f(n) = f(n-1) + 3;$ $f(1) = 4$	Explicit Equation
Create a context:	

4.

Table	Graph
Recursive Equation	Explicit Equation $f(n) = 4 + 5(n-1)$
Create a context:	

5.

Table	Graph
Recursive Equation	Explicit Equation
Create a context: Janet wants to know how many seats are in each row of the theater. Jamal lets her know that each row has 2 seats more than the row in front of it. The first row has 14 seats.	

Go

Topic: Writing explicit equations

Given the recursive equation for each arithmetic sequence, write the explicit equation.

6. $f(n) = f(n-1) - 2; f(1) = 8$

7. $f(n) = 5 + f(n-1); f(1) = 0$

8. $f(n) = f(n-1) + 1;\ f(1) = \frac{5}{3}$

Topics

1. Recognizing different kinds of patterns
   http://www.khanacademy.org/math/algebra/linear-equations-and-inequalitie/v/recognizing-linear-functions
2. Using a pattern to predict. http://www.khanacademy.org/math/algebra/solving-linear-equations/v/patterns-in-sequences-1
   http://www.khanacademy.org/math/algebra/solving-linear-equations/v/patterns-in-sequences-2
3. Writing Recursive and explicit equations
   http://www.khanacademy.org/math/algebra/algebra-functions/v/basic-linear-function
4. Functions and Mathematical Models

http://www.khanacademy.org/math/algebra/solving-linear-equations/v/equations-of-sequence-patterns

3. Functions and Mathematical Models

Objectives

- Categorize some realistic situations in terms of function families.
- Write a function to represent a situation.
- Use a function to determine key aspects of a situation.

Concept

Introduction

Throughout this chapter we have examined different kinds of functions and their behavior, and we have used functions to represent realistic situations. When we use a function to help us understand phenomena such as how to maximize the volume of a container or to minimize its surface area, we are engaging in mathematical modeling. In reality, scientists and social scientists use mathematical models to understand a wide variety of quantifiable phenomena, from the workings of subatomic particles, to how people will function in the economy.

In this lesson we will revisit some of the examples we have seen in previous lessons, in an effort to categorize models according to function families. We will look at several examples of models in depth, specifically in terms of how we can use a graphing calculator to help us analyze models.

Linear models

The very first example of a function in this chapter was a linear model. The equation $y=3x$ was used to represent how much money you would bring in if you sold x boxes of cookies for $3 per box. Many situations can be modeled with linear functions. The key idea is that some quantity in the situation has a constant rate of change. In the cookie-selling example, every box costs $3.00. Therefore the profits increase at a constant rate.

The cookie-selling model is an equation of the form $y=mx$. The function necessarily contains the point (0,0) : if we don't sell any cookies, we don't bring in any money!

Other models will be of the form $y=mx+b$. The constant b is the y-intercept of the function, and represents the value of the function when x is zero. For example, consider a situation in which you plan to save money at a constant rate of $20 per week. If you begin to save money after receiving a gift of $100, you can express the amount you have saved as a function of time: $S(t)=20t+100$, where t represents the number of weeks you have been saving. The function is linear because of the constant rate of change, that is, the constant savings of $20 per week.

Notice that in both of these examples we will only consider these functions for x values ≥ 0. In the first example, x represents the numbers of boxes of cookies, which cannot be negative. In the second example, x represents the number of weeks you have been saving money. In theory we could extend this situation back in time, but the given information does not indicate that the model would make sense. This is the case because you received $100 as a gift at a particular point in time. You didn't save that $100 at $20 per week.

Both of these examples also are linear functions with positive slope. In both situations, the function increases at a constant, or steady rate. We could also use a linear function to model a situation of constant decrease.

In sum, linear functions are used to model a situation of constant change, either increase or decrease. Next we will consider functions that can be used to model other kinds of situations.

Problem Set

1. Consider this situation: you run a business making birdhouses. You spend $600 to start your business, and it costs you $5.00 to make each birdhouse. a. Write a linear equation to represent this situation. b. State the domain of this function. c. What does the y-intercept represent? What does the slope represent?

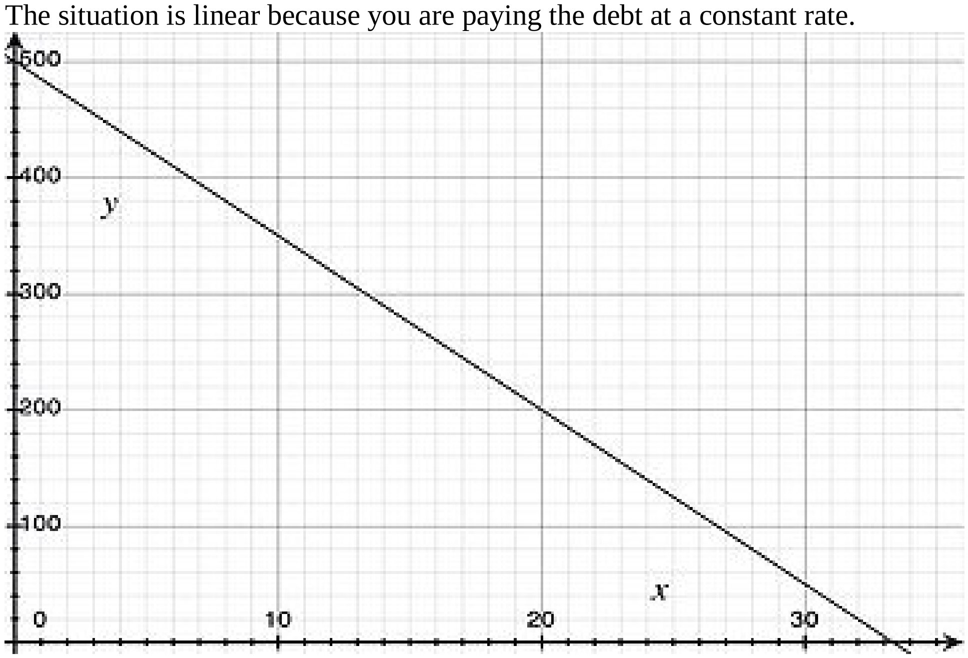
2. Consider this situation: you borrow $500 from a relative, and you agree to pay back the debt at a rate of $15 per month. a. Write a linear model to represent this situation. b. Explain why this situation is linear. c. Graph the function you wrote in part (a) and use the graph to determine the number of months it will take to pay off the debt.

3. Express the following situation as a composition of functions: You are running a small business making wooden jewelry boxes. It costs you $5.00 per unit to produce wooden boxes, plus an initial investment of $300 in other materials. It then costs you an additional $2.00 per box to decorate the boxes.

Answers

1. a. $y = 5x+600$
   b. D: All real numbers greater than or equal to 0.
   c. The y-intercept (0, 600) represents how much money your business has cost you before you have produced any birdhouses. The slope, 5, represents the cost per birdhouse.

2. a. $y = -15x + 500$
   b. The situation is linear because you are paying the debt at a constant rate.



c. It will take 33(1/3) months, or 34 months to pay off the debt.

3. Initial cost function: $C_1(x) = 5x + 300$ Second cost function $C_2(x) = 2x$ Composition: $C(x) = C_1(C_2(x)) = 5(2x) + 300 = 10x + 300$.