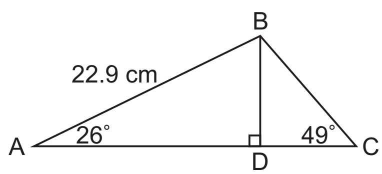
2.7 Solving Problems Involving More than One Right Triangle

FOCUS **Use trigonometric ratios to solve problems that involve more than one right triangle.**

When a problem involves more than one right triangle, we can use information from one triangle to solve the other triangle.

Example 1 Solving a Problem with Two Triangles

Find the length of BC to the nearest tenth of a centimetre.



To solve a right triangle we must know:
- *the lengths of two sides, or*
- *the length of one side and the measure of one acute angle*

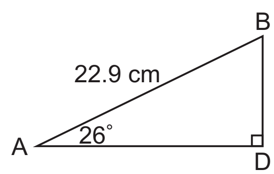
Solution

First use △ABD to find the length of BD.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{BD}{AB}$$

$$\sin 26° = \frac{BD}{22.9}$$



Side BD is common to both triangles.

$$22.9 \sin 26° = BD$$

$$BD = 10.0386\ldots$$

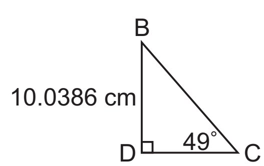
Do not clear the calculator screen.

In △BCD, find the length of BC.

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin C = \frac{BD}{BC}$$

$$\sin 49° = \frac{10.0386\ldots}{BC}$$



$$BC \sin 49° = 10.0386\ldots$$

$$BC = \frac{10.0386\ldots}{\sin 49°}$$

$$BC = 13.3014\ldots$$

BC is about 13.3 cm long.

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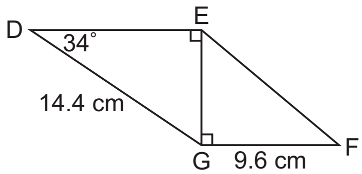
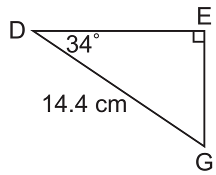
Check

1. Find the measure of ∠F to the nearest degree.



Use △DEG to find the length of EG.
Use the sine ratio.



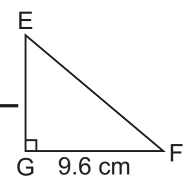
Side EG is common to both triangles.

sin D = ____________

sin D = ________

sin ______ = ________

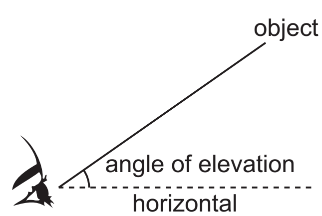
EG = ____________



In △EFG, use the __________ ratio to find ∠F.

The measure of ∠F is about _______.

The **angle of elevation** is the angle between the horizontal and a person's line of sight to an object above.



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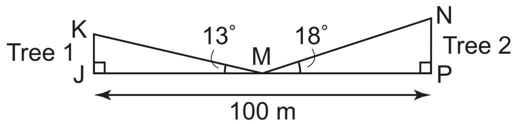
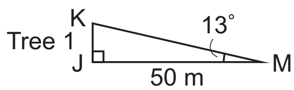
Example 2 Solving a Problem Involving Angle of Elevation

Jason is lying on the ground midway between two trees, 100 m apart.
The angles of elevation of the tops of the trees are 13° and 18°. How much taller is one tree than the other? Give the answer to the nearest tenth of a metre.



Solution

Jason is midway between the trees.
So, the distance from Jason to the base of each tree is: $\frac{100 \text{ m}}{2} = 50$ m
Use △JKM to find the length of JK.



We know ∠M = 13°.
JK is opposite ∠M.
JM is adjacent to ∠M.
Use the tangent ratio.

$$\tan M = \frac{\text{opposite}}{\text{adjacent}}$$

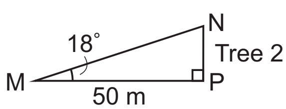
$$\tan M = \frac{JK}{JM}$$ Substitute: ∠M = 13° and JM = 50

$$\tan 13° = \frac{JK}{50}$$

$$50 \tan 13° = JK$$

$$JK = 11.5434\ldots$$

Use △MNP to find the length of NP.



We know ∠M = 18°.
NP is opposite ∠M.
MP is adjacent to ∠M.
Use the tangent ratio.

$$\tan M = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan M = \frac{NP}{MP}$$ Substitute: ∠M = 18° and MP = 50

$$\tan 18° = \frac{NP}{50}$$

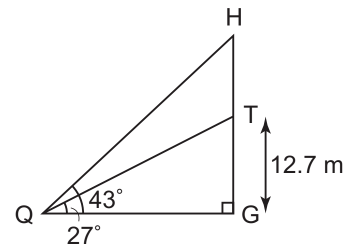
$$50 \tan 18° = NP$$

$$NP = 16.2459\ldots$$

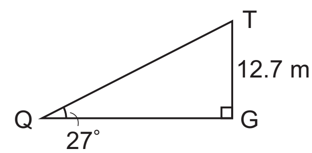
To find how much taller one tree is than the other, subtract:
16.2459… m − 11.5434… m = 4.7025… m
One tree is about 4.7 m taller than the other.

Check

1. The angle of elevation of the top of a tree, T, is 27°. From the same point on the ground, the angle of elevation of a hawk, H, flying directly above the tree is 43°. The tree is 12.7 m tall. How high is the hawk above the ground? Give your answer to the nearest tenth of a metre.



We want to find the length of HG.
Use △QTG to find the length of QG.
Use the tangent ratio.



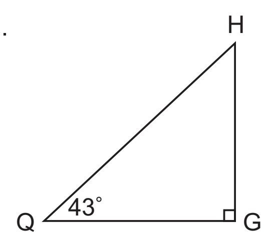
tan Q = ______

tan Q = ______ Substitute: ______ and ______

tan ______ = ______

QG = ______

In △QHG, use the tangent ratio to find HG.

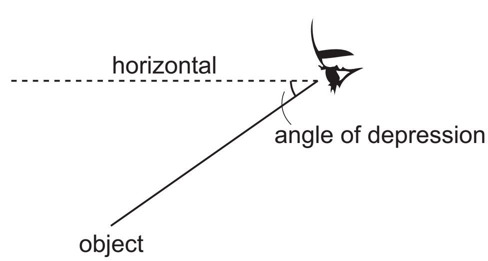


______

HG = ______

The hawk is about ______ above the ground.

The **angle of depression** is the angle between the horizontal and a person's line of sight to an object below.



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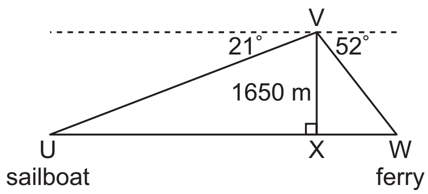
Example 3 Solving a Problem Involving Angle of Depression

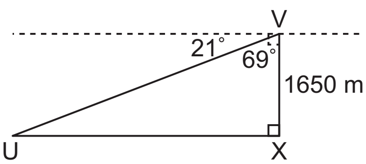
From a small plane, V, the angle of depression of a sailboat is 21°.
The angle of depression of a ferry on the other side of the plane is 52°.
The plane is flying at an altitude of 1650 m.
How far apart are the boats, to the nearest metre?



Solution

We want to find the length of UW.
The angle of depression of the sailboat is 21°.
So, in △UVX, ∠V = 90° − 21°, or 69°.

Use △UVX to find the length of UX.



We know ∠V = 69°.
UX is opposite ∠V.
VX is adjacent to ∠V.
So, use the tangent ratio.

$$\tan V = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan V = \frac{UX}{VX}$$ Substitute: ∠V = 69° and VX = 1650

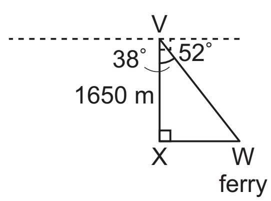
$$\tan 69° = \frac{UX}{1650}$$

$$1650 \tan 69° = UX$$

$$UX = 4298.3969\ldots$$

The angle of depression of the ferry is 52°.
So, ∠V in △VWX is: 90° − 52°, or 38°.

Use △VWX to find the length of WX.



We know ∠V = 38°.
WX is opposite ∠V.
VX is adjacent to ∠V.
So, use the tangent ratio.

$$\tan V = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan V = \frac{WX}{VX}$$ Substitute: ∠V = 38° and VX = 1650

$$\tan 38° = \frac{WX}{1650}$$

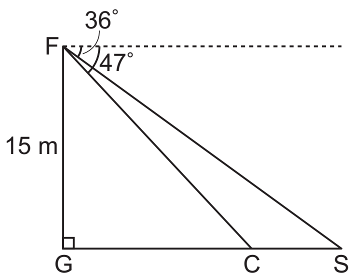
$$1650 \tan 38° = WX$$

$$WX = 1289.1212\ldots$$

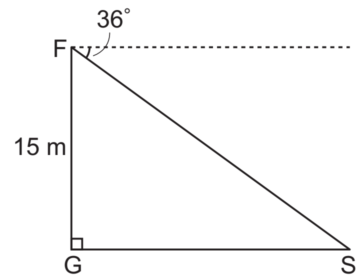
To find the distance between the boats, add:
4298.3969… m + 1289.1212… m = 5587.5182… m
The boats are about 5588 m apart.

Check

1. This diagram shows a falcon, F, on a tree, with a squirrel, S, and a chipmunk, C, on the ground. From the falcon, the angles of depression of the animals are 36° and 47°. How far apart are the animals on the ground to the nearest tenth of a metre?



We want to find the length of CS.
CS = GS − GC
The angle of depression of the squirrel is ______.
So, ∠F in △FSG is: 90° − ______, or ______.



Use △FSG to find the length of GS.

tan ____ = __________

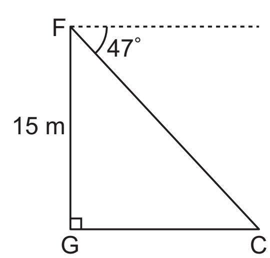
tan ____ = __________

tan _____ = __________

GS = ______________

The angle of depression of the chipmunk is ______.
So, ∠F in △FCG is: 90° − ______, or ______.

Use △FCG to find the length of GC.



GC = ______________

To find the distance between the animals, subtract:
_______________ − _______________ = _______________
The animals on the ground are about __________ apart.

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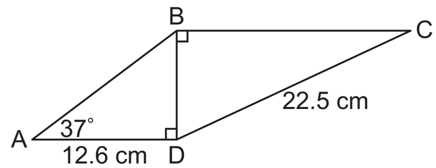
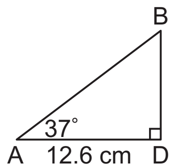
Practice

1. Find the measure of ∠C to the nearest degree.



Use △ABD to find the length of BD.



Use the tangent ratio.

tan A = ____________

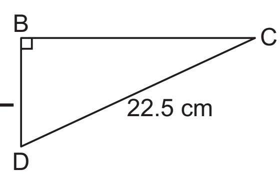
tan A = ___________

tan ______ = ________

BD = ____________

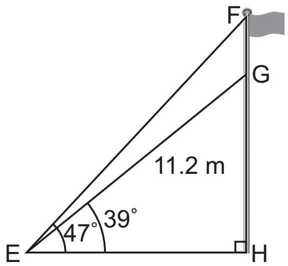
In △BCD, use the _______ ratio to find ∠C.

____



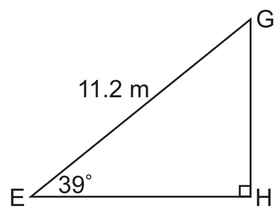
The measure of ∠C is about __________.

2. Two guy wires support a flagpole, FH. The first wire is 11.2 m long and has an angle of inclination of 39°. The second wire has an angle of inclination of 47°. How tall is the flagpole to the nearest tenth of a metre?



*Recall that the angle the wire makes with the ground is called the **angle of inclination**.*

We want to find the length of FH.
Use △EGH to find the length of EH.
Use the cosine ratio.



Side EH is common to both triangles.

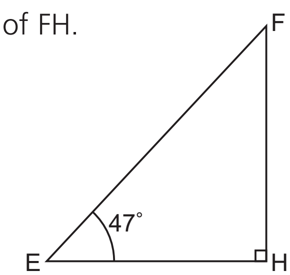
cos E = ______________

cos E = ______

cos ______ = ______

EH = ____________

In △EFH, use the __________ ratio to find the length of FH.



FH = __________

The flagpole is about _______ tall.

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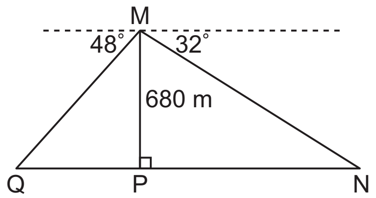
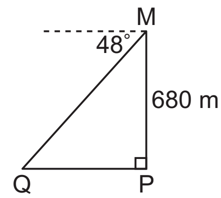
3. A mountain climber is on top of a mountain that is 680 m high. The angles of depression of two points on opposite sides of the mountain are 48° and 32°. How long would a tunnel be that runs between the two points? Give your answer to the nearest metre.



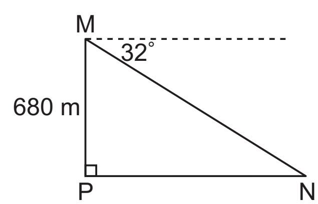
We want to find the length of QN.
The angle of depression of point Q is ______.
So, ∠M in △PQM is: 90° − ______, or ______.



Use △PQM to find the length of PQ.
Use the __________ ratio.

PQ = ____________

The angle of depression of point N is ______.
So, ∠M in △PMN is: 90° − ______, or ______.



Use △PMN to find the length of PN.
Use the ________ ratio.

NP = __________________

The length of the tunnel is: ______ = ______ + ______

QN = __________________

The tunnel would be about __________ long.