SOLVING WORD PROBLEMS

1. Read the problem all the way through quickly, to see what kind of word problem it is and what it is about.
2. Look for a question at the end of the problem. This is often a good way to find what you are solving for. Sometimes two or three things need to be found.
3. Start every problem with a "Let" statement, such as, "Let $x =$ something," letting the variable represent what you are trying to find (the *unknown)*. If there is more than one unknown to be solved, it is often easier to let your variable represent the smaller quantity.
4. Reread the problem, carefully analyzing it, using some or all of the following tools:
    a. Find Key words and phrases that can be translated into math symbols
    b. Draw a sketch and/or create a table
    c. Try to solve the problem using easier numbers
    d. Try to relate the problem to your own experience
5. Translate the word problem into an equation. Then solve the equation.
6. Write your answer(s), labeling with the appropriate units (inches, mph, pounds, etc.)
7. Reread the question to make sure your answer is sensible (i.e. Check your solution).

Number Problems

Example 1:
There are two numbers whose sum is 72. One number is twice the other. What are the numbers?

1. Read the problem. It is about numbers.
2. The question at the end asks, "What are the numbers?"
3. Create a let statement.
   Let $x =$ smaller number
   $2x =$ larger number
4. Reread the problem. It states that the sum of the two numbers is 72.
5. Translate the problem into an equation and solve.

$$x + 2x = 72$$
$$3x = 72$$
$$x = 24$$
$$2x = 48$$

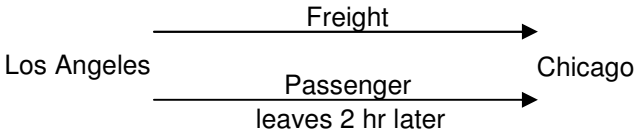
6. The answers are: The smaller number is 24 and the larger number is 48.
7. Is this sensible? Yes, because the sum of 24 and 48 is 72.

Time, Rate, & Distance Problems ($D = rt$)

Example 2:
A freight train starts from Los Angeles and heads for Chicago at 40 mph. Two hours later a passenger train leaves the same station for Chicago traveling 60 mph. How long before the passenger train overtakes the freight train?

1. Read the problem. It is a time, rate, and distance problem. (Use $D = rt$)
2. The question asks, "how long?" (which means time) for the passenger train.
3. Let $t =$ the time it takes the passenger train to overtake the freight train
   $t + 2 =$ the number of hours the freight train had traveled when overtaken.

4. Draw a sketch and make a table.



	Time	Rate	Distance
Freight	$t+2$	40 mph	$40(t+2)$
Passenger	t	60 mph	$60t$

5. Equation: When the passenger train overtakes the freight train, they have both traveled the same distance. Write each of their distances in terms of rate·time (as seen in the table above), then set them equal to each other to create an equation. Then solve the equation.

$$40(t+2) = 60t$$
$$40t + 80 = 60t$$
$$80 = 20t$$
$$t = 4$$

6. The answer: It takes the passenger train 4 hours to overtake the freight train.
7. Check by substituting 4 into the original equation.

$$40(4+2) = 60(4)$$
$$40(6) = 240$$
$$240 = 240 \quad \textbf{True}$$

Thus, 4 hours is the correct answer.

Mixture Problems

Mixture problems involve mixing two solutions of different percentages of a pure substance to get a solution with another percentage of the pure substance. They also can involve mixing two quantities with different prices per unit to get a mixture with another price per unit.
To solve mixture problems, remember these facts:

A. Amount in the first mixture plus amount in second mixture equals amount in total mixture.
B. Amount of pure substance in the first mixture plus amount of pure substance in the second mixture equals amount of pure substance in the total mixture.

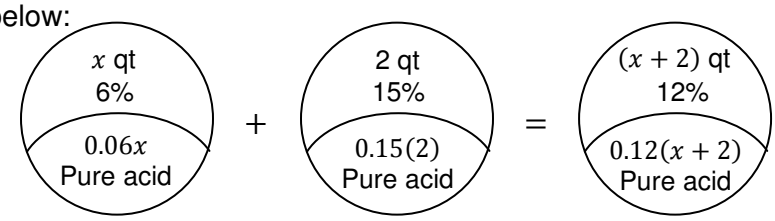
OR

The value of the first mixture plus the value of the second mixture equals the value of the total mixture.
These facts will help to set up the unknowns and the equation. It will look something like this: $(2\%)(12 \text{ quarts}) + (5\%)(x \text{ quarts}) = (3\%)[(12+x) \text{ quarts}]$
or $0.02(12) + 0.05x = 0.03(12+x)$

On the following example, a picture diagram will be used to help set up the equation.

Example 3:
A mixture containing 6% boric acid is to be mixed with 2 quarts of a mixture which is 15% acid in order to obtain a solution which is 12% acid. How much of the 6% solution must be used?

1. This is a mixture problem.
2. The question asks, "How much of the 6% solution".
3. Let $x =$ the number of quarts of 6% solution needed.
4. Draw a picture diagram to represent the 2 solutions being mixed to get the third as shown below:



1st mixture 2nd mixture Total mixture

5. Write the equation and solve.

$$0.06x + 0.15(2) = 0.12(x+2)$$
$$0.06x + 0.30 = 0.12x + 0.24$$
$$6x + 30 = 12x + 24$$
$$6 = 6x$$
$$x = 1$$

6. The answer: 1 qt of 6% boric acid solution must be used.

Money Problems

Example 4:
Michael has some coins in his pocket consisting of dimes, nickels, and pennies. He has two more nickels than dimes, and three times as many pennies as nickels. How many of each kind of coin does he have if the total value is $0.52?

1. This problem involves money.
2. The question asks, "How many of each kind of coin".
3. There is more than one unknown. So when doing the let statement, start with the smallest quantity, in this case, the number of dimes.
   Let $x =$ the number of dimes
   $x + 2 =$ the number of nickels
   $3(x+2)=$ the number of pennies
4. The value of all the dimes, $\$0.10x$, plus the value of all the nickels, $\$0.05(x+2)$, plus the value of all the pennies, $\$0.01(3)(x+2)$, equals the total value, $0.52.
5. Write the equation.

$$0.10x + 0.05(x+2) + 0.03(x+2) = 0.52$$
$$10x + 5(x+2) + 3(x+2) = 52$$
$$10x + 5x + 10 + 3x + 6 = 52$$
$$18x + 16 = 52$$
$$18x = 36$$
$$x = 2$$
$$x + 2 = 4$$
$$3(x+2) = 12$$

6. The answer: Michael has 2 dimes, 4 nickels, and 12 pennies.
7. Is the answer sensible? Yes, because 2 dimes, $0.20, plus 4 nickels, $0.20, plus 12 pennies, $0.12, equals $0.52.

Age Problems

Example 5:
Mary's father is four times as old as Mary. Five years ago he was seven times as old. How old is each now?

1. The problem involves age.
2. The question asks, "How old is each now?", which means it's asking for the ages of both Mary and her father.
3. Let x = Mary's current age
   $4x$ = Mary's father's current age
   $x - 5$ = Mary's age 5 years ago
   $4x - 5$ = Mary's father's age 5 years ago
4. The problem states that 5 years ago, Mary's father's age, $4x - 5$, was equal to seven times Mary's age, $7(x - 5)$.
5. Write the equation.

$$4x - 5 = 7(x - 5)$$
$$4x - 5 = 7x - 35$$
$$30 = 3x$$
$$x = 10$$
$$4x = 40$$

6. The answer: Mary's current age is 10 years old, and her father's age is 40 years old.
7. Is this answer sensible? Yes, because Mary's father's age five years ago was 35 which is seven times Mary's age five years ago, which was 5.