Rate of Return vs. Yield

Outline and Reading

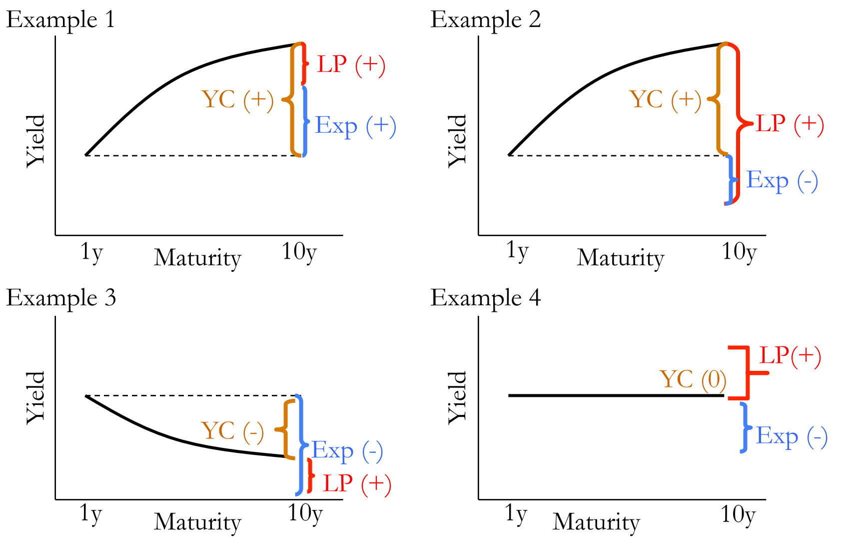
- Outline
  - Liquidity Preference Theory
  - Bond Rate of Return over a Holding Period
  - Yields vs. Returns
  - Yields vs. Expected Returns
  - Risk Premiums in Returns
- Reading
  - Tuckman and Serrat, Chapter 3

What Does the Shape of the Yield Curve Tell Us?

- The yield curve contains a mixture of
  1. information about expected returns on bonds of different maturities and
  2. forecasts of future yield changes
- In general, it is difficult to disentangle these two without a model of expected returns or interest rate forecasts.
- This lecture
  - clarifies the idea of rate of return,
  - distinguishes it from yield, and
  - quantifies the connection between yields, expected returns, and future yield forecasts.

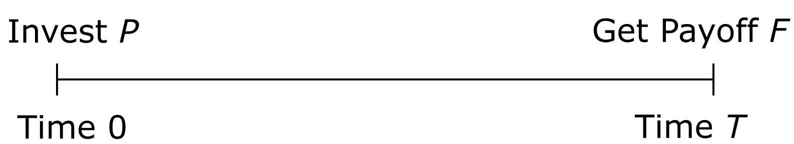
Summary - "Liquidity Preference" Theory

LP is yield spread due to bond risk premium; Exp is yield spread due to expected future rate changes; YC is total yield spread.



More formally, consider an investment over a holding period until horizon T…

- Consider an investment in any asset over a *holding period* from time 0 to time *T*.
- Suppose the amount invested at time 0 is *P* and the payoff at time *T* is *F*.
- *F* might not be known at time 0. In general, the payoff *F* is not known until the *investment horizon date T.*



Rate of Return over a Holding Period

- To compare the performance of different investments, and adjust for scale, one might consider the *gross* or *unannualized rate of return* (ROR) on the investment:
  - Unannualized ROR = F/P - 1.
- To adjust for differences in the length of the holding period as well, one might annualize the ROR. We'll use semi-annual compounding to be consistent with US bond market interest rate quote conventions.
  - The annualized ROR with semi-annual compounding is

    $R = 2[(F/P)^{(1/(2T))}-1]$

    so that $F/P=(1+R/2)^{2T}$

Example of Holding Period Return

- Suppose you invest \$100 in an asset at time 0 and at time 5 it is worth \$150.
- Your un-annualized ROR is 150/100-1=50%.
- **Class Problem:** What is your annualized ROR with semi-annual compounding?

Rate of Return on a Zero: Case 1) Maturity Equal to Investment Horizon

- If you buy a zero-coupon bond and hold it to maturity, the ROR on your investment is the zero rate at which you bought the bond:

  T = t, P = d_t, F = 1 so

  $R = 2[(1/d_t)^{(1/(2t))}-1]=r_t$

- Example: If you buy a 1-year zero at 5.25% and hold it to maturity your ROR over 1-year is

$$R = 2[(\frac{1}{1/(1+0.0525/2)^2})^{1/2} - 1] = 5.25\%$$

Rate of Return on a Zero: Case 2) Maturity Longer than Investment Horizon

- If you buy a t-year zero-coupon bond and sell it at time T<t, the ROR on your investment depends on market conditions at the selling time *T.*
- **Class Problem**: Suppose you buy the 1-year zero at 5.25% for a price of $1/(1+0.0525/2)^2 = 0.9495$ and sell it after 6 months at 6% for a price of $1/(1+0.06/2) = 0.9709$. What is your ROR over 6 months?
- **Class Problem**: Suppose you buy a 1-year zero at 5.25% and sell it after 6 months at 4%. What is your ROR over 6 months?

Zero Rates and Rates of Return are Different

- Zero rate and rate of return are simply different concepts.
- The zero rate is only for a zero, and is known at the time the zero is purchased.
- Rate of return is a concept that applies to any kind of investment, bond, stock, currency, etc. In general it is not known at the start of the investment—it is a random amount realized only at the end of the holding period.
- The only time the ROR is known in advance is when the investment is in the zero with maturity matching the horizon date. In that case, this known ROR is the zero rate.
- If the zero maturity differs from the horizon date, the ROR will generally differ from the rate at which the zero is purchased. It will be the same if the (longer) zero is sold at the same zero rate at which it was purchased or if the (shorter) zero payoff is reinvested at the same rate at which the zero was purchased.

Zero Rates Are Not Even Expected Returns

- The rate of return on an investment that is realized over a holding period is not generally known in advance.
- However, one could imagine that the market might form an *expectation* or forecast of what the ROR will be.
- By definition, this *expected rate of return* is known in advance.
- Does the zero rate equal the expected rate of return from investing in a zero?
- In general the answer is no…

Bond Market with Risk: Current and Future Possible Zero Rates

Let ${}_t r_u$ represent the zero rate at time t for maturity at time u. For example ${}_0 r_{0.5}$ means the 0.5-year rate at time 0 and ${}_{0.5} r_1$ means the 0.5-year rate at time 0.5. Consider the following current and future possible zero rates:

Time 0	Time 0.5
	${}_{0.5} r_1^u = 6.00\%$
${}_0 r_{0.5} = 5.00\%$ ${}_0 r_1 = 5.25\%$	
	${}_{0.5} r_1^d = 4.00\%$

Bond Market with Risk: Future Possible **0.5-Year** RORs

Now consider investing in this market for 6 months.

- You could buy a 0.5-year zero and earn 5% risklessly.
- Or you could buy a 1-year zero and sell it at time 0.5. The ROR from this strategy is risky—it would depend on market conditions at time 0.5:

Time 0: ${}_0r_{0.5}=5.00\%$, ${}_0r_1=5.25\%$

Time 0.5 Zero rate	ROR on 0.5-yr zero	ROR on 1-yr zero
${}_{0.5}r_1^u=6.00\%$	5.00%	?
${}_{0.5}r_1^d=4.00\%$	5.00%	?

Bond Market with Risk: Expected **0.5-Year** ROR

- Now suppose each possible time 0.5 interest rate state occurs with probability 50%.
- Then we can compute the expected ROR on each strategy:

Time 0: ${}_0r_{0.5}=5.00\%$, ${}_0r_1=5.25\%$

Time 0.5 Zero rate	0.5-Year ROR: ROR on 0.5-yr zero	ROR on 1-yr zero
${}_{0.5}r_1^u=6.00\%$	5.00%	?
${}_{0.5}r_1^d=4.00\%$	5.00%	?
Expected ROR:	5.00%	?

Different Bonds Can Have Different Expected Returns over the Same Holding Period

- This can be sustainable in equilibrium because different bonds have different risk profiles.
- Asset pricing: Given its future payoff distribution, what determines the price of an asset? I.e., what determines its expected return?
- Recall from CAPM theory that the specific risk of an asset isn't what's most important, because people don't hold securities in isolation. People hold portfolios, including real estate and labor income.
- What matters is how the asset contributes to the total portfolio risk—i.e, its covariance with other risks. Does it hedge other risks? Or add to them?
- Bond pricing: Roughly speaking, longer bonds are believed to have higher expected returns. Why? They do essentially have higher bond market "beta" because of higher duration, but why is the bond market risk premium positive? Is the average investor "long" the bond market, or "short" inflation risk, so that adding a bond adds variance? No simple story.

Rate of Return on a Zero: Case 3) Maturity Shorter than Investment Horizon

- If you buy a t-year zero-coupon bond, hold it to maturity, and then reinvest the payoff to some later date $T>t$, your ROR depends on your reinvestment rate from time t to T.
- **Class Problem**: Suppose your investment horizon is T = 1 year and you buy the 0.5-year zero at 5% and then roll it into another 0.5-year zero at 6%. What is your ROR over the year?
- **Class Problem**: Suppose you buy a 0.5-year zero at 5% for a price of 1/(1+0.05/2) = 0.97561 and then at time t = 0.5 you roll it into another 0.5-year zero at 4%. What is your ROR over the year?

Bond Market with Risk: Future Possible **1-Year** RORs

- To invest for 1 year, you could buy a 1-year zero and earn 5.25% risklessly.
- Or you could buy a 0.5-year zero and reinvest its payoff at time 0.5 in another 0.5-year zero. This strategy is subject to reinvestment risk.

Time 0	Time 0.5 Zero rate	1-Year ROR: ROR on 0.5-yr zero	1-Year ROR: ROR on 1-yr zero
$_0r_{0.5}$=5.00%, $_0r_1$=5.25%	$_{0.5}r_1^u$=6.00%	?	5.25%
	$_{0.5}r_1^d$=4.00%	?	5.25%
Expected:	5.00%	?	5.25%

Example of Bond Market with **Risk Premia**: The longer bond has a higher expected return

If the yield curve is upward-sloping, but rates are expected to stay the same, then longer bonds have higher expected return.

Time 0	Time 0.5 Zero rate	0.5-yr horizon: ROR on 0.5-yr z.	0.5-yr horizon: ROR on 1-yr z.	1-yr horizon: ROR on 0.5-yr z.	1-yr horizon: ROR on 1-yr z.
$_0r_{0.5}$=5.00%, $_0r_1$=5.25%	$_{0.5}r_1^u$=6.00%	5.00%	4.503%	5.499%	5.25%
	$_{0.5}r_1^d$=4.00%	5.00%	6.508%	4.499%	5.25%
Expected:	**5.00%**	**5.00%**	**5.50%**	**5.00%**	**5.25%**

Risk-Neutral Bond Market Example: Different bonds have the same expected return

If the yield curve is upward sloping, but expected returns across bonds are the same, it must be that rates are expected to rise.

Time 0	Time 0.5				
		0.5-yr horizon		1-yr horizon	
		ROR on	ROR on	ROR on	ROR on
	Zero rate	0.5-yr z.	1-yr z.	0.5-yr z.	1-yr z.
	${}_{0.5}r_1^u$=6.50%	5.00%	4.008%	5.749%	5.25%
${}_0r_{0.5}$=5.00% ${}_0r_1$=5.25%					
	${}_{0.5}r_1^d$=4.50%	5.00%	6.003%	4.750%	5.25%
Expected:	**5.50%**	**5.00%**	**5.00%**	**5.25%**	**5.25%**

Rate of Return Summary

1) Definition: ROR on any investment to time T is R s.t. $P = F/(1+R/2)^{2T}$.
2) Definition: Zero rate for a zero with maturity t is r_t s.t. $d_t = 1/(1+r_t/2)^{2t}$.
3) Bond math: The ROR from investing in a zero is different than the original zero rate when the zero maturity does not match the investment horizon. It depends on the rate at which the zero is sold, if the maturity is longer than the horizon, or reinvested, if shorter.
4) Bond math: When the zero maturity does not match the investment horizon, the zero rate may not even be the expected return.
5) Bond math: If the expected returns on different zeroes are equal (expectations hypothesis), then **the shape of the yield curve reflects expected changes in future interest rates**. For example, an upward-sloping yield curve would indicate an expectation that rates will rise. (More on this when we get to forward rates.)
6) Finance: Expected returns on different bonds over a given holding period are probably not equal, because the bonds have different risk profiles. Some evidence that longer bond expected returns are higher, but the story is more complicated than this.
7) Bond math: **The shape of the yield curve also reflects differences in expected returns across bonds**. For example, if longer bonds have higher expected returns than shorter bonds, the yield curve will slope upward by more than what expectations about future rates implies.