

## 2-1 Transformations and Rigid Motions

**Essential question:** How do you identify transformations that are rigid motions?

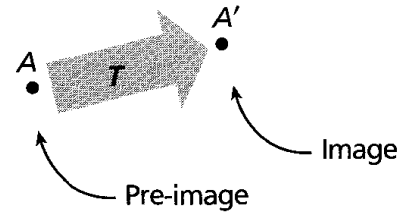
### ENGAGE 1 ~ Introducing Transformations

A **transformation** is a function that changes the position, shape, and/or size of a figure. The inputs for the function are points in the plane; the outputs are other points in the plane. A figure that is used as the input of a transformation is the **pre-image**. The output is the **image**.

For example, the transformation  $T$  moves point  $A$  to point  $A'$  (read "A prime"). Point  $A$  is the pre-image, and  $A'$  is the image. You can use function notation to write  $T(A) = A'$ .

Note that a transformation is sometimes called a *mapping*.

Transformation  $T$  maps point  $A$  to  $A'$ .



**Coordinate notation** is one way to write a rule for a transformation on a coordinate plane. The notation uses an arrow to show how the transformation changes the coordinates of a general point  $(x,y)$ .

$$(x, y) \rightarrow (\underline{\quad}, \underline{\quad})$$

**INVESTIGATE:** Given the coordinate notation for a transformation:  $(x, y) \rightarrow (x + 2, y - 3)$ . What do you think the image of the point  $(6,5)$  is? Explain your reasoning.

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### REFLECT

1a) Explain how to identify the pre-image and image in  $T(E) = F$ .

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1b) Consider the transformation given by the rule  $(x, y) \rightarrow (x + 1, y + 1)$ . What is the domain of this function? What is the range? Describe the transformation.

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1c) Transformation  $T$  maps points in the coordinate plane by moving them vertically up or down onto the  $x$ -axis. Points on the  $x$ -axis are unchanged by the transformation. Explain how to use coordinate notation to write a rule for transformation  $T$ . *Hint- if you're confused, make a coordinate plane and plot some points to see if that helps you figure it out.*

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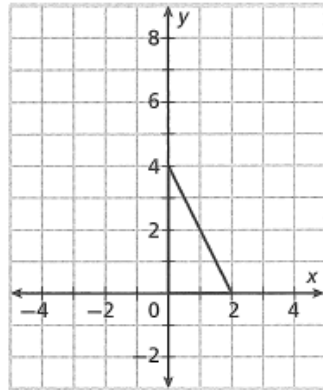
## Explore 2~ Classifying Transformations

Investigate the effects of various transformations on the given right triangle.

- Use coordinate notation to help you find the image of each vertex of the triangle.
- Plot the images of the vertices.
- Connect the images of the vertices to draw the image of the triangle.

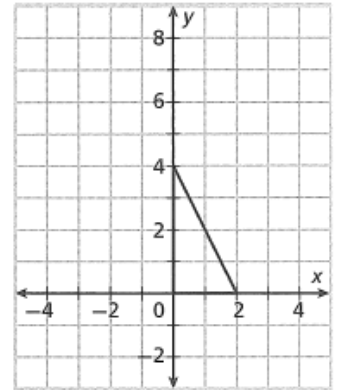
**A**  $(x, y) \rightarrow (x - 4, y + 3)$

Pre-image	→	Image
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )



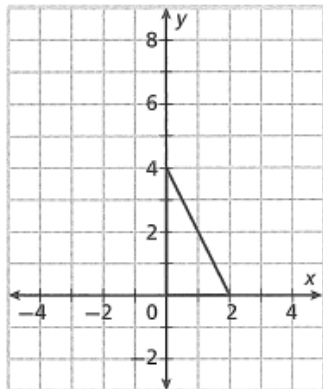
**B**  $(x, y) \rightarrow (-x, y)$

Pre-image	→	Image
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )



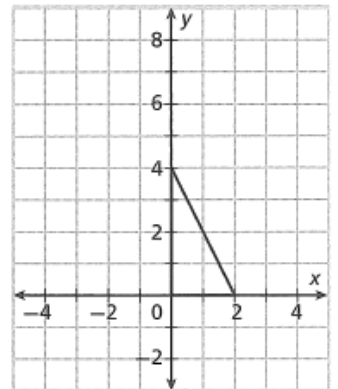
**C**  $(x, y) \rightarrow (-y, x)$

Pre-image	→	Image
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )



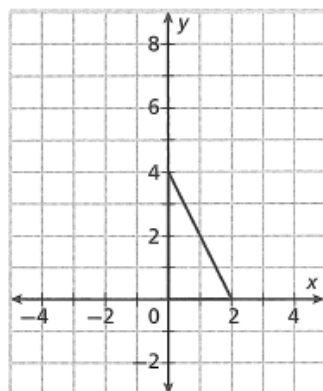
**D**  $(x, y) \rightarrow (2x, 2y)$

Pre-image	→	Image
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )



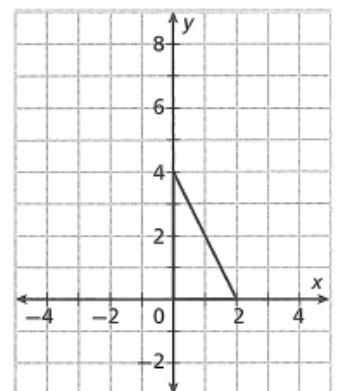
**E**  $(x, y) \rightarrow (2x, y)$

Pre-image	→	Image
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )



**F**  $(x, y) \rightarrow (x, \frac{1}{2}y)$

Pre-image	→	Image
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )
(   ,   )	→	(   ,   )



## REFLECT

2a) A transformation preserves distance if the distance between any two points of the pre-image equals the distance between the corresponding points of the image. Which of the above transformations preserve distance?

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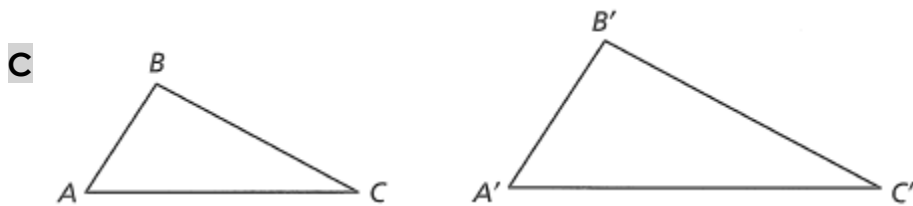
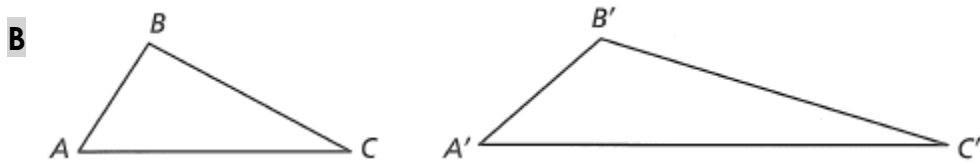
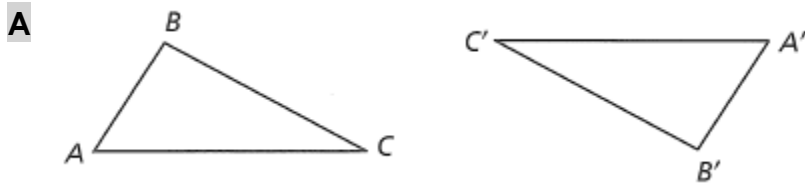
2b) A transformation preserves angle measure if the measure of any angle of the pre-image equals the measure of the corresponding angle of the image. Which of the above transformations preserve angle measure?

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A **rigid motion** (or isometry) is a transformation that changes the position of a figure without changing the size or shape of the figure.

### EXAMPLE 3 ~ Identifying Rigid Motions

The figures show the pre-image ( $\triangle ABC$ ) and image ( $\triangle A'B'C'$ ) under a transformation. Determine whether the transformation appears to be a rigid motion. Explain.



### REFLECT

3a) How could you use tracing paper to help identify rigid motions?

3b) Which of the transformations on the previous page appear to be rigid motions?

**REVIEW:** Rigid motions have some important properties. They are summarized to the right.

#### Properties of Rigid Motions (Isometries)

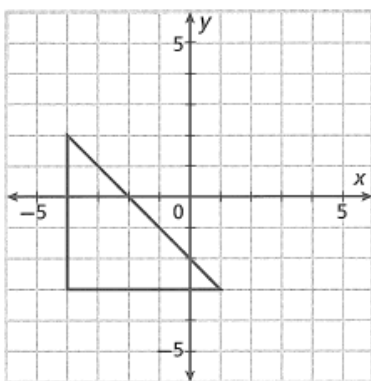
- \* Rigid motions preserve \_\_\_\_\_.
- \* Rigid motions preserve \_\_\_\_\_.
- \* Rigid motions preserve \_\_\_\_\_.
- \* Rigid motions preserve \_\_\_\_\_.

The above properties ensure that if a figure is determined by certain points, then its image after a rigid motion is also determined by those points. For example,  $\triangle ABC$  is determined by its vertices, points A, B, and C. The image of  $\triangle ABC$  after a rigid motion is the triangle determined by  $A'B'C'$ .

**Draw the image of the triangle under the given transformation. Then tell whether the transformation appears to be a rigid motion.**

**1.**  $(x, y) \rightarrow (x + 3, y)$

Pre-image $\rightarrow$ Image
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$



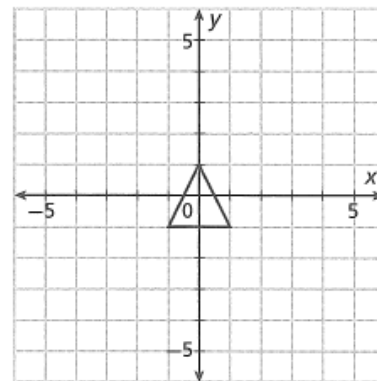

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**2.**  $(x, y) \rightarrow (3x, 3y)$

Pre-image $\rightarrow$ Image
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$



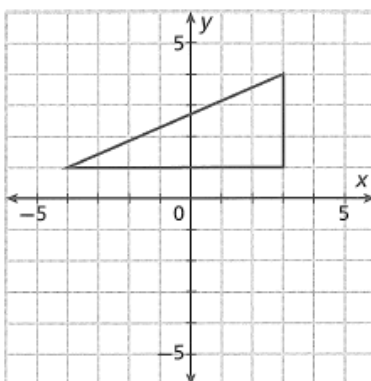

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**3.**  $(x, y) \rightarrow (x, -y)$

Pre-image $\rightarrow$ Image
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$



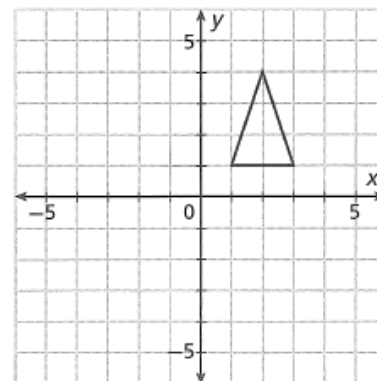

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**4.**  $(x, y) \rightarrow (-x, -y)$

Pre-image $\rightarrow$ Image
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$



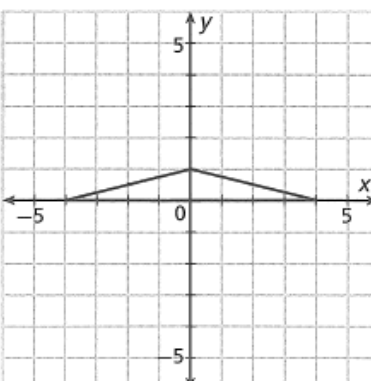

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**5.**  $(x, y) \rightarrow (x, 3y)$

Pre-image $\rightarrow$ Image
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$



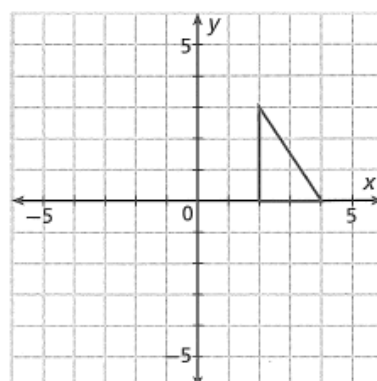

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**6.**  $(x, y) \rightarrow (x - 4, y - 4)$

Pre-image $\rightarrow$ Image
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$
$( \quad , \quad ) \rightarrow ( \quad , \quad )$



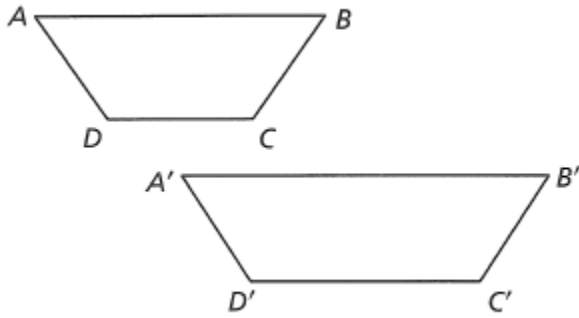

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The figures show the pre-image (ABCD) and image (A'B'C'D') under a transformation. Determine whether the transformation appears to be a rigid motion. Explain.

7.




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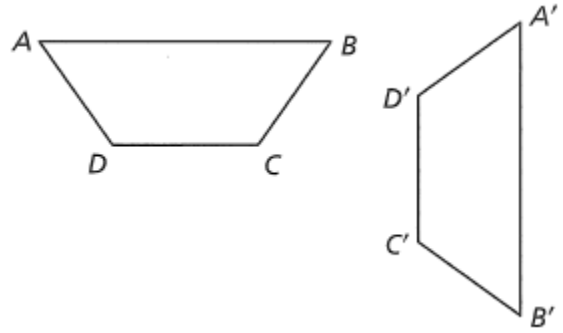


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8.




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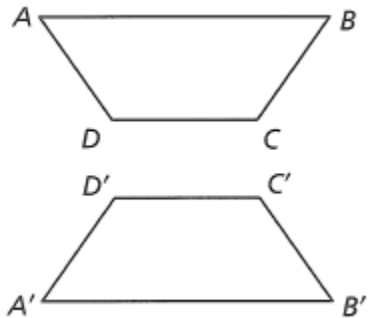


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9.




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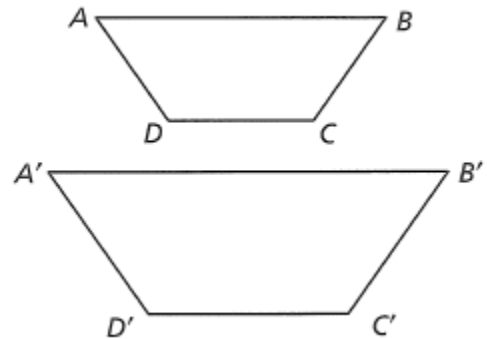


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10.




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In exercises 11-14, consider a transformation  $T$  that maps  $\triangle XYZ$  to  $\triangle X'Y'Z'$ .

11. What is the image of  $\overline{XY}$ ? \_\_\_\_\_

12. What is  $T(Z)$ ? \_\_\_\_\_

13. What is the pre-image of  $\angle Y'$ ? \_\_\_\_\_

14. Can you conclude that  $XY = X'Y'$ ? Why or why not?

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15. Point  $M$  is the midpoint of  $\overline{AB}$ . After a rigid motion, can you conclude that  $M'$  is the midpoint of  $\overline{A'B'}$ ? Why or why not?

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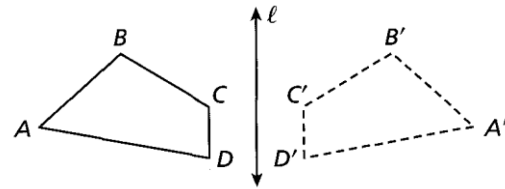


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## 2-2 Reflections

**Essential question:** How do you draw the image of a figure under a reflection?

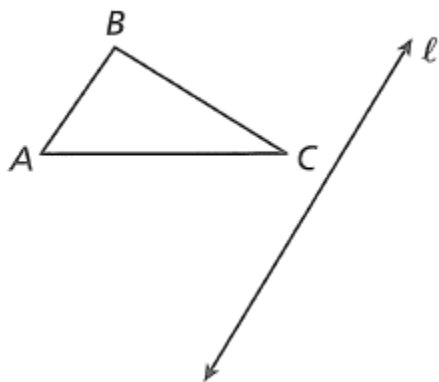
One type of rigid motion is a reflection. A **reflection** is a transformation that moves points by flipping them over a line called the line of reflection. The figure shows the reflection of quadrilateral ABCD across line  $\ell$ . Notice that the pre-image and image are mirror images of each other.



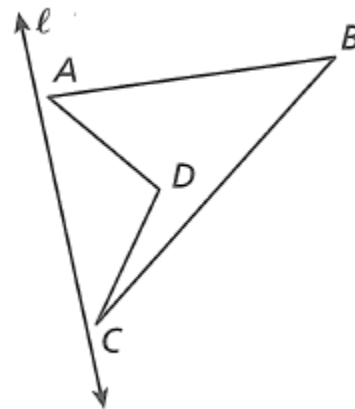
### EXPLORE 1 ~ Drawing a Reflection Image

Follow the steps below to draw the reflection image of each figure.

**A**



**B**



1. Place a sheet of tracing paper over the figure. Use a straightedge to help you trace the figure and the line of reflection with its arrowheads.
2. Flip the tracing paper over and move it so that line  $\ell$  lies on top of itself.
3. Trace the image of the figure on the tracing paper. Press firmly to make an impression on the page below.
4. Lift the tracing paper and draw the image of the figure. Label the vertices.

### REFLECT

1a) Make a conjecture about the relationship of the line of reflection to any segment drawn between a pre-image point and its image point.

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1b) Make a conjecture about the reflection image of a point that lies on the line of reflection.

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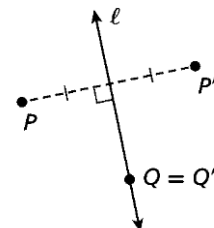
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Your work may help you understand the formal definition of a reflection.

A **reflection** across line  $\ell$  maps a point  $P$  to its image  $P'$  as follows.

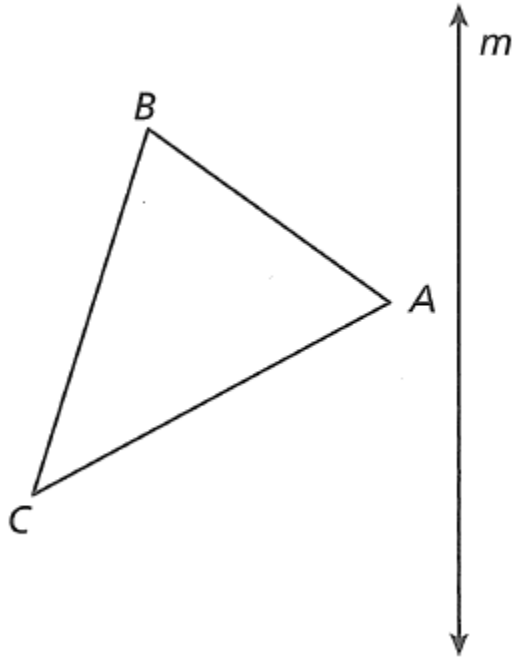
- If  $P$  is not on line  $\ell$ , then  $\ell$  is the \_\_\_\_\_ of  $\overline{PP'}$ .
- If  $P$  is on line  $\ell$ , then  $P = P'$ .

The notation of  $r_\ell(P) = P'$  says that the image of \_\_\_\_\_ after a reflection across \_\_\_\_\_ is \_\_\_\_\_.

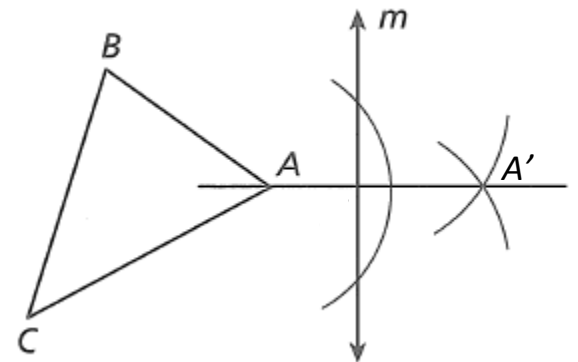


## EXAMPLE 2 ~ Constructing a Reflection Image

Work directly on the figure below and follow the given steps to construct the image of  $\triangle ABC$  after a reflection across line  $m$ .



- Start with point A. Construct a perpendicular to line  $m$  that passes through point A. Do the entire construction without changing your compass setting.
- Label the point where the last two arcs intersect point  $A'$ .
- Repeat the steps for the other vertices of  $\triangle ABC$ . (It may be helpful to extend line  $m$  in order to construct perpendiculars from points B and C.)



## REFLECT

2a) Reflections have all the properties of rigid motions. For example, reflections preserve distance and angle measure. Use a ruler and protractor, or tracing paper, to check this in your construction.

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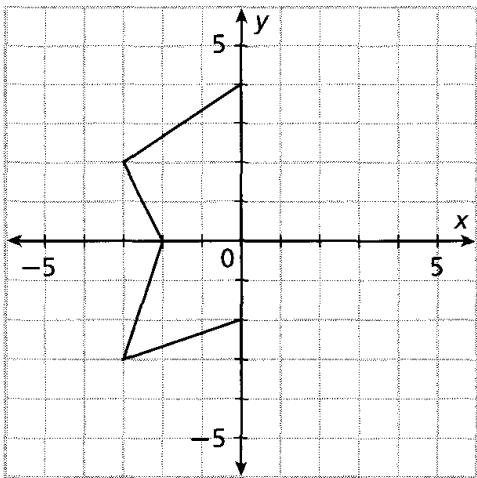
The table provides coordinate notation for reflections in a coordinate plane.

Rules for Reflections in a Coordinate Plane	
Reflection across the x-axis	$(x, y) \rightarrow (x, -y)$
Reflection across the y-axis	$(x, y) \rightarrow (-x, y)$
Reflection across the line $y = x$	$(x, y) \rightarrow (y, x)$
Reflection across the line $y = -x$	$(x, y) \rightarrow (-y, -x)$

**EXAMPLE 3 ~ Drawing a Reflection in a Coordinate Plane**

You are designing a logo for a bank. The left half of the logo is shown. You will complete the logo by reflecting this figure across the y-axis.

**A** In the space below, sketch your prediction of what the completed logo will look like.



**B** In the table at right, list the vertices of the left half of the logo. Then use the rule for a reflection across the y-axis to write the vertices of the right half of the logo.

**C** Plot the vertices of the right half of the logo. Then connect the vertices to complete the logo. Compare the vertices based on the rule to your prediction.

Left Half (x, y)	Right Half (-x, y)
(0, 4)	
(-3, 2)	(3, 2)
(-2, 0)	

**REFLECT**

3a) Explain how your prediction compares to the completed logo.

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3b) How can you use paper folding to check that you completed the logo correctly?

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3c) What does the rule  $(x, y) \rightarrow (-x, y)$  mean?

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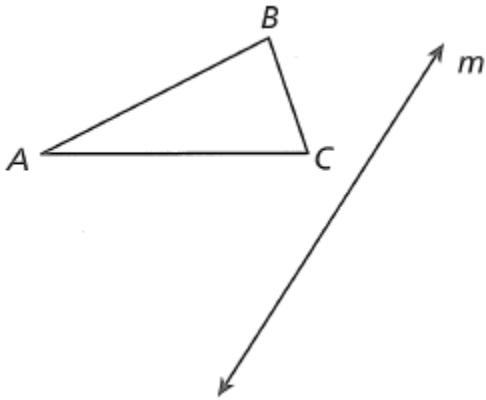


**PRACTICE ~ Reflections**

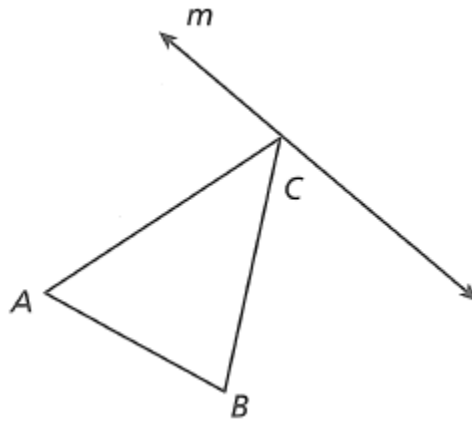
Name \_\_\_\_\_

Use tracing paper to help you draw the reflection image of each figure across line  $m$ . Label the vertices of the image using prime notation.

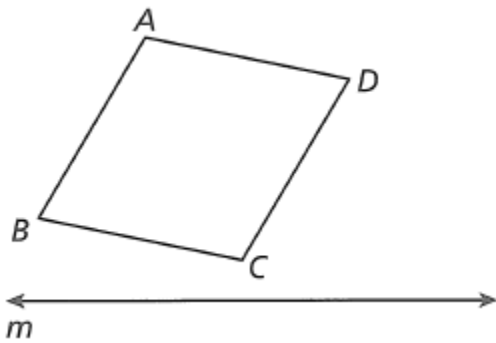
1.



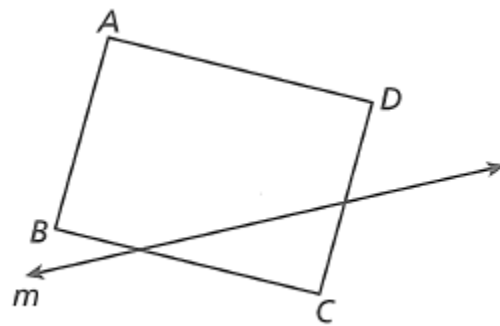
2.



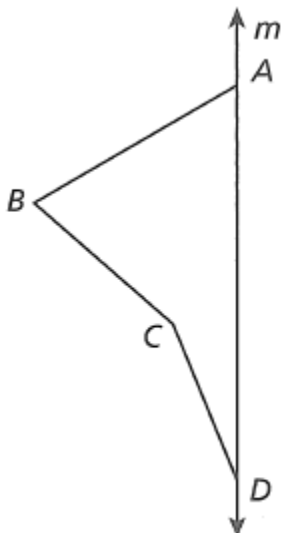
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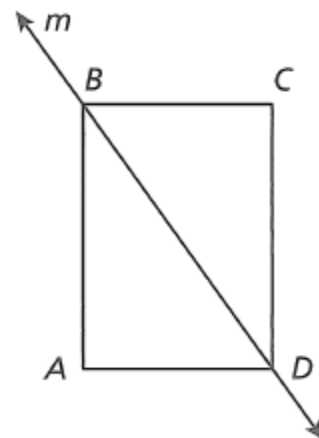
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5.

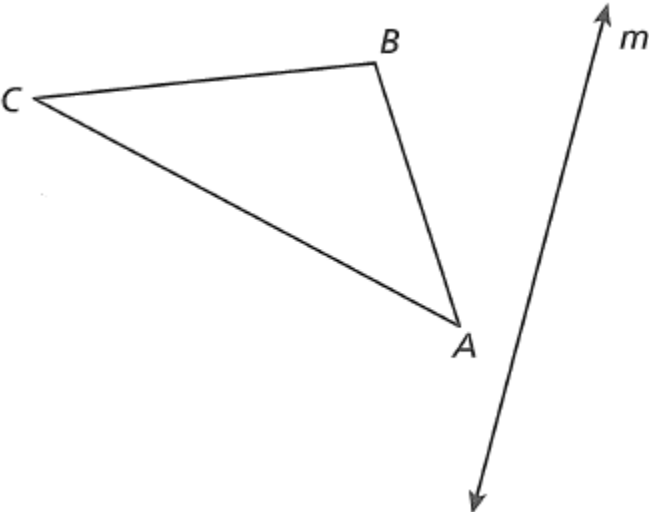


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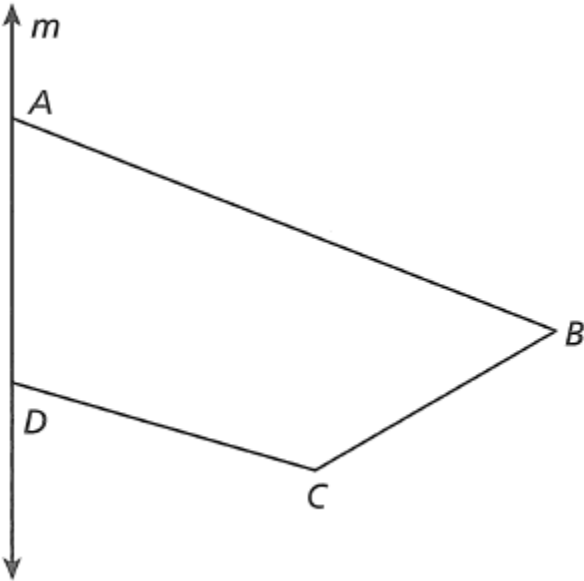


Use a compass and straightedge to construct the reflection image of each figure across line  $m$ . Label the vertices of the image using prime notation.

7.



8.



Give the image of each point after a reflection across the given line.

9.  $(3, 1)$ ; x-axis

\_\_\_\_\_

10.  $(-6, -3)$ ; y-axis

\_\_\_\_\_

11.  $(0, -2)$ ;  $y = x$

\_\_\_\_\_

12.  $(-4, 3)$ ; y-axis

\_\_\_\_\_

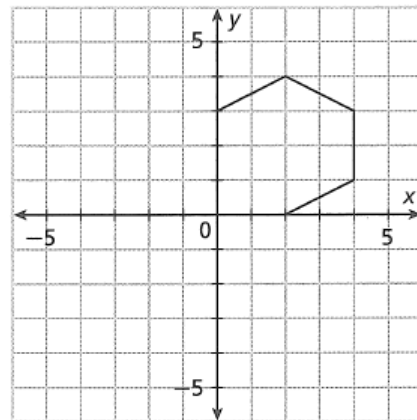
13.  $(5, 5)$ ;  $y = x$

\_\_\_\_\_

14.  $(-7, 0)$ ; x-axis

\_\_\_\_\_

15. As the first step in designing a logo, you draw the figure shown in the first quadrant of the coordinate plane. Then you reflect the figure across the x-axis. You complete the design by reflecting the original figure and its image across the y-axis. Draw the completed design.



16. When point P is reflected across the y-axis, its image lies in Quadrant IV. When point P is reflected across the line of  $y=x$ , its position does not change. What can you say about the coordinate of point P?

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\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Use the diagram to name the image of  $\Delta 1$  after the reflection.

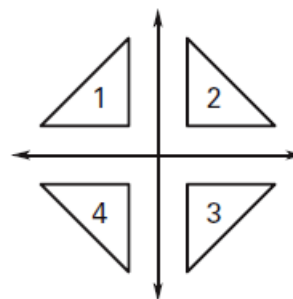
17. Reflection across the x – axis. \_\_\_\_\_

18. Reflection across the y – axis. \_\_\_\_\_

19. Reflection across the line  $y = x$ . \_\_\_\_\_

20. Reflection across the line  $y = -x$ . \_\_\_\_\_

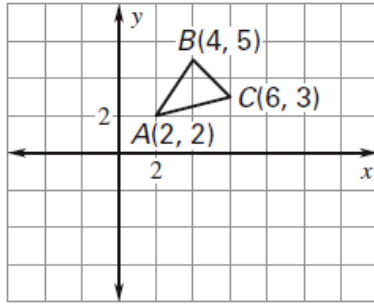
21. Reflection across the y – axis, followed by a reflection across the x – axis. \_\_\_\_\_



Graph the reflection of the polygon in the given line. Label the vertices of the image using prime notation.

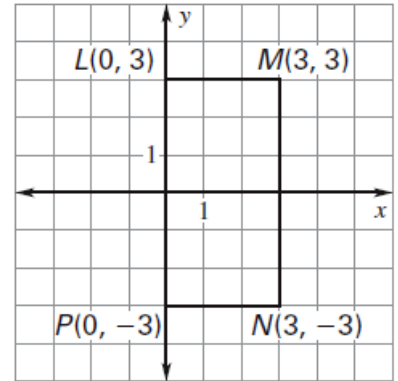
21. x-axis

$(x, y)$	Rule: $(x, y) \rightarrow ( \quad , \quad )$



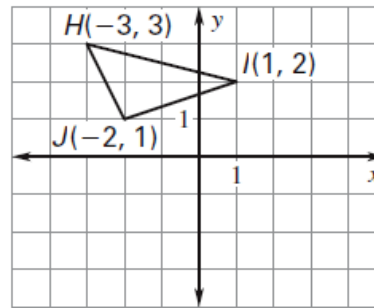
22. y-axis

$(x, y)$	Rule: $(x, y) \rightarrow ( \quad , \quad )$



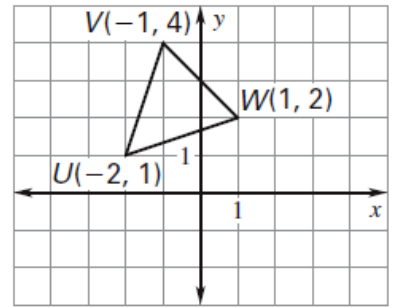
23.  $y = x$

$(x, y)$	Rule: $(x, y) \rightarrow ( \quad , \quad )$



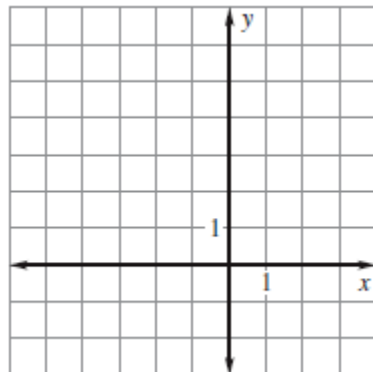
24.  $y = -x$

$(x, y)$	Rule: $(x, y) \rightarrow ( \quad , \quad )$

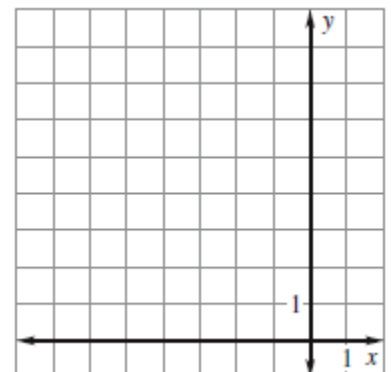


The vertices of  $\triangle ABC$  are  $A(-4, 4)$ ,  $B(0, 7)$ , and  $C(-1, 3)$ . Reflect  $\triangle ABC$  in the first line. Then reflect  $\triangle A'B'C'$  in the second line. Graph  $\triangle A'B'C'$  and  $\triangle A''B''C''$

25. In  $y = 4$ , then in  $x = -1$



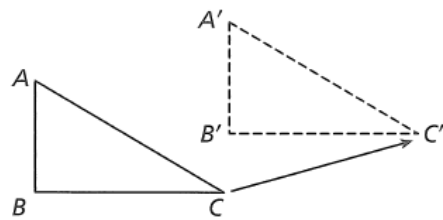
26. In  $x = -3$ , then in  $y = 5$



## 2-3 Translations

**Essential question:** How do you draw the image of a figure under a translation?

You have seen that a reflection is one type of rigid motion. A *translation* is another type of rigid motion. A translation slides all points of a figure the same distance in the same direction. The figure shows a translation of a triangle.

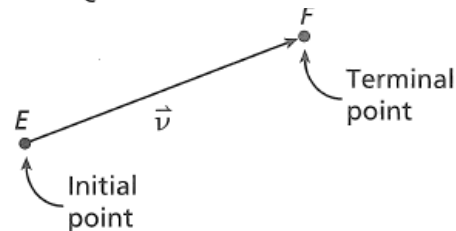


It is convenient to describe translations using the language of vectors.

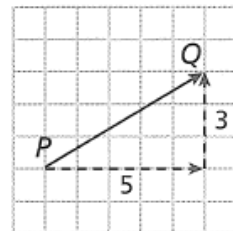
A **vector** is a quantity that has both direction and magnitude.

The **initial point** of a vector is the starting point.

The **terminal point** of a vector is the ending point. The vector at right may be named  $\overrightarrow{EF}$  or  $\vec{v}$ .



A vector can also be named using **component form**,  $\langle a, b \rangle$ , which specifies the horizontal change  $a$  and the vertical change  $b$  from the initial point to the terminal point. The component form for  $\overrightarrow{PQ}$  is  $\langle 5, 3 \rangle$ .



### EXAMPLE 1 ~ Naming a Vector

Name the vector and write it in component form.

**A** To name the vector, identify the initial point and the terminal point.

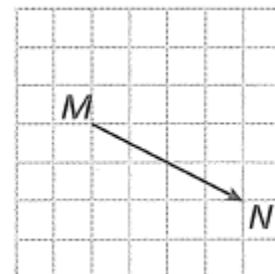
The initial point is \_\_\_\_\_. The terminal point is \_\_\_\_\_.

The name of the vector is \_\_\_\_\_.

**B** To write the vector in component form, identify the horizontal change and vertical change from the initial point to the terminal point.

The horizontal change is \_\_\_\_\_. The vertical change is \_\_\_\_\_.

The component form for the vector is \_\_\_\_\_.



### REFLECT

1a) Is  $\overrightarrow{XY}$  the same as  $\overrightarrow{YX}$ ? Why or why not?

---



---

1b) How is  $\overrightarrow{AB}$  different from  $\overline{AB}$ ?

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1c) How is  $\langle 4, -2 \rangle$  different from  $(4, -2)$ ?

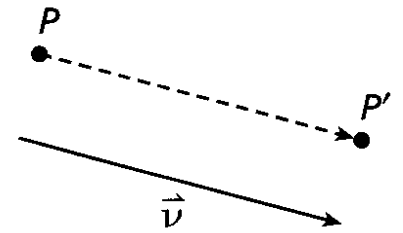
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You can use vectors to give a formal definition of a translation.

A **translation** is a transformation along a vector such that the segment joining a point and its image has the same length as the vector and is parallel to the vector.

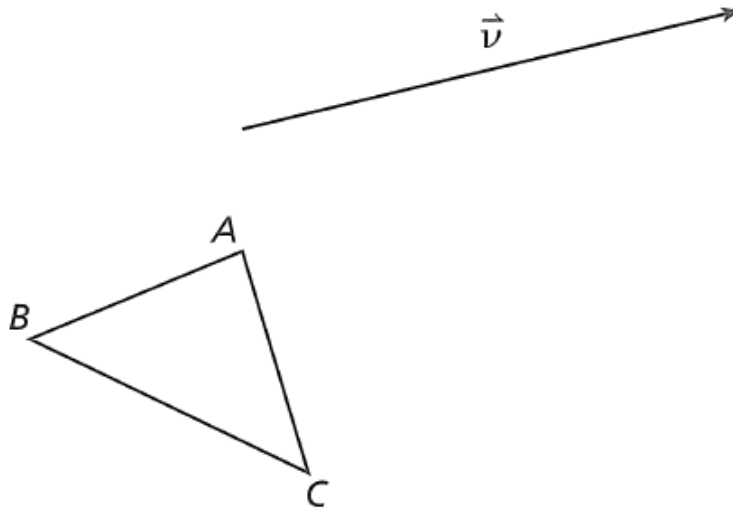


The notation  $T_{\vec{v}}(P) = P'$  says that the image of point  $P$  after a translation along vector  $\vec{v}$  is  $P'$ .

### EXAMPLE 2 ~ Constructing a Translation Image

Work directly on the figure below and follow the given steps to construct the image of  $\triangle ABC$  after a translation along vector  $\vec{v}$ .

- A** Place the point of your compass on the initial point of  $\vec{v}$  and open the compass to the length of  $\vec{v}$ . Then move the point of the compass to point  $A$  and make an arc in the same direction as  $\vec{v}$ .
- B** Place the point of your compass on the initial point of  $\vec{v}$  and open the compass to the length of initial point to point  $A$ . Then move the point of the compass to terminal point of  $\vec{v}$  and make an arc that intersects the arc made in part A. Label the intersection of the arcs  $A'$ .
- C** Repeat the process from points  $B$  &  $C$  to locate  $B'$  and  $C'$ .



### REFLECT

Why do you begin by constructing a line parallel to  $\vec{v}$ ?

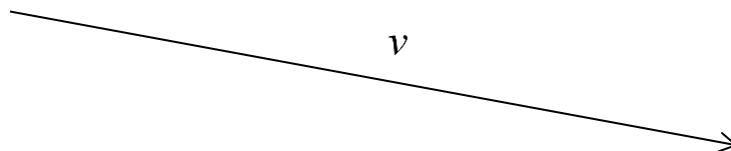
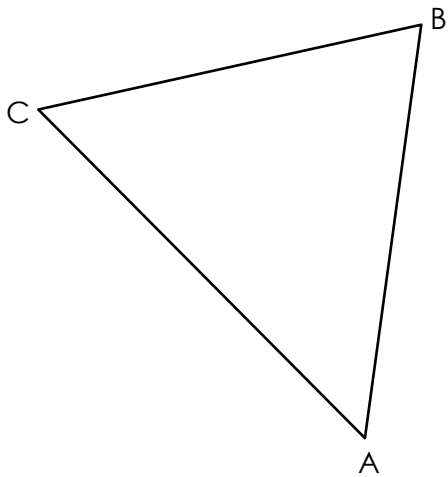
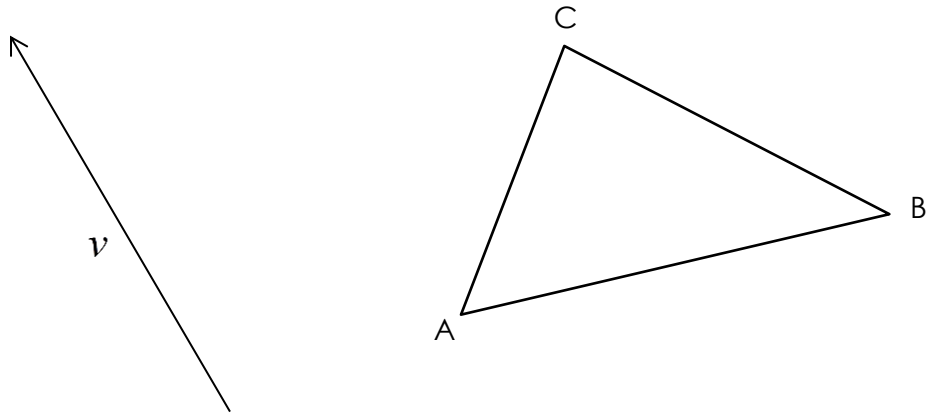
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### Try ~ Construct a Translation Image

Use a compass and straight edge to construct the image of the triangle after a translation along  $v$ . Label the vertices of the image using prime notation.



## Vector $\langle r, s \rangle$

Vectors use brackets for its component form. For the vector  $\langle r, s \rangle$ ,  $r$  shows the horizontal change and  $s$  shows the vertical change.

A translation in a coordinate plane can be specified by the component form of a vector. For example, the translation along  $\langle 3, -4 \rangle$  moves each point in the coordinate plane 3 units to the right and 4 units down.

More generally, a translation along vector  $\langle a, b \rangle$  in the coordinate plane can be written in coordinate notation as  $(x, y) \rightarrow (x + a, y + b)$ .

### EXAMPLE 3 ~ Drawing a Translation in a Coordinate Plane

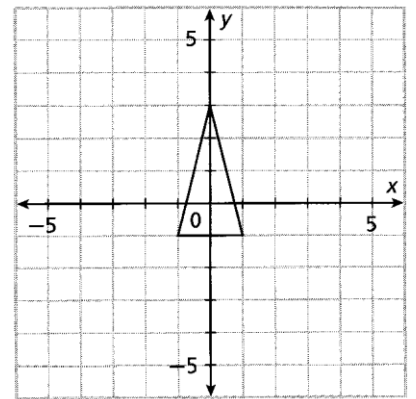
Draw the image of the triangle under a translation along  $\langle -3, 2 \rangle$ .

**A** Write the vector using coordinate notation.

---

**B** In the table below, list the vertices of the triangle. Then use the rule for the translation to write the vertices of the image.

Pre-Image ( $x, y$ )	Image ( $x - 3, y + 2$ )



**C** Plot the vertices of the image. Then connect the vertices to complete the image. Compare the completed image to your prediction.

### REFLECT

3a) Give an example of a translation that would move the original triangle into Quadrant IV.

---

3b) Suppose you translate the original triangle along  $\langle -10, -10 \rangle$  and then reflect the image across the  $y$ -axis. In which quadrant would the final image lie? Explain.

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3c) Suppose you transform a figure with a translation along  $\langle -8, 0 \rangle$ . How will the location of the image compare to the location of the pre-image?

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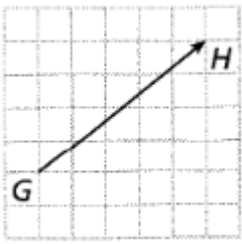
3d) How can you write the translation along  $\langle -8, 0 \rangle$  in coordinate notation?

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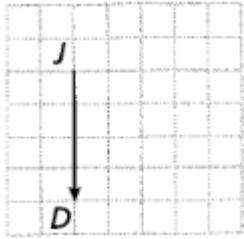


Name the vector and write it in component form.

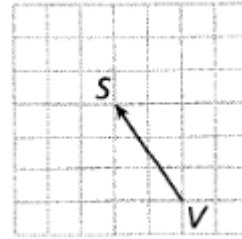
1.



2.

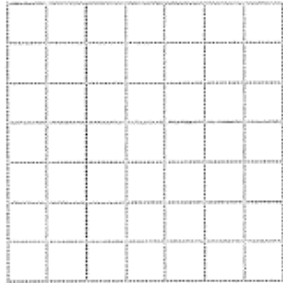


3.

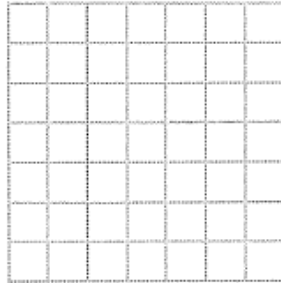


Draw and label a vector with the given name and component form.

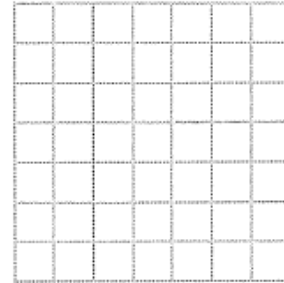
4.  $\overrightarrow{MP}; \langle 3, -1 \rangle$



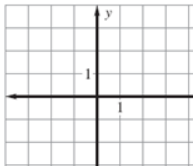
5.  $\overrightarrow{CB}; \langle 3, 0 \rangle$



6.  $\overrightarrow{HK}; \langle -5, 4 \rangle$

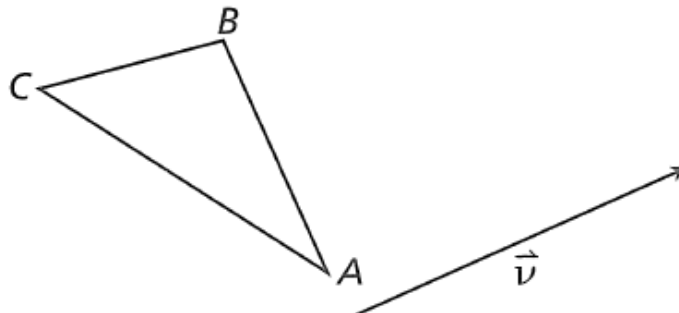


7. A vector has initial point  $(-2, 2)$  and a terminal point  $(2, -1)$ . Write the vector in component form. Then find the magnitude of the vector by using the distance formula.



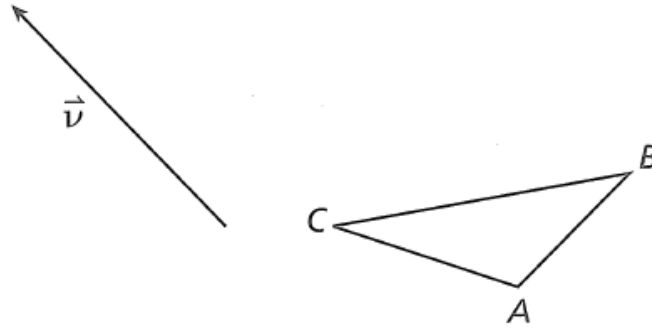
Use a compass and straightedge to construct the image of each triangle after a translation along  $\vec{v}$ . Label the vertices of the image.

8.



Use a compass and straightedge to construct the image of each triangle after a translation along  $\vec{v}$ . Label the vertices of the image.

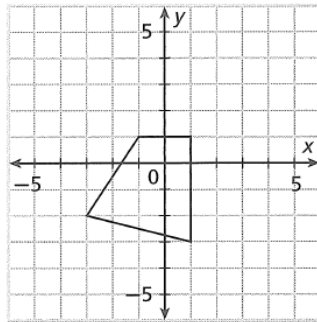
9.



Draw the image of the figure under the given translation.

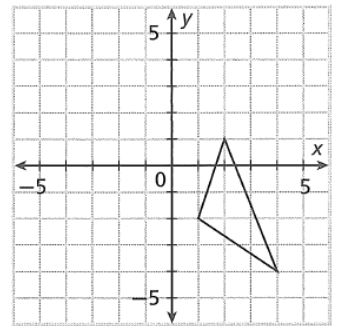
10.  $\langle 3, -2 \rangle$

$(x, y)$	Coordinate Notation $(x, y) \rightarrow ( \quad , \quad )$



11.  $\langle -4, 4 \rangle$

$(x, y)$	Coordinate Notation $(x, y) \rightarrow ( \quad , \quad )$

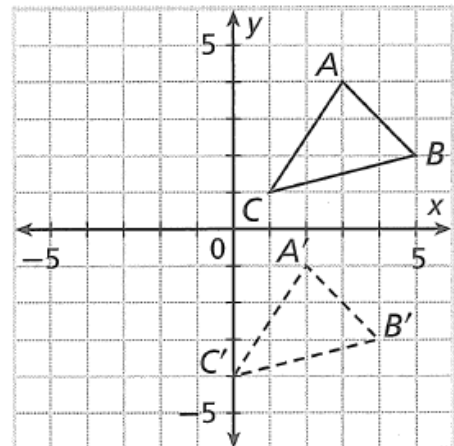


12. Use coordinate notation to name the translation that maps  $\triangle ABC$  to  $\triangle A'B'C'$ .

\_\_\_\_\_

13. What distance does each point move under this translation?

\_\_\_\_\_

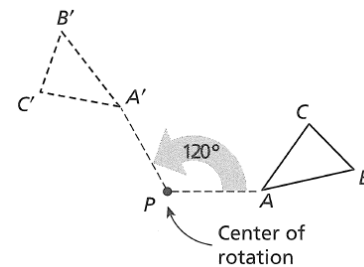


## 2-4 Rotations

**Essential question:** How do you draw the image of a figure under a rotation?

You have seen that reflections and translations are two types of rigid motions. The final rigid motion you will consider is a *rotation*. A rotation turns all points of the plane around a point called the **center of rotation**. The **angle of rotation** tells you the number of degrees through which points rotate around the center of rotation.

The figure shows a  $120^\circ$  counterclockwise rotation around point  $P$ . When no direction is specified, you can assume the rotation is in the counterclockwise direction.



### EXPLORE 1 ~ Investigating Rotations

Use the TI-nspire to investigate properties of rotations.

**A** Plot a point and label it  $P$ . (Menu, Points & Lines, Point)

**B** Draw a triangle, labeling the vertices  $A$ ,  $B$ , and  $C$ .  
(Menu, Shapes, Triangles)

**C** Using the text tool type the number 90 to represent the angle of rotation.  
(Menu, Actions, Text)

**D** Rotate triangle  $ABC$   $90^\circ$ . (Menu, Transformation, Rotation)  
(Click on the triangle, then point  $P$ , and then the angle of rotation,  $90^\circ$ )

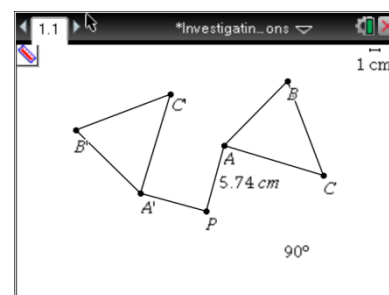
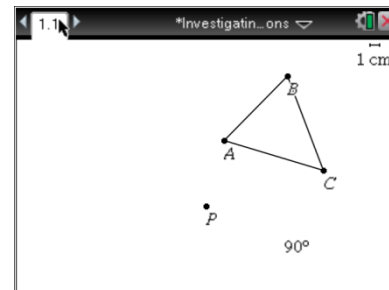
**E** Label the vertices of the image  $A'$ ,  $B'$ , and  $C'$ . (move the arrow over a point, ctrl, menu, Label)(Press [?]►] to get prime)

**F** Draw segment  $PA$  and segment  $PA'$ . (Menu, Points & Lines, Segment)

**G** Find the Length of  $PA$  and  $PA'$  (Menu, Measurement, Length)

$PA =$  \_\_\_\_\_       $PA' =$  \_\_\_\_\_

**H** Modify the shape or location of  $\triangle ABC$  and notice what changes and what remains the same.

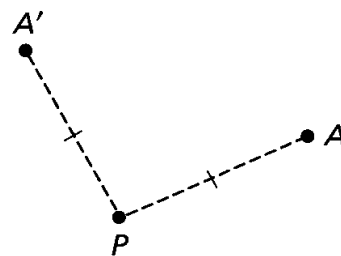


### REFLECT

1a) Make a conjecture about the distance of a point and its image from the center of rotation.

1b) What are the advantages of using geometry software rather than tracing paper or a compass and straightedge to investigate rotations?

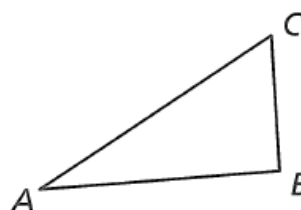
A **rotation** is a transformation about a point  $P$  such that  
 (1) every point and its image are the same distance from  $P$  and  
 (2) all angles with vertex  $P$  formed by a point and its image have the same measure.



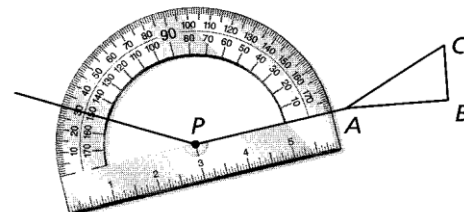
The notation  $R_{P,m^\circ}(A) = A'$  says that the image of point  $A$  after a rotation of  $m^\circ$  about point  $P$  is  $A'$ .

### EXAMPLE 2 ~ Drawing a Rotation Image

Work directly on the figure below and follow the given steps to draw the image of  $\triangle ABC$  after a  $150^\circ$  rotation about point  $P$ .

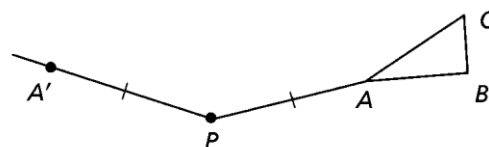


**A** Draw  $\overline{PA}$ . Then use a protractor to draw a ray that forms a  $150^\circ$  angle with  $\overline{PA}$ .



**B** Use a ruler or compass to mark point  $A'$  along the ray so that  $PA' = PA$ .

**C** Repeat the process for points  $B$  and  $C$  to locate  $B'$  and  $C'$ . Connect the image points to form  $\triangle A'B'C'$ .



### REFLECT

2a) What is  $m\angle CPC'$ ? \_\_\_\_\_

2b) What segment has the same length as  $\overline{BP}$ ? \_\_\_\_\_

2c) Would it be possible to draw the rotation of image  $\triangle ABC$  using only a compass and straightedge (traditional construction tools). Why or why not?

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The table provides coordinate notation for rotations in a coordinate plane. You can assume that all rotations in a coordinate plane are rotations about the origin. Also, note that  $270^\circ$  rotation is equivalent to turning  $\frac{3}{4}$  of a complete circle.

Rules for Rotations in a Coordinate Plane	
Rotation of $90^\circ$	$(x, y) \rightarrow (-y, x)$
Rotation of $180^\circ$	$(x, y) \rightarrow (-x, -y)$
Rotation of $270^\circ$	$(x, y) \rightarrow (y, -x)$

### EXAMPLE 3 ~ Drawing a Rotation in a Coordinate Plane

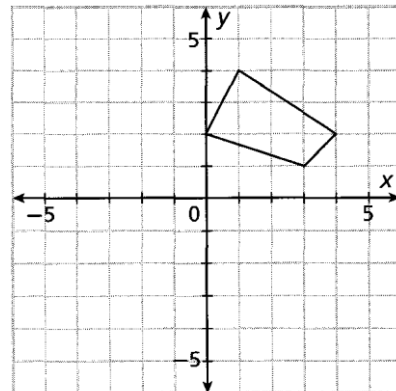
Draw the image of the quadrilateral under a  $270^\circ$  rotation.

**A** Before drawing the image, predict the quadrant in which the image will lie.

---

**B** In the table below, list the vertices of the quadrilateral. Then use the rule for the rotation to write the vertices of the image.

Pre-Image $(x, y)$	Image $(x, y) \rightarrow ( \quad , \quad )$



**C** Plot the vertices of the image. Then connect the vertices to complete the image. Compare the completed image to your prediction.

### REFLECT

3a) What would happen if you rotated the image of the quadrilateral an additional  $90^\circ$  about the origin? Why does this make sense?

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---

3b) Suppose you rotate the original quadrilateral  $810^\circ$ . In which quadrant will the image lie? Explain.

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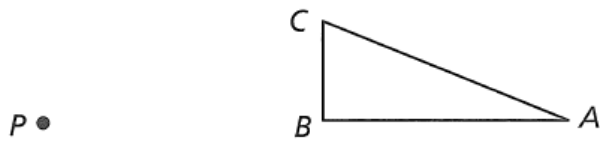
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**PRACTICE ~ Rotations**

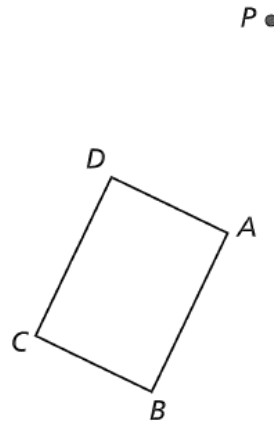
Name \_\_\_\_\_

Use a ruler and protractor to draw the image of each figure after a rotation about point P by the given number of degrees. Label the vertices of the image.

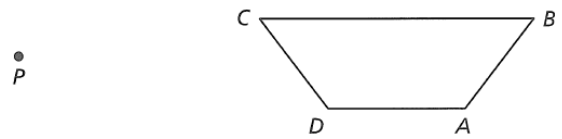
1.  $50^\circ$



2.  $80^\circ$



3.  $160^\circ$

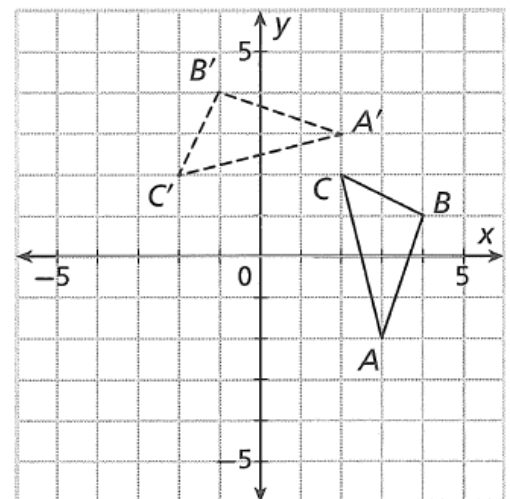


4. a. Use coordinate notation to write a rule for the rotation that maps  $\triangle ABC$  to  $\triangle A'B'C'$

\_\_\_\_\_

b. What is the angle of rotation?

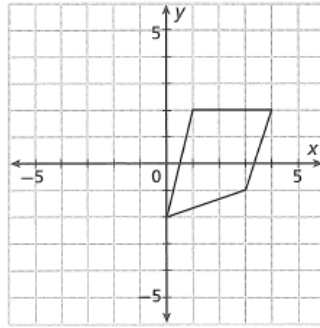
\_\_\_\_\_



Draw the image of the figure after the given rotation.

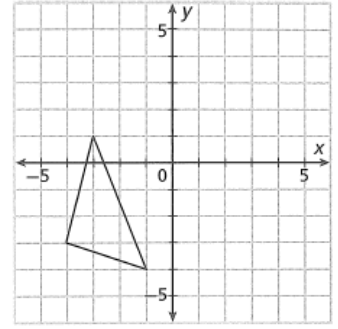
5.  $180^\circ$

$(x, y)$	Coordinate Notation $(x, y) \rightarrow ( \quad , \quad )$



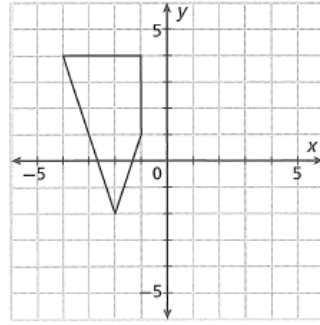
6.  $90^\circ$

$(x, y)$	Coordinate Notation $(x, y) \rightarrow ( \quad , \quad )$



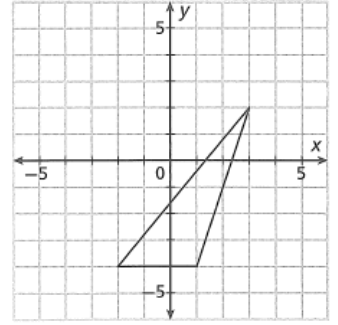
7.  $270^\circ$

$(x, y)$	Coordinate Notation $(x, y) \rightarrow ( \quad , \quad )$



8.  $180^\circ$

$(x, y)$	Coordinate Notation $(x, y) \rightarrow ( \quad , \quad )$



9. **a.** Reflect  $\triangle JKL$  across the  $x$ -axis. Then reflect the image across the  $y$ -axis. Draw the final image of the triangle and label it  $\triangle J'K'L'$

**b.** Describe a single rotation that maps  $\triangle JKL$  to  $\triangle J'K'L'$ .

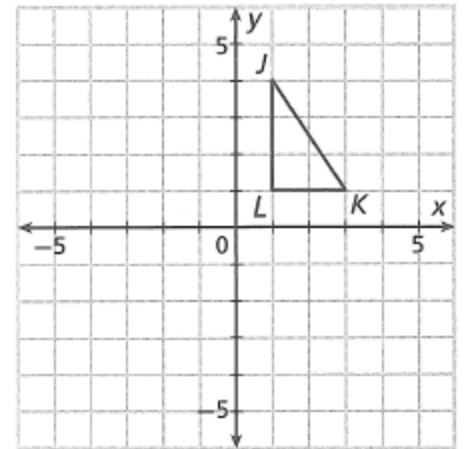
\_\_\_\_\_

**c.** Use coordinate notation to show that your answer to part **b** is correct.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_



10. **Error Analysis** A student was asked to use coordinate notation to describe the result of  $180^\circ$  rotation followed by a translation 3 units to the right and 5 units up. The student wrote this notation:  $(x, y) \rightarrow (-[x + 3], -[y + 5])$ . Describe and correct the student's error.

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

## 2-5 Perpendicular Bisectors

**Essential question:** What are the key theorems about perpendicular bisectors?

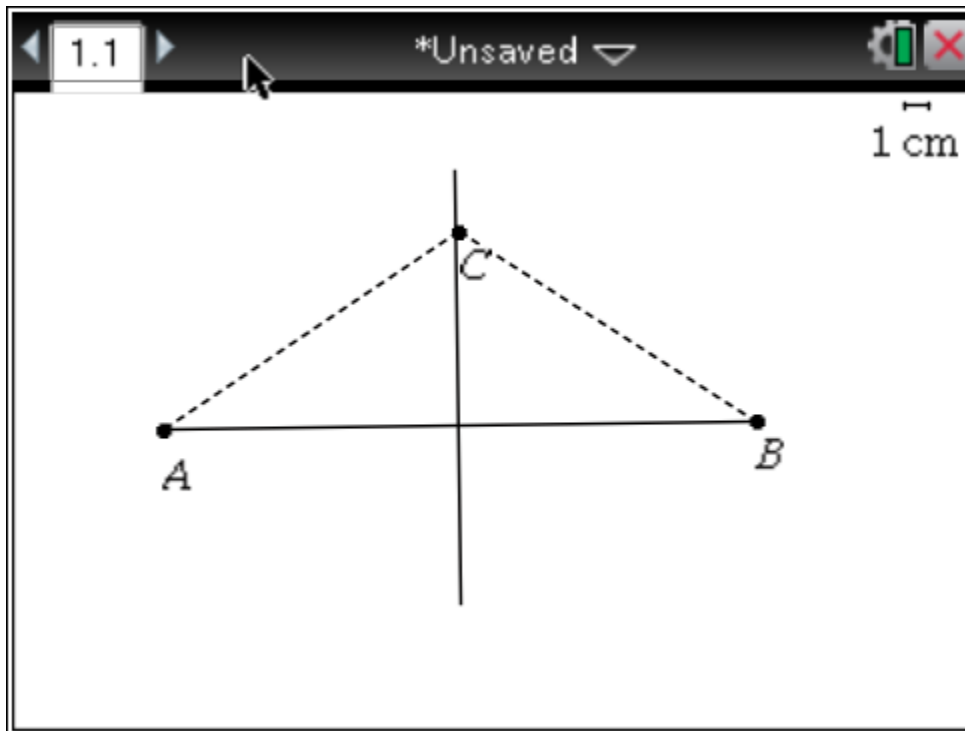
You can use reflections and their properties to prove theorems about perpendicular bisectors. These theorems will be very useful in proofs later on.

### EXPLORE ~ Perpendicular Bisectors

A segment, ray, line or plane that is perpendicular to a segment at its midpoint is called a **perpendicular bisector**.

Complete the following on a TI-nspire Calculator

- Go to the main screen [ON] and create a New Document [1]
- If the calculator asks if you would like to save the previous document select "No"
- Add a Geometry Page [3]
- Draw  $\overline{AB}$  (Menu, Points & Lines, Segment. Drop the 1<sup>st</sup> point and press shift A, move to a new location, drop the 2<sup>nd</sup> point and press shift B.)
- Construct the perpendicular bisector of  $\overline{AB}$  (Menu, Construction, Perpendicular Bisector, Click on the segment)
- Place a point C anywhere on the perpendicular bisector (Menu, Points & Lines, Point On. Press Shift C after dropping the point)
- Draw segments AC and BC. (Menu, Points & Lines, Segment)
- Find the length of segments AC and BC (Menu, Measurement, Length. Place the arrow over the segment, click to pick up the measurement and move it to the location you want. Click again to drop the measurement.)  
AC = \_\_\_\_\_ BC = \_\_\_\_\_
- Grab point C and move it to a new location. What do you notice? \_\_\_\_\_
- Compare your results above with your group. Describe the relationship between the points on a perpendicular bisector of a segment and the endpoints of the segment.



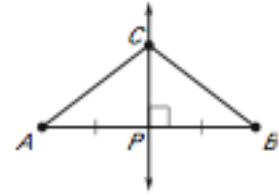


### Perpendicular Bisector Theorem

In a plane, if a point is on the perpendicular bisector of a segment, then it is \_\_\_\_\_ from the endpoints of the segment.

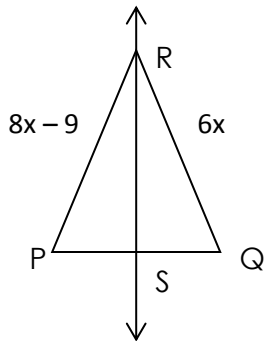
GIVEN:

CONCLUSION:

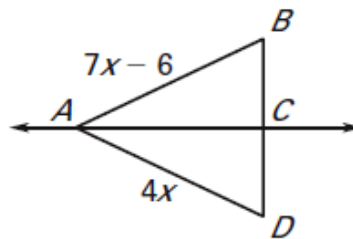


\_\_\_\_\_

1.  $\overleftrightarrow{RS}$  is the perpendicular bisector of  $\overline{PQ}$ . Find PR.

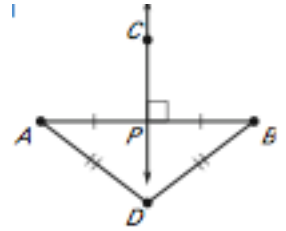


2.  $\overleftrightarrow{AC}$  is the perpendicular bisector of  $\overline{BD}$ . Find AD.



**Converse of the Perpendicular Bisector Theorem**

In a plane, if a point is equidistant from the endpoints of a segment, then it is on the \_\_\_\_\_ of the segment.



Given:

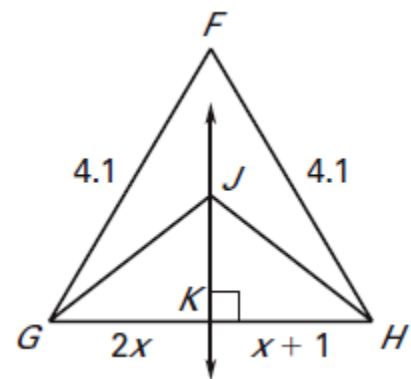
Conclusion:

\_\_\_\_\_

3. In the diagram,  $\overleftrightarrow{JK}$  is the perpendicular bisector of  $\overline{GH}$ .

a. Which lengths in the diagram are equal?

b. Find GH.



c. Is F on  $\overleftrightarrow{JK}$ ? Why or why not?