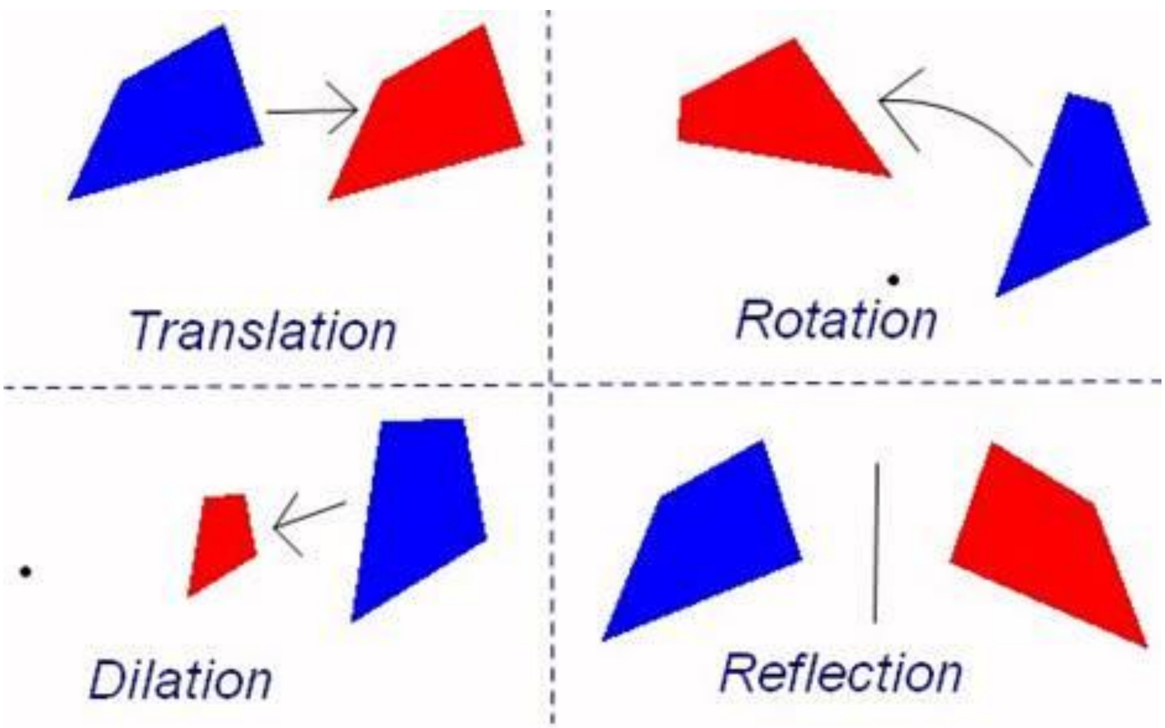
Chapter 2:
Transformations



Unit 2: Vocabulary

1)	transformation	
2)	pre-image	
3)	image	
4)	map(ping)	
5)	rigid motion (isometry)	
6)	orientation	
7)	line reflection	
8)	line of reflection	
9)	translation	
10)	vector	
11)	rotation	
12)	center of rotation	
13)	angle of rotation	
14)	point reflection	

15)	dilation	
16)	center of dilation	
17)	scale factor	
18)	enlargement	
19)	reduction	

Day 1: Line Reflections

G.CO.2 Represent transformations in the plane, e.g., using transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch.)

G.CO.4. Develop definitions of reflections, translations, and rotations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, e.g. using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry one figure onto another.

Warm-Up

If $f(x) = 3x + 4$, find $f(1)$.

A **transformation** is a change in the position, size, or shape of a figure. A transformation takes points in the plane and maps them to other points in the plane.

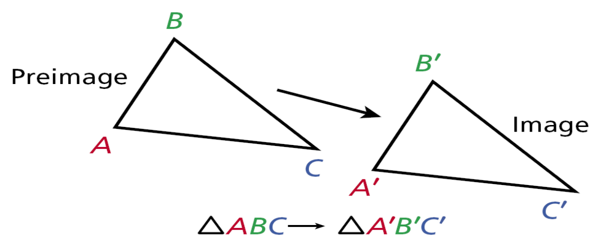
The original figure (the inputs for the transformation) is called the **preimage**.

The resulting figure (the outputs) is called the **image**.

We can represent transformations in a number of ways.

1) Mapping Notation

A transformation is sometimes called a **mapping**. The transformation *maps* the preimage to the image. In mapping notation, arrow notation ($\rightarrow$) is used to describe a transformation, and primes ($'$) are used to label the image.



2) Function Notation

The notation $\boldsymbol{T(A) = A'}$ means that a transformation T maps a point A onto its image, A'.

3) Coordinate Notation

Coordinate notation will tell you how to change the coordinates of a general point (x, y) to get the coordinates of its image. For example, $(x, y) \rightarrow (x + 5, y - 3)$ means you get the image point by adding 5 to each x and subtracting 3 from each y.

Exercise

1) Given the transformation: $T(B) = C$

Which point is the pre-image? ____________ Which point is the image? _________________

2) Given a transformation F: $(x, y) \to (x + 1, y + 1)$

a) Describe what this transformation is going to do to a point in the plane. _________________

____

b) Transformations are *functions* because each *input* in the domain is mapped to a unique *output* in the range.

How would you describe the domain of F? ____

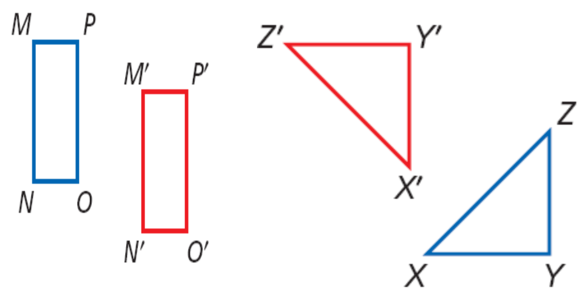
The range of F? ____

Rigid Motions

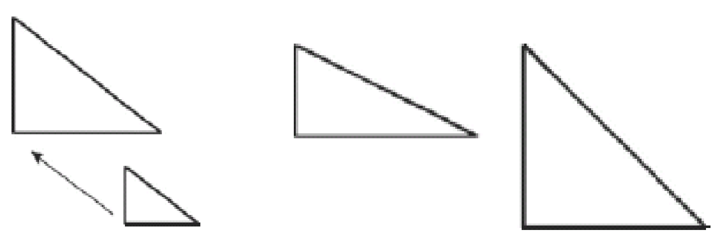
A **rigid motion** is the action of taking an object and moving it to a different location without altering its shape or size. Reflections, rotations, translations, and glide reflections are all examples of rigid motions. In fact, every rigid motion is one of these four kinds.

Rigid motions are also called *isometries*. Rigid motions are therefore called isometric transformations.

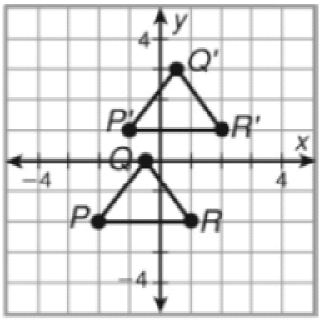
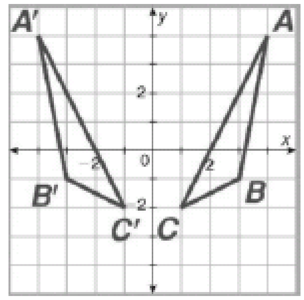
Examples of rigid motions:



NOT rigid motions:

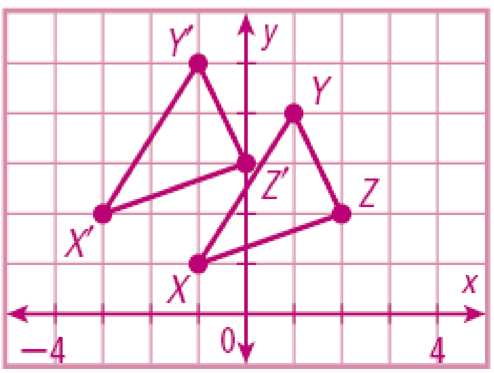


The **orientation** of a figure is the arrangement of points around a figure. Orientation can be clockwise or counterclockwise. There are two types of rigid motions.

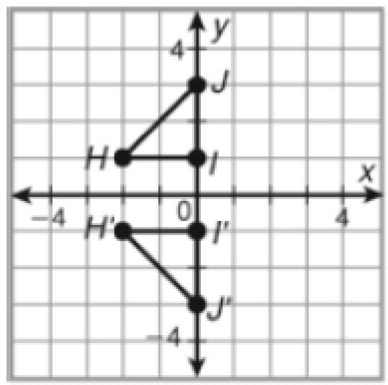
*A **proper rigid motion** **preserves** orientation.* *(It keeps it the same.)*	*An **improper rigid motion** changes orientation.*
	
Both the figure and its image have clockwise orientations. This rigid motion preserves orientation.	ΔABC has a clockwise orientation but $\Delta A'B'C'$ has a counterclockwise orientation. Orientation changes, and this is an improper rigid motion.

Exercise Are the following transformations **rigid motions**? If so, do they preserve or change **orientation**?

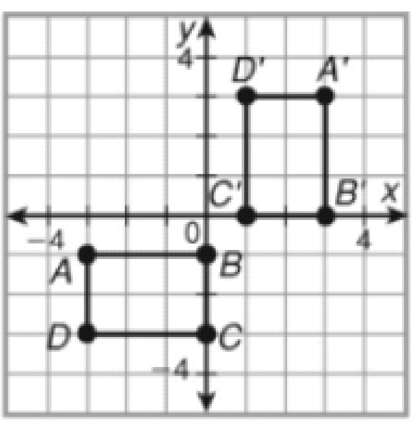
1)



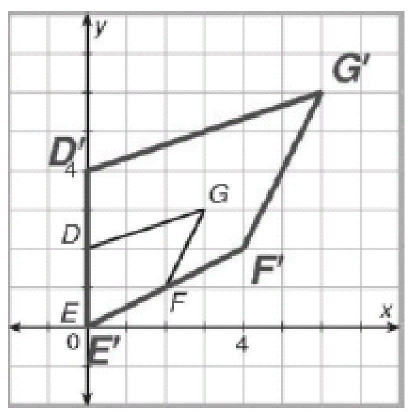
2)



3)

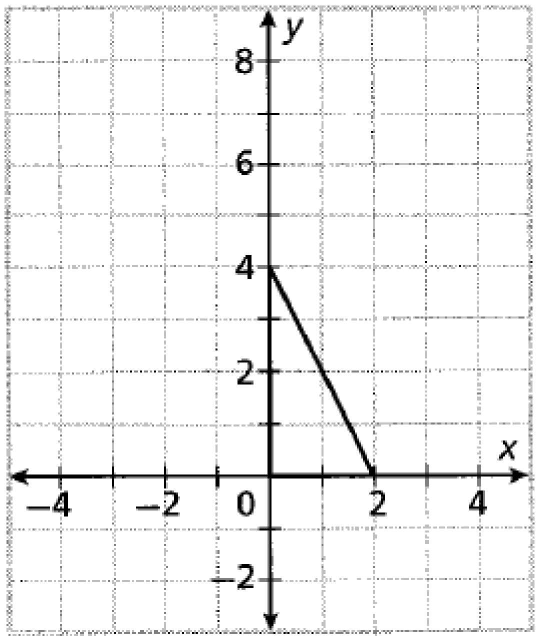


4)

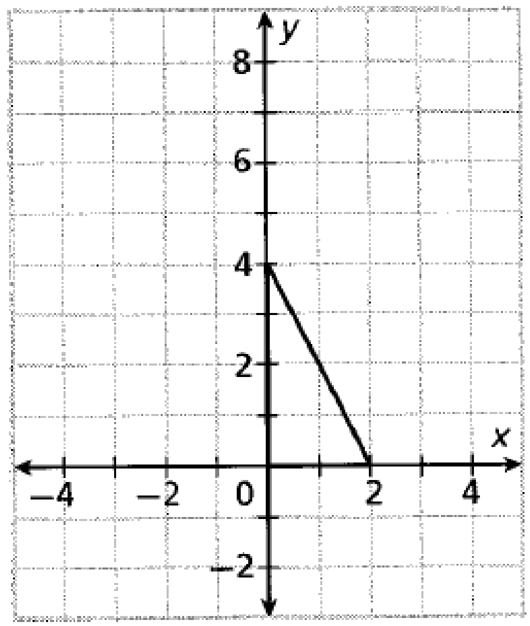


For #5-6, a transformation is mapped below in coordinate notation. Graph the image on the same set of axes. Then state whether the transformation is a rigid motion.

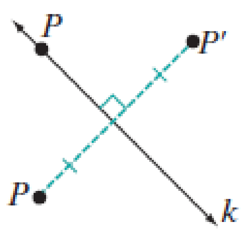
5) $(x, y) \rightarrow (x - 4, y + 3)$



6) $(x, y) \rightarrow (2x, y)$



Line Reflections



DEFINITION

A **reflection in line k** is a transformation in a plane such that:

1. If point P is not on k, then the image of P is P' where k is the perpendicular bisector of $\overline{PP'}$.
2. If point P is on k, the image of P is P.

So, a line reflection is a "flip" across a line. This line is called the *line of reflection*.

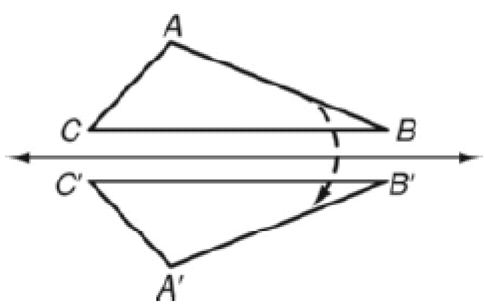
The line of reflection is the **perpendicular bisector** of the segment connecting each point and its image.

If a point is on the line of reflection, its image is the original point.

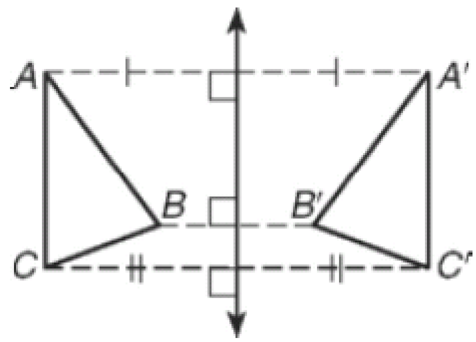
Other properties of line reflections:

- Reflections are improper rigid motions. They preserve distance, but change orientation.
- Applying the same reflection twice gives the identity motion. That is, the figure goes back to its original position.
- Angle measure, midpoint, and collinearity are also preserved.

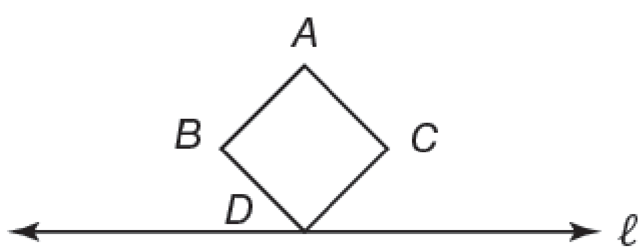
Examples of line reflections:



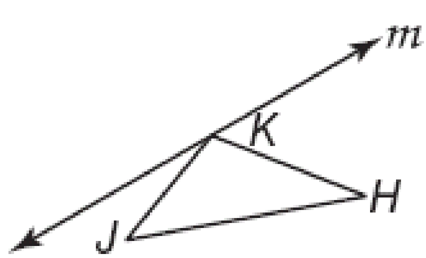
$\triangle ABC \rightarrow \triangle A'B'C'$

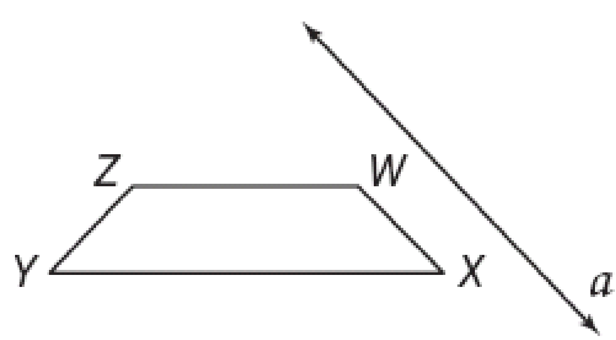


Model Problem Use a ruler or tracing paper to sketch the reflection of each figure in the line provided.



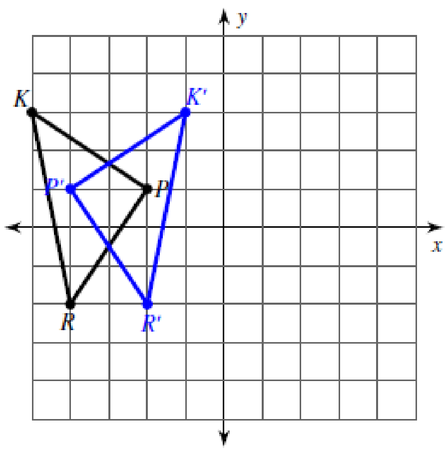
Exercise Use a ruler or tracing paper to sketch the reflection of each figure in the line provided.

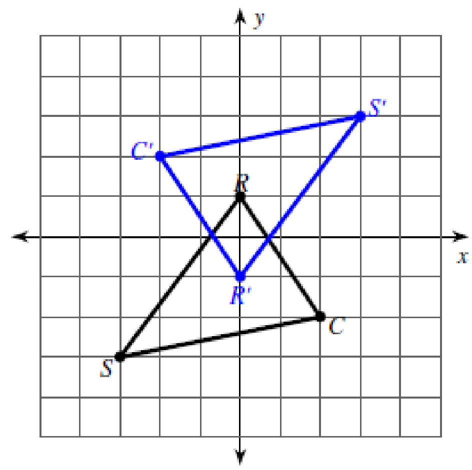




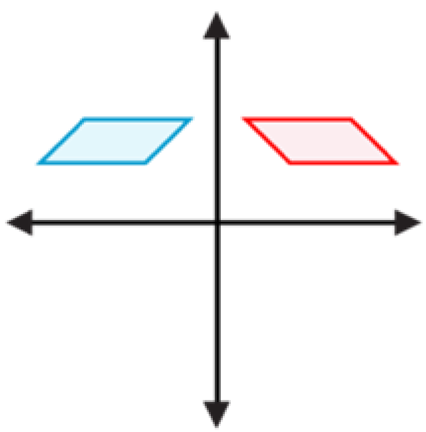
Identifying Line Reflections

Model Problem Is the transformation shown a line reflection? Justify your answer.

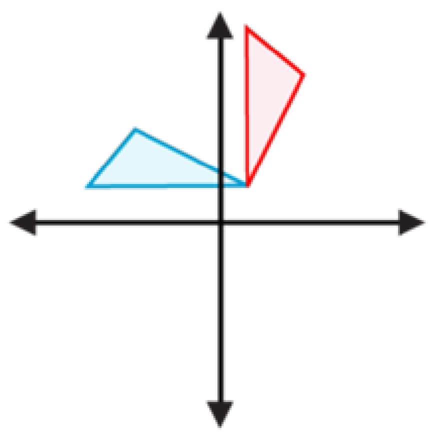




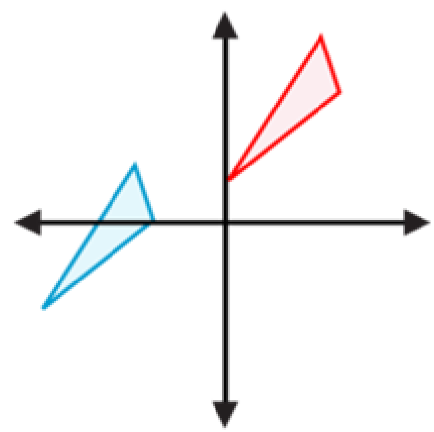
Exercise Is the transformation shown a line reflection? Justify your answer.



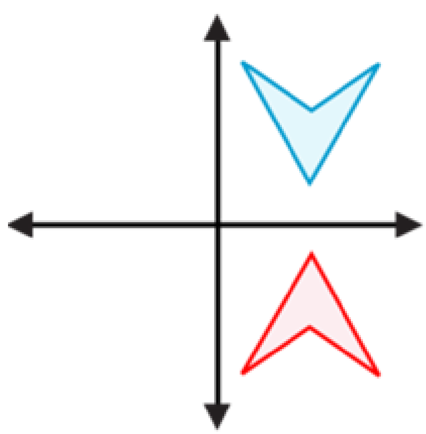
(1)



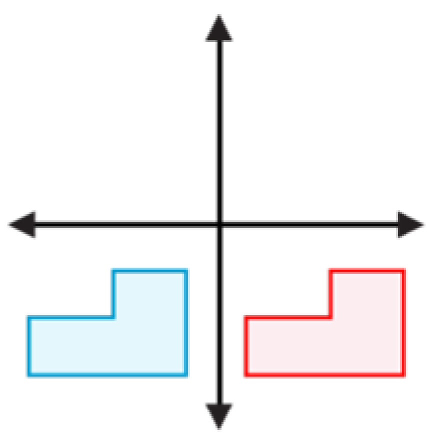
(2)



(3)



(4)



(5)

Summary

A **transformation** is a change in the position, size, or shape of a figure.

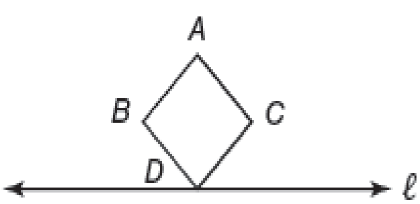
A **rigid motion** is the action of taking an object and moving it to a different location without altering its shape or size. Reflections, rotations, translations, and glide reflections are all examples of rigid motions. Rigid motions preserve distance, angle measure, collinearity, parallelism, and midpoint.

A **line reflection** is a transformation where the line of reflection is the perpendicular bisector of each segment containing a point and its image. It is described as a "flip" over a line.

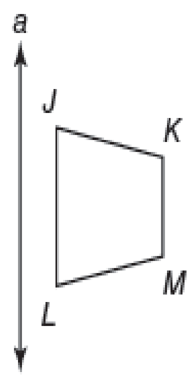
Homework

Use the given figure and line of reflection. Draw the image in this line using a ruler.

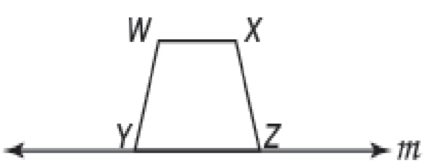
1.



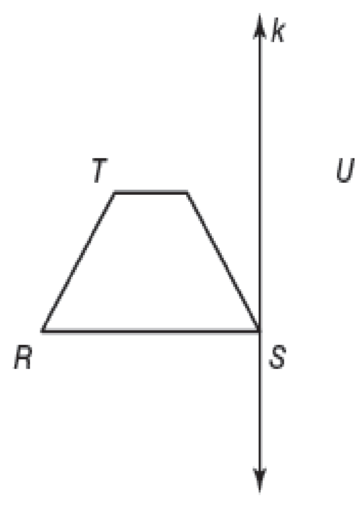
2.



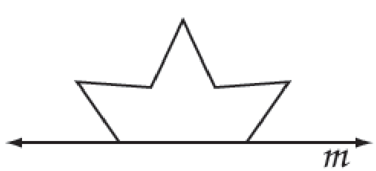
3.



4.



REFLECTIONS Vincent is making a star. Complete the star by drawing the reflected image of the figure in line m.



6) Consider the line reflection at right, where $r_k(ABCD) = A'B'C'D'$.

Answer TRUE or FALSE.

a) $\angle A \cong \angle A'$ ____________________________

b) $\overline{BC} \cong \overline{B'C'}$ ___________________________

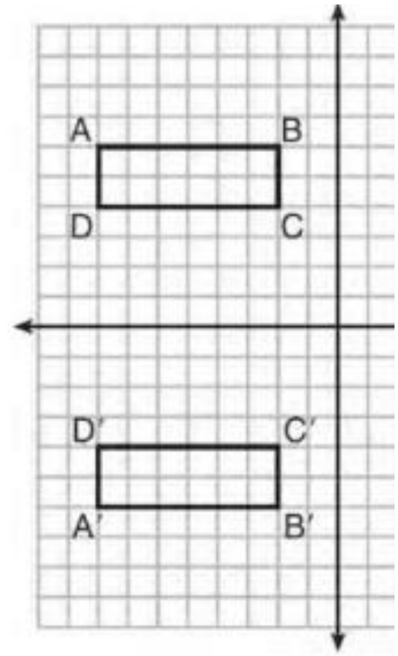
c) $\overline{BC} \cong \overline{A'D'}$ ___________________________

d) $If\ \overline{AB} \parallel \overline{CD}$, then $\overline{A'B'} \parallel \overline{C'D'}$. ____________________

e) $If\ \overline{AB} \perp \overline{BC}$, then $\overline{A'B'} \perp \overline{B'C'}$. ____________________

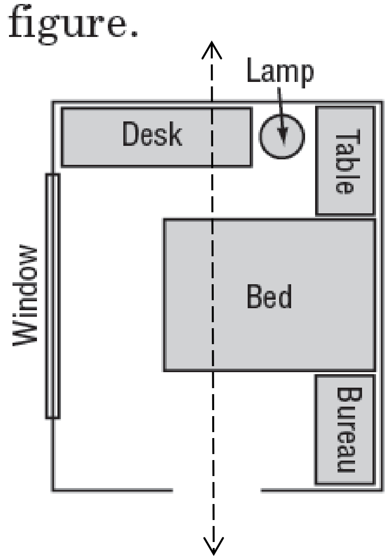
f) Quadrilateral $ABCD$ and quadrilateral $A'B'C'D'$ have the same orientation. ______________________________________

k



g) Explain why the transformation that maps $ABCD$ onto $A'B'C'D'$ is a line reflection. You may want to use words like **perpendicular bisector, image**, and **segment**, among others.

INTERIOR DESIGN Wilfred hired an interior designer to layout the furniture in his bedroom. The designer produced the plan shown in the figure. Unfortunately, Wilfred's window is located on the opposite wall from the plan. Wilfred decided to just reflect the plan over the vertical line through the center of the room. Draw the reflected plan.



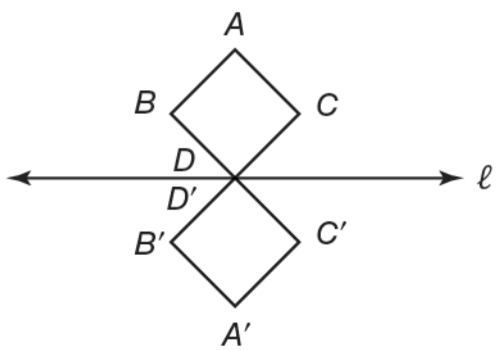
Day 2: More on Line Reflections: Graphing and Constructions

G.CO.2 Represent transformations in the plane, e.g., using transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch.)

G.CO.4. Develop definitions of reflections, translations, and rotations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, e.g. using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry one figure onto another.

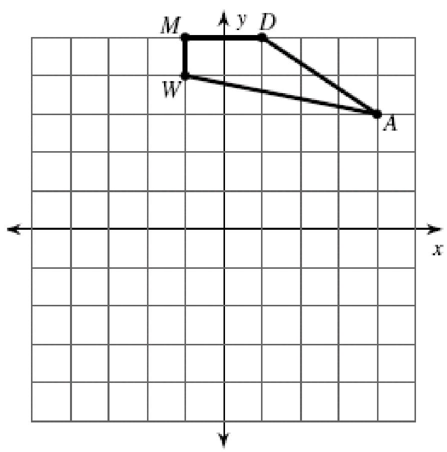
Warm-Up Is this a line reflection? Justify your answer using appropriate mathematical vocabulary.



Special Line Reflections

Some line reflections have special rules. We can develop these rules by performing them on points in the coordinate plane and examining the coordinates of their image.

1) Reflection in the x-axis (r_{x-axis})



M(-1, 5) → $M'($ $)$

D (1, 5) → $D'($ $)$

A (4, 3) → $A'($ $)$

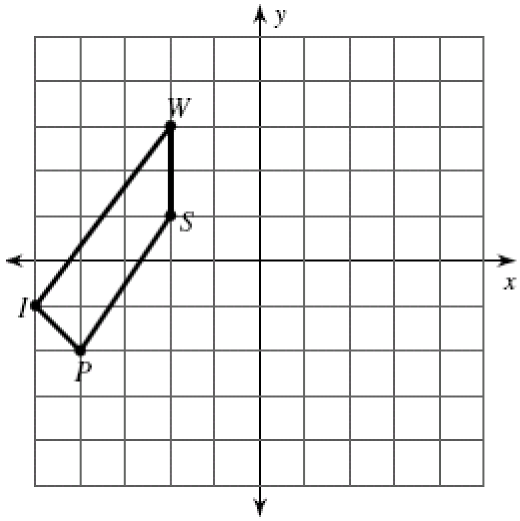
W (-1, 4) → $W'($ $)$

Rule for Reflection in the x-axis

$(x, y) \rightarrow (\quad , \quad)$

To reflect in the x-axis, ____

2) <u>Reflection in the y-axis</u> (r_{y-axis})



W(-2, 3) → W'(   )

S (-2, 1) → S'(   )

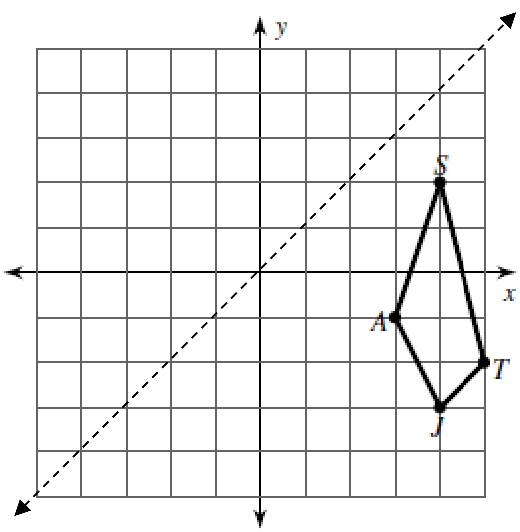
I (-5, -1) → I'(   )

P (-4, -2) → P'(   )

<u>Rule for Reflection in the y-axis</u>
$(x, y) \rightarrow (\quad , \quad)$

To reflect in the y-axis, ____

3) <u>Reflection in the line y = x</u> ($r_{y=x}$)



J(4, -3) → J'(   )

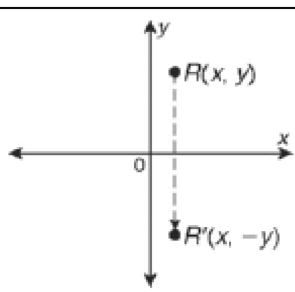
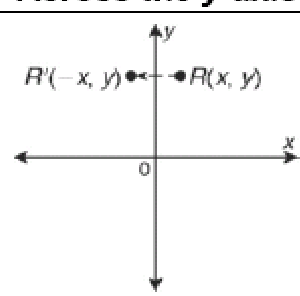
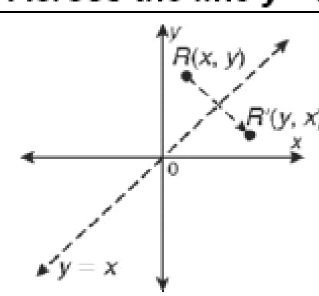
A (3, -1) → A'(   )

T (5, -2) → T'(   )

S (4, 2) → S'(   )

<u>Rule for Reflection in the line y = x</u>
$(x, y) \rightarrow (\quad , \quad)$

To reflect in the line y = x , ____

Reflections in the Coordinate Plane		
Across the *x*-axis	**Across the *y*-axis**	**Across the line *y* = *x***
		
$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$	$(x, y) \rightarrow (y, x)$

Practice

Point A is located at $(4,-7)$. The point is reflected in the x-axis. Its image is located at
1) $(-4,7)$
2) $(-4,-7)$
3) $(4,7)$
4) $(7,-4)$

Reflecting $(5,1)$ in the y-axis yields an image of
1) $(5,-1)$
2) $(-5,-1)$
3) $(5,1)$
4) $(-5,1)$

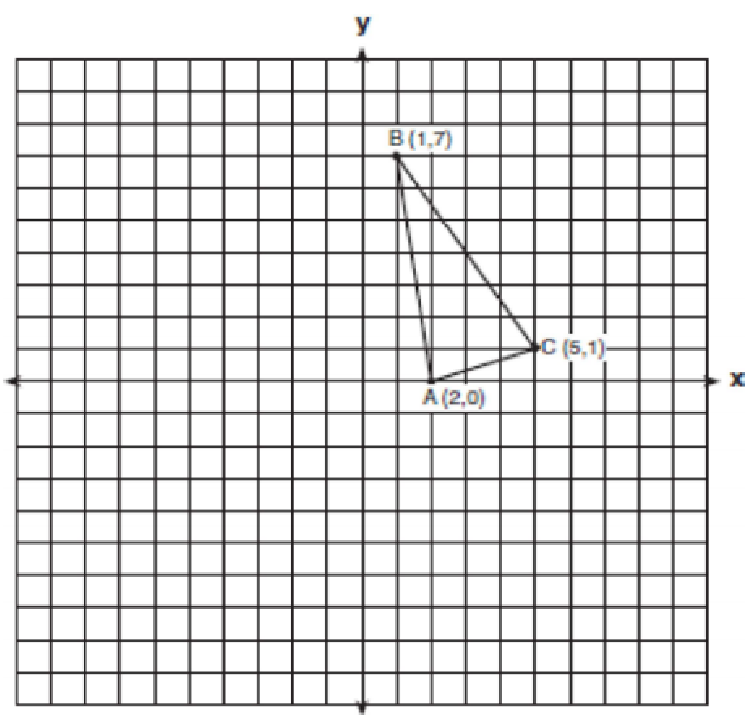
When the point $(2,-5)$ is reflected in the x-axis, what are the coordinates of its image?
1) $(-5,2)$
2) $(-2,5)$
3) $(2,5)$
4) $(5,2)$

The image of point $(3,4)$ when reflected in the y-axis is
1) $(-3,-4)$
2) $(-3,4)$
3) $(3,-4)$
4) $(4,3)$

What is the image of point $(-3,7)$ after a reflection in the x-axis?
1) $(3,7)$
2) $(-3,-7)$
3) $(3,-7)$
4) $(7,-3)$

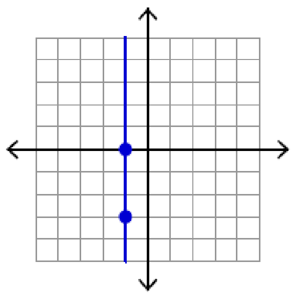
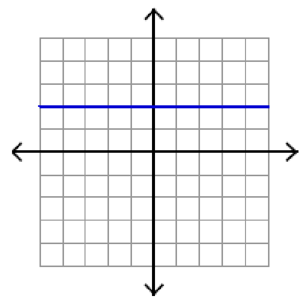
What is the image of the point $(2,-3)$ after the transformation $r_{y-\text{axis}}$?
1) $(2,3)$
2) $(-2,-3)$
3) $(-2,3)$
4) $(-3,2)$

Triangle ABC has coordinates $A(2,0)$, $B(1,7)$, and $C(5,1)$. On the accompanying set of axes, graph, label, and state the coordinates of $\triangle A'B'C'$, the reflection of $\triangle ABC$ in the y-axis.



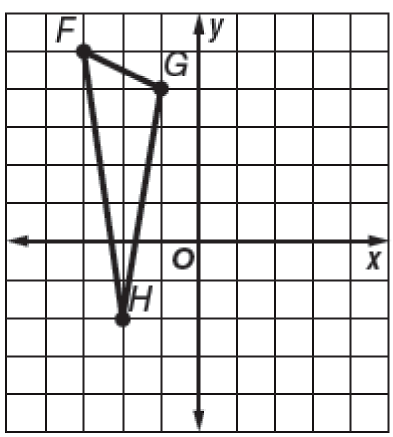
Reflections across Vertical and Horizontal Lines

Other lines can be used as lines of reflection as well.

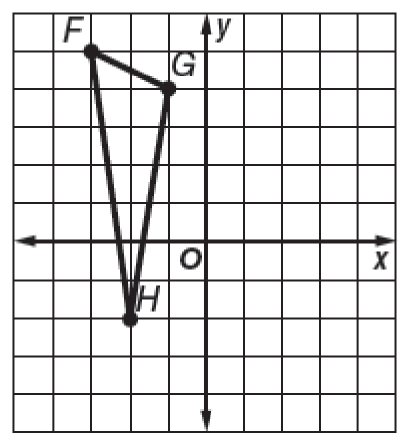
<table>
<tr><td colspan="2">Reminder! Equations of Horizontal and Vertical Lines</td></tr>
<tr><td>x = a number</td><td>y = a number</td></tr>
<tr><td></td><td></td></tr>
</table>

Model Problems — Sketch the reflection of the figure in the line indicated. What are the coordinates of the image?

1) Reflection in x = -1

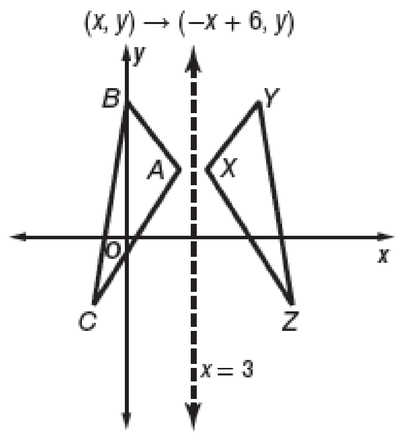


2) Reflection in y = 1



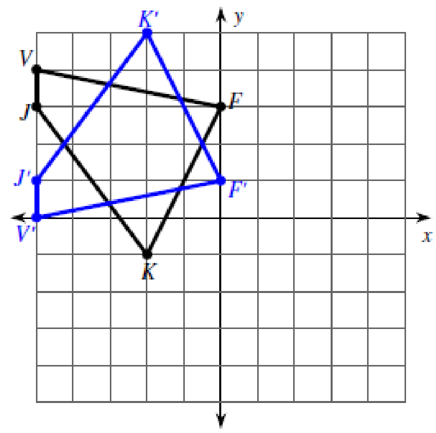
3) In the diagram at right, ΔABC is mapped onto ΔXYZ by the transformation $(x, y) \rightarrow (-x + 6, y)$. Notice that this transformation is equivalent to a reflection in the line $x = 3$.

Prove that the line $x = 3$ is the perpendicular bisector of the segment with endpoints (x, y) *and* $(-x + 6, y)$. (Hint: Use the midpoint formula.)



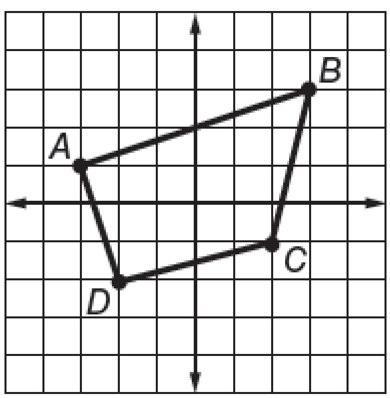
4) Sketch the line of reflection on the diagram below. Then write a rule for the reflection. Be sure to include the name of the transformation and the equation of the line of reflection.

Rule: ____

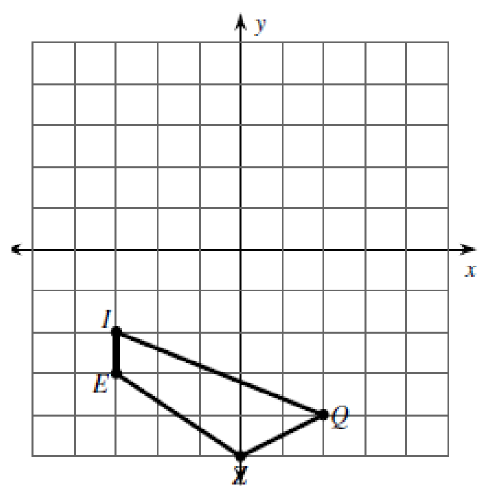


Exercise Sketch the reflection of each figure in the line indicated.

1) *Reflection across x = 1*



2) reflection across $y = -2$

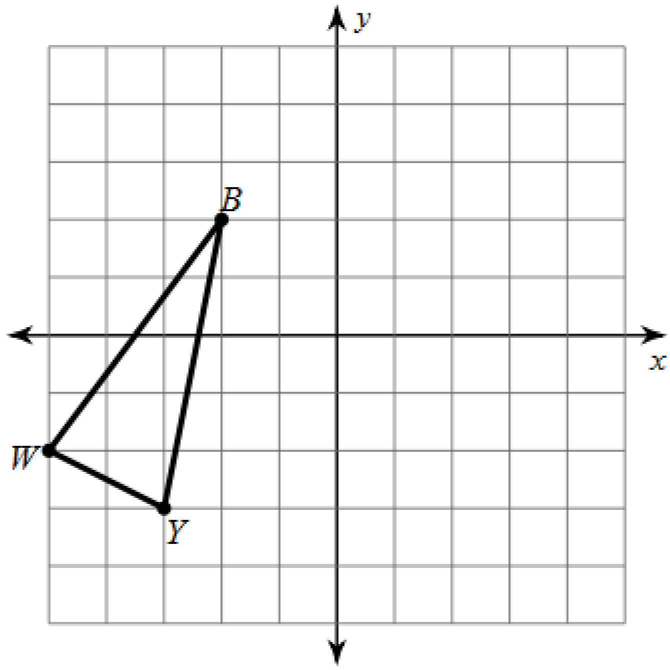


3) A transformation T is given by the rule $(x, y) \rightarrow (-x - 4, y)$.

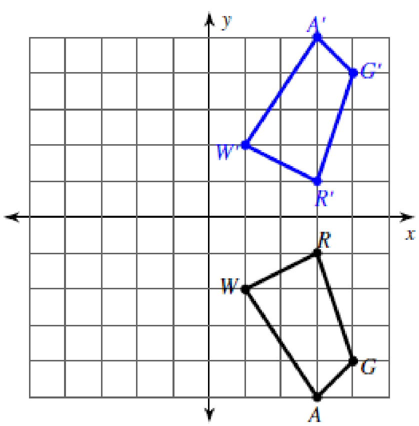
a) Graph and state the coordinates of the image of the figure below under transformation T.

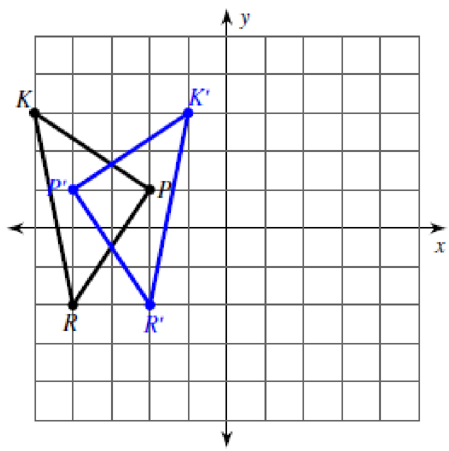
b) Show that transformation T is a line reflection.

c) State the equation of the line of reflection.



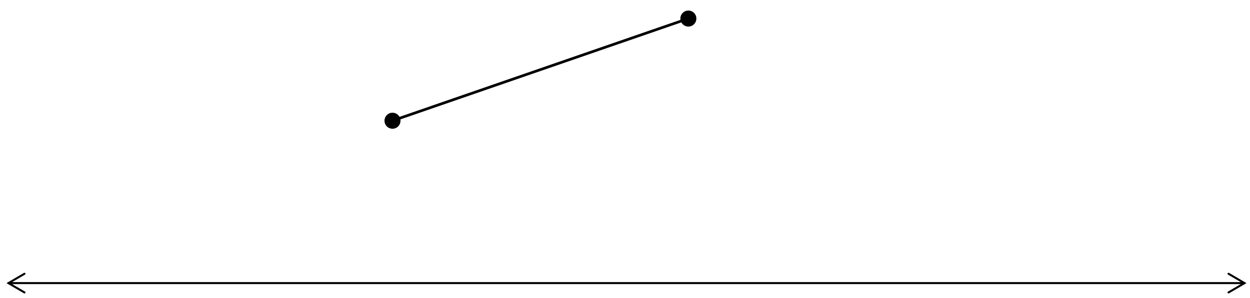
4) Write a rule for each reflection.



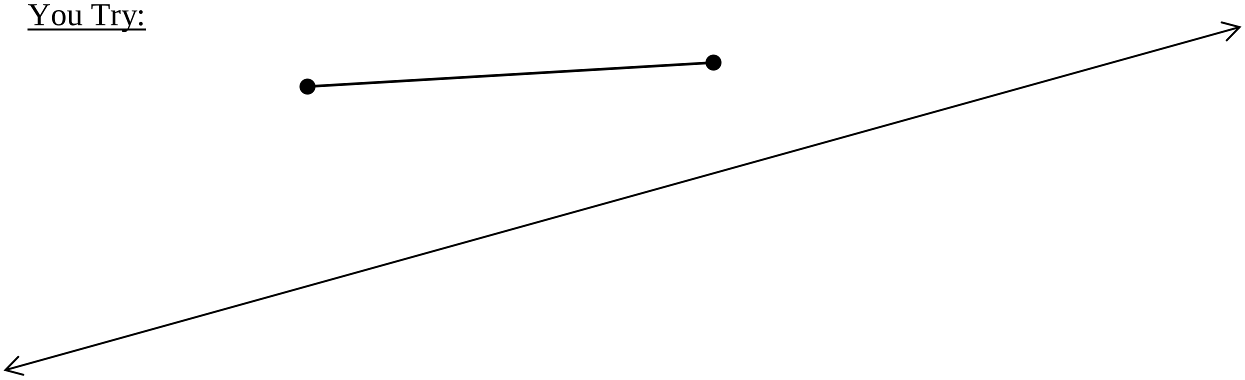


Constructing a Line Reflection

Task: Construct the image of the line segment after a reflection in the given line.



You Try:



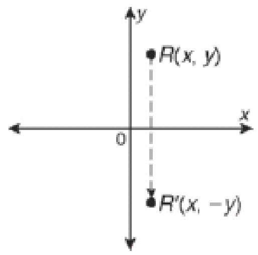
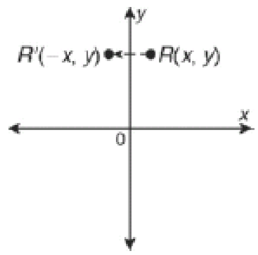
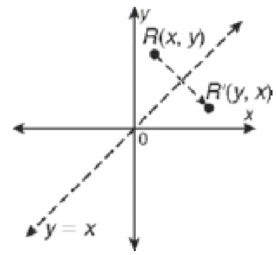
Justify this construction (*Fill in the blanks):*

1. We construct a line ________________________________ to the given line passing through the given point and its image.

2. Then we construct a segment where the line of reflection goes through that segment's ______________.

3. Therefore, the given line is the ____________________________ ________________________ of the segment containing each given point and its image, making it a line of reflection.

Summary

Reflections in the Coordinate Plane		
Across the *x*-axis	**Across the *y*-axis**	**Across the line $y = x$**
		
$(x, y) \rightarrow (x, -y)$	$(x, y) \rightarrow (-x, y)$	$(x, y) \rightarrow (y, x)$

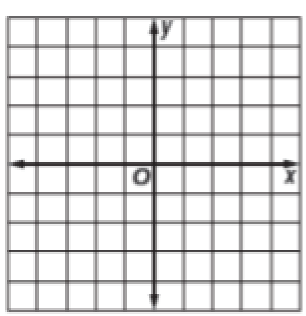
- To reflect a figure across a horizontal or vertical line, sketch the line of reflection, and sketch the image of each point individually.

- To describe a rule for a line reflection, observe the line of symmetry between the original figure and the image.

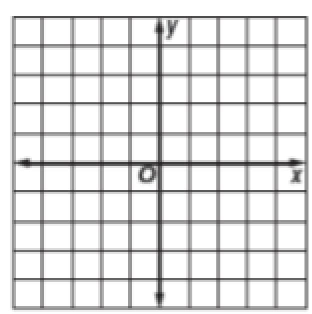
Homework

COORDINATE GEOMETRY Graph each figure and its image under the given reflection.

$\triangle ABC$ with vertices $A(-3, 2)$, $B(0, 1)$, and $C(-2, -3)$ in the line $y = x$

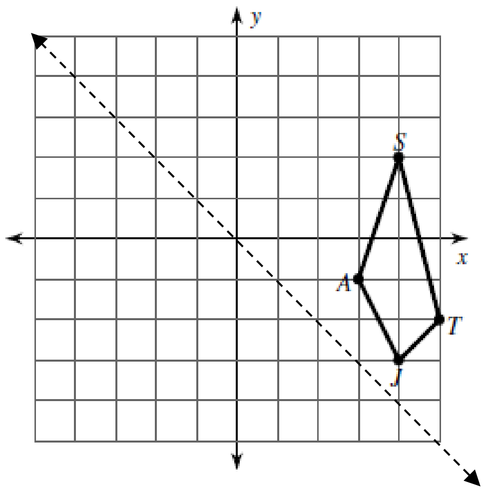


trapezoid $DEFG$ with vertices $D(0, -3)$, $E(1, 3)$, $F(3, 3)$, and $G(4, -3)$ in the y-axis

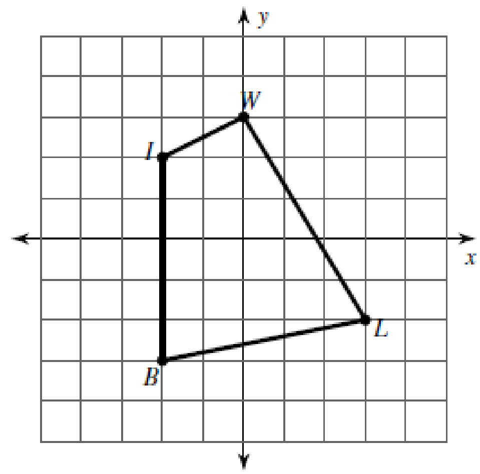


Sketch the image of the figure shown in the line indicated.

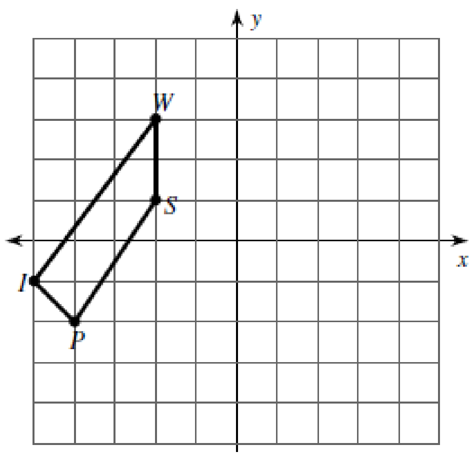
reflection across $y = -x$



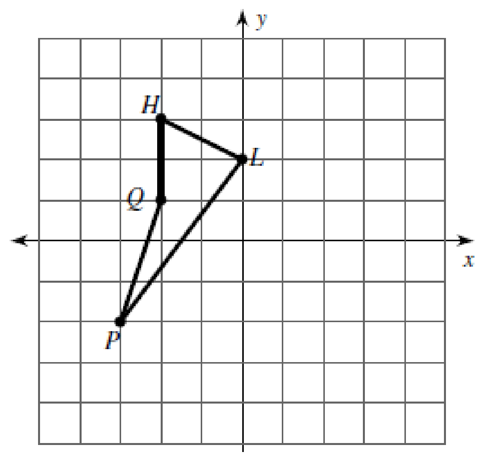
reflection across $y = -1$



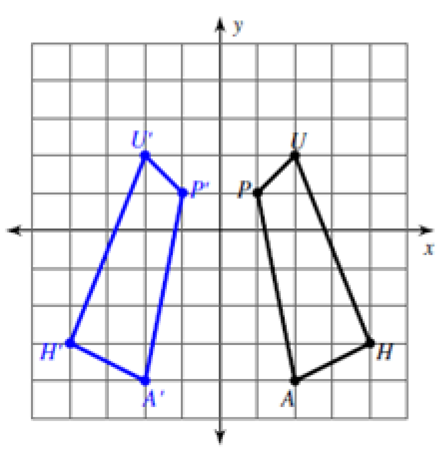
reflection across $x = -3$

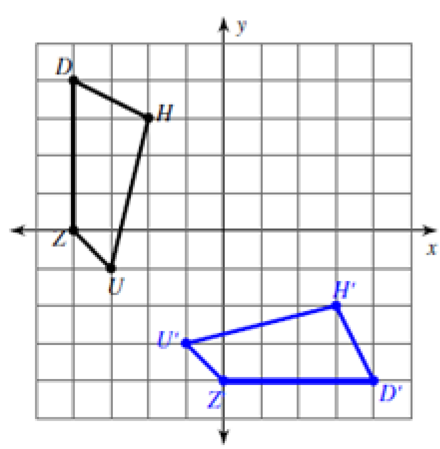


reflection across $y = x$



Write the rule for the transformation shown.





Day 3: Translations

G.CO.2 Represent transformations in the plane, e.g., using transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch.)

G.CO.4. Develop definitions of reflections, translations, and rotations in terms of angles, circles, perpendicular lines, parallel lines, and line segments

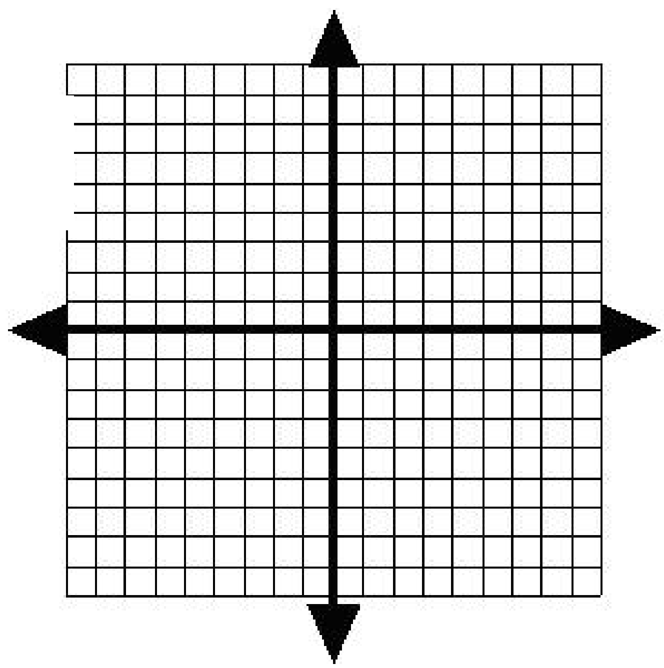
G.CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, e.g. using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry one figure onto another.

Warm-up

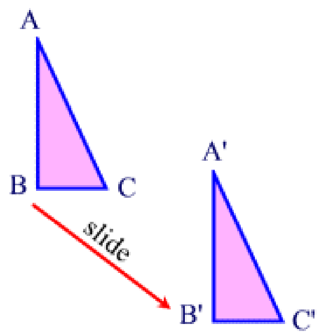
What is the image of (1,–3) when reflected in the y-axis?

(1) (–1,3)
(2) (3,–1)
(3) (–3,–1)
(4) (–1,–3)

What are the coordinates of the image of point $A(3,-1)$ after a reflection in the line $x = 2$?



Translations

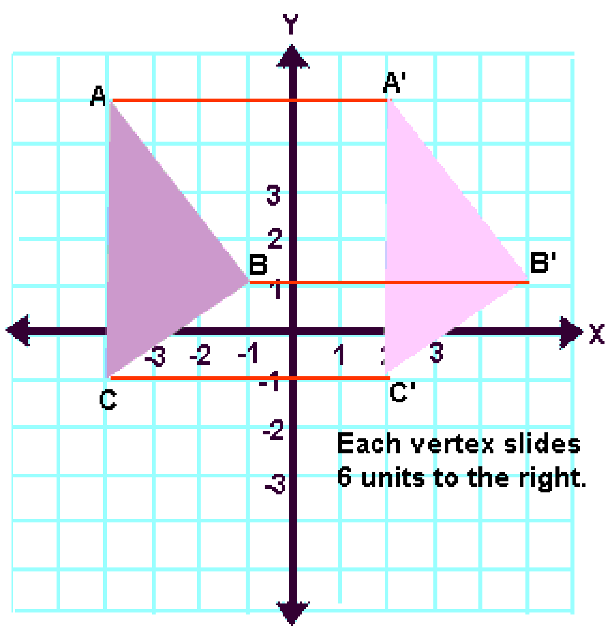


$$T_{a,b}(x, y) = (x + a, y + b)$$

Definition: A translation (notation $T_{a,b}$) is a transformation of the plane that slides every point of a figure the same distance in the same direction.

So a translation "slides" an object a fixed distance in a given direction.

In the example below, notice how each vertex moves the same distance in the same direction.



Check all properties that are preserved under a **translation:**

_________ distance _______ orientation _______ angle measure

_______ collinearity _______ midpoint _______ parallelism

_______ perpendicularity

Notations:

$T_{-7,-3}$: The -7 tells you to subtract 7 from the x-coordinates and the -3 tells you to subtract 3 from the y-coordinates

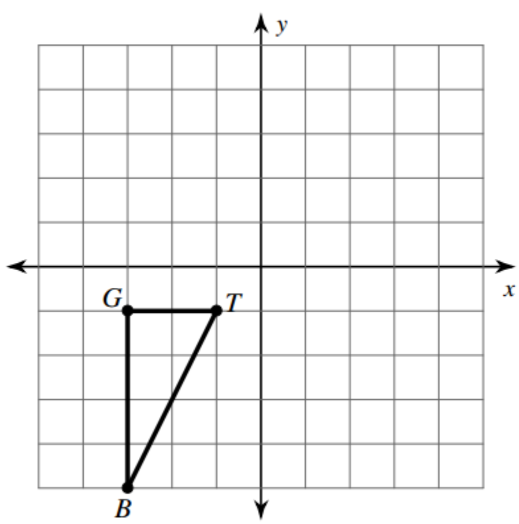
$(x, y) \rightarrow (x-7, y-3)$: This is read as "the x and y coordinates will be translated into x-7 and y-3". x-coordinates moves 7 units to the left, y-coordinates moves 3 units down

$\vec{v} = \langle 4, -9 \rangle$: This is a vector. A vector is a directed line segment and may also be used to show the movement of a translation. This vector would translate a pre-image 4 units to the right (positive direction) and 9 units down (negative direction).

Rules:

x+a : the x-coordinates moves in the positive (right) direction	**y+b** : the y-coordinates moves in the positive (up) direction
x-a : the x-coordinates moves in the negative (left) direction	**y-b** : the y-coordinates moves in the negative (down) direction

Model Problem $T_{5,1}$



B : (,) → **B'**: (+5, +1)

B' : ________________

G : (,) → **G'**: (+5, +1)

G' : ________________

T : (,) → **T'**: (+5, +1)

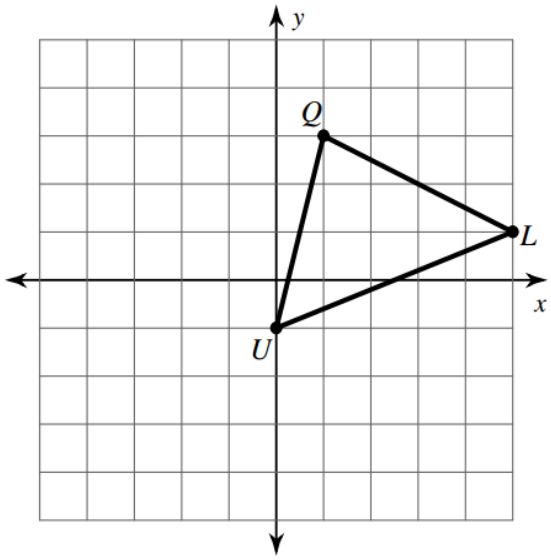
T' : ________________

Which way did the pre-image move?

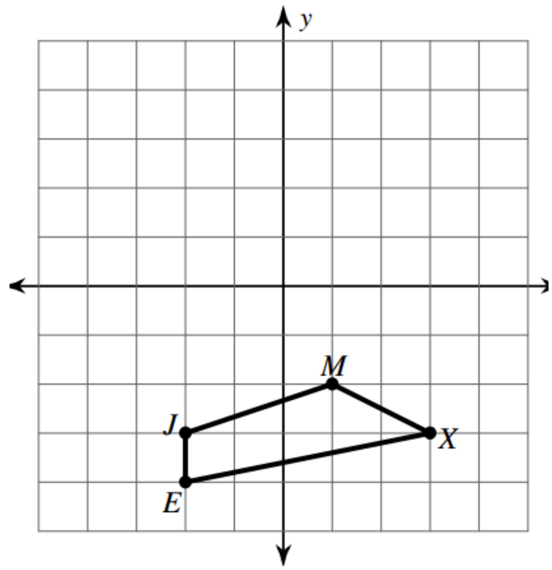
______________ and ______________

Exercise

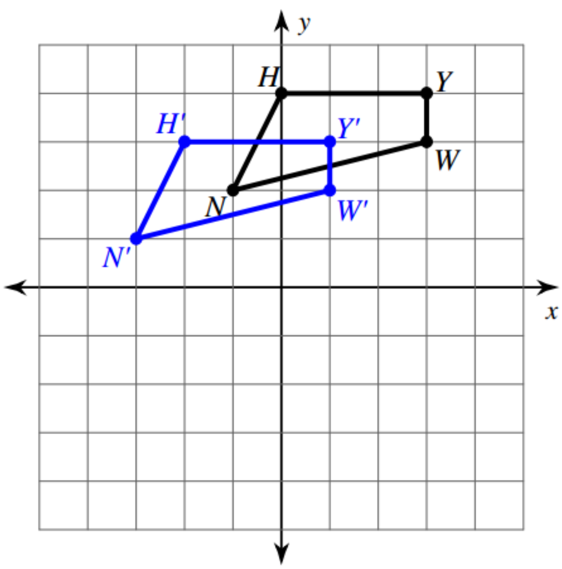
(x,y) $\longrightarrow$ (x-1, y-4)



$\vec{v} = \langle -2,5 \rangle$

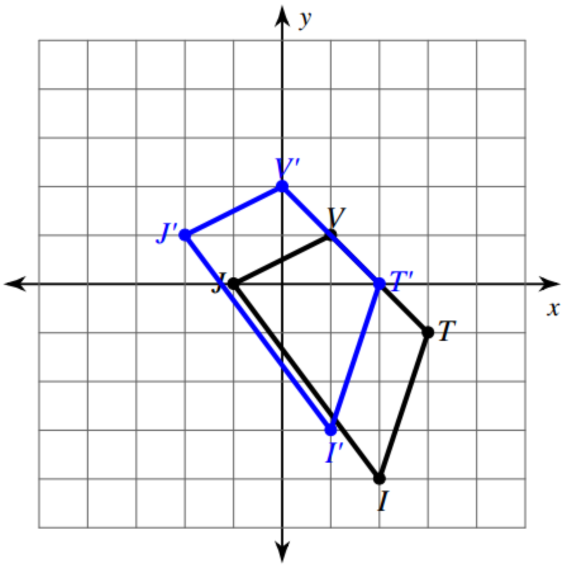


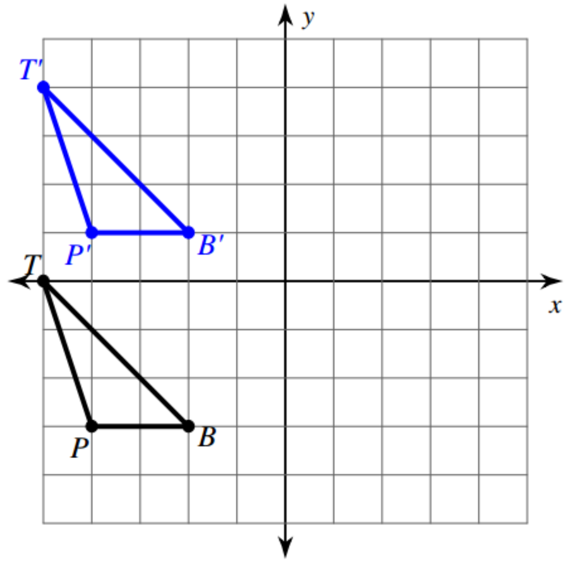
Model Problem Write the rule for the translation shown below



Rule: ________________________________

Exercise: Write the rule for the translation in two ways ($T_{a,b}$, (x,y), or $\vec{v}$)





Rule: ______________________________

____

Rule: ______________________________

____

Given a point under translation T, find the coordinates of another point under the same translation.

Model Problems

A translation moves $P(3,5)$ to $P'(6,1)$. What are the coordinates of the image of point $(-3,-5)$ under the same translation?

1) $(0,-9)$
2) $(-5,-3)$
3) $(-6,-1)$
4) $(-6,-9)$

The image of point $(-2,3)$ under translation T is $(3,-1)$. What is the image of point $(4,2)$ under the same translation?

1) $(-1,6)$
2) $(0,7)$
3) $(5,4)$
4) $(9,-2)$

Practice

Triangle ABC has vertices $A(1,3)$, $B(0,1)$, and $C(4,0)$. Under a translation, A', the image point of A, is located at $(4,4)$. Under this same translation, point C' is located at

1) $(7,1)$
2) $(5,3)$
3) $(3,2)$
4) $(1,-1)$

A design was constructed by using two rectangles $ABDC$ and $A'B'C'D'$. Rectangle $A'B'C'D'$ is the result of a translation of rectangle $ABDC$. The table of translations is shown below. Find the coordinates of points B and D'.

Rectangle *ABDC*	Rectangle *A'B'D'C'*
A (2,4)	A' (3,1)
B	B' (-5,1)
C (2,-1)	C' (3,-4)
D (-6,-1)	D'

Write two translations as a single translation.

Rule:

S: (x,y) → (x+a, y+b)

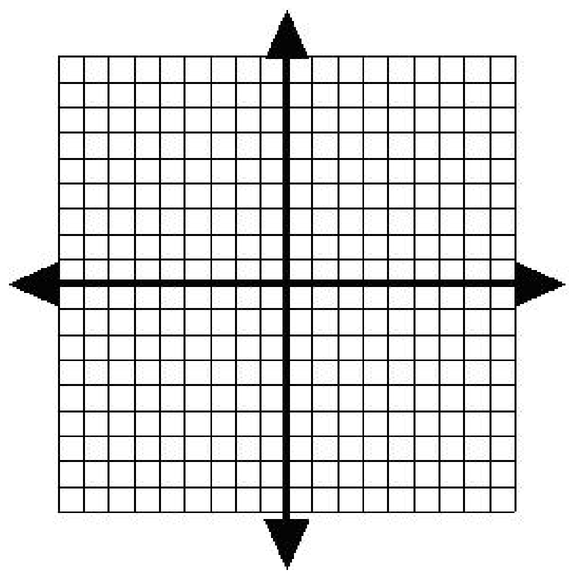
= (x,y) → (x+(a+c), y+(b+d))

T: (x,y) → (x+c, y+d)

Note: watch signs!

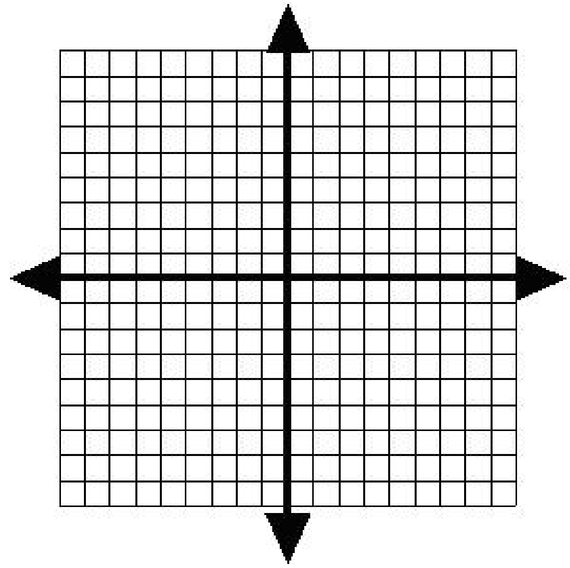
<u>Model Problem</u>

A TALE OF TWO TRANSLATIONS
Lacy performs the translation $(x, y) \rightarrow (x + 5, y + 3)$ to an object in the coordinate plane. Kyle performs the translation $(x, y) \rightarrow (x - 4, y + 2)$ to the same object after Lacy. What single translation could have been done to achieve the same effect as Lacy and Kyle's combined translations? Would the result have been different if Kyle did his translation first?



Exercise:

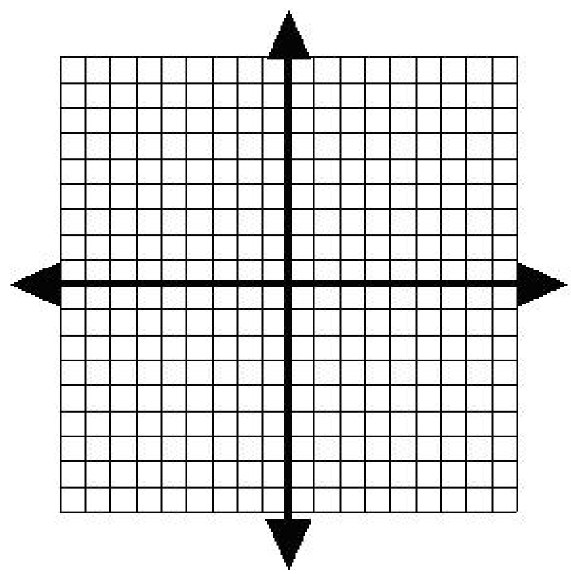
1) A group of hikers walks 2 miles east and then 1 mile north. After taking a break, they then hike 4 miles east and another mile north to set up camp. What vector describes their hike from their starting position to camp? Let 1 unit represent 1 mile.



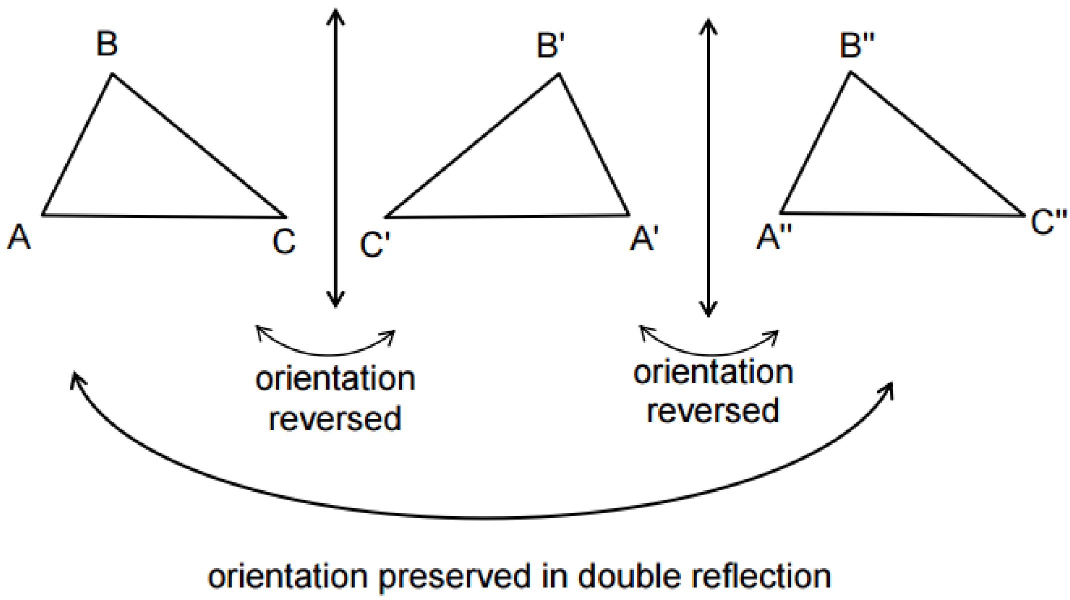
Use the graph to the right. If the hikers started at **P** (-4, 2), what will be there final position?

2) In a video game, a character at **P** (8, 3) moves three times, as described by the translations shown at the right. What is the final position of the character after three moves?

Move 1:	$T_{2,7}$
Move 2:	$T_{-10,-4}$
Move 3:	$T_{1,-5}$



A translation can be defined in terms of reflections:



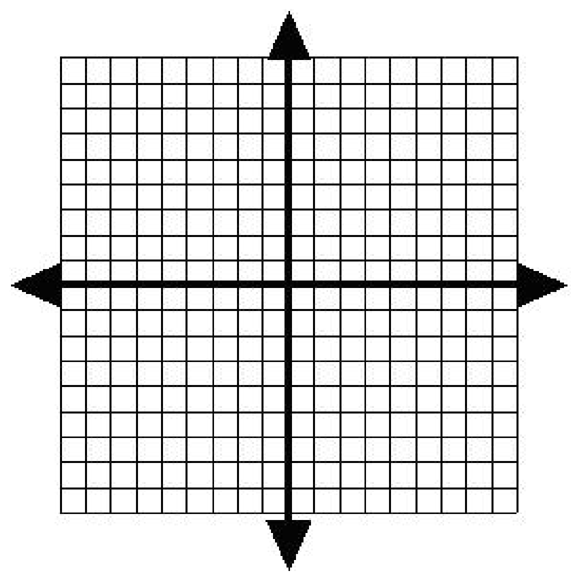
<u>Note</u>: The lines must be parallel.

<u>Model Problem</u>

Graph ΔABC with A (2,-3), B (2,-6), C (5,-3)

Use ΔABC to answer the question below.

REFLECTIONS Gus reflects an object twice. The first step is to reflect it over the line $y = -1$. Then Gus completes the composite reflection by reflecting it over the line $y = 1$. The net effect is a translation of the object. Describe this translation.



Summary

Three notations for translations: $T_{a,b}$, (x, y) → (x+a, y+b), $\langle a, b\rangle$

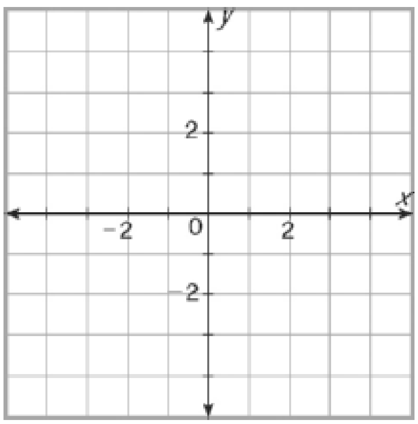
x+a : the x-coordinates moves in the positive (right) direction	**y+b** : the y-coordinates moves in the positive (up) direction
x-a : the x-coordinates moves in the negative (left) direction	**y-b** : the y-coordinates moves in the negative (down) direction

Homework

Write a rule for each transformation described.

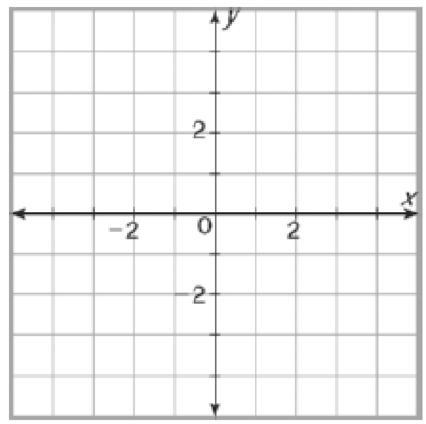
1) Two units left and 5 units up ________________________
2) Eight units right and 4 units down ___________________
3) 7 units down _________________________
4) 2 units right _____________________

5. $E(-2, -4)$, $F(3, 0)$, $G(3, -4)$



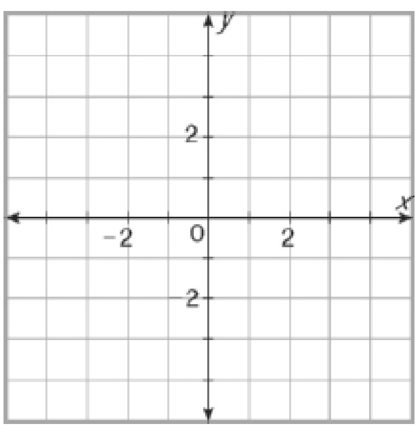
T (0, 3)

6. $P(-4, -1)$, $Q(-1, 3)$, $R(0, -4)$



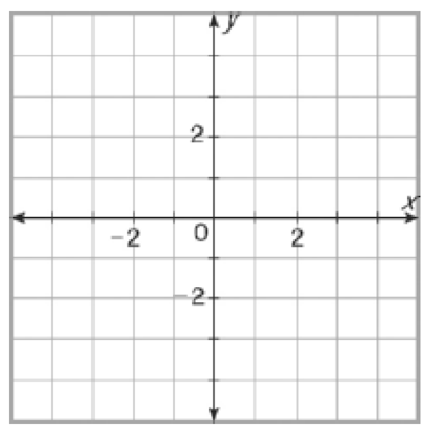
T (4, 1)

7. $A(1, -2)$, $B(1, 0)$, $C(3, 1)$, $D(4, -3)$



(x, y) --> (x - 5, y + 3)

8. $G(-3, 4)$, $H(4, 3)$, $J(1, 2)$



(x, y) --> (x - 1, y - 6)

1) If translation T maps point A(-3,1) onto point A' (5,5), what is the translation T?

A) $T(2,4)$ B) $T(8,6)$ C) $T(8,4)$ D) $T(2,6)$

2) If a translation maps point A(-3,1) to point A' (5,5), the translation can be represented by

A) $(x+8, y+4)$ B) $(x+8, y+6)$ C) $(x+2, y+4)$ D) $(x+2, y+6)$

3) What is the image of the point (3,-4) under the translation $T(-2,0)$?

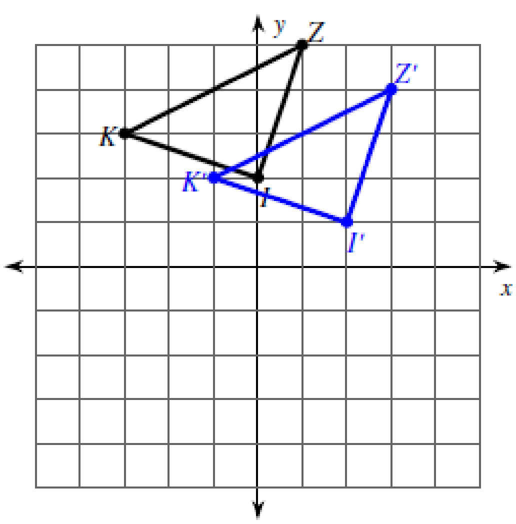
4) Under a translation, the image of point (3,2) is (-1,3). What are the coordinates of the image of point (-2,6) under the same translation?

5) A translation maps A(-3,4) onto A' (2,-6). Find the coordinates of B', the image of B(-4,0) under the same translation.

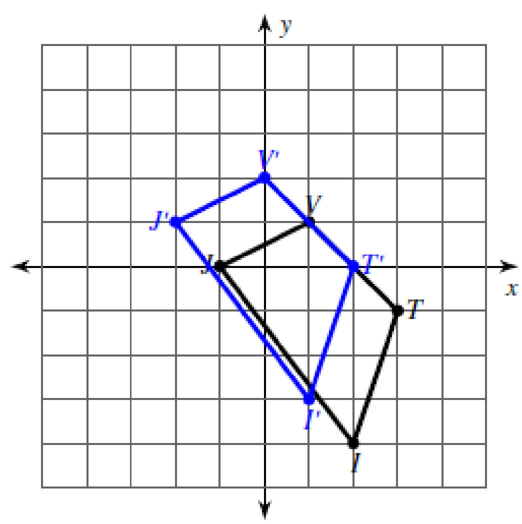
6) If a translation maps $(x, y) \longrightarrow (x+2, y+3)$, what are the coordinates of B', the image of point B(-3,5) after this translation?

Write the rule for the translations pictured in the graphs below:

7)



8)



Day 4: Rotations

G.CO.2 Represent transformations in the plane, e.g., using transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g. translation vs. horizontal stretch.)

G.CO.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.4. Develop definitions of reflections, translations, and rotations in terms of angles, circles, perpendicular lines, parallel lines, and line segments

G.CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, e.g. using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry one figure onto another.

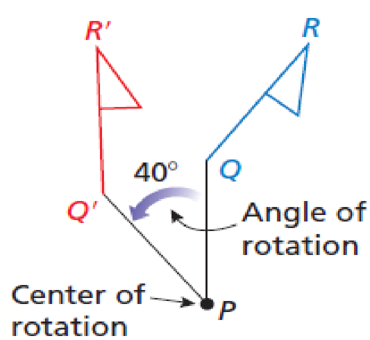
Warm-Up

Under a translation, the image of point (-3, 7) is (2, 5). Find the image of point (-4, 2) under the same translation.

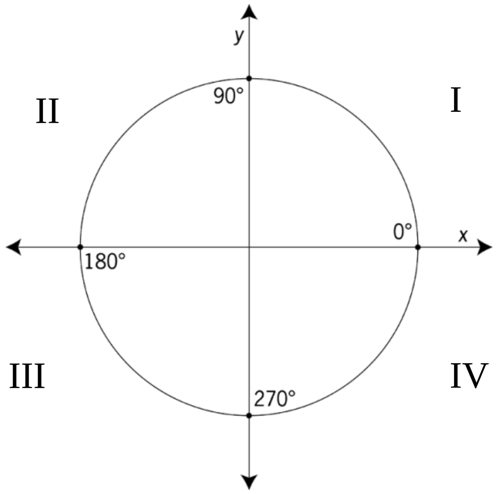
What is a rotation?

A **rotation** is a transformation in which a figure is turned about a fixed point, called the *center of rotation*. Notice that each point and its image are the same distance from the center of rotation.

The angle that a point and its image make with the center of rotation is called the angle of rotation, as shown below.



Because a full rotation is 360°, angles of rotation are commonly measured in multiples of 90°.



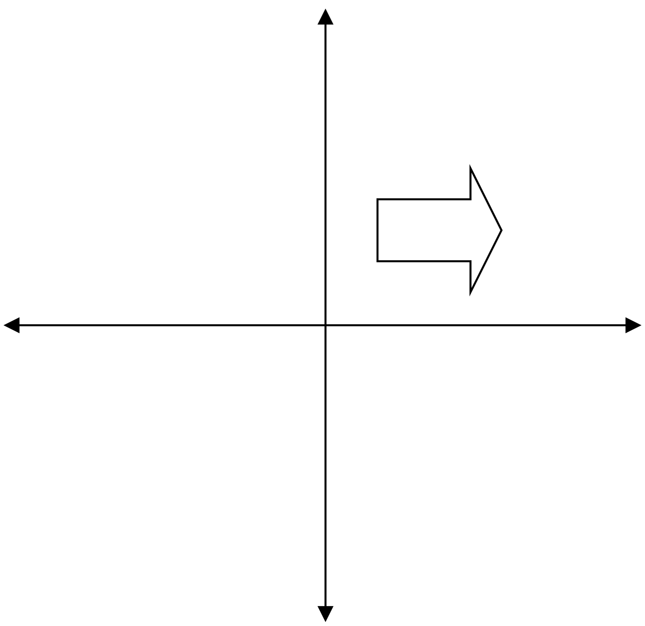
Notice that the angles are increasing in a *counterclockwise* direction. This is because **positive rotations are counterclockwise**, while negative rotations are clockwise.

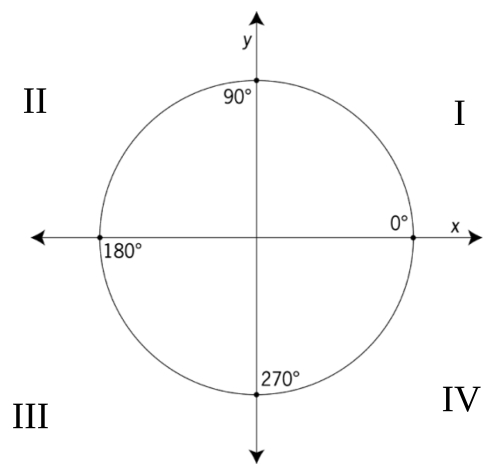
Visualizing Rotations

A figure is graphed below on a set of coordinate axes. Sketch how the figure would look after:

a) one rotation of 90°
b) two rotations of 90°
c) three rotations of 90°

Name the quadrant in which each image is located.





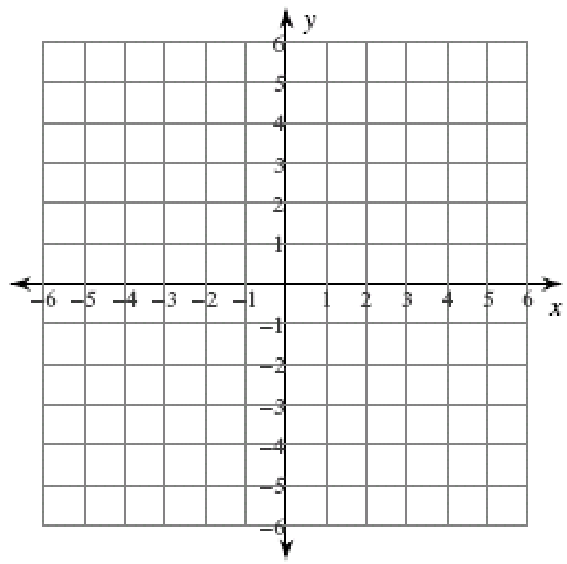
Coordinate Rule

The rule for a counter-clockwise rotation of 90° around the origin is given by:

$$(x, y) \rightarrow (-y, x)$$

Model Problem

ΔABC has coordinates A(3,1), B(6, 1) and C(6, 3). Sketch and label ΔABC and its image after a rotation of **90**°. State the coordinates of the image of ΔABC.



In which quadrant did the preimage lie? ________________ The image? __________________

Is a rotation a **rigid motion**? ____________________

Check all properties that are preserved under a **rotation:**

_______ distance

_______ orientation

_______ angle measure

_______ collinearity

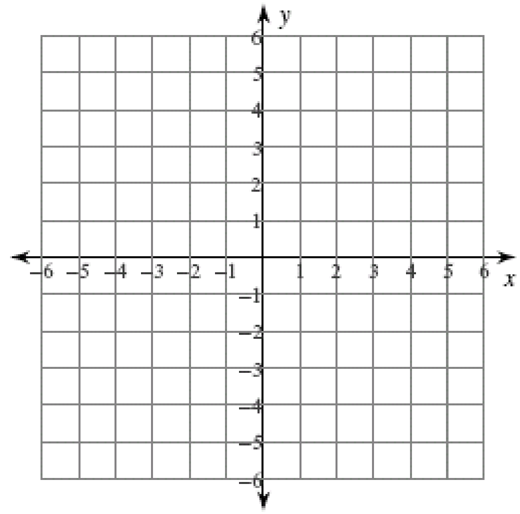
_______ midpoint

_______ parallelism

_______ perpendicularity

Exercise

Graph trapezoid FGHI with vertices F(1,2), G(5,0), H(3,3), and I(4,2).
Rotate trapezoid FGHI 90°.



Rotations of 180°, 270°, and 360° can be performed by doing the rotation of 90° multiple times.

	R$_{90}$°	**R$_{180}$°**	**R$_{270}$°**
$\boldsymbol{A(2,1)} \xrightarrow{R_{90°}} A'($	$) \xrightarrow{R_{90°}} A''($	$) \xrightarrow{R_{90°}} A'''($	$)$
$\boldsymbol{B(6,1)} \xrightarrow{R_{90°}} B'($	$) \xrightarrow{R_{90°}} B''($	$) \xrightarrow{R_{90°}} B'''($	$)$
$\boldsymbol{C(6,4)} \xrightarrow{R_{90°}} C'($	$) \xrightarrow{R_{90°}} C''($	$) \xrightarrow{R_{90°}} C'''($	$)$

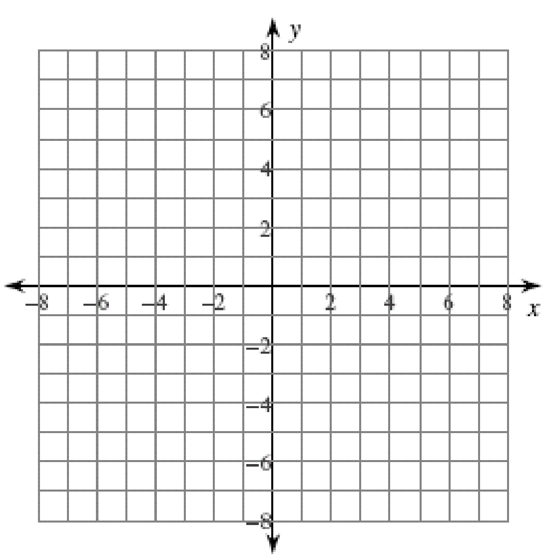
R$_{90}$° (x, y) → (   )

R$_{180}$° (x, y) → (   )

R$_{270}$° (x, y) → (   )

Exercise

ΔABC has coordinates A(2,1), B(6, 1) and C(6, 4). Sketch and state the coordinates of ΔABC and its image after rotations of 90°, 180°, and 270°. Then write a rule for these rotations.



Reminder!		
R$_{90}$° (x, y) → (-y, x)	**R**$_{180}$° (x, y) → (-x, -y)	**R**$_{270}$° (x, y) → (y, -x)

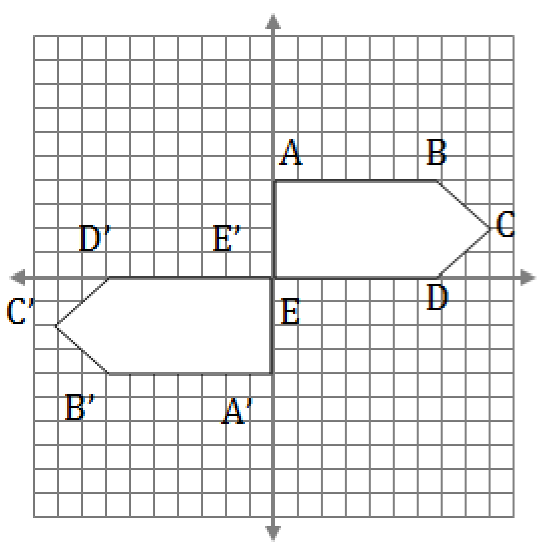
Practice

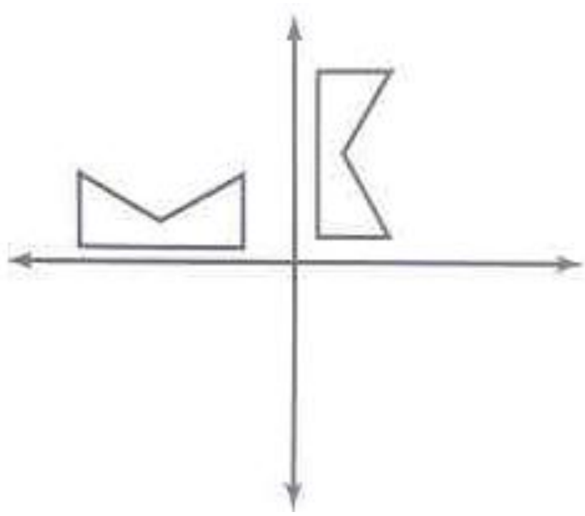
1) What is the image of each point under a counterclockwise rotation of 90°?
   a) (7, 1) b) (4, -3) c) (-5, 0)

2) What is the image of each point under a counterclockwise rotation of 180°?
   a) (-9, 6) b) (4, -12) c) (0, 8)

3) What is the image of each point under a counterclockwise rotation of 270°?
   a) (0, 5) b) (-10, 7) c) (5, -1)

<u>Write the Rule for Rotations</u>

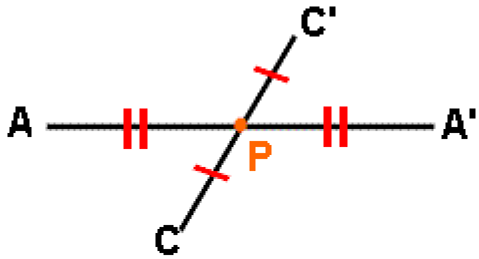


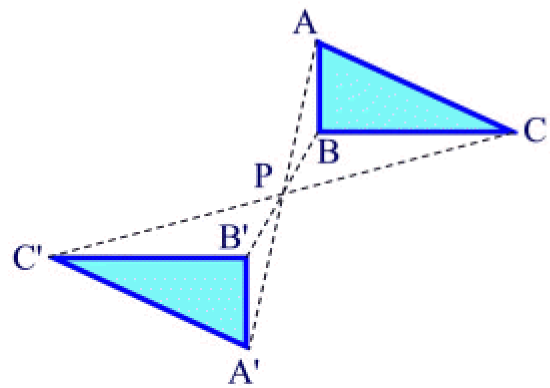


Rule:____________________________ Rule:____________________________

Point Reflection and Reflection in the Origin

A **point reflection** exists when a fixed point, called a point of reflection, is the midpoint of a given point and its image. In the diagram at right, P is the midpoint of $\overline{CC'}, \overline{BB'},\ and\ \overline{AA'}$.





Notice that reflection in a point has the same effect as turning a figure upside down.

In other words, this transformation has the same result as a rotation of 180°.

Therefore, it makes sense that the rule for reflection in the origin (r_{origin}) should have the same rule as a rotation of 180°, in other words:

$\mathbf{r_{origin}\ (x, y) \rightarrow (-x, -y)}$
The rule for a reflection in the origin is the same as the rule for a rotation of 180°.

Exercise

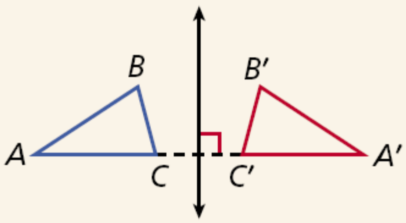
Find the image of each point under $\mathbf{r_{origin}}$:

a) (-4, 3) b) (2, 6) c) (0, -8)

Summary

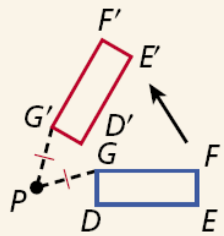
Transformations

REFLECTION



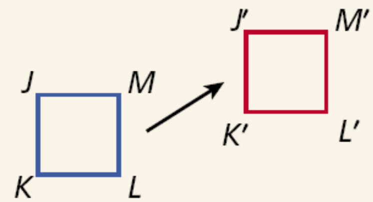
A **reflection** (or *flip*) is a transformation across a line, called the line of reflection. Each point and its image are the same distance from the line of reflection.

ROTATION



A **rotation** (or *turn*) is a transformation about a point *P*, called the center of rotation. Each point and its image are the same distance from *P*.

TRANSLATION



A **translation** (or *slide*) is a transformation in which all the points of a figure move the same distance in the same direction.

Rules to Memorize:

Line Reflections:

$$r_{x-axis}(x, y) = (x, -y)$$
$$r_{y-axis}(x, y) = (-x, y)$$
$$r_{y=x}(x, y) = (y, x)$$

Point Reflection:

$$R_{180^\circ}(x, y) = (-x, -y)$$

Rotations:

$$R_{90^\circ}(x, y) = (-y, x)$$
$$R_{180^\circ}(x, y) = (-x, -y)$$
$$R_{270^\circ}(x, y) = (y, -x)$$
$$R_{-90^\circ}(x, y) = (y, -x)$$

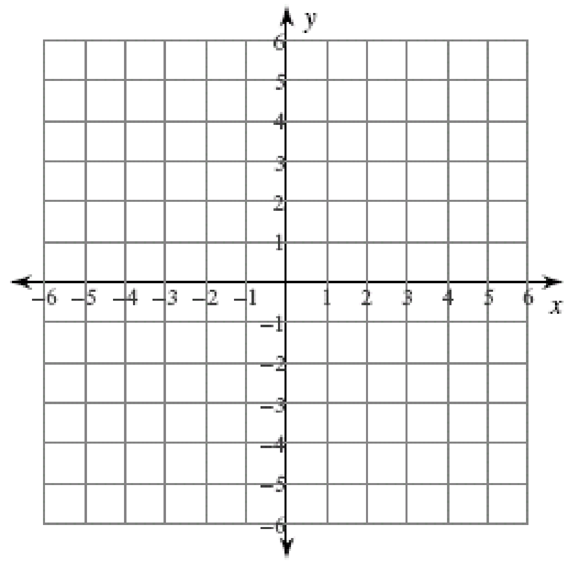
Translation:

$$T_{a,b}(x, y) = (x + a, y + b)$$

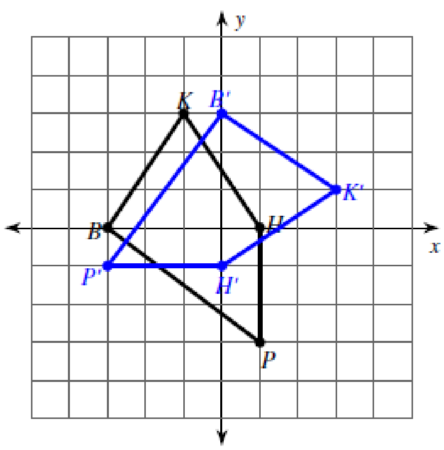
Homework

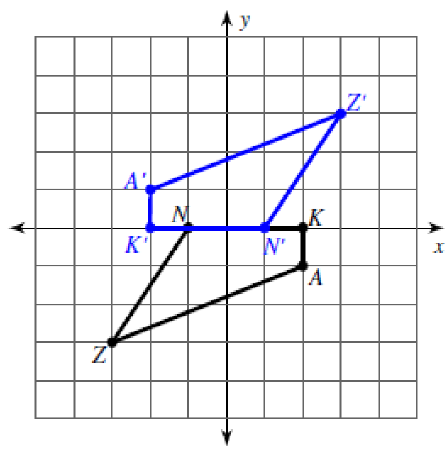
1) When a point (a, b) is rotated counterclockwise about the origin, $(a, b) \to (b, -a)$ is the result of a rotation of _______.

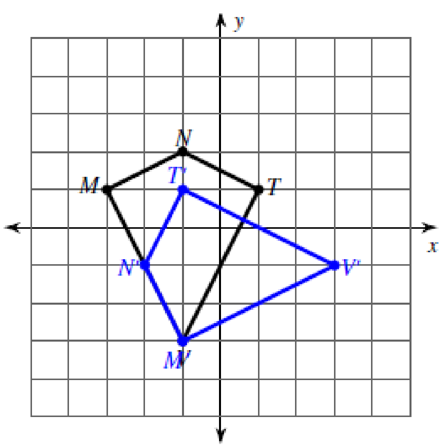
2) Graph trapezoid FGHI with vertices F(1,2), G(5,0), H(3,3), and I(4,2). Rotate trapezoid FGHI 180°.

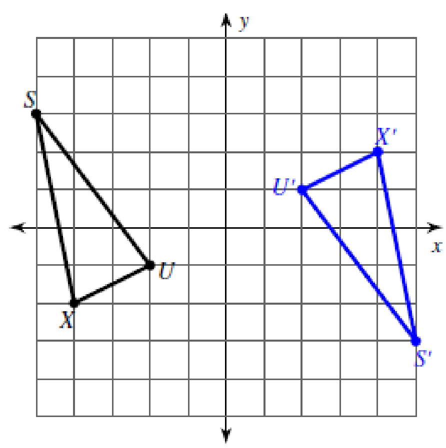


3) ***Write a rule for the rotations below.***









4) ***Write the coordinates of the given point after a reflection in the origin:***

a) (-9, -8) b) (4, 14) c) (4, 6)

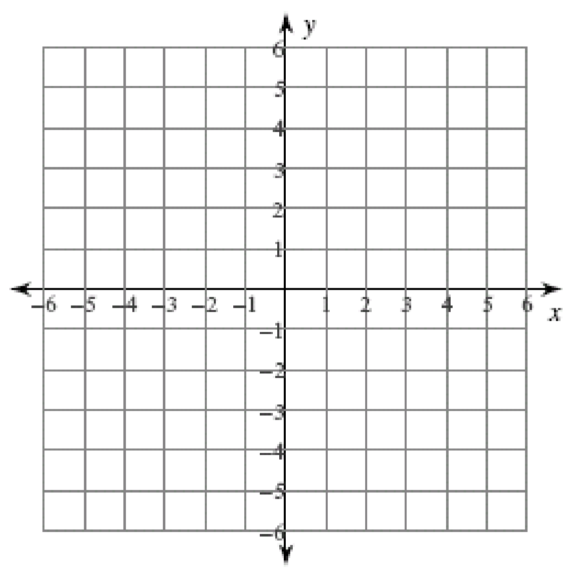
5) **REASONING** You enter the revolving door at a hotel.

a. You rotate the door 180°. What does this mean in the context of the situation? Explain.

b. You rotate the door 360°. What does this mean in the context of the situation? Explain.

6)

REASONING Use the coordinate rules for counterclockwise rotations about the origin to write coordinate rules for clockwise rotations of 90°, 180°, or 270° about the origin.



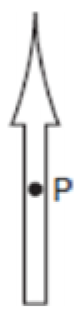
7) What is the image of $A(5,2)$ under R_{90°?

1) $(-5,2)$
2) $(5,-2)$
3) $(2,5)$
4) $(-2,5)$

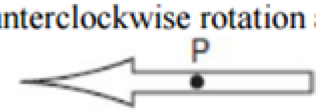
8) If the letter **P** is rotated 180 degrees, which is the resulting figure?

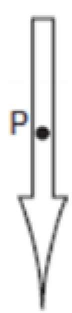
1) 
2) 
3) 
4) 

9) The accompanying diagram shows the starting position of the spinner on a board game.

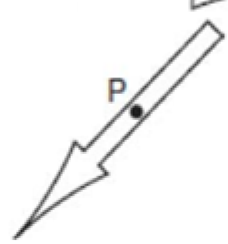


How does this spinner appear after a 270° counterclockwise rotation about point P?

1) 

2) 

3) 

4) 

Day 5: Dilations

G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Warm-Up

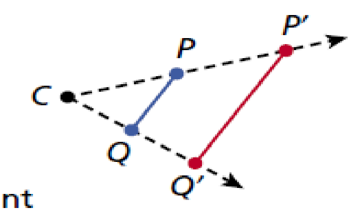
Find the image of (-5, 3) after each transformation.

	$r_{x\text{-axis}}$	$r_{y\text{-axis}}$	$r_{y=x}$	$T_{-3,2}$	R_{90}	R_{180}	R_{270}
(-5, 3)							

Dilations

A dilation, or *similarity transformation*, is a transformation in which the lines connecting every point P with its image P' all intersect at a point C, called the **center of dilation**. $\frac{CP'}{CP}$ is the same for every point P.

The scale factor k of a dilation is the ratio of a linear measurement of the image to a corresponding measurement of the preimage. In the figure, $k = \frac{P'Q'}{PQ}$.



A dilation enlarges or reduces all dimensions proportionally. A dilation with a scale factor greater than 1 is an **enlargement**, or *expansion*. A dilation with a scale factor greater than 0 but less than 1 is a **reduction**, or *contraction*.

When the scale factor $k < 0$, the dilation will also contain a rotation of 180°.

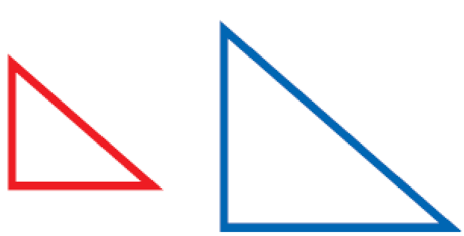
A dilation of scale factor k is performed using the rule:

$$D_k(x, y) \rightarrow (kx, ky)$$

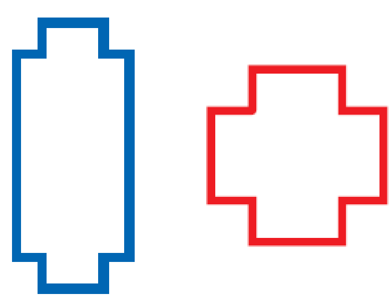
Important Facts

- A dilation is NOT a rigid motion. Its image is not congruent to the original figure.
- Since the ratio of the lengths of segments in the figure and its image is a constant, the figure and its image are always *similar*.

This is a dilation. The figures are similar:



This is NOT a dilation. The figures are NOT similar:



Exploration

$\overline{AB}$ has coordinates $A(3,1)$ and $B(-1,4)$.

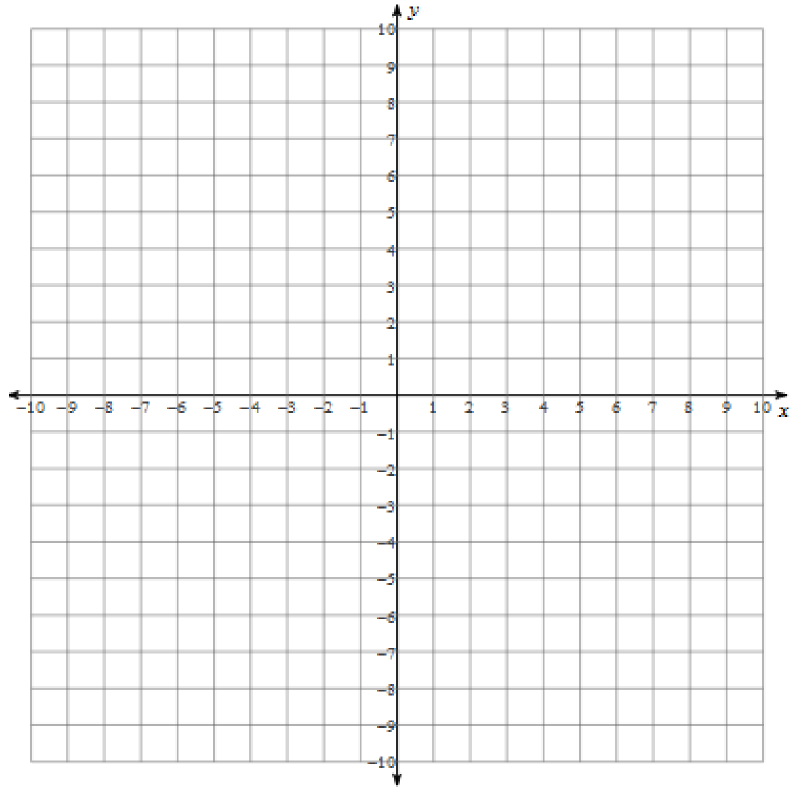
1) Graph and label $\overline{AB}$.

2) Determine and state the coordinates of the image of $\overline{AB}$ under D_2 with respect to the origin.

$A(3,1) \rightarrow A'(\qquad)$

$B(\qquad) \rightarrow B'(\qquad)$

3) Justify informally that the transformation you performed is a dilation.



Verifying the Properties of a Dilation

*Use the **distance formula** to find:*

AB	$A'B'$
What do you notice?	

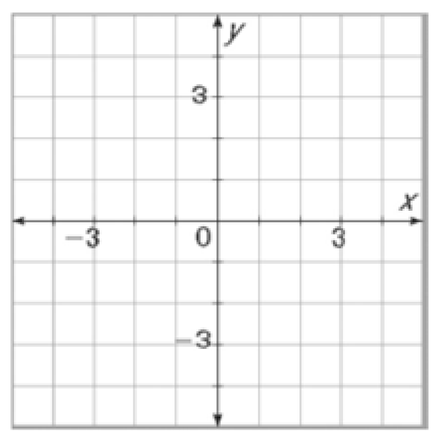
Parallel lines have the same slope. Use the ***slope formula*** *to find the slope of* $\overline{AB}$ *and* $\overline{A'B'}$.

$slope\ of\ \overline{AB}$	$slope\ of\ \overline{A'B'}$
What conclusion can you draw about these line segments?	

Model Problems

Graph and state the coordinates of the image of each dilation, using the given scale factor. Each dilation is with respect to the origin.

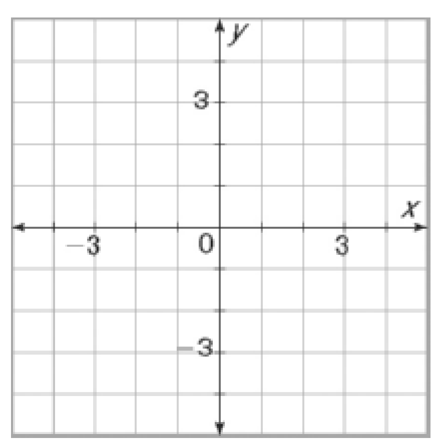
1) $J(0, 2)$, $K(-2, 1)$, $L(0, -2)$, $M(2, -1)$; scale factor: 2



Exercise — Graph and state the coordinates of the image of each dilation, using the given scale factor. Each dilation is with respect to the origin.

2) $D(0, 0)$, $E(-1, 0)$, $F(-1, -1)$;

Scale factor: -3

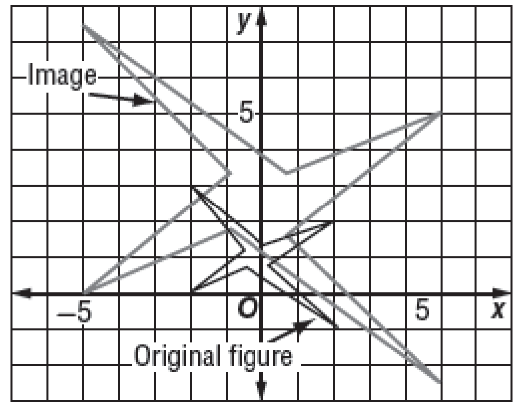


Model Problems

Find the scale factor in each dilation given.

a) (-6, 4) → (-9, 6)

b)



Exercise

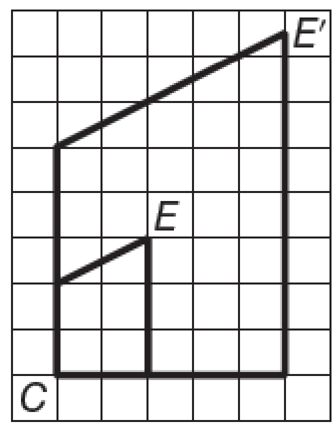
Find the scale factor in each dilation.

1) (4,8) → (-2, -4)

2) (4, 5) → (12, 15)

3) (-5, 4) → (10, -8)

4) This dilation where C is the center of dilation:



Homework

1) What is the image of the point (9, 1) under the dilation D_2?

2) Find the image of the point (-3, 2) under the dilation (3x, 3y).

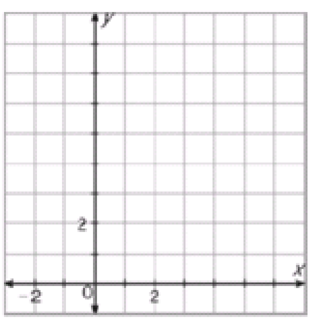
3) A dilation maps point A (2, 6) onto A' (1, 3). What is the scale factor?

4) A triangle has sides of length 12, 14, and 16. Find the perimeter of this triangle after a dilation D_2.

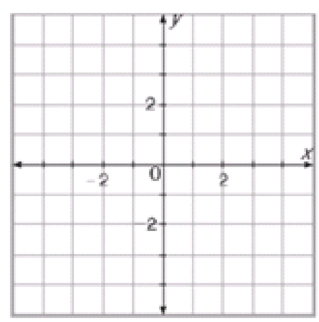
5) Point B' (14, 21) is the image of point B(2, 3) under a certain dilation. Find the image of point (-1, 0) under the same dilation.

Draw the image of the figure with the given vertices under a dilation with the given scale factor and centered at the origin.

6. $J(0, 0)$, $K(-1, 2)$, $L(3, 4)$; scale factor: 2



7. $G(2, 0)$, $H(0, 4)$, $I(4, 2)$; scale factor: $-\frac{1}{2}$



Review Write the image of each point under the transformation indicated.

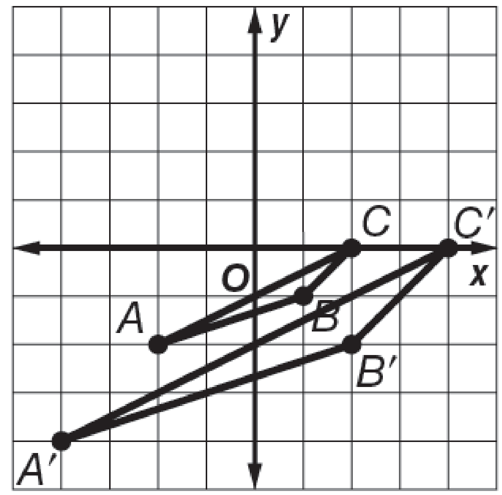
	$r_{x\text{-axis}}$	$r_{y\text{-axis}}$	$r_{y=x}$	$r_{x=3}$	R_{90°	R_{180°	$T_{3,0}$	D_2
(3, 4)								
(-2, 6)								
(4, 0)								

Day 6: More on Dilations

G-SRT.1 Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

Warm-Up

Is this transformation a dilation? Justify your answer mathematically.

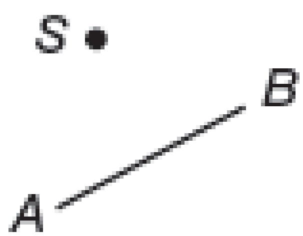


Sketching a Dilation   Center is Outside the Figure

The center of dilation and the scale factor of a dilation is given. Using a compass and a ruler, construct the image of the figure after the dilation.

Label it $\overline{A'B'}$.

$r = 2$



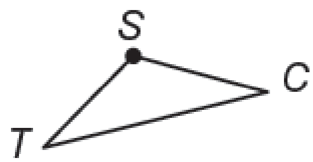
Questions for Thought:

What is the ratio of $A'B'$ to AB? __________ of SB' to SB? _________ of SA' to SA? ______________

Is parallelism preserved under this transformation? _________________________________

Center is On the Figure

$r = 3$



Questions for Thought

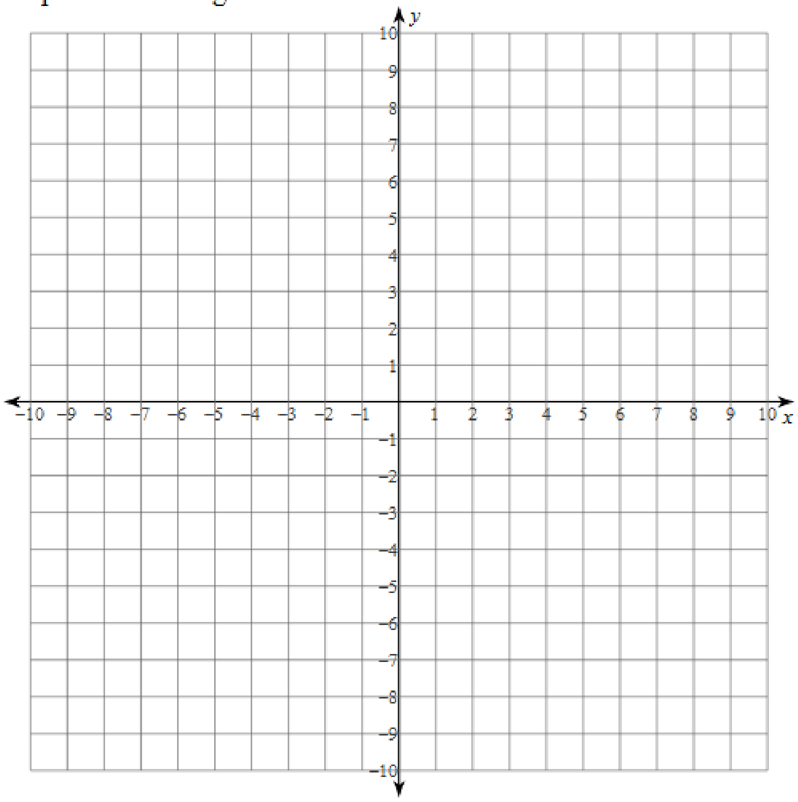
1) How is this dilation different from the previous one?

2) Do any points remain invariant (the same) under this dilation? Why do you think that is?

3) Is $\overline{SC} \parallel \overline{SC'}$? If parallelism is preserved under a dilation, how do you explain this result?

Dilating a Line

1) Graph the line $y = -\frac{2}{3}x + 2$ on the axes below. Then graph its image after a dilation of 2 with respect to the origin.



2) What is the equation of the new line? ____

3) Plot the point (3, 0) on the same set of axes. Using (3, 0) as the center of dilation, let's attempt to do the same dilation of 2.

a) What is the image of (3, 0) under this dilation? ______________ Why?

b) What is the image of (0, 2) under this dilation? ______________

What mathematical term describes the relationship between (0, 2) and its image? ______________

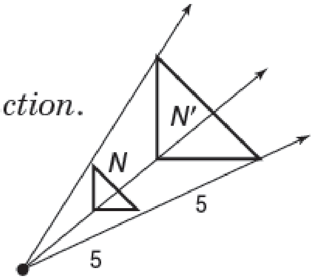
c) What is the equation of the new line this time? ____

d) Complete this sentence: When a dilation is performed on a line with respect to a point on that same line, the equation is ________________________________.

Homework *Dilations and Review*

For Questions 1 and 2, use the dilation.

1. Determine whether the dilation from Figure N to N' is an *enlargement* or a *reduction*. **1.** ____________



2. Find the scale factor of the dilation. **2.** ____________

3. $\triangle ABC$ has vertices $A(2, 2)$, $B(3, 4)$, and $C(5, 2)$. Find the coordinates of the image of the triangle after a dilation centered at the origin with a scale factor of 2.5. **3.** ____________

4. Name a property that is *not* preserved by a dilation. **4.** ____________

5) If $\triangle ABC$ is dilated by a scale factor of 3, which statement is true of the image $\triangle A'B'C'$?

1 $3A'B' = AB$

2 $B'C' = 3BC$

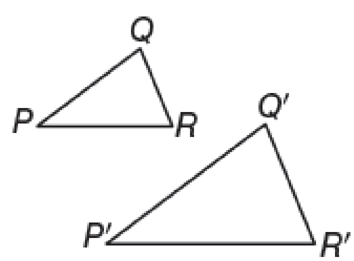
3 $m\angle A' = 3(m\angle A)$

4 $3(m\angle C') = m\angle C$

6. Name the coordinates of $S(-7, 1)$ under a reflection in the y-axis.

7. $DEFG$ is a square with vertices at $D(1, 1)$, $E(1, 6)$, $F(6, 6)$, and $G(6, 1)$. $DEFG$ is reflected in the line $x = 1$. Find the coordinates of D', E', F', and G'.

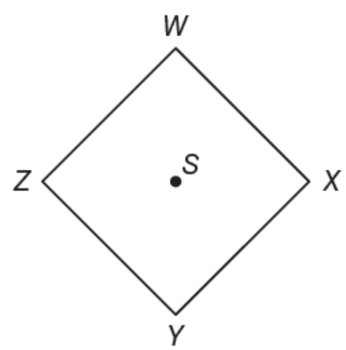
8. Determine whether $\triangle P'Q'R'$ is a translation image of $\triangle PQR$. Explain.



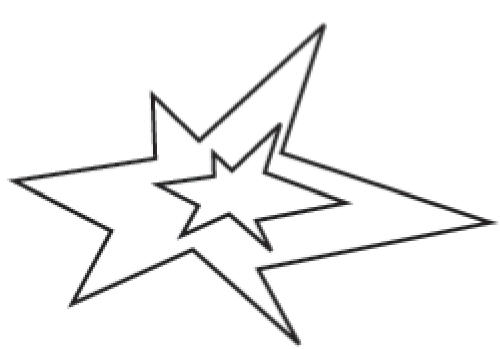
9. Find the image of $B(4, 7)$ reflected in the y-axis.

10. The function notation $(x, y) \rightarrow (-x, -y)$ describes the rotation of how many degrees about the origin?

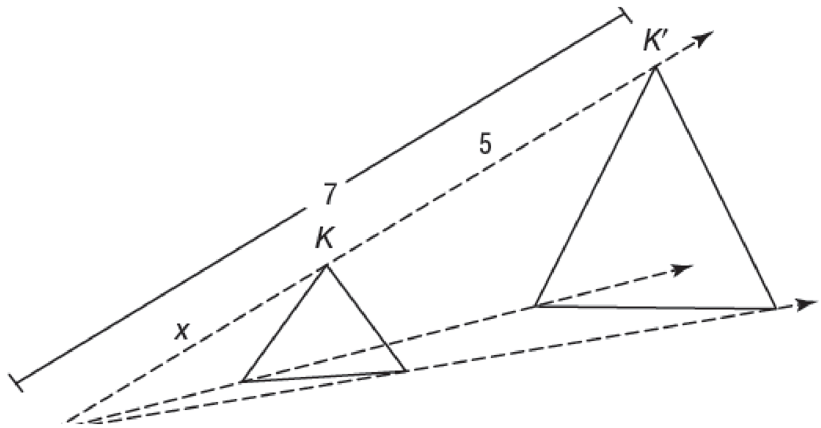
11) *Dilation with Center Inside the Figure.* Sketch the dilation of the figure below if the center of dilation is at S and the scale factor is 2.



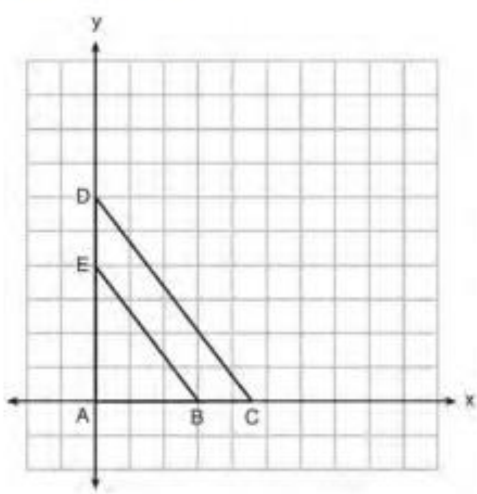
12) Margot superimposed the image of the dilation of a figure on its original figure as shown. Identify the center of this dilation. Explain how you found it.



13) Find the value of x and the scale factor of the dilation below.



14) In the diagram below, $\triangle ABE$ is the image of $\triangle ACD$ after a dilation centered at the origin. The coordinates of the vertices are $A(0,0)$, $B(3,0)$, $C(4.5,0)$, $D(0,6)$, and $E(0,4)$.



The ratio of the lengths of $\overline{BE}$ to $\overline{CD}$ is

1 $\frac{2}{3}$

2 $\frac{3}{2}$

3 $\frac{3}{4}$

4 $\frac{4}{3}$

Day 7: Composition of Transformations

G.CO.5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure, e.g. using graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry one figure onto another.

Warm-Up

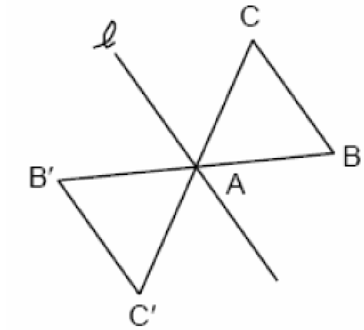
The transformation of ΔABC to $\Delta AB'C'$ is shown in the accompanying diagram.

This transformation is an example of a

[A] translation [B] dilation

[C] line reflection in line ℓ

[D] rotation about point A



What is a composition?

When two or more transformations are combined to form a new transformation, the result is called a **composition** of transformations.

In a composition:

1) The first transformation is performed.
2) The second transformation is performed on the *image* of the first.

The symbol for a composition of transformations is an open circle. ($\circ$)

Example: $r_{x\text{-}axis} \circ T_{3,4}$

Read: "reflection in the x-axis ON a translation of (3, 4)"

➢ **Important Note:** Compositions are always done in ***reverse***!!

$r_{x\text{-}axis} \circ T_{3,4}$

Do SECOND!! **Do FIRST!!!**

Model Problem A

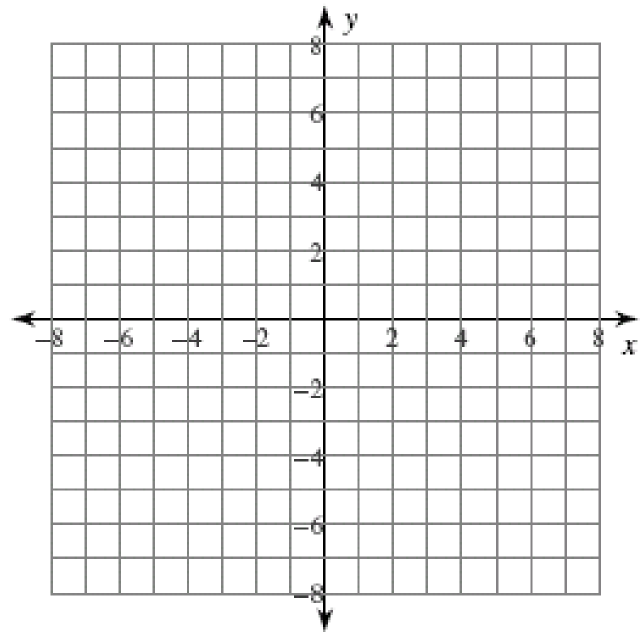
ΔABC has coordinates $A(-6,0), B(-3,5), and\ C(0,2)$. Find the coordinates of image of $\Delta A''B''C''$ under the composition $\boldsymbol{r_{x-axis} \circ T_{2,3}}$. Sketch ΔABC and $\Delta A''B''C''$ on the axes below.

a) Which transformation goes first? ____

A(-6,0) $\xrightarrow{T_{2,3}} A'(\quad,\quad) \xrightarrow{r_{x-axis}} A''(\quad,\quad)$

B(-3,5) $\xrightarrow{T_{2,3}} B'(\quad,\quad) \xrightarrow{r_{x-axis}} B''(\quad,\quad)$

C(0, 2) $\xrightarrow{T_{2,3}} C'(\quad,\quad) \xrightarrow{r_{x-axis}} C''(\quad,\quad)$



b) Which statement is ***not*** true?
(1) $\angle A \cong \angle A''$
(2) $\overline{BC} \cong \overline{B''C''}$
(3) $\overline{BC} \parallel \overline{B''C''}$
(4) $\overline{AC} \cong \overline{A''C''}$

Exercise

1) What is the image of point A(4, 3) under the composition $D_2 \circ r_{y=x}$?

(4,3) $\xrightarrow{r_{y=x}} (\quad,\quad) \xrightarrow{D_2} (\quad,\quad)$

Remember to work from <u>right to left</u>!

2) What is the image of (3,-1) under the composition $T_{2,5} \circ r_{y-axis}$?

3) Find the image of (-1, 8) under the composition:

a) $R_{90^\circ} \circ r_{y-axis}$

b) $r_{y-axis} \circ R_{90^\circ}$

c) Is the composition of transformations commutative?
(i.e. Does changing the order of the composition keep the same image?)

4) After a composition of transformations, the coordinates $A(4,2)$, $B(4,6)$, and $C(2,6)$ become $A''(-2,-1)$, $B''(-2,-3)$, and $C''(-1,-3)$, as shown on the set of axes below.

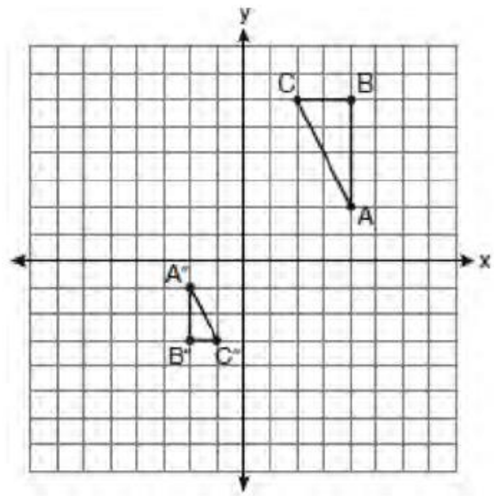
Which composition of transformations was used?

1 $R_{180°} \circ D_2$

2 $R_{90°} \circ D_2$

3 $D_{\frac{1}{2}} \circ R_{180°}$

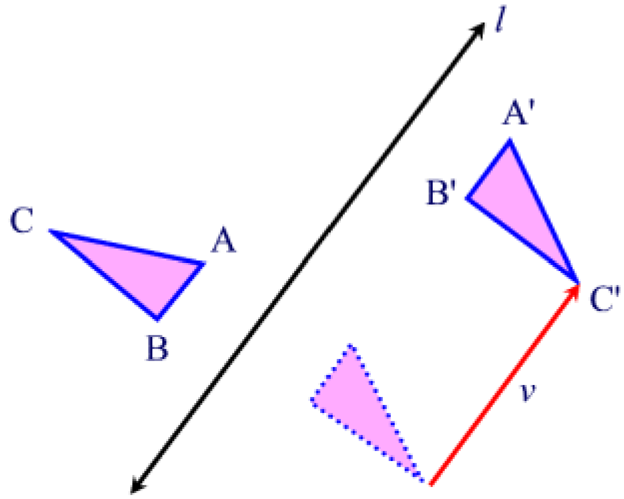
4 $D_{\frac{1}{2}} \circ R_{90°}$



5) In Question #4, is $\overline{BC} \cong \overline{B''C''}$? Explain why or why not.

The Glide Reflection

Definition: A **glide reflection** is a transformation in the plane that is the composition of a line reflection and a translation through a line (a vector) parallel to that line of reflection.



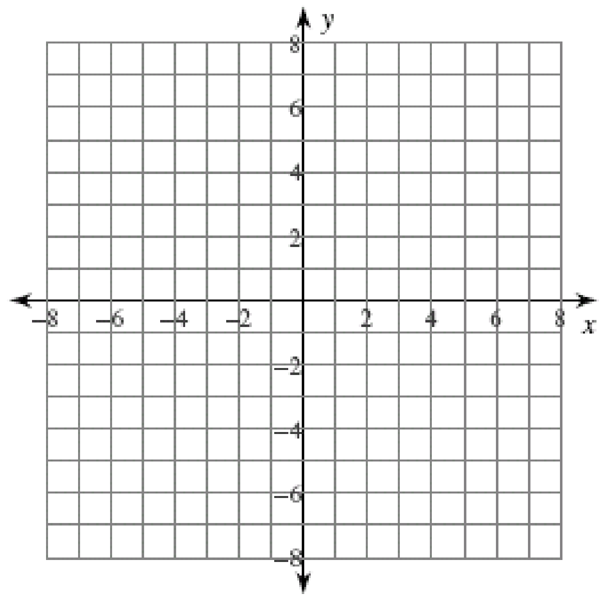
Example: $\Delta A'B'C'$ is the image of ΔABC under a glide reflection that is a composition of a reflection over the line l and a translation through the vector v.

Recognizing Glide Reflections

- Look for a line reflection and a translation.
- The translation should go in the same direction as the line of reflection runs.

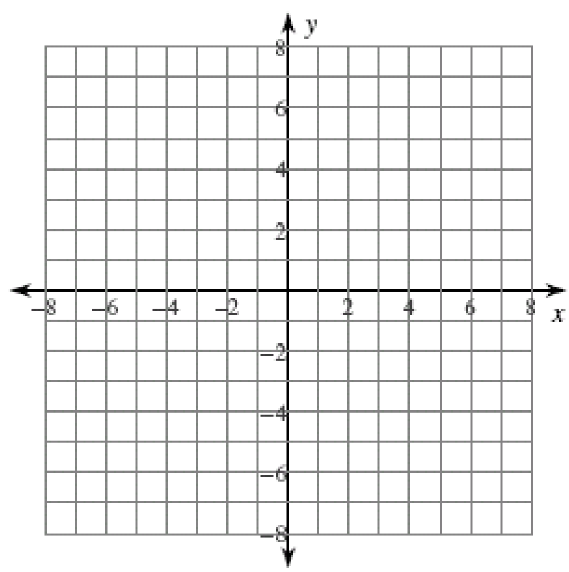
Example ΔABC has coordinates A(0, 2), B(4, 2), and C(4, 5).

Sketch ΔABC and its image under $r_{x=2} \circ T_{0,3}$



Exercise ΔABC has coordinates A(0, 2), B(4, 2), and C(4, 5).

Sketch ΔABC and its image under $T_{-2,0} \circ r_{x-axis}$



Notes on Glide Reflections:

1) A glide reflection is an improper rigid motion.

(A translation is a *proper rigid motion* and a reflection is an *improper rigid motion*. Their composition is an improper rigid motion.)

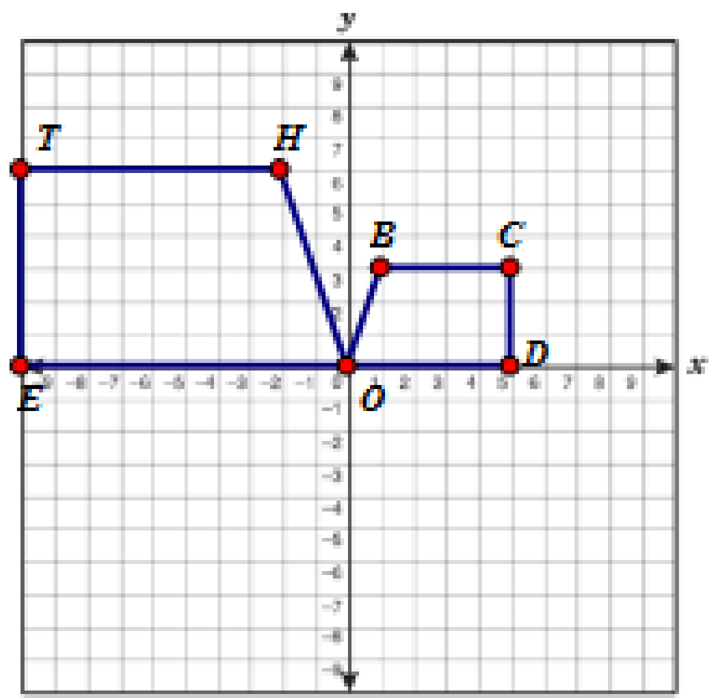
2) Properties preserved under a glide reflection:

- Distance
- Angle measure
- Parallelism and perpendicularity
- Collinearity
- Midpoint

Describing a Composition

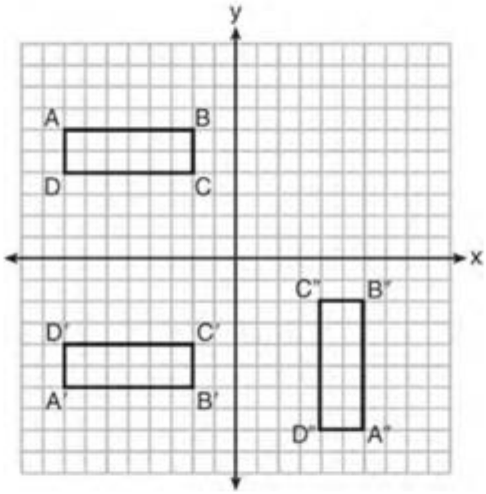
Model Problem

Describe a series of transformations that will map OBCD onto OHTE. Are the figures congruent or similar? How do you know?



Exercise

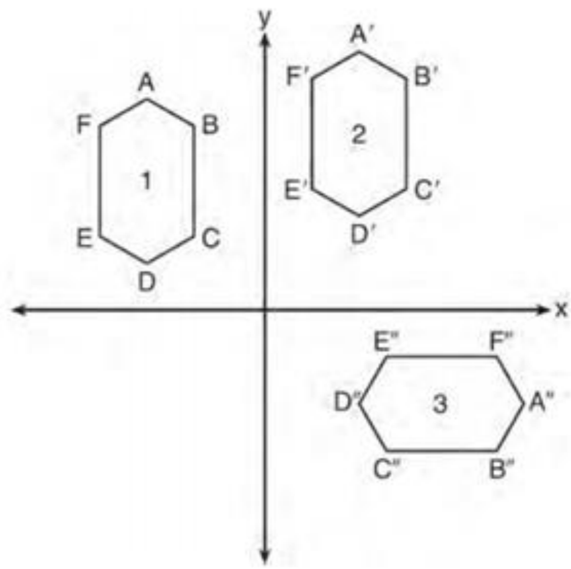
A sequence of transformations maps rectangle *ABCD* onto rectangle *A"B"C"D"*, as shown in the diagram below.



Which sequence of transformations maps *ABCD* onto *A'B'C'D'* and then maps *A'B'C'D'* onto *A"B"C"D"*?

1 a reflection followed by a rotation
2 a reflection followed by a translation
3 a translation followed by a rotation
4 a translation followed by a reflection

In the diagram below, congruent figures 1, 2, and 3 are drawn.



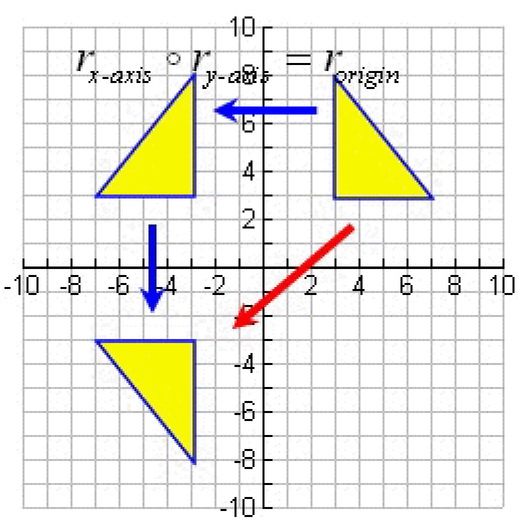
Which sequence of transformations maps figure 1 onto figure 2 and then figure 2 onto figure 3?

1 a reflection followed by a translation
2 a rotation followed by a translation
3 a translation followed by a reflection
4 a translation followed by a rotation

Describing Compositions as a Single Transformation

It is possible that a composition of two transformations may be renamed using one single transformation.

For example, the composition of a line reflection in the *y*-axis followed by a line reflection in the *x*-axis could be described as a single transformation of a reflection in the origin.



We can prove this using coordinate geometry using (x, y):

$$(x, y) \xrightarrow{r_{y-axis}} (\quad , \quad) \xrightarrow{r_{x-axis}} (\quad , \quad)$$

Exercise — Transform the point (x, y) using each composition. Then demonstrate the composition graphically using the sample coordinates. Answer all questions that follow.

1) $\boldsymbol{r_{y-axis} \circ r_{y=x}}$

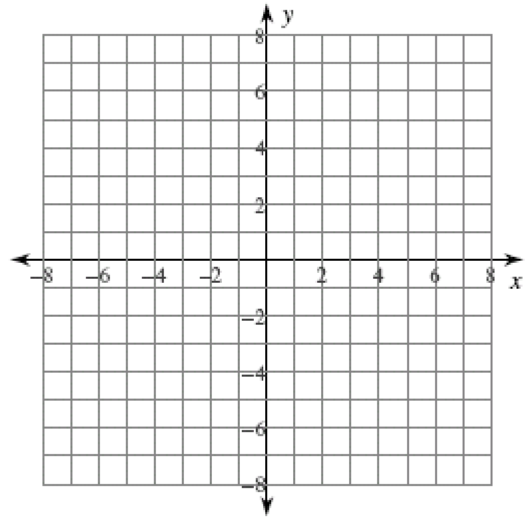
$$(x, y) \xrightarrow{r_{y=x}} (\quad , \quad) \xrightarrow{r_{y-axis}} (\quad , \quad)$$

This composition is equivalent to:

(1) A reflection in the origin
(2) A rotation of 90°
(3) A vertical translation
(4) A reflection in the x-axis

Are the figure and its image congruent or similar? How do you know?

Demonstrate graphically using $\overline{AB}$ where A(6, 2) and B(2,4)



2) $\boldsymbol{R_{y-axis} \circ R_{90^\circ}}$

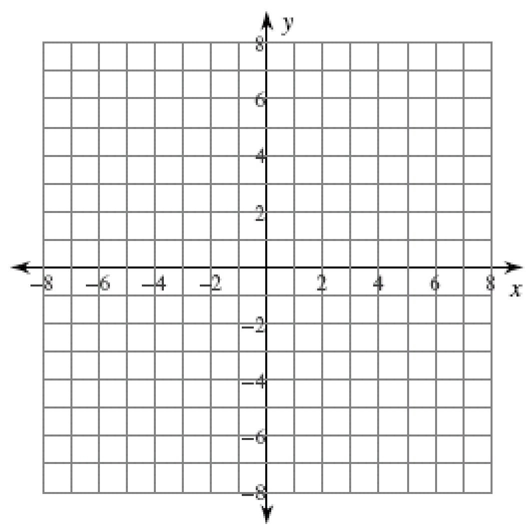
$$(x, y) \xrightarrow{R_{90^\circ}} (\quad , \quad) \xrightarrow{r_{y-axis}} (\quad , \quad)$$

This composition is equivalent to:

(1) A reflection in the line $y = x$
(2) A rotation of 270°
(3) A translation of <2, 3>
(4) A reflection in the x-axis

Does this transformation preserve distance? Explain.

Demonstrate graphically using $\overline{AB}$ where A(6, 2) and B(2,4)



3) $\boldsymbol{T_{4,-1} \circ T_{6,8}}$

$$(x, y) \xrightarrow{T_{6,8}} (\quad , \quad) \xrightarrow{T_{4,-1}} (\quad , \quad)$$

Write a single translation rule for this composition:

(x, y) → (,)

4) $\boldsymbol{R_{90^\circ} \circ R_{180^\circ}}$

$$(x, y) \xrightarrow{R_{180^\circ}} (\quad , \quad) \xrightarrow{R_{90^\circ}} (\quad , \quad)$$

This composition is equivalent to a rotation of:

Homework

1) What is the image of point A(7, 8) under the composition $T_{2,-1} \circ r_{y-axis}$?

$$(7,8) \xrightarrow{r_{y-axis}} (\quad , \quad) \xrightarrow{T_{2,-1}} (\quad , \quad)$$

2) What is the image of (0,4) under the composition $T_{-1,-3} \circ r_{x-axis}$?

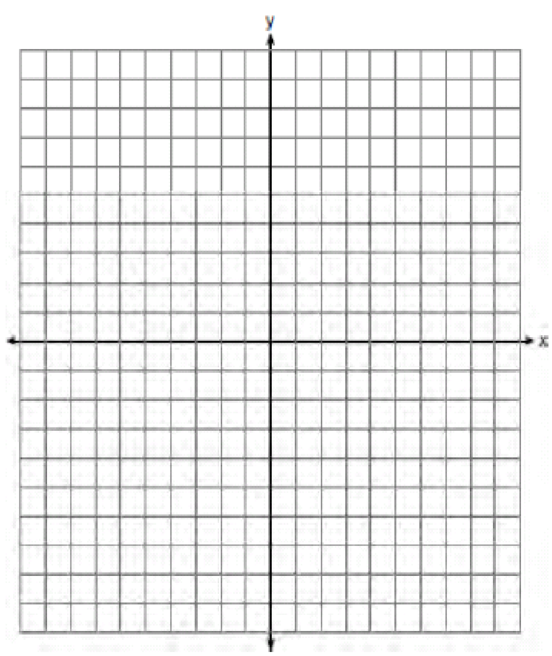
3) Find the image of (-1, 2) under the composition:

a) $R_{90^\circ} \circ r_{y=x}$

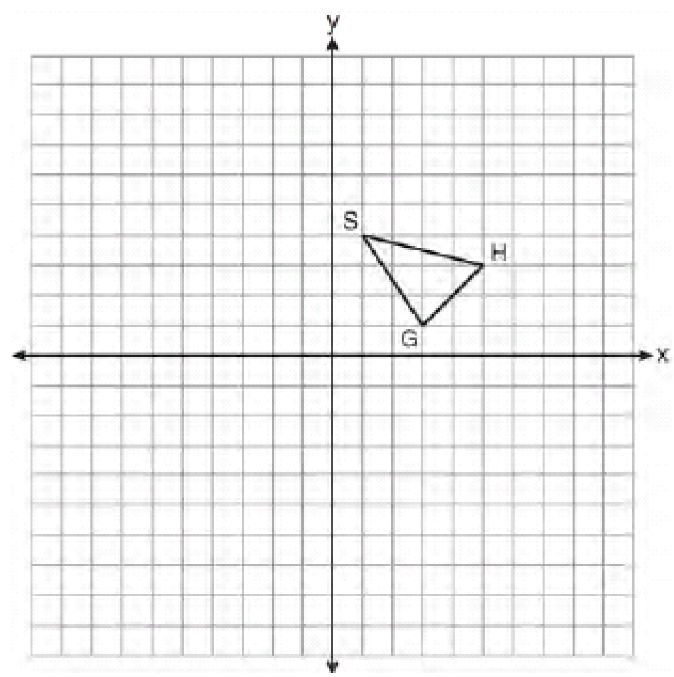
b) $r_o \circ R_{180^\circ}$

4) Find the image of (-2, 1) under the composition $r_{y-axis} \circ R_{180^\circ} \circ r_o \circ D_2$

5) The coordinates of the vertices of parallelogram $ABCD$ are $A(-2,2)$, $B(3,5)$, $C(4,2)$, and $D(-1,-1)$. State the coordinates of the vertices of parallelogram $A''B''C''D''$ that result from the transformation $r_{y-axis} \circ T_{2,-3}$. [The use of the set of axes below is optional.]



6) As shown on the set of axes below, $\triangle GHS$ has vertices $G(3,1)$, $H(5,3)$, and $S(1,4)$. Graph and state the coordinates of $\triangle G''H''S''$, the image of $\triangle GHS$ after the transformation $T_{-3,1} \circ D_2$.



7) Is the following composition a direct or an opposite isometry? Explain your reasoning.

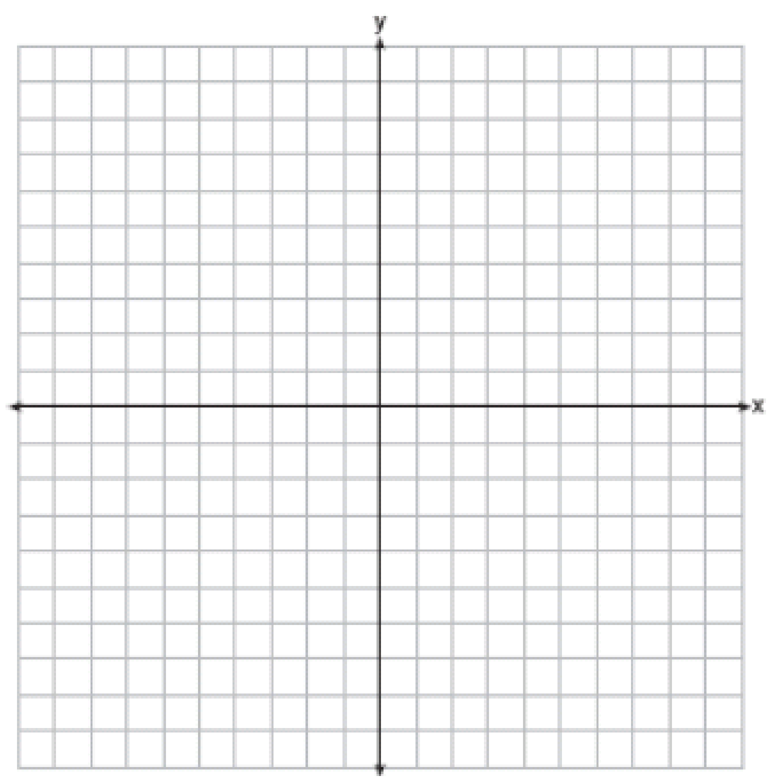
$$\boldsymbol{r_{x=2} \circ r_{x=4} \circ r_{x=6} \circ r_{x=8}}$$

8) ΔABC has coordinates $A(3, 1), B(6, 1)$, and $C(6, 6)$.

a) Graph and label ΔABC on the axes below.

b) Graph and state the coordinates of $\Delta A''B''C''$, the image of ΔABC under the composition $\boldsymbol{r_{x=-3} \circ r_{x=2}}$.

c) Show that the composition $r_{x=-3} \circ r_{x=2}$ is equivalent to a translation of $T_{-10,0}$.



Day 8: Symmetry

G-CO.3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

Warm-Up

$R_{90^\circ} \circ R_{180^\circ}$

$(x, y) \rightarrow (\quad , \quad) \rightarrow (\quad , \quad)$

This composition is equivalent to a rotation of: ________

Symmetry

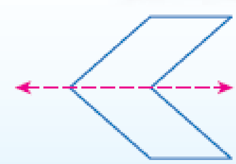
To carry a shape onto itself is another way of saying that a shape has symmetry.

There are three types of symmetries that a shape can have: line symmetry, rotation symmetry, and point symmetry.

Line Symmetry (or Reflection Symmetry)

Key Concept **Line Symmetry** For Your FOLDABLE

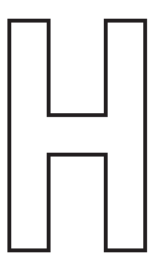
A figure in the plane has **line symmetry** (or *reflection symmetry*) if the figure can be mapped onto itself by a reflection in a line, called a **line of symmetry** (or *axis of symmetry*).



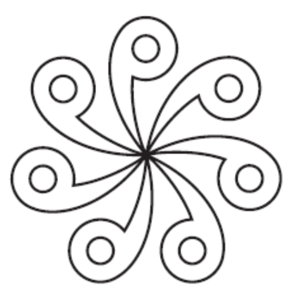
Identify Line Symmetry

State whether the figure has line symmetry. Write *yes* or *no*. If so, copy the figure, draw all lines of symmetry, and state their number.

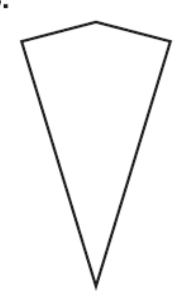
1.



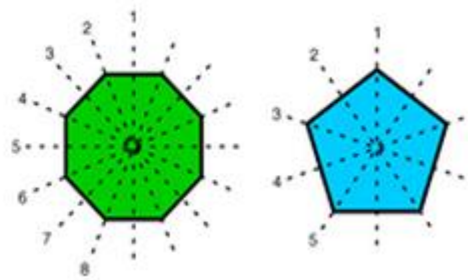
2.



3.



The maximum lines of symmetry that a polygon can have are equal to its number of sides. The maximum is always found in the regular polygon, because all sides and all angles are congruent.

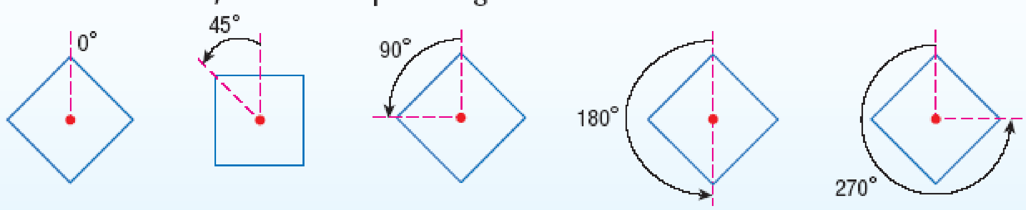


Rotational Symmetry

Key Concept — **Rotational Symmetry** — For Your FOLDABLE

A figure in the plane has **rotational symmetry** (or *radial symmetry*) if the figure can be mapped onto itself by a rotation between 0° and 360° about the center of the figure, called the **center of symmetry** (or *point of symmetry*).

Examples The figure below has rotational symmetry because a rotation of 90°, 180°, or 270° maps the figure onto itself.



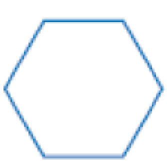
The number of times a figure maps onto itself as it rotates from 0° to 360° is called the **order of symmetry**. The **magnitude of symmetry** (or angle of rotation) is the smallest angle through which a figure can be rotated so that it maps onto itself. The order and magnitude of a rotation are related by the following equation.

$$\text{magnitude} = 360° \div \text{order}$$

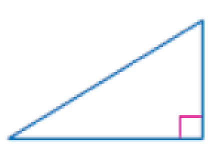
The figure above has rotational symmetry of order 4 and magnitude 90°.

Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry.

a.



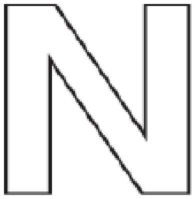
b.

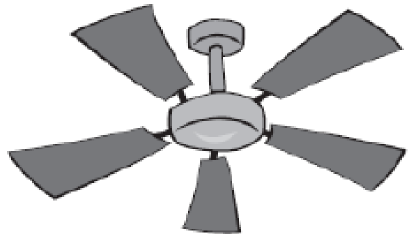


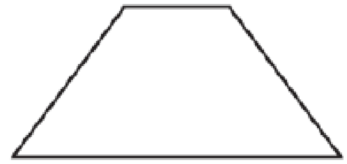
c.

Check for Understanding

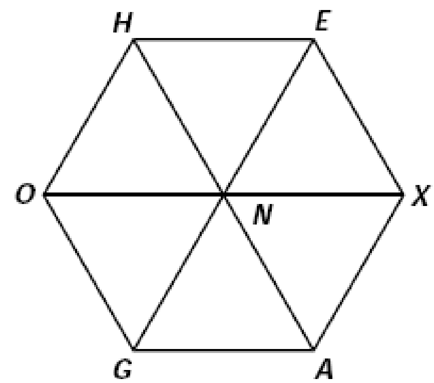
Tell whether each figure has rotational symmetry. If so, give the angle of rotational symmetry.

4. 

5. 

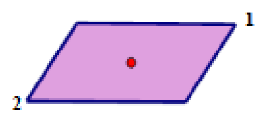
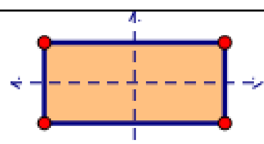
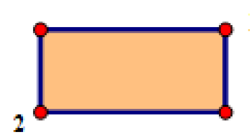
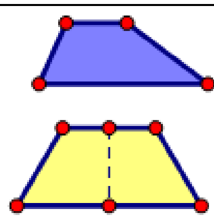
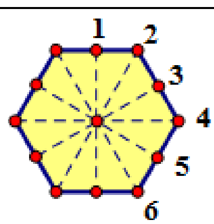
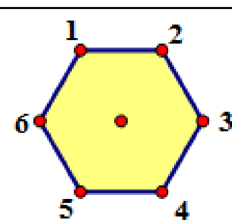
6. 

Regular hexagon HEXAGO is divided into six congruent triangles.



4. Name the image of E under a 60° rotation about N.
5. Name the image of X under a 180° rotation about N.
6. Name the image of O under a 120° rotation about N.
7. Name the image of A under a 240° rotation about N.
8. Name the image of H under a 300° rotation about N.
9. Name the image of G under a 360° rotation about N.

Summary

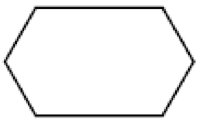
Name	Line Symmetry Diagram	Line Symmetry Count	Rotation Symmetry Diagram	Rotation Symmetry Order
Parallelogram	None	0		2
Rectangle		2		2
Trapezoid		0/1	None	0
Regular Polygon		Equal to the number of sides of the regular polygon		Equal to the number of sides of the regular polygon

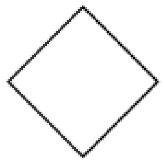
Homework

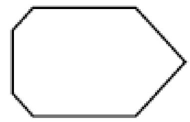
Example Exercises

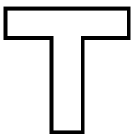
Example 1

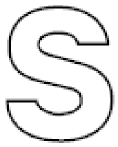
Draw the lines of symmetry for each figure.

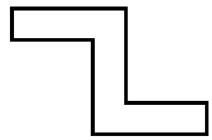
1. 

2. 

3. 

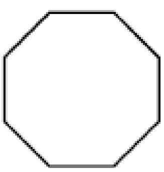
4. 

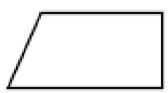
5. 

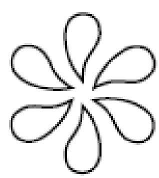
6. 

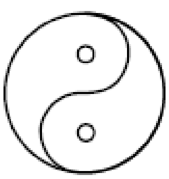
Example 3

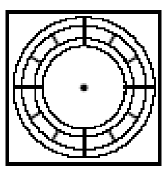
Which figures have rotational symmetry? For those that do, give the angle of rotation.

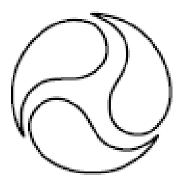
10. 

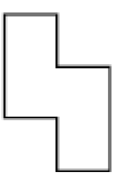
11. 

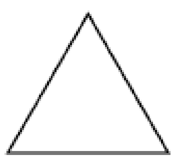
12. 

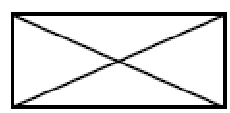
13. 

14. 

15. 

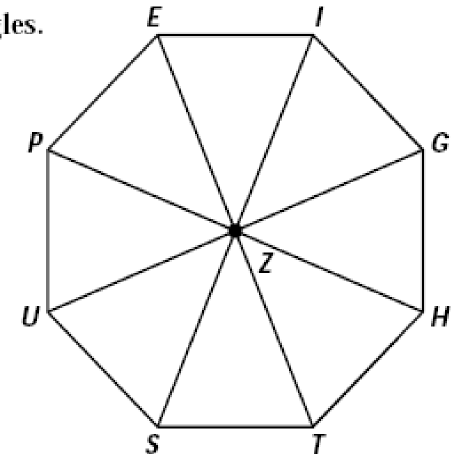
16. 

17. 

18. 

Example 4

Regular octagon *EIGHTSUP* is divided into eight congruent triangles. Find the image of each point or segment.



1. 45° rotation of G about Z
2. 225° rotation of U about Z
3. 315° rotation of E about Z
4. 270° rotation of $\overline{EI}$ about Z
5. 135° rotation of S about Z
6. 360° rotation of $\overline{ST}$ about Z

<u>Chapter 2 Test Review</u>

Read each question carefully and answer all parts to each question.

1) What is the image of the following points after a reflection in the x-axis?

a) (x, y) b) (0, 12) c) (4, -6)

2) What is the image of the following points after a reflection in the y-axis?

a) (x, y) b) (-4, 0) c) (5, -10)

3) What is the image of the following points after a reflection in the line y=x?

a) (x, y) b) (1, 9) c) (-4, 3)

4) What is the image of P (7, 4) after a $T_{-8,3}$?

5) What translation moves Q (5, 1) to Q' (0, -6)?

6) What is the image of each point after a R_{90°?

a) (x, y) b) (0, 8) c) (-9, 6)

7) What is the image of each point after a R_{180°?

a) (x, y) b) (4, -8) c) (6, 0)

8) What is the image of each point after a R_{270°?

a) (x, y) b) (0, 7) c) (12, 14)

9) What is the image of (9, -6) after a r_0?

10) What is the image of the point (1, 8) under the dilation D_3?

11) Circle the transformation you perform first in the following composition?

$$T_{-2,3} \circ r_{x-axis}$$

Chapter 2 Test Review

Read each question carefully and answer all parts to each question.

1) Which of the following transformations is **NOT** a rigid motion?
(a) Line Reflection
(b) Dilation
(c) Rotation
(d) Translation

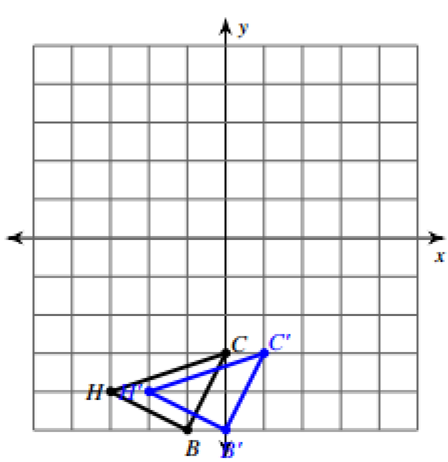
2) Which of the following transformations **DOES NOT** preserve orientation?
(a) Line Reflection
(b) Dilation
(c) Rotation
(d) Translation

3) The line of reflection is the ________________ of the segment connecting each point and its image.
(a) Mapping
(b) Scale Factor
(c) Perpendicular Bisector
(d) Average

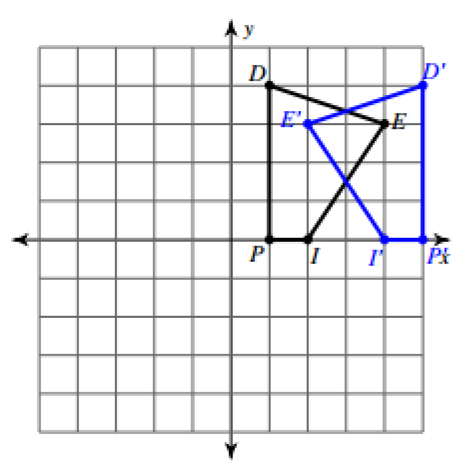
4) After a figure is dilated with a **negative** scale factor, the image is a dilation with:
(a) a translation
(b) a rotation of 180°
(c) a rotation of 90°
(d) a reflection over the line x=2

For the following graphs, write the rule for the transformation shown:

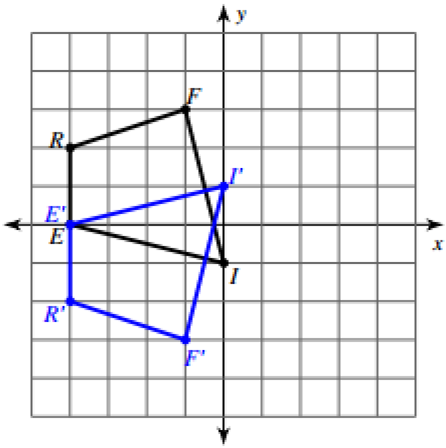
5)



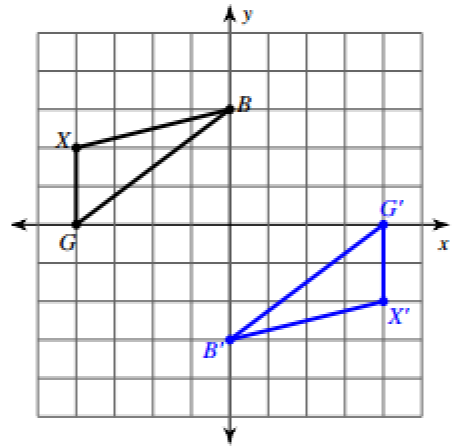
6)



7)



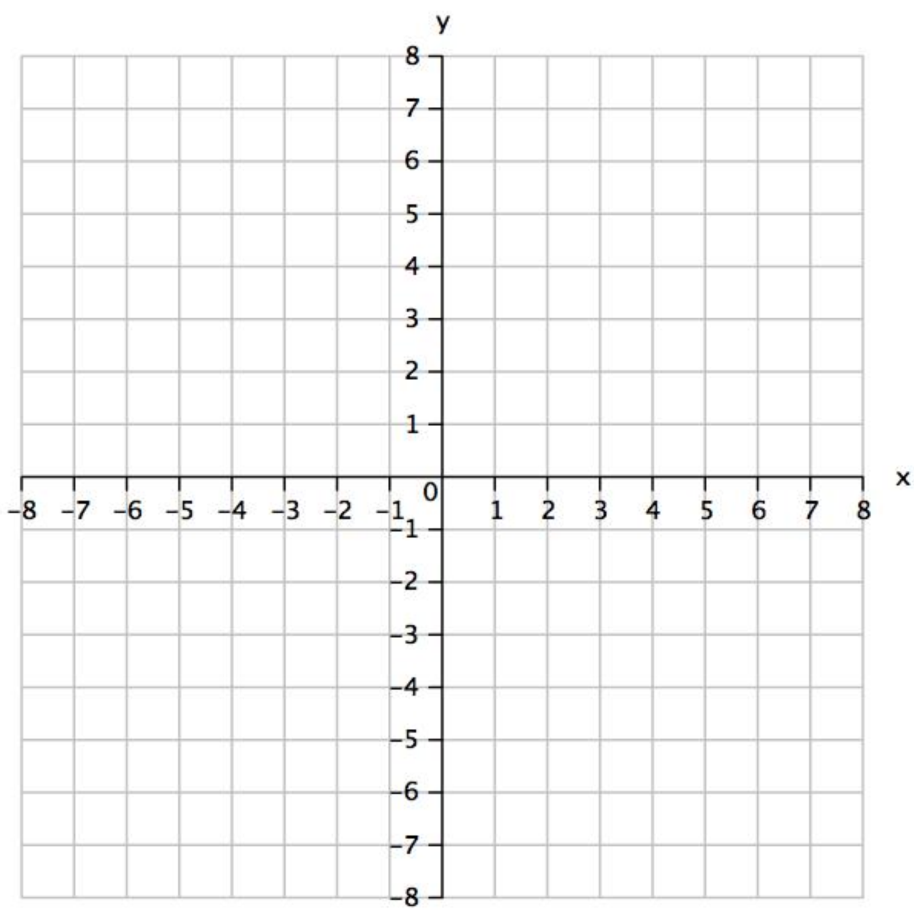
8)



9) Graph and state the coordinates of the image of each dilation , using the given scale factor. Each dilation is with respect to the origin.

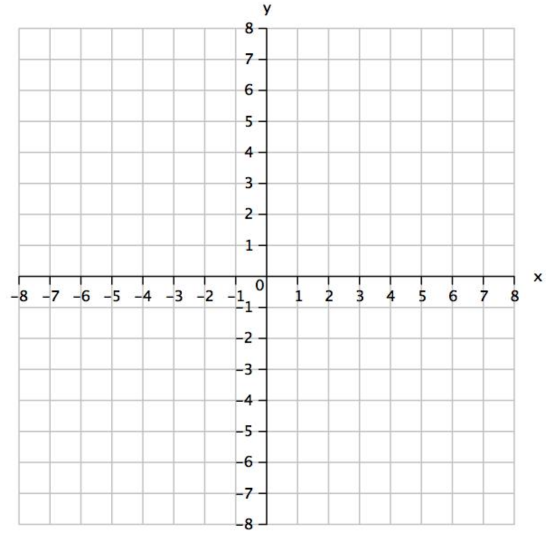
D (0, 0), E (-4, 0), F (-4, -4)

Scale factor: - $\frac{1}{2}$



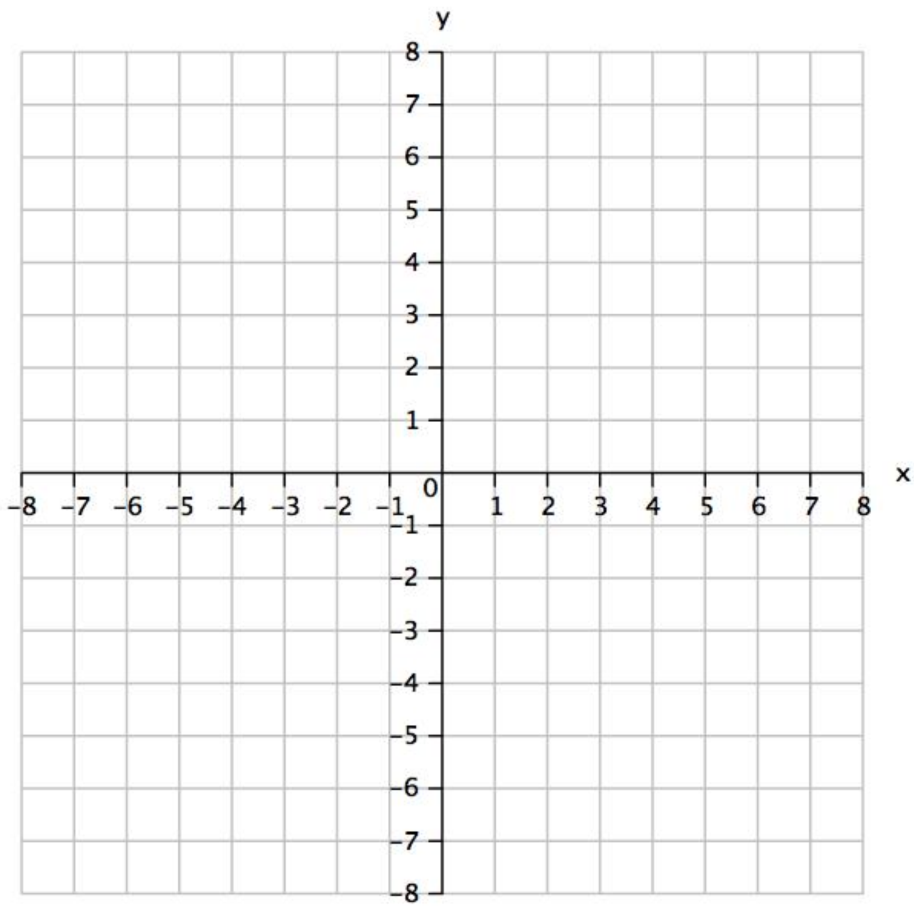
10) The line $y = 2x - 4$ is dilated by a scale factor of $\frac{3}{2}$ and centered at the origin. Which equation represents the image of the line after the dilation?

1 $y = 2x - 4$
2 $y = 2x - 6$
3 $y = 3x - 4$
4 $y = 3x - 6$



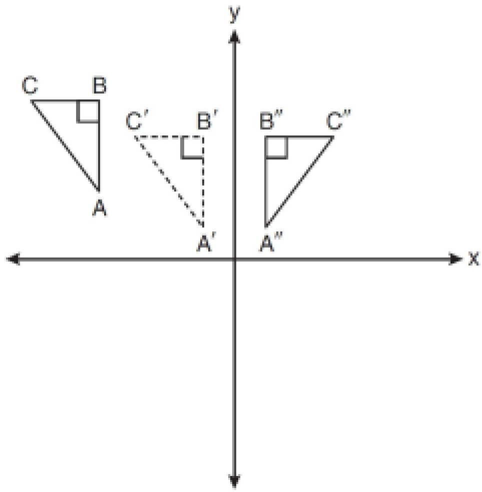
11) Graph and state the coordinates of the image of ΔABC with A (3, -2), B (6, -2), C (3, -7) after the composition

$r_{y-axis} \circ R_{90^\circ}$



Is this composition a rigid motion? Justify.

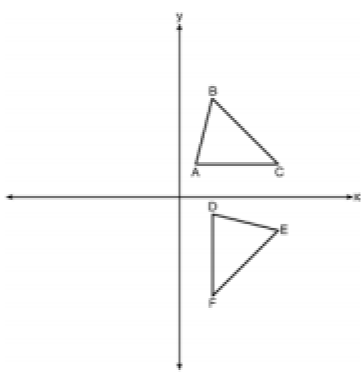
12) In the diagram below, $\triangle A'B'C'$ is a transformation of $\triangle ABC$, and $\triangle A''B''C''$ is a transformation of $\triangle A'B'C'$.



The composite transformation of $\triangle ABC$ to $\triangle A''B''C''$ is an example of a

1) reflection followed by a rotation
2) reflection followed by a translation
3) translation followed by a rotation
4) translation followed by a reflection

13) The image of $\triangle ABC$ after a rotation of 90º clockwise about the origin is $\triangle DEF$, as shown below.



Which statement is true?

1 $\overline{BC} \cong \overline{DE}$
2 $\overline{AB} \cong \overline{DF}$
3 $\angle C \cong \angle E$
4 $\angle A \cong \angle D$

Chapter 2 Test Review

Read each question carefully and answer all parts to each question.

1) **True or False**: After a figure is dilated with a scale factor of 2, the area of the figure doubles.

2) **True or False**: After a figure is dilated with a scale factor of -3, the figure and its image are similar.

3) Which of the following rules is the composition of a dilation of scale factor 2 following a translation of 3 units to the right?

1) (2x + 3, 2y)
2) (2x + 6, 2y)
3) (2x – 3, 2y)
4) (2x – 6, 2y)

4) The image of (-2,6) after a dilation with respect to the origin is (-10,30). What is the constant of the dilation?

5) Which of the following is **NOT** invariant under a dilation?

(a) angle measure
(b) orientation
(c) distance
(d) parallelism

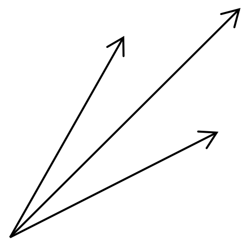
6) Is the following composition a direct or an opposite isometry? Explain your reasoning.

$$T_{-2,3} \circ r_{x-axis} \circ R_{90^\circ} \circ r_{x=4}$$

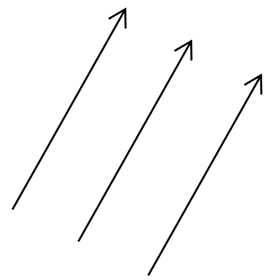
7) How are a point reflection and a line reflection the same? How are they different? Explain.

8) Certain transformations are represented with the use of vectors. State the transformation that would require the use of each set of vectors.

a)



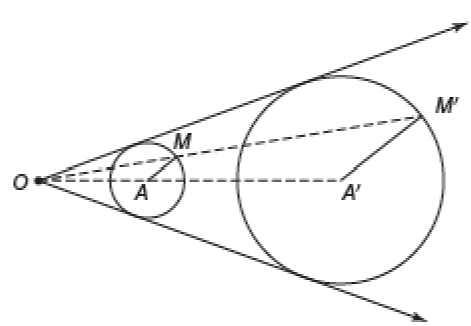
b)



9) Every transformation of the form $(x, y) \rightarrow (-x + 2h, y)$ is a reflection with respect to the vertical line with equation $x = h$. What kind of transformation is $(x, y) \rightarrow (x, -y + 2k)$?

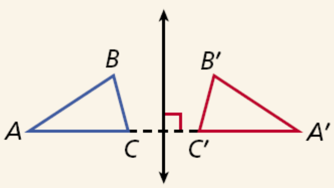
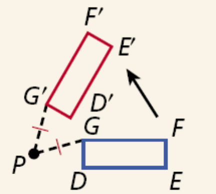
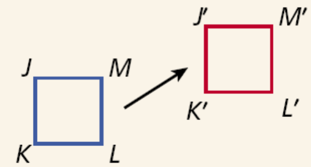
10) **REASONING** When a rotation and a reflection are performed as a composition of transformations on an image, does the order of the transformations affect the location of the final image *sometimes*, *always* or *never*? Explain.

11) Here is diagram that suggests one way to map a smaller circle onto a larger one using a dilation. The circles are given. The lines suggest how to find the center for the dilation. Describe how the center is found. Use segments in the diagram to name the scale factor.



The Rigid Motions and Their Properties *Congruence Transformations*

Transformations

REFLECTION	ROTATION	TRANSLATION
		
A **reflection** (or *flip*) is a transformation across a line, called the line of reflection. Each point and its image are the same distance from the line of reflection.	A **rotation** (or *turn*) is a transformation about a point *P*, called the center of rotation. Each point and its image are the same distance from *P*.	A **translation** (or *slide*) is a transformation in which all the points of a figure move the same distance in the same direction.

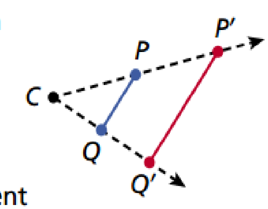
Properties Preserved:

- Distance
- Angle measure
- Parallelism and perpendicularity
- Collinearity
- Midpoint

Dilations *Similarity Transformation*

Dilations

A dilation, or *similarity transformation*, is a transformation in which the lines connecting every point *P* with its image *P′* all intersect at a point *C*, called the **center of dilation**. $\frac{CP'}{CP}$ is the same for every point *P*.
The scale factor *k* of a dilation is the ratio of a linear measurement of the image to a corresponding measurement of the preimage. In the figure, $k = \frac{P'Q'}{PQ}$.



Distance is NOT preserved under a dilation!

Line Reflections:

$$r_{x-axis}(x,y) = (x,-y)$$
$$r_{y-axis}(x,y) = (-x,y)$$
$$r_{y=x}(x,y) = (y,x)$$

Point Reflection:

$$R_{180^\circ}(x,y) = (-x,-y)$$

Rotations:

$$R_{90^\circ}(x,y) = (-y,x)$$
$$R_{180^\circ}(x,y) = (-x,-y)$$
$$R_{270^\circ}(x,y) = (y,-x)$$
$$R_{-90^\circ}(x,y) = (y,-x)$$

Translation:

$$T_{a,b}(x,y) = (x+a, y+b)$$

Compositions

$$r_{x-axis} \circ T_{3,4}$$

Do SECOND!! **Do FIRST!!!**