11.5 Inscribed Angles and Polygons

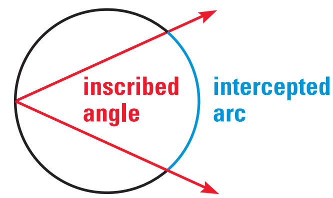
Goal

Use properties of inscribed angles.

Key Words

- inscribed angle
- intercepted arc
- inscribed
- circumscribed

An **inscribed angle** is an angle whose vertex is on a circle and whose sides contain chords of the circle.



The arc that lies in the interior of an inscribed angle and has endpoints on the angle is called the **intercepted arc** of the angle.

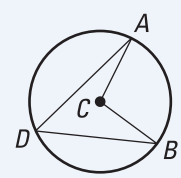
Activity 11.5 shows the relationship between an inscribed angle and its intercepted arc.

THEOREM 11.7

Measure of an Inscribed Angle

Words If an angle is inscribed in a circle, then its measure is half the measure of its intercepted arc.

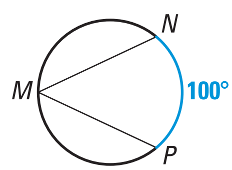
Symbols $m\angle ADB = \frac{1}{2}m\overset{\frown}{AB}$



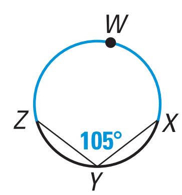
EXAMPLE 1 Find Measures of Inscribed Angles and Arcs

Find the measure of the inscribed angle or the intercepted arc.

a.



b.



Solution

a. $m\angle NMP = \frac{1}{2}m\overset{\frown}{NP}$ — The measure of an inscribed angle is half the measure of its intercepted arc.

$= \frac{1}{2}(100°)$ — Substitute 100° for $m\overset{\frown}{NP}$.

$= 50°$ — Simplify.

b. $m\angle ZYX = \frac{1}{2}m\overset{\frown}{ZWX}$ — The measure of an inscribed angle is half the measure of its intercepted arc.

$105° = \frac{1}{2}m\overset{\frown}{ZWX}$ — Substitute 105° for $m\angle ZYX$.

$210° = m\overset{\frown}{ZWX}$ — Multiply each side by 2.

Student Help
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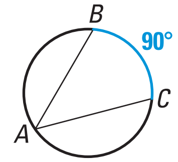
MORE EXAMPLES
More examples at classzone.com

Checkpoint Find Measures of Inscribed Angles and Arcs

Find the measure of the inscribed angle or the intercepted arc.

1.



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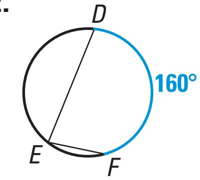
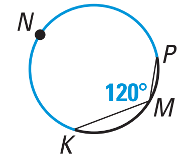
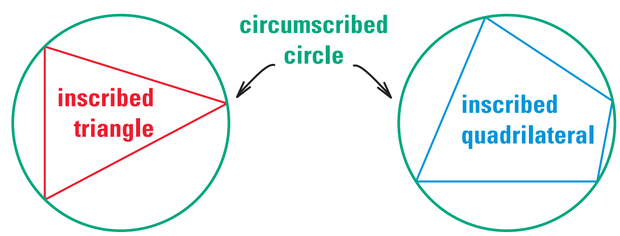


3.



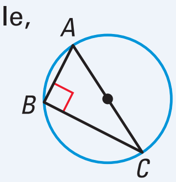
Inscribed and Circumscribed If all the vertices of a polygon lie on a circle, the polygon is **inscribed** in the circle and the circle is **circumscribed** about the polygon. The polygon is an *inscribed polygon* and the circle is a *circumscribed circle.*



THEOREM 11.8

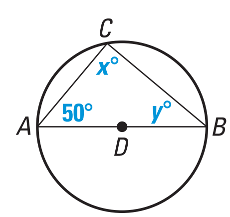
Words If a triangle inscribed in a circle is a right triangle, then the hypotenuse is a diameter of the circle.

If a side of a triangle inscribed in a circle is a diameter of the circle, then the triangle is a right triangle.



EXAMPLE 2 Find Angle Measures

Find the values of x and y.



Solution

Because $\triangle ABC$ is inscribed in a circle and $\overline{AB}$ is a diameter, it follows from Theorem 11.8 that $\triangle ABC$ is a right triangle with hypotenuse $\overline{AB}$.

Therefore, $x = 90$. Because $\angle A$ and $\angle B$ are acute angles of a right triangle, $y = 90 - 50 = 40$.

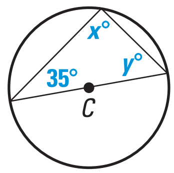
Student Help

Look Back
To review the Corollary of the Triangle Sum Theorem, see p. 180.

Checkpoint Find Angle Measures

Find the values of *x* and *y* in ⊙*C*.

4.



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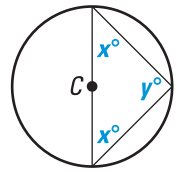

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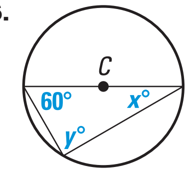
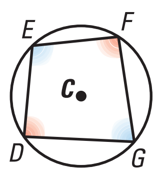


Visualize It!

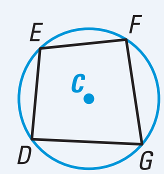
∠*D* and ∠*F* are *opposite* angles. ∠*E* and ∠*G* are *opposite* angles.



THEOREM 11.9

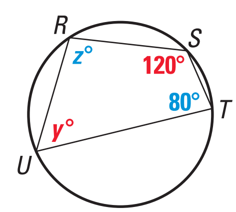
Words If a quadrilateral can be inscribed in a circle, then its opposite angles are supplementary.

If the opposite angles of a quadrilateral are supplementary, then the quadrilateral can be inscribed in a circle.



EXAMPLE 3 Find Angle Measures

Find the values of y and z.



Solution

Because *RSTU* is inscribed in a circle, by Theorem 11.9 opposite angles must be supplementary.

$\angle S$ and $\angle U$ are opposite angles.

$m\angle S + m\angle U = 180°$

$120° + y° = 180°$

$y = 60$

$\angle R$ and $\angle T$ are opposite angles.

$m\angle R + m\angle T = 180°$

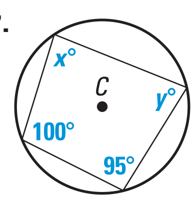
$z° + 80° = 180°$

$z = 100$

Checkpoint Find Angle Measures

Find the values of *x* and *y* in ⊙*C*.

7.



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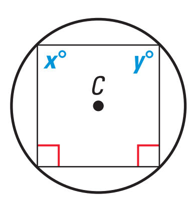

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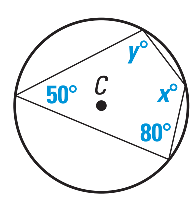
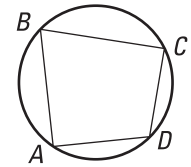


11.5 Exercises

Guided Practice

Vocabulary Check

In Exercises 1 and 2, use the diagram at the right.



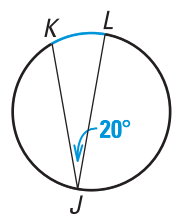
1. Name the *inscribed angles.*

2. Identify the two pairs of *opposite angles* in the inscribed quadrilateral.

Skill Check

Find the measure of the blue intercepted arc.

3.



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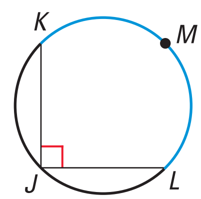

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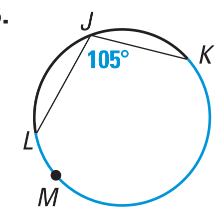
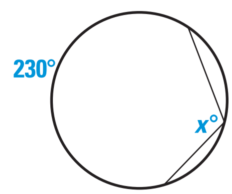


Find the value of each variable.

6.



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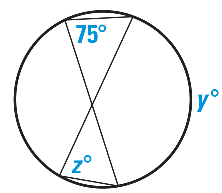

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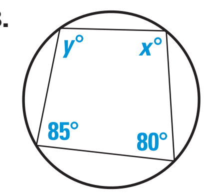


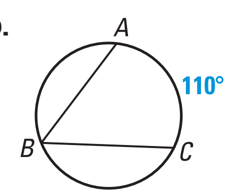
Practice and Applications

Extra Practice

See p. 696.

Angle Measures **Find the measure of the inscribed angle.**

9.



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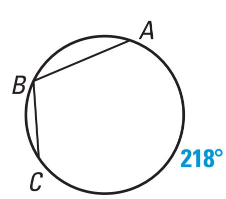

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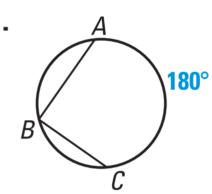
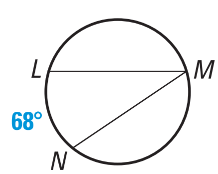


Homework Help

Example 1: Exs. 9–27
Example 2: Exs. 28–31
Example 3: Exs. 32–38

12.



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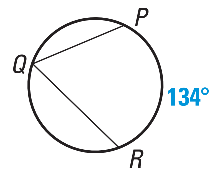

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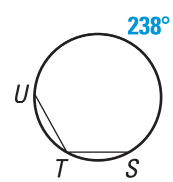
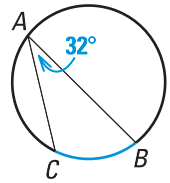
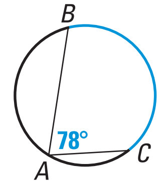


Arc Measures **Find the measure of the blue intercepted arc.**

15.



16.



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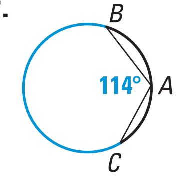

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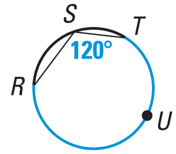

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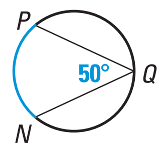

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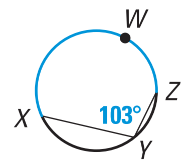


Student Help

VISUAL STRATEGY

In Exs. 21–26, copy the diagram and add information to it as you solve the exercises, as shown on p. 588.

Arc and Angle Measures **In Exercises 21–26, use the diagram below to find the intercepted arc or inscribed angle.**

21. $m\overset{\frown}{BE}$

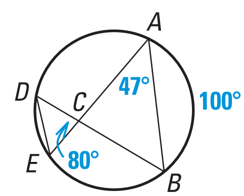
22. $m\angle BDE$

23. $m\angle AED$

24. $m\overset{\frown}{AD}$

25. $m\angle ABD$

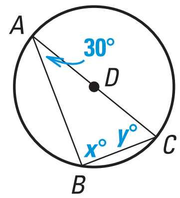
26. $m\overset{\frown}{DE}$



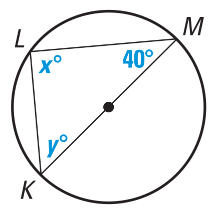
27. Are $\triangle ABC$ and $\triangle DEC$ similar? Explain your reasoning.

Inscribed Right Triangles **Find the value of each variable. Explain your reasoning.**

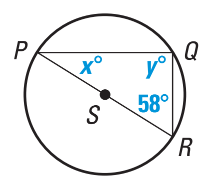
28.



29.



30.



31. **Carpenter's Square** A carpenter's square is an L-shaped tool used to draw right angles. Suppose you are making a toy truck. To make the wheels you trace a circle on a piece of wood. How could you use a carpenter's square to find the center of the circle?

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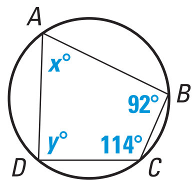
HOMEWORK HELP

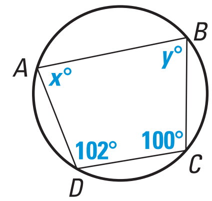
Extra help with problem solving in Ex. 31 is at classzone.com

Inscribed Quadrilaterals **Find the values of x and y.**

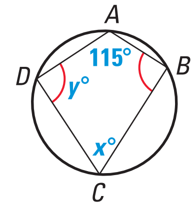
32.



33.



34.



You be the Judge **Can the quadrilateral always be inscribed in a circle? Explain your answer.**

35. square

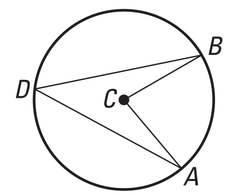
36. isosceles trapezoid

37. rhombus

38. rectangle

Standardized Test Practice

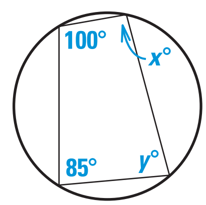
39. Multiple Choice In the diagram at the right, if $\angle ACB$ is a central angle and $m\angle ACB = 80°$, what is $m\angle ADB$?



Ⓐ 20°　Ⓑ 40°

Ⓒ 80°　Ⓓ 160°

40. Multiple Choice In the diagram at the right, what are the values of x and y?



Ⓕ $x = 80, y = 95$　Ⓖ $x = 85, y = 100$

Ⓗ $x = 95, y = 80$　Ⓙ $x = 95, y = 85$

Mixed Review

Multiplying Radicals **Multiply the radicals. Then simplify if possible.** *(Lesson 10.1)*

41. $\sqrt{5} \cdot \sqrt{7}$

42. $\sqrt{2} \cdot \sqrt{2}$

43. $\sqrt{6} \cdot \sqrt{14}$

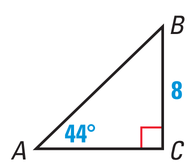
44. $(8\sqrt{2})^2$

45. $(3\sqrt{3})^2$

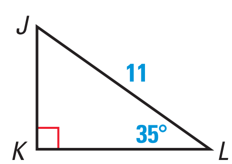
46. $2\sqrt{5} \cdot \sqrt{10}$

Solving Right Triangles **Solve the right triangle. Round decimals to the nearest tenth.** *(Lesson 10.6)*

47.



48.



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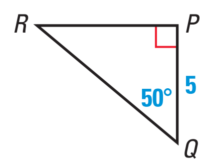

Algebra Skills

Evaluating Expressions **Evaluate the expression when $x = 2$.** *(Skills Review, p. 670)*

50. $3x + 5$

51. $8x - 7$

52. $x^2 + 9$

53. $(x + 4)(x - 4)$

54. $x^2 + 3x - 2$

55. $x^3 + x^2$