3.7 Graphing Absolute Value Functions

Essential Question

How do the values of a, h, and k affect the graph of the absolute value function $g(x) = a|x - h| + k$?

The parent absolute value function is

$f(x) = |x|$. Parent absolute value function

The graph of f is V-shaped.

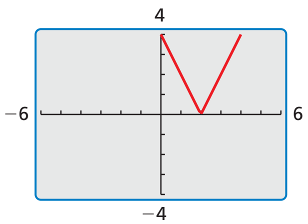
EXPLORATION 1 Identifying Graphs of Absolute Value Functions

Work with a partner. Match each absolute value function with its graph. Then use a graphing calculator to verify your answers.

a. $g(x) = -|x - 2|$ **b.** $g(x) = |x - 2| + 2$ **c.** $g(x) = -|x + 2| - 2$

d. $g(x) = |x - 2| - 2$ **e.** $g(x) = 2|x - 2|$ **f.** $g(x) = -|x + 2| + 2$

A.



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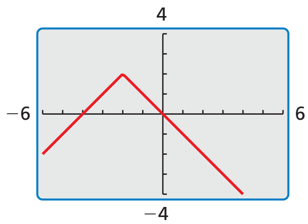

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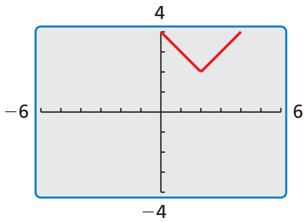

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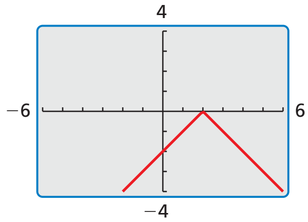

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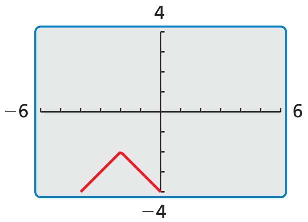

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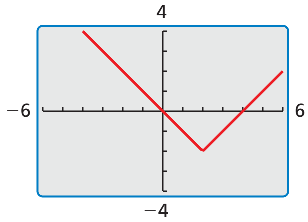


LOOKING FOR STRUCTURE

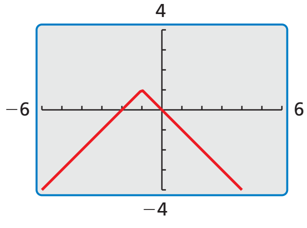
To be proficient in math, you need to look closely to discern a pattern or structure.

Communicate Your Answer

2. How do the values of a, h, and k affect the graph of the absolute value function

$g(x) = a|x - h| + k$?

3. Write the equation of the absolute value function whose graph is shown. Use a graphing calculator to verify your equation.



3.7 Lesson

What You Will Learn

- Translate graphs of absolute value functions.
- Stretch, shrink, and reflect graphs of absolute value functions.
- Combine transformations of graphs of absolute value functions.

Core Vocabulary

absolute value function, *p. 156*
vertex, *p. 156*
vertex form, *p. 158*

Previous
domain
range

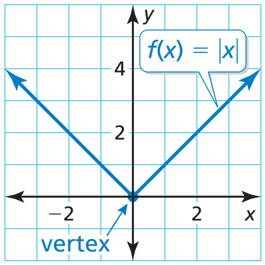
Translating Graphs of Absolute Value Functions

Core Concept

Absolute Value Function

An **absolute value function** is a function that contains an absolute value expression. The parent absolute value function is $f(x) = |x|$. The graph of $f(x) = |x|$ is V-shaped and symmetric about the y-axis. The **vertex** is the point where the graph changes direction. The vertex of the graph of $f(x) = |x|$ is (0, 0).

The domain of $f(x) = |x|$ is all real numbers. The range is $y \geq 0$.



The graphs of all other absolute value functions are transformations of the graph of the parent function $f(x) = |x|$. The transformations presented in Section 3.6 also apply to absolute value functions.

EXAMPLE 1 Graphing $g(x) = |x| + k$ and $g(x) = |x - h|$

Graph each function. Compare each graph to the graph of $f(x) = |x|$. Describe the domain and range.

a. $g(x) = |x| + 3$ **b.** $m(x) = |x - 2|$

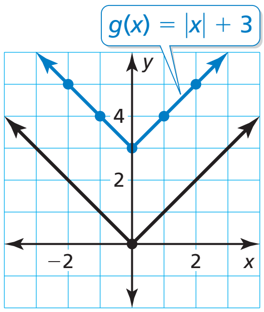
SOLUTION

a. Step 1 Make a table of values.

x	−2	−1	0	1	2
$g(x)$	5	4	3	4	5

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.



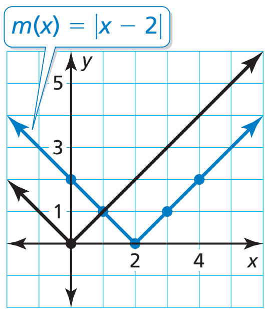
The function g is of the form $y = f(x) + k$, where $k = 3$. So, the graph of g is a vertical translation 3 units up of the graph of f. The domain is all real numbers. The range is $y \geq 3$.

b. Step 1 Make a table of values.

x	0	1	2	3	4
$m(x)$	2	1	0	1	2

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.



The function m is of the form $y = f(x - h)$, where $h = 2$. So, the graph of m is a horizontal translation 2 units right of the graph of f. The domain is all real numbers. The range is $y \geq 0$.

Monitoring Progress

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Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range.

1. $h(x) = |x| - 1$ **2.** $n(x) = |x + 4|$

Stretching, Shrinking, and Reflecting

EXAMPLE 2 **Graphing $g(x) = a|x|$**

Graph each function. Compare each graph to the graph of $f(x) = |x|$. Describe the domain and range.

a. $q(x) = 2|x|$ **b.** $p(x) = -\frac{1}{2}|x|$

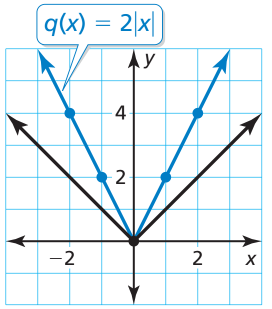
SOLUTION

a. Step 1 Make a table of values.

x	-2	-1	0	1	2
$q(x)$	4	2	0	2	4

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.



STUDY TIP

A vertical stretch of the graph of $f(x) = |x|$ is *narrower* than the graph of $f(x) = |x|$.

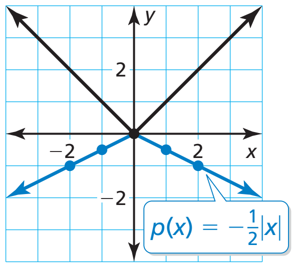
▶ The function q is of the form $y = a \cdot f(x)$, where $a = 2$. So, the graph of q is a vertical stretch of the graph of f by a factor of 2. The domain is all real numbers. The range is $y \geq 0$.

b. Step 1 Make a table of values.

x	-2	-1	0	1	2
$p(x)$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-1

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.



STUDY TIP

A vertical shrink of the graph of $f(x) = |x|$ is *wider* than the graph of $f(x) = |x|$.

▶ The function p is of the form $y = -a \cdot f(x)$, where $a = \frac{1}{2}$. So, the graph of p is a vertical shrink of the graph of f by a factor of $\frac{1}{2}$ and a reflection in the x-axis. The domain is all real numbers. The range is $y \leq 0$.

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Graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range.

3. $t(x) = -3|x|$ **4.** $v(x) = \frac{1}{4}|x|$

Core Concept

Vertex Form of an Absolute Value Function

An absolute value function written in the form $g(x) = a|x - h| + k$, where $a \neq 0$, is in **vertex form**. The vertex of the graph of g is (h, k).

Any absolute value function can be written in vertex form, and its graph is symmetric about the line $x = h$.

STUDY TIP

The function g is *not* in vertex form because the x variable does not have a coefficient of 1.

EXAMPLE 3 Graphing $f(x) = |x - h| + k$ and $g(x) = f(ax)$

Graph $f(x) = |x + 2| - 3$ and $g(x) = |2x + 2| - 3$. Compare the graph of g to the graph of f.

SOLUTION

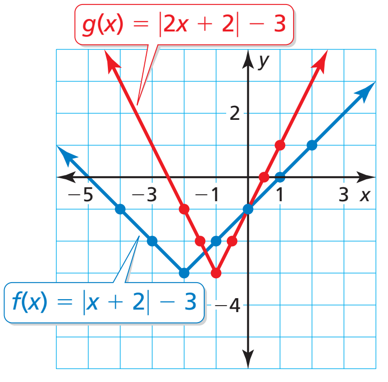
Step 1 Make a table of values for each function.

x	-4	-3	-2	-1	0	1	2
$f(x)$	-1	-2	-3	-2	-1	0	1

x	-2	-1.5	-1	-0.5	0	0.5	1
$g(x)$	-1	-2	-3	-2	-1	0	1

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph of each function. Notice that the vertex of the graph of f is $(-2, -3)$ and the graph is symmetric about $x = -2$.



Note that you can rewrite g as $g(x) = f(2x)$, which is of the form $y = f(ax)$, where $a = 2$. So, the graph of g is a horizontal shrink of the graph of f by a factor of $\frac{1}{2}$. The y-intercept is the same for both graphs. The points on the graph of f move halfway closer to the y-axis, resulting in the graph of g. When the input values of f are 2 times the input values of g, the output values of f and g are the same.

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5. Graph $f(x) = |x - 1|$ and $g(x) = \left|\frac{1}{2}x - 1\right|$. Compare the graph of g to the graph of f.

6. Graph $f(x) = |x + 2| + 2$ and $g(x) = |-4x + 2| + 2$. Compare the graph of g to the graph of f.

Combining Transformations

EXAMPLE 4 Graphing $g(x) = a|x - h| + k$

REMEMBER

You can obtain the graph of $y = a \cdot f(x - h) + k$ from the graph of $y = f(x)$ using the steps you learned in Section 3.6.

Let $g(x) = -2|x - 1| + 3$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.

SOLUTION

a. Step 1 Translate the graph of f horizontally 1 unit right to get the graph of $t(x) = |x - 1|$.

Step 2 Stretch the graph of t vertically by a factor of 2 to get the graph of $h(x) = 2|x - 1|$.

Step 3 Reflect the graph of h in the x-axis to get the graph of $r(x) = -2|x - 1|$.

Step 4 Translate the graph of r vertically 3 units up to get the graph of $g(x) = -2|x - 1| + 3$.

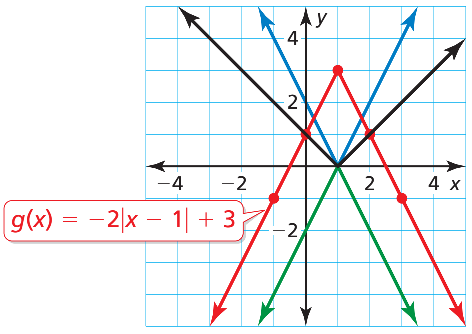
b. Method 1

Step 1 Make a table of values.

x	-1	0	1	2	3
$g(x)$	-1	1	3	1	-1

Step 2 Plot the ordered pairs.

Step 3 Draw the V-shaped graph.

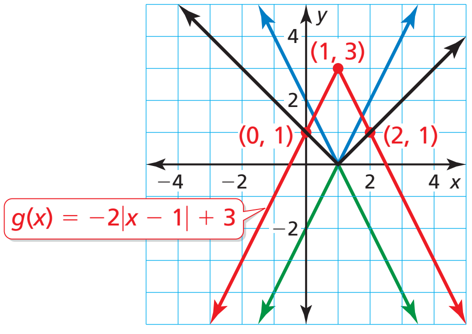


Method 2

Step 1 Identify and plot the vertex. $(h, k) = (1, 3)$

Step 2 Plot another point on the graph, such as $(2, 1)$. Because the graph is symmetric about the line $x = 1$, you can use symmetry to plot a third point, $(0, 1)$.

Step 3 Draw the V-shaped graph.



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7. Let $g(x) = \left|-\frac{1}{2}x + 2\right| + 1$. (a) Describe the transformations from the graph of $f(x) = |x|$ to the graph of g. (b) Graph g.

3.7 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The point $(1, -4)$ is the ________ of the graph of $f(x) = -3|x - 1| - 4$.

2. **USING STRUCTURE** How do you know whether the graph of $f(x) = a|x - h| + k$ is a vertical stretch or a vertical shrink of the graph of $f(x) = |x|$?

3. **WRITING** Describe three different types of transformations of the graph of an absolute value function.

4. **REASONING** The graph of which function has the same y-intercept as the graph of $f(x) = |x - 2| + 5$? Explain.

$g(x) = |3x - 2| + 5$ $\qquad$ $h(x) = 3|x - 2| + 5$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, graph the function. Compare the graph to the graph of $f(x) = |x|$. Describe the domain and range. *(See Examples 1 and 2.)*

5. $d(x) = |x| - 4$
6. $r(x) = |x| + 5$
7. $m(x) = |x + 1|$
8. $v(x) = |x - 3|$
9. $p(x) = \frac{1}{3}|x|$
10. $j(x) = 3|x|$
11. $a(x) = -5|x|$
12. $q(x) = -\frac{3}{2}|x|$

In Exercises 13–16, graph the function. Compare the graph to the graph of $f(x) = |x - 6|$.

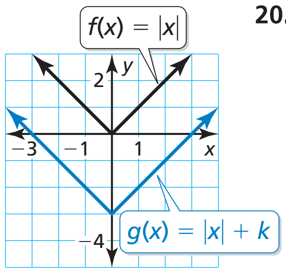
13. $h(x) = |x - 6| + 2$
14. $n(x) = \frac{1}{2}|x - 6|$
15. $k(x) = -3|x - 6|$
16. $g(x) = |x - 1|$

In Exercises 17 and 18, graph the function. Compare the graph to the graph of $f(x) = |x + 3| - 2$.

17. $y(x) = |x + 4| - 2$
18. $b(x) = |x + 3| + 3$

In Exercises 19–22, compare the graphs. Find the value of h, k, or a.

19.



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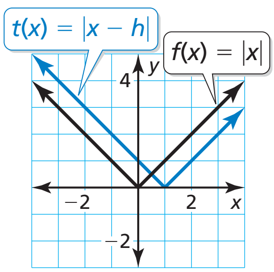

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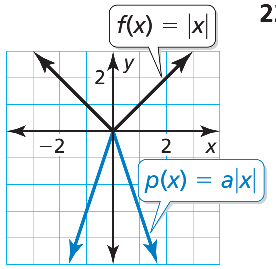

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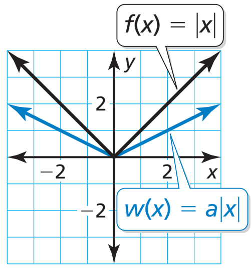


In Exercises 23–26, write an equation that represents the given transformation(s) of the graph of $g(x) = |x|$.

23. vertical translation 7 units down
24. horizontal translation 10 units left
25. vertical shrink by a factor of $\frac{1}{4}$
26. vertical stretch by a factor of 3 and a reflection in the x-axis

In Exercises 27–32, graph and compare the two functions. *(See Example 3.)*

27. $f(x) = |x - 4|$; $g(x) = |3x - 4|$
28. $h(x) = |x + 5|$; $t(x) = |2x + 5|$
29. $p(x) = |x + 1| - 2$; $q(x) = \left|\frac{1}{4}x + 1\right| - 2$
30. $w(x) = |x - 3| + 4$; $y(x) = |5x - 3| + 4$
31. $a(x) = |x + 2| + 3$; $b(x) = |-4x + 2| + 3$
32. $u(x) = |x - 1| + 2$; $v(x) = \left|-\frac{1}{2}x - 1\right| + 2$

In Exercises 33–40, describe the transformations from the graph of $f(x) = |x|$ to the graph of the given function. Then graph the given function. *(See Example 4.)*

33. $r(x) = |x + 2| - 6$ **34.** $c(x) = |x + 4| + 4$

35. $d(x) = -|x - 3| + 5$ **36.** $v(x) = -3|x + 1| + 4$

37. $m(x) = \frac{1}{2}|x + 4| - 1$ **38.** $s(x) = |2x - 2| - 3$

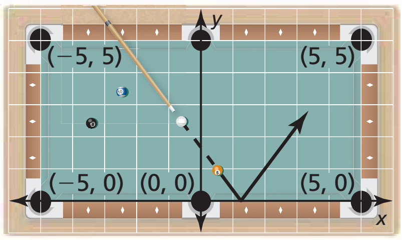
39. $j(x) = |-x + 1| - 5$ **40.** $n(x) = \left|-\frac{1}{3}x + 1\right| + 2$

41. MODELING WITH MATHEMATICS The number of pairs of shoes sold s (in thousands) increases and then decreases as described by the function $s(t) = -2|t - 15| + 50$, where t is the time (in weeks).

a. Graph the function.

b. What is the greatest number of pairs of shoes sold in 1 week?

42. MODELING WITH MATHEMATICS On the pool table shown, you bank the five ball off the side represented by the x-axis. The path of the ball is described by the function $p(x) = \frac{4}{3}\left|x - \frac{5}{4}\right|$.



a. At what point does the five ball bank off the side?

b. Do you make the shot? Explain your reasoning.

43. USING TRANSFORMATIONS The points $A\left(-\frac{1}{2}, 3\right)$, $B(1, 0)$, and $C(-4, -2)$ lie on the graph of the absolute value function f. Find the coordinates of the points corresponding to A, B, and C on the graph of each function.

a. $g(x) = f(x) - 5$ b. $h(x) = f(x - 3)$

c. $j(x) = -f(x)$ d. $k(x) = 4f(x)$

44. USING STRUCTURE Explain how the graph of each function compares to the graph of $y = |x|$ for positive and negative values of k, h, and a.

a. $y = |x| + k$

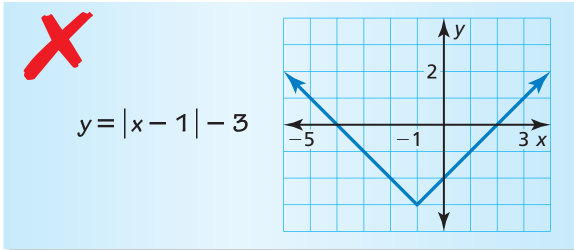
b. $y = |x - h|$

c. $y = a|x|$

d. $y = |ax|$

ERROR ANALYSIS In Exercises 45 and 46, describe and correct the error in graphing the function.

45.



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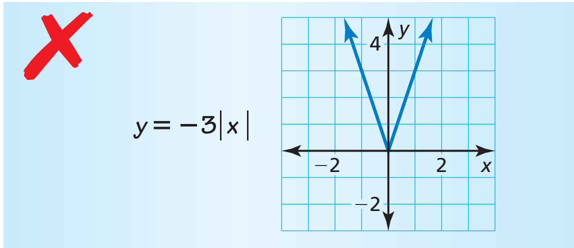
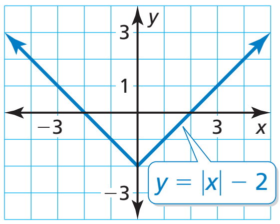


MATHEMATICAL CONNECTIONS In Exercises 47 and 48, write an absolute value function whose graph forms a square with the given graph.

47.



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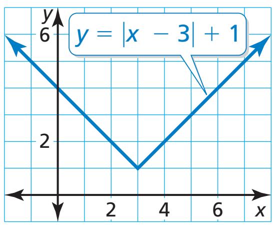
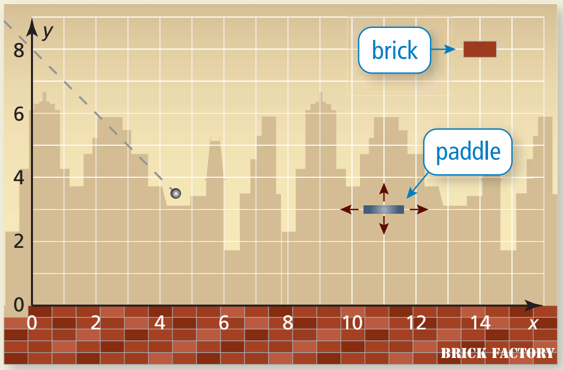


49. WRITING Compare the graphs of $p(x) = |x - 6|$ and $q(x) = |x| - 6$.

50. **HOW DO YOU SEE IT?** The object of a computer game is to break bricks by deflecting a ball toward them using a paddle. The graph shows the current path of the ball and the location of the last brick.



a. You can move the paddle up, down, left, and right. At what coordinates should you place the paddle to break the last brick? Assume the ball deflects at a right angle.

b. You move the paddle to the coordinates in part (a), and the ball is deflected. How can you write an absolute value function that describes the path of the ball?

In Exercises 51–54, graph the function. Then rewrite the absolute value function as two linear functions, one that has the domain $x < 0$ and one that has the domain $x \geq 0$.

51. $y = |x|$

52. $y = |x| - 3$

53. $y = -|x| + 9$

54. $y = -4|x|$

In Exercises 55–58, graph and compare the two functions.

55. $f(x) = |x - 1| + 2$; $g(x) = 4|x - 1| + 8$

56. $s(x) = |2x - 5| - 6$; $t(x) = \frac{1}{2}|2x - 5| - 3$

57. $v(x) = -2|3x + 1| + 4$; $w(x) = 3|3x + 1| - 6$

58. $c(x) = 4|x + 3| - 1$; $d(x) = -\frac{4}{3}|x + 3| + \frac{1}{3}$

59. **REASONING** Describe the transformations from the graph of $g(x) = -2|x + 1| + 4$ to the graph of $h(x) = |x|$. Explain your reasoning.

60. **THOUGHT PROVOKING** Graph an absolute value function f that represents the route a wide receiver runs in a football game. Let the x-axis represent distance (in yards) across the field horizontally. Let the y-axis represent distance (in yards) down the field. Be sure to limit the domain so the route is realistic.

61. **SOLVING BY GRAPHING** Graph $y = 2|x + 2| - 6$ and $y = -2$ in the same coordinate plane. Use the graph to solve the equation $2|x + 2| - 6 = -2$. Check your solutions.

62. **MAKING AN ARGUMENT** Let p be a positive constant. Your friend says that because the graph of $y = |x| + p$ is a *positive* vertical translation of the graph of $y = |x|$, the graph of $y = |x + p|$ is a *positive* horizontal translation of the graph of $y = |x|$. Is your friend correct? Explain.

63. **ABSTRACT REASONING** Write the vertex of the absolute value function $f(x) = |ax - h| + k$ in terms of a, h, and k.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Solve the inequality. *(Section 2.4)*

64. $8a - 7 \leq 2(3a - 1)$

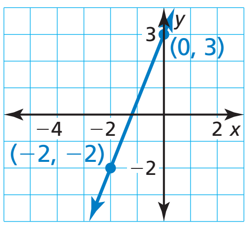
65. $-3(2p + 4) > -6p - 5$

66. $4(3h + 1.5) \geq 6(2h - 2)$

67. $-4(x + 6) < 2(2x - 9)$

Find the slope of the line. *(Section 3.5)*

68.



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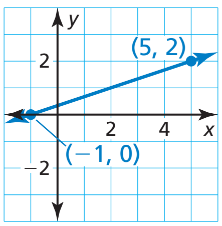

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