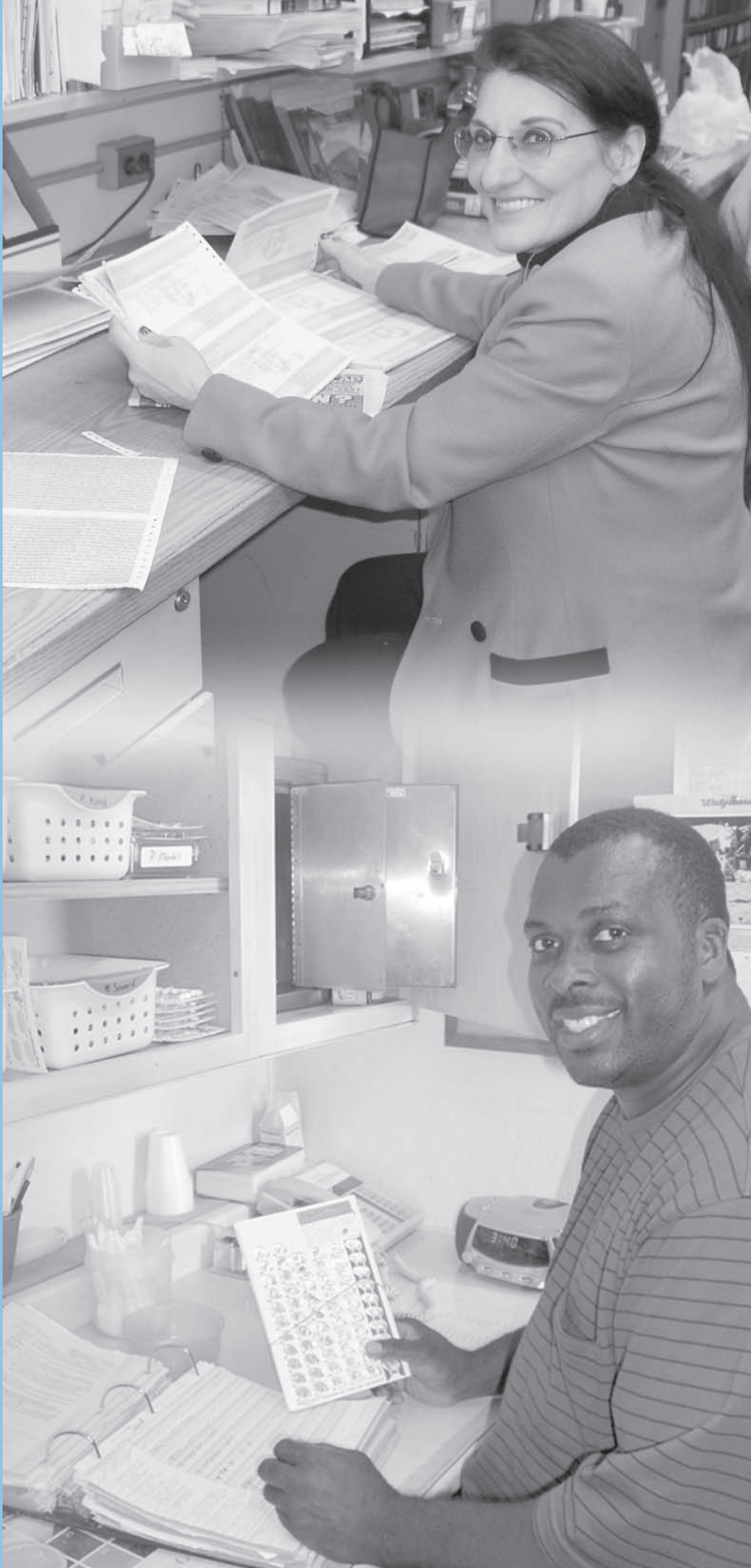
Math Refresher

Book #2



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Contents

Math Refresher

This booklet is designed to refresh your understanding of basic math operations involving fractions, decimals, percents, and ratios. The first part of this booklet contains explanations of some basic math concepts. The second part contains practice questions that will test your ability to apply these concepts to a variety of math problems. Answers and explanations for all of the problems are included in the back of the book.

The best way to master this material is to work with the practice questions. Some of these questions may seem very difficult at first, especially if it has been years since you've worked with math problems. But if you study the explanations in the back of the book, you will learn how to approach these problems and you will gain a better understanding of the basic math concepts they involve.

Good luck!

Basic Math Concepts

1. Fractions

A fraction represents part of a whole. For example, if you divided a pie into four equal pieces, each piece would be $\frac{1}{4}$ of the whole.

The top number in a fraction is called the numerator. The bottom number is called the denominator.

In the fraction $\frac{1}{2}$ the numerator is 1 and the denominator is 2.

Any whole number (1, 2, 3, etc) can be written as a fraction with a denominator of 1.

$$\frac{2}{1} = 2$$

$$\frac{50}{1} = 50$$

Adding Fractions

It is easy to add fractions that have the same denominator. You simply add the numerators and keep the denominators the same. For example:

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Adding fractions becomes a little more complicated when the denominators are different. For example:

$$\frac{1}{2} + \frac{1}{4} = ?$$

In these cases, you need to find a *common denominator*, that is, a denominator that you can use for both fractions.

One way to find a common denominator is to multiply the denominators together. To find a common denominator for $\frac{1}{2}$ and $\frac{1}{4}$, you could multiply 2 times 4.

$$2 \times 4 = 8$$

So you know that these two fractions have a common denominator of 8.

When you convert the original fractions into fractions with a common denominator, you have to be sure to keep the values of the fractions the same. You can do this by multiplying both the numerator and the denominator of the fractions by the same number.

To convert $\frac{1}{2}$ to a fraction with a denominator of 8, you would have to multiply both the numerator and the denominator by 4

$$\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$$

To convert $\frac{1}{4}$ to a fraction with a denominator of 8, you would have to multiply both the numerator and the denominator by 2

$$\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}$$

So now the two fractions have the same denominator, and to add them you can simply add the numerators:

$$\frac{4}{8} + \frac{2}{8} = \frac{6}{8}$$

When you come up with an answer like $\frac{6}{8}$, you should *simplify* the answer. To simplify a fraction, you look for the largest number that you can divide evenly into both the numerator and the denominator. In the fraction $\frac{6}{8}$, you can divide both the numerator and the denominator by 2.

$$6 \div 2 = 3$$
$$8 \div 2 = 4$$

So you can simplify $\frac{6}{8}$ to $\frac{3}{4}$.

You can also find a common denominator by mentally reviewing your multiplication tables. Look for the smallest number that can be divided evenly by both denominators. For example:

$$\frac{1}{6}+\frac{3}{8}=?$$

Both 6 and 8 will divide evenly into 24:

$$24 \div 6 = 4$$
$$24 \div 8 = 3$$

To convert the denominators of these two fractions to 24, multiply both the numerators and denominators by the same number.

To convert $\frac{1}{6}$ to a fraction with a denominator of 24, multiply both the numerator and the denominator by 4:

$$\frac{1}{6}\times\frac{4}{4}=\frac{4}{24}$$

To convert $\frac{3}{8}$ to a fraction with a denominator of 24, multiply both the numerator and the denominator by 3:

$$\frac{3}{8}\times\frac{3}{3}=\frac{9}{24}$$

So now the two fractions have the same denominator, and to add them you can simply add the numerators:

$$\frac{4}{24}+\frac{9}{24}=\frac{13}{24}$$

Subtracting Fractions

You can use the same basic procedure for subtracting fractions. If the denominators are the same, subtract the numerators and leave the denominators the same. For example:

$$\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

If the denominators are different, you will have to find a common denominator:

$$\frac{1}{2}-\frac{1}{4}=?$$

If you use a common denominator, you come up with:

$$\frac{2}{4}-\frac{1}{4}=\frac{1}{4}$$

Multiplying Fractions

To multiply fractions, you simply multiply the numerators together and multiply the denominators together. For example:

$$\frac{1}{2}\times\frac{3}{4}=\frac{3}{8}$$

In this example, you multiply the numerators ($1\times 3=3$) and the denominators ($2\times 4=8$). You would follow the same process to multiply three or more fractions. For example:

$$\frac{1}{2}\times\frac{3}{4}\times\frac{2}{3}=\frac{6}{24}$$

When you come up with an answer like $\frac{6}{24}$, you should *simplify* the answer to $\frac{1}{4}$.

Dividing Fractions

In a division problem, the *divisor* is the number we are dividing by. In the problem

$$\frac{2}{3} \div \frac{1}{4} = ?$$

the divisor is $\frac{1}{4}$.

To divide fractions, invert the divisor and then multiply. In the problem

$$\frac{2}{3} \div \frac{1}{4} = ?$$

Invert $\frac{1}{4}$ to $\frac{4}{1}$ and multiply.

$$\frac{2}{3} \times \frac{4}{1} = \frac{8}{3}$$

Remember: to multiply fractions, multiply the numerators together and multiply the denominators together. In this problem, multiply the numerators ($2 \times 4 = 8$) and the denominators ($3 \times 1 = 3$).

Mixed Numbers

A mixed number is a whole number and a fraction, for example $2\frac{1}{3}$. To work with mixed numbers, you often need to convert them to fractions. To do this, you need to perform two steps.

First, convert the whole number to a fraction that has the same denominator as the fractional part of the mixed number. For the mixed number $2\frac{1}{3}$, convert the whole number 2 to a fraction that has a denominator of 3.

$$2 = \frac{?}{3}$$

To find the numerator, multiply the whole number by the denominator:

$$2 = \frac{6}{3}$$

Now add $\frac{6}{3}$ and the fractional part of the mixed number:

$$\frac{6}{3} + \frac{1}{3} = \frac{7}{3}$$

and you find that $2\frac{1}{3} = \frac{7}{3}$.

You can convert $\frac{7}{3}$ back to a mixed number by dividing the numerator by the denominator. The number of times the denominator goes into the numerator *evenly* is the whole number part of the mixed number. The remainder (the amount left over after dividing) is the new numerator of the fractional part. In this example, 7 divided by 3 equals 2 with a remainder of 1, so

$$\frac{7}{3} = 2\frac{1}{3}$$

Multiplying and Dividing Mixed Numbers

To multiply mixed numbers, you first need to convert them to fractions. For example, to multiply $2\frac{1}{3}$ by $1\frac{3}{4}$, you would first have to convert these numbers to fractions:

$$2\frac{1}{3} = \frac{7}{3}$$

$$1\frac{3}{4} = \frac{7}{4}$$

Then you would follow the usual procedure for multiplying fractions. That is, you would multiply the numerators together and multiply the denominators together:

$$\frac{7}{3}\times\frac{7}{4}=\frac{49}{12}$$

Then you would convert $\frac{49}{12}$ to a mixed number:

$$\frac{49}{12}=4\frac{1}{12}$$

To divide mixed numbers, you would again have to convert them to fractions. For example, to divide $2\frac{2}{3}$ by $1\frac{2}{5}$ you would first have to convert these numbers to fractions:

$$2\frac{2}{3}=\frac{8}{3}$$

$$1\frac{2}{5}=\frac{7}{5}$$

So now you are dividing $\frac{8}{3}$ by $\frac{7}{5}$. Remember that to divide fractions you invert the divisor and multiply.

$$\frac{8}{3}\div\frac{7}{5} \text{ is the same as } \frac{8}{3}\times\frac{5}{7}$$

So now you can complete the problem:

$$\frac{8}{3} \times \frac{5}{7} = \frac{40}{21}$$

Then convert $\frac{40}{21}$ to a mixed number:

$$\frac{40}{21} = 1\frac{19}{21}$$

Adding and Subtracting Mixed Numbers

When adding mixed numbers, add the whole numbers together and add the fractions together. It is not necessary to convert the mixed numbers to fractions. For example, to add $4\frac{1}{3}$ and $2\frac{1}{3}$, add 4 and 2 and then add $\frac{1}{3}$ and $\frac{1}{3}$.

$$4\frac{1}{3} + 2\frac{1}{3} = 6\frac{2}{3}$$

Remember: if the fractional parts of whole numbers have different denominators, you have to find a common denominator before you can add them. For example:

$$4\frac{3}{8} + 6\frac{1}{12} = ?$$

The smallest common denominator is 24, so this problem would become:

$$4\frac{9}{24} + 6\frac{2}{24} = 10\frac{11}{24}$$

In a subtraction problem you can often follow a similar procedure. Subtract the whole numbers from each other and the fractions from each other. For example:

$$6\frac{3}{5} - 3\frac{2}{5} = 3\frac{1}{5}$$

Sometimes, though, you need to convert the mixed numbers to fractions before you subtract. For example, consider this problem:

$$2\frac{1}{3} - 1\frac{2}{3} = ?$$

Notice that the fraction in $1\frac{2}{3}$ is larger than the fraction in $2\frac{1}{3}$. The easiest way to handle a problem like this is to convert the mixed numbers to fractions:

$$\frac{7}{3} - \frac{5}{3} = \frac{2}{3}$$

2. Decimals

Decimal notation is a way of representing fractions that have denominators of 10 or a multiple of 10 (100, 1,000, 10,000, etc.). To work with decimals, you need to understand the concept of *place value*. Let's use the number 1,234.5678 to illustrate this concept.

1,234.5678

In this number:

1 is in the ***thousands*** place

2 is in the ***hundreds*** place

3 is in the ***tens*** place

4 is in the ***ones*** place

5 is in the ***tenths*** place

6 is in the ***hundredths*** place

7 is in the ***thousandths*** place

8 is in the ***ten thousandths*** place

Our money system illustrates how the decimal system works. One cent is $\frac{1}{100}$ of a dollar. It can be written as \$.01. When ten one-cent pieces (pennies) are added together, we move into the next place value. This can be represented as $\frac{10}{100}$, $\frac{1}{10}$ of a dollar, or \$.10. One hundred pennies equals one dollar, which is written as \$1.00.

Converting Fractions to Decimals

To convert a fraction to a decimal, simply divide the numerator (the top number) by the denominator (the bottom number). For example, to convert $\frac{3}{8}$ to a decimal, divide 3 by 8. The answer is .375.

Adding and Subtracting Decimals

Adding or subtracting decimals is fairly simple. You use the same basic process that you would use to add or subtract money. For example, if you had one dollar and you spent 73¢, how much would you have left? You set up this problem like this:

$$\begin{array}{r} \$1.00 \\ -.73 \\ \hline .27 \end{array}$$

The answer is .27 or 27¢.

The key to adding or subtracting decimals, if you are not using a calculator, is to line up the decimal points. Once the decimals are in line, work from right to left. Here is an example:

$$.532 + .219 + .90 + .0002 = ?$$

Line up the decimal points:

$$\begin{array}{r} .532 \\ .219 \\ .90 \\ +\ .0002 \\ 1.6512 \end{array}$$

The same procedure applies to subtraction. For example:

$$.103 - .091 = ?$$

Line up the decimal points:

$$\begin{array}{r} .103 \\ -\ .091 \\ .012 \end{array}$$

Multiplying Decimals

The only difficult thing about multiplying decimals is getting the decimal point in the right place in the answer. If you are using a calculator, you do not need to worry about this. The answer will have the decimal point in the right place.

If you are not using a calculator, begin by multiplying the numbers and ignoring the decimal points. Then count the total number of places (digits) to the right of the decimal point in both original numbers. This will be the total number of places to the right of the decimal point in the answer. Starting from the digit all the way to the right, count this number of places to the left and place the decimal point there.

Example:

$$1.2 \times .09 = ?$$

First multiply the numbers, ignoring the decimal points:

$$12 \times 9 = 108$$

Now count the number of places to the right of the decimal point in the original numbers. There are a total of three places to the right of the decimal point in the numbers 1.2 and .09. So you will need to have three places to the right of the decimal point in the answer. The answer is .108.

If there are not enough digits in the answer, add zeros to the left of the answer (to fill in the place values).

Example:

$$.0002 \times .03 = ?$$

Begin by multiplying 2 times 3. The answer is 6. Now count the number of places to the right of the decimal point in the original numbers. There are a total of six places to the right of the decimal point in the numbers .0002 and .03. To have six places to the right of the decimal point in the answer, we need to add five zeros.

The answer is .000006.

Dividing Decimals

If you are using a calculator to divide decimals, you do not need to worry about where to put the decimal point in the answer. The answer will have the decimal point in the right place.

If you are not using a calculator, use the following procedure.

To divide decimals when the divisor (the number you are dividing *by*) is not a decimal, carry out the division as usual without removing the decimal point from the dividend (the number being divided). Place the decimal point in the answer (the quotient) directly above the decimal point in the dividend. Let’s look at an example:

$$.4963 \div 7 = ?$$

First set up the problem for division. Place the decimal point in the quotient directly above the decimal point in the dividend:

$$\begin{array}{r} . \\ 7\overline{).4963} \end{array}$$

Since 7 divides into 49, not 4, place a zero in the first place to the right of the decimal point as a placeholder. Now divide as usual.

$$\begin{array}{r} .0709 \\ 7\overline{).4963} \\ \underline{49} \\ 063 \\ \underline{63} \\ 0 \end{array}$$

To divide when the *divisor* is a decimal, move the decimal point in the divisor to the *right end* of the number. Now move the decimal point in the *dividend* the same number of places to the *right*. Put the decimal point for the quotient directly above the newly placed decimal in the dividend.

For example:

$$361.6 \div .08 = ?$$

$$.08\overline{)361.60}$$

$$\begin{array}{r} 4520 \\ 08\overline{)36160} \\ \underline{32} \\ 41 \\ \underline{40} \\ 16 \\ \underline{16} \\ 0 \end{array}$$

Move the decimal point in the divisor (.08) two places to the right. Do the same in the dividend (361.6) You will have to add a zero to the right of the 6 in order to move this decimal point two places. Place the decimal point in the quotient and divide. Do not forget to include the zero between the 2 and the decimal point in the answer.

3. Percentages

In order to solve problems that involve percentages, you need to understand the relationship between percents and decimals.

Let's begin by thinking about 100%. If you have 100% of something, you have all of it. Another way of saying this is:

100% = 1

or

100% = 1.0 (1.0 is the same as 1)

If you have less than 100%, you have less than 1. Think about sales tax. Suppose you have to pay a sales tax of 8%. How would you express this as a decimal? You know that a tax of 8% is the same as 8¢ on the dollar.

8% = 8¢ on the dollar

or

8% = .08

Here is another example. You see a sweater on sale for 20% off. You know this means that you will save 20¢ off every dollar of the original price. In other words, you will be saving $.20 off every dollar of the original price. So

20% = .20

If you look at these two examples, you will see that you convert a percentage to a decimal by moving the decimal point two places to the ***left***.

20% = .20

You convert a decimal to a percentage by doing just the opposite: You move the decimal point two places to the ***right.***

.20 = 20%

Let's see how we can approach some sample questions by using the relationship between percentages and decimals.

Question. A coat that was originally priced at $100 is on sale for 25% off. How much do you save?

Answer. To answer this question, you need to convert the percentage to a decimal. You convert a percentage to a decimal by moving the decimal point two places to the ***left***.

25% = .25

Now multiply the original price by the decimal

$100 x .25 = $25

You save $25 on the coat. This question asked you to use a percentage to calculate an amount. The next question asks you to do just the opposite. It asks you to use an amount to calculate a percentage.

Question. Between 1995 and 2000, the average price of a house in Middlebury went up from \$100,000 to \$120,000. What was the percentage increase in the average price of a house?

Answer. To answer this question, you need to know the actual amount of the increase.

	\$120,000	average price in 2000
–	\$100,000	average price in 1995
	\$20,000	increase

Now divide the increase by the original price:

$$20{,}000 \div 100{,}000 = .20$$

This is one of the basic types of percent questions. When a question asks,

What was the percentage increase or decrease?

you divide the amount of the increase or decrease by the original number.

Now you have to convert .20 to a percentage. To do this, you move the decimal point two places to the ***right***.

$$.20 = 20\%$$

So the answer to this question is 20%.

Question. In Middlebury High School, there are 330 freshmen, 300 sophomores, 290 juniors and 280 seniors. What percentage of the total student body are sophomores?

Answer. To answer this question, you need to know two numbers: the number of sophomores and the total number of students. You are told in the question that there are 300 sophomores. To find out the total number of students, add the numbers in each class. The total number of students is 1,200.

To find the percentage of the student body that is sophomores, divide the number of sophomores by the total number of students.

$$300 \div 1{,}200 = .25$$

This is another one of the basic types of percentage questions. When a question asks,

What percentage of the total is represented by a certain part?

you divide *the part* by the total.

Now you have to convert .25 to a percentage. To convert .25 to a percentage, move the decimal point two places to the ***right***.

$$.25 = 25\%$$

So the answer to this question is 25%.

4. Ratios

Ratios are used to express relationships. Let's say you spend $400 a month on food and $800 a month on rent. For every dollar you spend on food, how many dollars do you spend on rent? In ratio language, this question is:

400 is to 800 as 1 is to ?

There are two ways to set up this problem. One looks like this:

$$\frac{1}{x} = \frac{400}{800}$$

The letter x is used to represent the "unknown" (the number you are trying to find).

To solve this problem, you "cross multiply." That is, multiply 400 times x and multiply 800 times 1.

$$\frac{1}{x} \quad \frac{400}{800}$$

400x = 800

To solve for x, divide both sides of the equation by 400.

$$\frac{400x}{400} = \frac{800}{400}$$

$$x = 2$$

So for every dollar you spend on food, you spend two dollars on rent.

Another way to solve this problem is to write the ratio in sentence form:

food is to rent as 1 is to x

or

400 is to 800 as 1 is to x

Now multiply the two inside numbers (800 and 1) and the two outside numbers (400 and x).

400 is to 800 as 1 is to x

400x = 800

x = 2

This is really the same as "cross multiplying," but this way of setting up the problem might be easier for you to remember.

Sample Questions

1. A package of 6 tomato seedlings costs $3.25. How much would 3 dozen seedlings cost?
   a. $6.75
   b. $39.00
   c. $58.50
   d. $19.50

2. One steel rod measures $4\frac{3}{16}$ inches long and another steel rod measures $3\frac{3}{4}$ inches long. Together their length is
   a. $7\frac{3}{8}$ inches
   b. $7\frac{15}{16}$ inches
   c. $8\frac{1}{16}$ inches
   d. $7\frac{7}{8}$ inches

3. Men’s socks cost $8.00 for a package of three. How much would two dozen socks cost?
   a. $64.00
   b. $32.00
   c. $48.00
   d. $80.00

4. A pole casts a shadow 20 feet long. Another pole 3 feet high casts a shadow 5 feet long. If the heights and shadows are in proportion, how high is the first pole?
   a. 10 feet
   b. 12 feet
   c. 15 feet
   d. 16 feet

5. It costs $360 for Office X's service contract with a photocopier company to service 18 copiers for six months. At this same rate, how much would it cost Office X to service six copiers for three months?
   a. $80
   b. $75
   c. $90
   d. $60

6. Between 2002 and 2005, the average price of a home in Pleasantville went up from $150,000 to $165,000. What was the percentage increase in the price of a home?
   a. 10%
   b. 15%
   c. 5%
   d. 20%

7. A lab can run 126 tests in 14 days. If it continues to run tests at this rate, how many can it run in 30 days?
   a. 256
   b. 290
   c. 248
   d. 270

8. A fax machine and a printer cost a total of $840. If the printer costs $360 more than the fax machine, how much did the fax machine cost?
   a. $480
   b. $440
   c. $240
   d. $280

9. A coat that originally cost $135 is on sale at 20% off. How much does the coat cost?
    a. $115
    b. $108
    c. $112
    d. $100

10. If a sofa costs $640 after a 20% discount, how much did it originally cost?
    a. $768
    b. $512
    c. $800
    d. $780

11. If one of every eight junior year students at a high school takes French, approximately what percent of a junior year class of 650 takes French?
    a. 6
    b. 14
    c. 81
    d. 13

12. If the sum of two numbers is 280, and their ratio is 7:3, then the smaller number is:
    a. 28
    b. 84
    c. 56
    d. 196

13. An alloy consists of copper and tin in the ratio of 10:1 by weight. If an object made of this alloy weighs 66 pounds, how many pounds of tin does it contain?
    a. 6.6
    b. 10
    c. 6
    d. 12

14. If a per diem worker earns $1,050 dollars in 16 days, the amount he will earn in 115 days is most nearly

   a. $10,540

   b. $7,547

   c. $17,333

   d. $8,942

15. On a blueprint, $\frac{1}{4}$ inch equals 1 foot. How long is a wall that is represented by a

   $3\frac{1}{2}$ inch line on the blueprint?

   a. 14 feet

   b. 24 feet

   c. 1.4 feet

   d. 16 feet

16. Al, Bob, and Chris invest $3,600, $2,400, and $1,200 respectively. They make $900 on the investment, and they divide their profits in proportion to the amount each person invested. Bob uses his share of the profits to pay off a debt of $185. How much of his share does he have left?

   a. $70

   b. $115

   c. $40

   d. $35

17. If the outer diameter of a metal pipe is 2.76 inches and the inner diameter is 2.34 inches, the thickness of the metal is

   a. .21 inches

   b. 5.1 inches

   c. .42 inches

   d. .124 inches

18. To the nearest cent, 345 bolts at $4.15 per hundred will cost

    a. $.14

    b. $1.43

    c. $14.32

    d. $143.20

19. Alex can paint a room in 4 hours, and Frank can paint the same room in 7 hours. How long will it take them to paint the room if they work together?

    a. 4.4 hours

    b. 3.2 hours

    c. 5.2 hours

    d. 2.4 hours

20. A car travels 50 miles an hour and a plane travels 10 miles a minute. How far will the car travel when the plane travels 500 miles?

    a. 50.4 miles

    b. 37.5 miles

    c. 41.6 miles

    d. 39.7 miles

21. If Kevin must have a score of 80% to pass a test of 35 questions, the largest number of questions he can miss and still pass the test is

    a. 7

    b. 8

    c. 11

    d. 28

22. Maria bought a TV set on sale for 20% off the list price. She paid $237.60. What was the list price?

    a. $297.00

    b, $285.20

    c. $277.68

    d. $190.08

23. A box contains 400 coins. Of these 10 percent are dimes, 30 percent are nickels, and the rest are quarters. The amount of money in the box is

a. less than $75

b. between $75 and $150

c. between $151 and $225

d. more than $225

24. Four gallons of fuel that contains 80% gasoline and 20% ethanol are mixed with five gallons of fuel that contains 90% gasoline and 10% ethanol. What is the percentage of ethanol in the mixture?

a. 14.4%

b, 15%

c. 16%

d. 15.2%

25. At Jefferson High School, 46 percent of the students are boys. On Tuesday 80% of the boys were present. There were 736 boys present on Tuesday. What is the total enrollment of the school?

a. 2,400

b. 2,000

c. 1,880

d. 2,012

26. The number of half-pound packages of rice that can be weighed out of a container that holds $9\frac{1}{2}$ pounds of rice is

a. 18

b. 28

c. 19

d. 21

27. Mary and Alice jog three miles each evening. If they run at a constant rate and it takes Mary 40 minutes while Alice finishes in half an hour, how much distance does Mary have left when Alice finishes?

   a. 1 mile

   b. 3/4 mile

   c. 2/3 mile

   d. 1.33 miles

28. Karen drives her car until the gas gauge is down to $\frac{1}{8}$ full. Then she fills the tank by adding 14 gallons. What is the capacity of the gas tank?

   a. 14 gallons

   b. 15 gallons

   c. 16 gallons

   d. 17 gallons

29. A lab technician uses one third of the alcohol in a bottle on Monday and three fourths of the remainder on Tuesday. What fraction of the original contents of the bottle is left at the end of the day Tuesday?

   a. 5/12

   b. 7/12s

   c. 1/6

   d. 1/2

30. If 10 kilometers equal 6.2 miles, then how many miles are there in 35 kilometers?

   a. 31

   b. 12.2

   c. 20

   d. 21.7

31. Jerry bought a book, a calculator, a poster, and a DVD player. The DVD player cost four times what the poster cost. The calculator cost three times what the poster cost. The book cost twice what the poster cost. Jerry paid a total of $160 for all four items. How much did the book cost?

a. $16

b. $8

c. $32

d. $24

32. If $16\frac{1}{2}$ feet equal 1 rod, how many feet are there in 4 rods?

a. 22

b. 66

c. 792

d. 2,376

33. On a map, 1 centimeter represents 70 kilometers. Two cities 245 kilometers apart would be separated on the map by how many centimeters?

a. 2.5

b. 3.5

c. 4.5

d. 120

34. How many paintings were displayed at the Hamilton Gallery if 20% of them were by Picasso and Picasso was represented by 22 paintings?

a. 44

b. 110

c. 88

d. 98

35. If Jean’s weekly income doubled, she would be making $120 more than Barbara. Jean’s weekly income is $80 more than half of Betty’s. Betty makes $400 a week. How much does Barbara make?

    a. $280
    b. $400
    c. $440
    d. $560

36. A conference with 3600 participants gathers in Albany. One out of every twelve people attending the conference who have ordered meals have special dietary needs. Half of those attending the conference signed up for meals. How many have special dietary needs?

    a. 266
    b. 133
    c. 150
    d. 300

37. Catherine bought an equal number of $60, $50, and $40 tickets for a concert. She spent $600 for all of the tickets. How many of each did she buy?

    a. 12
    b. 4
    c. 6
    d. cannot be determined from the information given.

38. A gardener combines ingredients x, y, and z in a ratio of 1:2:7 to produce a special fertilizer. How many pounds of the second ingredient, y, are needed to make 12 pounds of fertilizer?

    a. 2.4
    b, 8.4
    c. 1.2
    d. 3.6

39. The attendance at a weekly training program in the month of January averaged 116 people. If there were 105 people attending the first week, 106 the second, and 125 the third, how many people attended the fourth week?

    a. 118

    b, 128

    c. 130

    d. 124

40. The population of Metro County in 2005 was 130% of its population in 2000. The population in 2000 was 145,000. What was the population in 2005?

    a. 196,425

    b, 174,612

    c. 111,539

    d. 188,500

Answer Key

1.	d	11.	d	21.	a	31.	a
2.	b	12.	b	22.	a	32.	b
3.	a	13.	c	23.	a	33.	b
4.	b	14.	b	24.	a	34.	a
5.	d	15.	a	25.	b	35.	c
6.	a	16.	b	26.	c	36.	c
7.	d	17.	b	27.	b	37.	b
8.	c	18.	c	28.	c	38.	a
9.	b	19.	d	29.	c	39.	c
10.	c	20.	c	30.	d	40.	d

Answers and Explanations

1. The answer is choice d. There are a few ways to approach this problem. Perhaps the simplest is to calculate how many packages of 6 tomato seedlings you would need to buy to have 3 dozen seedlings. You would need 2 packages to have a dozen seedlings, so to have 3 dozen you would need 3 times 2. So you would need 6 packages. Now multiply the number of packages times the price per package:

   6 x \$3.25 = \$19.50

2. The answer is choice b. This problem involves adding "mixed numbers." A mixed number is a combination of a whole number and a fraction. One way to approach a problem like this is to add the whole numbers and the fractions separately. In this problem you would add 4 and 3 and then add the fractions $\frac{3}{16}$ and $\frac{3}{4}$.

   In order to add these fractions, you need to find a common denominator. One way to do this is to see if one denominator can be divided evenly into the other. You see that 4 can be divided evenly into 16, so you can use 16 as a common denominator.

   To convert $\frac{3}{4}$ to a fraction with a denominator of 16, multiply both the numerator and the denominator by 4

   $$\frac{3}{4} \times \frac{4}{4} = \frac{12}{16}$$

   Now you can add the fractions.

   $$\frac{3}{16} + \frac{12}{16} = \frac{15}{16}$$

   The whole numbers add up to 7 and the fractions add up to $\frac{15}{16}$, so the answer is $7\frac{15}{16}$.

3. The answer is choice a. This question is similar to question 1. Begin by calculating how many packages of 3 you would need to have 2 dozen. You would need 4 packages of 3 to have 1 dozen, so you would need 8 packages of 3 to have 2 dozen. Now multiply the number of packages times the cost per package.

   8 x \$8.00 = \$64.00

4. The answer is choice b. This is a ratio problem. In ratio language, you could state the problem like this:

   3 is to 5 as x is to 20

   The letter x is used to represent the number you are trying to find. You can set up this ratio as an equation.

   $$\frac{3}{5} = \frac{x}{20}$$

   To find x, you cross multiply. You multiply 5 times x and 3 times 20. You get:

   $$5x = 60$$

   To solve for x, divide both sides of the equation by 5.

   $$\frac{5x}{5} = \frac{60}{5}$$

   $$x = 12$$

   Another way to solve this problem is to write the ratio in sentence form:

   3 is to 5 as x is to 20

   Now multiply the two inside numbers (5 and x) and the two outside numbers (3 and 20).

   3 is to 5 as x is to 20

   $$5x = 60$$

   $$x = 12$$

   This is really the same as "cross multiplying," but this way of setting up the problem might be easier for you to remember.

5. The answer is choice d. The simplest way to solve this problem is to begin by figuring out how much it costs to service one copier for six months. If it costs $360 to service 18 copiers, then it costs $20 for each copier.

   $$\$360 \div 18 = \$20$$

   The question asks about six copiers, so multiply $20 times six. It would cost $120 to service six copiers for six months. But the question asks about three months, not six. In other words, half as long. If we divide $120 by two, we see that it would cost $60 to service six copiers for three months.

6. The answer is choice a. This is a fairly common type of percentage problem. If a problem asks about percent increase or decrease, you first have to find the amount of the increase or decrease. If the average price of a home went up from $150,000 to $165,000, that is an increase of $15,000.

   Now to find the percent increase, divide the amount of the increase by the original amount. $15,000 divided by $150,000 is .1. To convert a decimal to a percent, we move the decimal point two places to the right. So .1 equals 10%.

7. The answer is choice d. The easiest way to approach this problem is to determine how many tests the lab runs per day. It runs 256 tests in 14 days, so to determine how many it runs per day, just divide 256 by 14. The lab can run 9 tests per day. Now to see how many tests it can run in 30 days, just multiply 9 times 30. The answer is 270.

8. The answer is choice c. Let's set this problem up as an equation. We'll say that the cost of the fax machine is x. The printer costs $360 more than the fax machine, so it costs x + $360.

   So now we have:

   fax machine = x

   printer = x +360

   Together the fax machine and the printer cost $840, so:

   x + x + 360 = 840

   or

   2x + 360 = 840

   To solve this problem, subtract 360 from both sides of the equation. You are left with:

   2x = 480

   Now to find x, divide both sides of the equation by 2, and you get

   x = 240

9. The answer is choice b. To solve a problem involving percentages, convert the percentage to a decimal. You do this by moving the decimal point two places to the left.

   20% = .2

   Now multiply .20 times the original price:

   $$.20 \times \$135 = \$27$$

   This is how much you save. Subtract $27 from $135, the original price, and you find that the coat costs $108.

10. The answer is choice c. The sofa is on sale at 20% off. This means that the sale price is 80% of the original price. You could set this up as a ratio:

$$\frac{640}{x} = \frac{80}{100}$$

When you cross multiply, you get

$$80x = 64000$$

To find x, divide both sides of the equation by 80.

$$x = 800$$

11. The answer is choice d. This question contains a piece of information that is meant just as a distraction — the number of students in the junior class. This number has nothing to do with the question.

You are told that 1 in 8 students in the junior class takes French. In other words, $\frac{1}{8}$ of the students take French.

Begin by converting this fraction to a decimal. To convert a fraction to a decimal, divide the numerator by the denominator.

$$\frac{1}{8} = .125$$

To convert .125 to a percentage, move the decimal point two places to the right.

$$.125 = 12.5\%$$

The question asks for an *approximate* percentage, so the answer is 13%.

12. The answer is choice b. This is a question about ratios. Two numbers add up to 280. The ratio of the two numbers is 7:3. So the larger number is 7/10 of 280 and the smaller number is 3/10 of 280.

To find the smaller number, multiply 3/10 times 280:

$$\frac{3}{10} \times \frac{280}{1} = \frac{840}{10} = 84$$

13. The answer is choice c. This is another ratio problem. Let's use x to represent the amount of tin in this object. The ratio of copper to tin is 10:1, so the amount of copper in the object is 10x. We know that the amount of copper plus the amount of tin equals 66 pounds. So

$$10x + x = 66$$

or

$$11x = 66$$

To find x, divide both sides of the equation by 11.

$$x = 6$$

14. The answer is choice b. You could set this problem up as a ratio, but it might be simpler just to calculate how much the worker makes per day and then multiply this amount times the number of days worked.

The worker earns $1,050 in 16 days. Divide $1,050 by 16 and you get $65.625. Multiply this amount by 115 days and, after rounding, you get the answer, $7,547.

15. The answer is choice a. This is another ratio problem, but it is a little more complicated because it involves fractions. You could set up the problem this way:

$$\frac{1}{4} \text{ is to 1 as } 3\frac{1}{2} \text{ is to x.}$$

Multiply the two inside numbers and the two outside numbers and you get

$$\frac{1}{4}x = 3\frac{1}{2}$$

Now to find x, you have to divide both sides of the equation by $\frac{1}{4}$. If you divide $\frac{1}{4}x$ by $\frac{1}{4}$, you get x. But how do you divide $3\frac{1}{2}$ by $\frac{1}{4}$?

First convert the mixed number $3\frac{1}{2}$ to a fraction.

$$3\frac{1}{2} = \frac{7}{2}$$

Now to divide fractions, invert the divisor (the number you are dividing by) and *multiply*.

$$\frac{7}{2} \div \frac{1}{4} \text{ is the same as } \frac{7}{2} \times \frac{4}{1}$$

$$\frac{7}{2} \times \frac{4}{1} = \frac{28}{2}$$

$$\frac{28}{2} = 14$$

16. The answer is choice b. To answer this question, you need to know Bob's share of the profits. The profits are divided in proportion to how much each person invested. Begin by calculating the total investment.

$3,600	Al
$2,400	Bob
+ $1,200	Chris
$7,200	Total

Now set up a ratio to see what portion of the profits Bob received.

$$\frac{2400}{7200} = \frac{x}{900}$$

Now cross multiply.

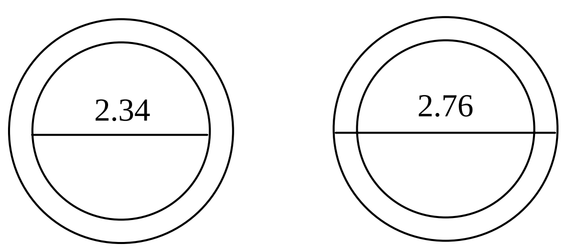
$$7200x = 2{,}160{,}000$$

To find x, divide both sides of the equation by 7200.

$$x = 300$$

Bob's share of the profits was $300. He used $185 to pay off a loan ($300 – $185), so he has $115 left.

17. The answer is choice b. This is basically a subtraction problem. To find the thickness of the metal, first subtract the inner diameter from the outer diameter.



$$\begin{array}{r} 2.76 \\ -\ 2.34 \\ \hline .42 \end{array}$$

Now if you study the diagram on the previous page, you will see that .42 represents ***two*** sections of the pipe. To find the answer, divide .42 by 2.

$$.42 \div 2 = .21$$

18. The answer is choice c. This is a ratio problem. You could set it up like this:

$$\frac{4.15}{100} = \frac{x}{345}$$

When you cross multiply you get 100x = 1431.75. To find x, divide both side of the equation by 100. You get x = 14.3175. To the nearest cent, this is $14.32.

19. The answer is choice d. This is a difficult problem. Go through this solution one step at a time until you're sure you know how to solve it.

One way to approach a problem like this is to think about what part of the job each person can do in an hour.

If Alex can paint the room in 4 hours, then in 1 hour he can do 1/4 of the job.

If Frank can paint the room in 6 hours, then in 1 hour he can do 1/6 of the job.

Let's find a common denominator for these two fractions. One way to find a common denominator is to multiply the two denominators.

$$4 \times 6 = 24$$

Now convert 1/4 and 1/6 to fractions with a denominator of 24. You do this by multiplying the numerator and denominator of each fraction by the same number.

To convert 1/4 to a fraction with a denominator of 24, multiply the numerator and the denominator by 6:

$$\frac{1}{4} = \frac{6}{24}$$

To convert 1/6 to a fraction with a denominator of 24, multiply the numerator and the denominator by 4:

$$\frac{1}{6} = \frac{4}{24}$$

Now to find out how much of the job the two men can do in an hour working together, add the two fractions:

$$\frac{6}{24} + \frac{4}{24} = \frac{10}{24}$$

Now we know that in one hour the two men can do $\frac{10}{24}$ of the job.

The whole job can be shown as $\frac{24}{24}$. If the men do 24 twenty-fourths of the job, they have done the whole job.

To find how long it will take the two men to do the whole job, divide the whole job by the amount they can do in an hour:

$$\frac{24}{24} \div \frac{10}{24} = ?$$

To divide fractions, invert the divisor, $\frac{10}{24}$, and multiply:

$$\frac{24}{24} \times \frac{24}{10} = \frac{576}{240}$$

The answers are expressed as decimals. To convert this fraction to a decimal, divide 576 by 240:

$$576 \div 240 = 2.4$$

So the two men working together can complete the job in 2.4 hours.

20. The answer is choice c. The first step in solving this problem is to find how long it takes the plane to travel 500 miles. The plane travels 10 miles per minute. So to find how many minutes it will take for the plane to travel 500 miles, divide 500 by 10.

$$500 \div 10 = 50$$

It will take the plane 50 minutes to travel 500 miles. We know that the car travels 50 miles in 60 minutes. So how far can it travel in 50 minutes? Let's set this up as a ratio:

$$\frac{50}{60} = \frac{x}{50}$$

If we cross multiply, we get

$$60x = 2500$$

To find x, divide both sides of the question by 60

$$\frac{60x}{60} = \frac{2500}{60}$$

$$x = 41.6$$

21. The answer is choice a. This question is basically asking, what is 80% of 35? Begin by converting 80% to a decimal. You do this by moving the decimal point two places to the left:

$$80\% = .80$$

Now multiply .80 times 35.

$$.80 \times 35 = 28$$

Kevin must answer at least 28 questions right to pass the test. There are 35 questions on the test, so the largest number of questions he can miss is 7.

22. The answer is choice a. You can set up this problem as a ratio. Maria paid $237.60 for a TV. This was 20% off the list price, so in effect she paid 80% of the list price. As a ratio this is

$$\frac{237.60}{x} = \frac{80}{100}$$

The letter x here represents the list price. When you cross multiply, you get

$$80x = 23760$$

To find x, divide both sides of the equation by 80.

$$x = 297$$

23. The answer is choice a. This question might seem puzzling at first, but it is really not difficult.

You know that 10% of the 400 coins are dimes. Convert 10% to a decimal.

$$10\% = .10$$

and multiply .10 times 400. There are 40 dimes. Each dime is worth $.10, so the value of the dimes is $4.00.

You know that 30% of the 400 coins are nickels. Convert 30% to a decimal.

$$30\% = .30$$

and multiply .30 times 400. There are 120 nickels. Each nickel is worth $.05, so the value of the nickels is $6.00.

The remaining 60% of the 400 coins are quarters. Convert 60% to a decimal.

$$60\% = .60$$

and multiply .60 times 400. There are 240 quarters. Each quarter is worth $.25, so the value of the quarters is $60.00.

When you add the values of the nickels, dimes, and quarters, you get $70.

24. The answer is choice a. Once you understand the first step in this question, the rest is fairly easy. Begin by calculating how many gallons of ethanol are in each type of fuel.

The first type of fuel has 20% ethanol. Convert 20% to a decimal

$$20\% = .20$$

and multiply .20 times 4 gallons

$$.20 \times 4 \text{ gallons} = .8 \text{ gallons}$$

The second type of fuel has 10% ethanol. Convert 10% to a decimal

$$10\% = .10$$

and multiply .10 times 5 gallons

$$.10 \times 5 \text{ gallons} = .5 \text{ gallons}$$

Now add the amount of ethanol in the two types of fuel:

$$.8 \text{ gallons} + .5 \text{ gallons} = 1.3 \text{ gallons}$$

What percent of 9 gallons is 1.3 gallons? In this type of question, you divide the part by the whole.

$$1.3 \div 9 = .144$$

Convert this decimal to a percent by moving the decimal point two places to the right.

$$.144 = 14.4\%$$

25. The answer is choice b. Begin by calculating how many boys there are in the school. We know that 736 is 80% of the total number of boys. So you can set up this ratio:

$$\frac{736}{x} = \frac{80}{100}$$

When you cross multiply, you get

$$80x = 73600$$

To find x, divide both sides of the equation by 80

$$\frac{80x}{80} = \frac{73600}{80}$$

$$x = 920$$

There are 920 boys in the school. Now to answer the question, you need to set up another ratio. At this school, 46 percent of the students are boys. So you can write

$$\frac{920}{x} = \frac{46}{100}$$

When you cross multiply you get

$$46x = 92000$$

To find x, divide both sides of the equation by 46.

$$\frac{46x}{46} = \frac{92000}{46}$$

$$x = 2000$$

The total enrollment of the school is 2000.

26. The answer is choice c. This question is fairly simple. Each pound of rice makes two half-pound packages. So 9 pounds of rice make 18 packages. In fact you have $9\frac{1}{2}$ pounds of rice, so this will make 19 packages.

27. The answer is choice b. The first step in solving this problem is to calculate Mary's speed. To find her speed, divide the distance she travels, 3 miles, by her time, 40 minutes:

$$3 \div 40 = .075$$

So her speed is .075 miles per minute. We know that when Alice finishes, Mary still has 10 minutes left to run. so multiply 10 minutes times her speed:

$$10 \times .075 = .75$$

So after 30 minutes, Mary still has .75 or 3/4 of a mile left to run.

28. The answer is choice c. Karen drives her car until the gas tank is $\frac{1}{8}$ full. This means that it is $\frac{7}{8}$ empty. She fills it by adding 14 gallons. In other words, 14 gallons is $\frac{7}{8}$ of the tank's capacity. Use the letter x to represent the capacity of the tank. Then

$$\frac{7}{8}x = 14$$

To find x, divide both side of the equation by $\frac{7}{8}$. When you divide $\frac{7}{8}$ by $\frac{7}{8}$, you get 1. But how do you divide 14 by $\frac{7}{8}$?

Remember, to divide fractions, invert the divisor and *multiply*.

$$14 \div \frac{7}{8} \text{ is the same as } 14 \times \frac{8}{7}$$

$$\frac{14}{1} \times \frac{8}{7} = \frac{112}{7}$$

$$\frac{112}{7} = 16$$

29. The answer is choice c. The technician uses $\frac{1}{3}$ of the alcohol in a bottle on Monday. So on Tuesday there is $\frac{2}{3}$ of the original amount. He uses $\frac{3}{4}$ of the remainder on Tuesday, so $\frac{1}{4}$ of the remainder is left. To find what fraction of the original amount this is, multiply $\frac{2}{3}$ times $\frac{1}{4}$

$$\frac{2}{3} \times \frac{1}{4} = \frac{2}{12}$$

$$\frac{2}{12} = \frac{1}{6}$$

30. The answer is choice d. This is a fairly simple ratio problem:

$$\frac{6.2}{x} = \frac{10}{35}$$

When you cross multiply, you get

$$10x = 217$$

To find x, divide both sides of the equation by 10.

$$x = 21.7$$

31. The answer is choice a. The cost of everything in this question is related to the cost of a poster. Let's use x to represent the cost of a poster. Then 2x is the cost of a book, 3x is the cost of a calculator, and 4x is the cost of a DVD player. The cost of all four items is $160.

We can express these amounts in an equation.

$$x + 2x + 3x + 4x = 160$$

When we add the x's, we get

$$10x = 160$$

To find x, divide both sides of the equation by 10.

$$x = 16$$

This is the cost of a poster. But the question asks about the cost of the book. The book costs twice as much as the poster, so the book costs $32.

32. The answer is choice b. This is a multiplication problem. Begin by converting the mixed number $16\frac{1}{2}$ to a fraction with a denominator of 2. To find the numerator of this fraction, multiply 16 times 2 and add the numerator in the mixed number, 1.

$$16\frac{1}{2} = \frac{33}{2}$$

There are $\frac{33}{2}$ feet in a rod. To find how many feet there are in 4 rods, multiply $\frac{33}{2}$ by 4

$$\frac{33}{2} \times \frac{4}{1} = \frac{132}{2}$$

$$\frac{132}{2} = 66$$

33. The answer is choice b. This is another ratio problem. You can set it up this way.

$$\frac{1}{x} = \frac{70}{245}$$

When you cross multiply, you get

$$70x = 245.$$

To find x, divide both sides of the equation by 70.

$$\frac{70x}{70} = \frac{245}{70}$$

$$x = 3.5$$

34. The answer is choice a. This is another ratio problem. Use x to represent the total number of paintings.

$$\frac{22}{x} = \frac{20}{100}$$

When you cross multiply, you get

$$20x = 2200$$

To find x, divide both sides of the equation by 20.

$$\frac{20x}{20} = \frac{2200}{20}$$

$$x = 110$$

35. The answer is choice c. This kind of question is difficult unless you break it down into parts and solve it step-by-step. The question asks about Barbara's salary, but we can see that we can't find Barbara's salary until we find Jean's.

Jean's weekly salary is $80 more than half of Betty's. We can write this in equation form like this:

Jean = $80 + 1/2 Betty.

We are told that Betty makes $400, so now the equation is:

Jean = $80 + 1/2 ($400)

or

Jean = $80 + $200.

So Jean’s salary is $280. Now we have to figure Barbara’s salary. The first sentence tells us that if Jean’s weekly salary doubled, she would be making $120 more per week than Barbara. You can write this in equation form like this:

2 x Jean = Barbara + $120

Two times Jean’s salary of $280 would be $560. So

$560 = Barbara plus $120

$560 minus $120 is $440, and that is Barbara’s salary.

36. The answer is choice c. Here are the facts you are given. There are 3600 people attending the conference, but only half of the 3600 have ordered meals. When you divide 3600 by 2, you see that 1800 people have ordered meals.

Now 1 in 12, or one twelfth, of these 1800 people have special dietary needs. So to find out how many have special dietary needs, multiply 1800 times one twelfth.

$$\frac{1}{12} \times 1800$$

The first step in multiplying numbers like these is to convert all the numbers to fractions. 1800 is the same as 1800 over 1.

$$\frac{1}{12} \times \frac{1800}{1}$$

The top numbers in these fractions are called the numerators. The bottom numbers are called the denominators. To multiply fractions, you multiply the numerators by each other and the denominators by each other.

$$\frac{1}{12} \times \frac{1800}{12} = \frac{1800}{12}$$

When you divide 1800 by 12, you get 150, and that’s the answer.

37. The answer is choice b. Let’s set up this problem as an equation. Use x to represent the number of tickets Catherine bought at each price level.

60x + 50x + 40x = 600

or

150x = 600

To find x, divide both sides of the equation by 150:

$$\frac{150x}{150} = \frac{600}{150}$$
$$x = 4$$

38. The answer is choice a. Notice that the numbers in this ratio (1:2:7) add up to 10. Ingredient x represents 1/10 of the mixture. Ingredient y represents 2/10. Ingredient z represents 7/10.

The part represented by y, 2/10, can be written in decimal form as .2. To find out how many pounds of ingredient y will be in 12 pounds of fertilizer, multiply .2 times 12

.2 x 12 = 2.4

39. The answer is choice c. This is a question about averages. We are told that the average attendance at four weekly training sessions was 116. So the total attendance for all four sessions was 4 times 116.

4 x 116 = 464 total attendance

Now calculate the number of people who attended the sessions during the first three weeks.

105	week 1
106	week 2
+ 125	week 3
336	total for first three weeks

We know that the total attendance at all four sessions was 464. So to find how many attended the session in week 4, subtract the number who attended the first three sessions from the total attendance.

$$\begin{array}{r} 464 \\ -\ 336 \\ \hline 128 \end{array}$$

40. The answer is choice d. This is another percentage question. The population of Metro County in 2005 was 130% of its population in 2000. Its population in 2000 was 145,000. So its population in 2005 was 130% times 145,000.

Convert 130% to a decimal by moving the decimal point two places to the left.

130% = 1.30

Now multiply 1.30 times 145,000.

1.30 x 145,000 = 188,500