Fractions
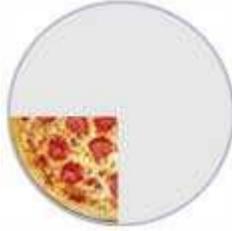
A fraction is a part of a whole

Slice a pizza, and you will have fractions:
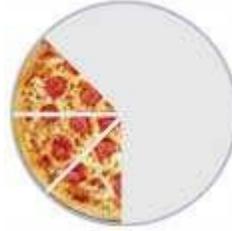
$^1/_2$

(One-Half)

$^1/_4$

(One-Quarter)

$^3/_8$

(Three-Eighths)

The top number tells how many slices you **have**
The bottom number tells how many slices the pizza was **cut into**.

Numerator / Denominator

We call the top number the **Numerator**, it is the number of parts you have.
We call the bottom number the **Denominator**, it is the number of parts the whole is divided into.
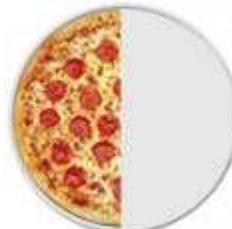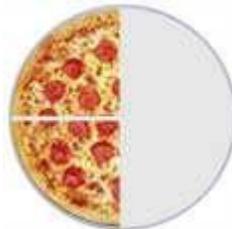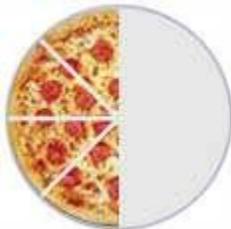
Numerator

Denominator

You just have to remember those names! (If you forget just think "Down"-ominator)

Equivalent Fractions

Some fractions may look different, but are really the same, for example:

$^4/_8$ = $^2/_4$ = $^1/_2$

(Four-Eighths)   Two-Quarters)   (One-Half)

=   =

It is usually best to show an answer using the simplest fraction ($^1/_2$ in this case). That is called *Simplifying*, or *Reducing* the Fraction

Equivalent Fractions

Equivalent Fractions have the same value, even though they may look different.

These fractions are really the same:

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

Why are they the same? Because when you multiply or divide **both** the top and bottom by the same number, the fraction keeps it's value.
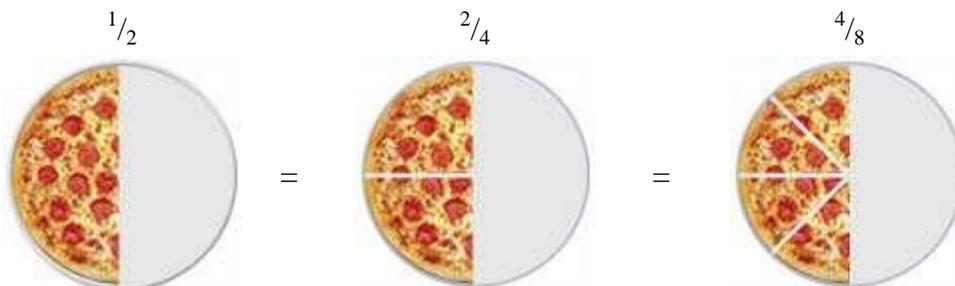
The rule to remember is:

What you do to the top of the fraction
you must also do to the bottom of the fraction !

So, here is why those fractions are really the same:

$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

And visually it looks like this:

Dividing

Here are some more equivalent fractions, this time by dividing:

$$\frac{18}{36} = \frac{6}{12} = \frac{1}{2}$$

÷ 3 ÷ 6

÷ 3 ÷ 6

If we keep dividing until we can't go any further, then we have **simplified** the fraction (made it as simple as possible).

Important:

- The top and bottom of the fraction must always be a **whole number**.
- So, what you divide by must **divide evenly** (ie no remainders) for both the top and bottom numbers.
- You only multiply or divide, **never add or subtract**, to get an equivalent fraction.

Simplifying Fractions

*To simplify a fraction, divide the top and bottom by the **highest number** that can divide into both numbers exactly.*

Simplifying Fractions

Simplifying (or *reducing*) fractions means to make the fraction as simple as possible. Why say four-eighths ($^4/_8$) when you really mean half ($^1/_2$) ?

$^4/_8$ ==> $^2/_4$ ==> $^1/_2$

(Four-Eighths)   (Two-Quarters)   (One-Half)
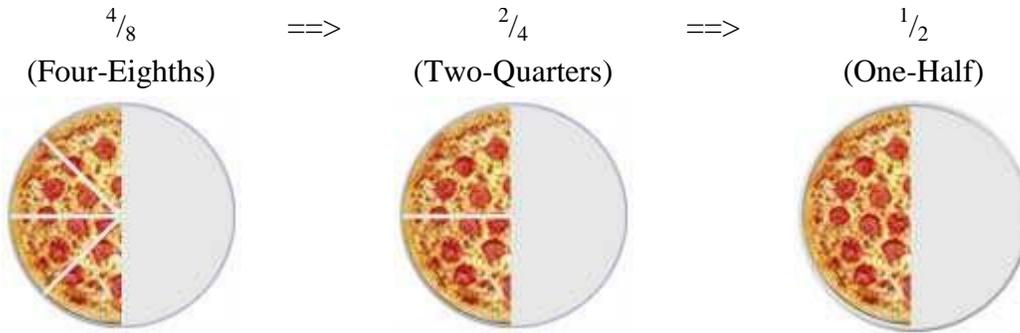
How do I Simplify a Fraction ?

There are two ways to simplify a fraction:

Method 1

Try dividing both the top and bottom of the fraction until you can't go any further (try dividing by 2,3,5,7,... etc).

Example: Simplify the fraction $^{24}/_{108}$:

$$\frac{24}{108} \xrightarrow{\div 2} = \frac{12}{54} \xrightarrow{\div 2} = \frac{6}{27} \xrightarrow{\div 3} = \frac{2}{9}$$

Method 2

Divide both the top and bottom of the fraction by the Greatest Common Factor, (you have to work it out first!).

Example: Simplify the fraction $^8/_{12}$:

1. The largest number that goes exactly into both 8 and 12 is 4, so *the Greatest Common Factor is 4*.

2. Divide both top and bottom by 4:

$$\div 4$$

$$\frac{8}{12} = \frac{2}{3}$$

$$\div 4$$

And the answer is: $^2/_3$

Decimals

A Decimal Number (*based on the number 10*) contains a **Decimal Point.**

Place Value

To understand decimal numbers you must first know about Place Value.

When we write numbers, the **position** (or "**place**") of each number is important.

In the number 327:

- the "7" is in the **Units** position, meaning just 7 (or 7 "1"s),
- the "2" is in the **Tens** position meaning 2 tens (or twenty),
- and the "3" is in the **Hundreds** position, meaning 3 hundreds.

Units (1s)
Tens (10s)
Hundreds (100s)

327

100s 10s 1s

10x Bigger
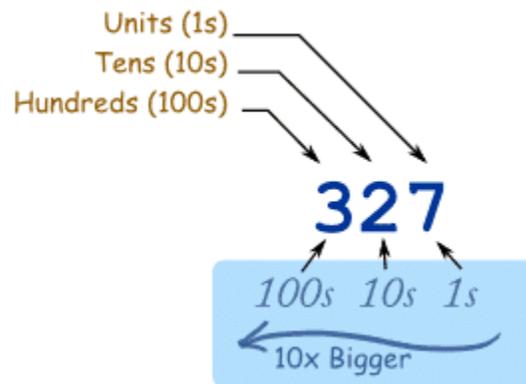
"Three Hundred Twenty Seven"

As we move left, each position is 10 times bigger!

From Units, to Tens, to Hundreds

... and ...

As we move right, each position is 10 times **smaller**.

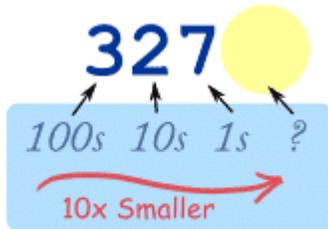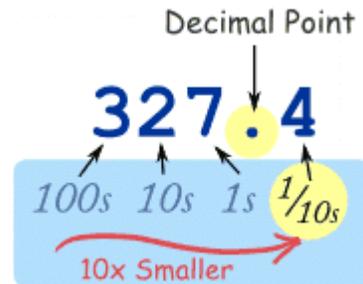From Hundreds, to Tens, to Units

327

100s 10s 1s **?**

10x Smaller

But what if we continue past Units?

What is **10 times smaller** than Units?

$^1/_{10}$ ths (Tenths) are!

But we must first write a **decimal point**,
so we know exactly where the Units position is:

Decimal Point

327.4

100s 10s 1s $^1/_{10}s$

10x Smaller

"three hundred twenty seven **and four tenths**"

but we usually just say "three hundred twenty seven **point four**"
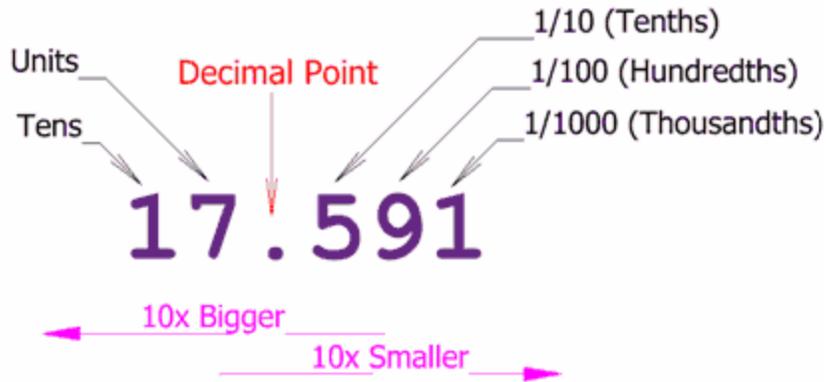
And **that** is a Decimal Number!

Decimal Point

The **decimal point** is the most important part of a Decimal Number. It is exactly to the right of the Units position. Without it, we would be lost ... and not know what each position meant.

Now we can continue with smaller and smaller values, from **tenths**, to **hundredths**, and so on, like in this example:

Large and Small

So, our Decimal System lets us write numbers as large or as small as we want, using the decimal point. Numbers can be placed to the left or right of a decimal point, to indicate values greater than one or less than one.

17.591

The number to the left of the decimal point is a whole number (17 for example)

As we move further left, every number place gets **10 times bigger**.

The first digit on the right means **tenths** (1/10).

As we move further right, every number place gets **10 times smaller** (one tenth as big).
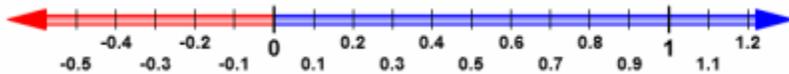
Definition of Decimal

The word "Decimal" really means "based on 10" (From Latin *decima*: *a tenth part*).

We sometimes say "decimal" when we mean anything to do with our numbering system, but a "Decimal Number" usually means there is a Decimal Point.

Ways to think about Decimal Numbers ...

... as a Whole Number Plus Tenths, Hundredths, etc

You could think of a decimal number as a whole number plus tenths, hundredths, etc:

Example 1: What is 2.3 ?

- On the left side is "2", that is the whole number part.
- The 3 is in the "tenths" position, meaning "3 tenths", or 3/10
- So, 2.3 is "2 and 3 tenths"

Example 2: What is 13.76 ?

- On the left side is "13", that is the whole number part.
- There are two digits on the right side, the 7 is in the "tenths" position, and the 6 is the "hundredths" position
- So, 13.76 is "13 and 7 tenths and 6 hundredths"

... as a Decimal Fraction

Or, you could think of a decimal number as a Decimal Fraction.

A Decimal Fraction is a fraction where the denominator (the bottom number) is a number such as 10, 100, 1000, etc (in other words a [power of ten](#))

So "2.3" would look like this: $\dfrac{23}{10}$

And "13.76" would look like this: $\dfrac{1376}{100}$

... as a Whole Number and Decimal Fraction

Or, you could think of a decimal number as a Whole Number plus a Decimal Fraction.
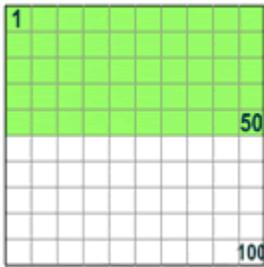
So "2.3" would look like this: $2 \text{ and } \dfrac{3}{10}$

And "13.76" would look like this: **13 and** $\frac{76}{100}$

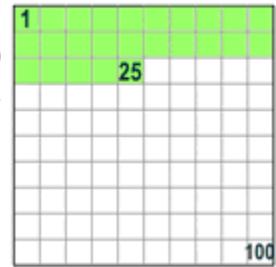Those are all good ways to think of decimal numbers.

Percentages (%)

When you say "Percent" you are really saying "per 100"

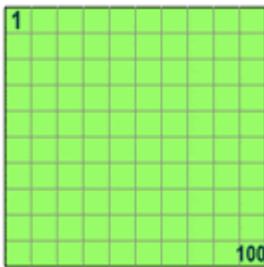So **50%** means 50 per 100
(50% of this box is green)

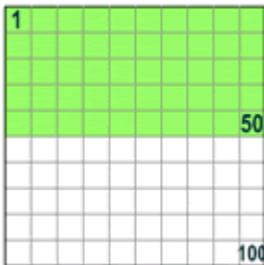And **25%** means 25 per 100
(25% of this box is green)

Examples:

100% means all.

Example:

100% of **80** is $^{100}/_{100} \times 80 = 80$

50% means half.

Example:

50% of **80** is $^{50}/_{100} \times 80 = 40$
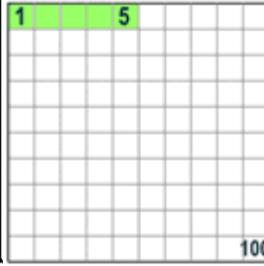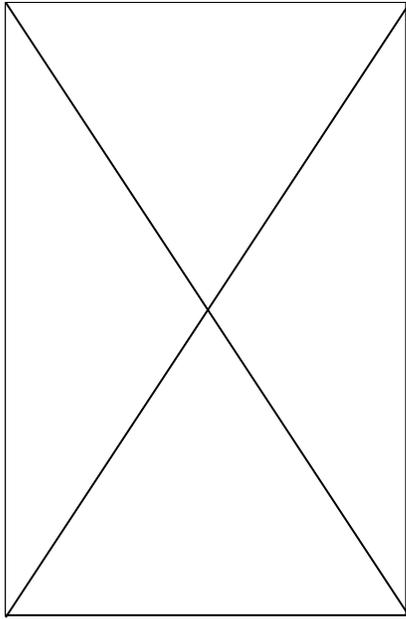
5% means $^5/_{100\text{ths}}$.

Example:

5% of **80** is $^5/_{100} \times 80 = 4$

Using Percent

Because "Percent" means "per 100" you should think "this should always be divided by 100"

So **75%** really means $^{75}/_{100}$

And **100%** is $^{100}/_{100}$, or exactly **1** (100% of any number is just the number, unchanged)

And **200%** is $^{200}/_{100}$, or exactly **2** (200% of any number is twice the number)

Use the slider on the left and try some different numbers (example, what is 40% of 80?)

A Percent can also be expressed as a *Decimal* or a *Fraction*

A Half can be written...

As a percentage: 50%
As a decimal: 0.5
As a fraction: $^1/_2$

Example: Calculate 25% of 80

$25\% = {}^{25}\!/_{100}$

$({}^{25}\!/_{100}) \times 80 = 20$

So 25% of 80 is **20**

Example: 15% of 200 apples were bad. How many apples were bad?

$15\% = {}^{15}\!/_{100}$

$({}^{15}\!/_{100}) \times 200 = 15 \times 2 = 30$ apples

30 apples were bad

Example: if only 10 of the 200 apples were bad, what percent is that?

As a fraction, $10/200 = 0.05$

As a percentage it is: $(10/200) \times 100 = 5\%$

5% of those apples were bad

Example: A Skateboard is reduced 25% in price in a sale. The old price was $120. Find the new price

First, find 25% of $120:

$25\% = {}^{25}\!/_{100}$

$({}^{25}\!/_{100}) \times \$120 = \$30$

25% of $120 is $30

So the **reduction** is $30

Take the reduction from the original price

$120 - $30 = $90

The Price of the Skateboard in the sale is **$90**

The Word

"Percent" comes from the latin *Per Centum*. The latin word *Centum* means 100, for example a Century is 100 years.

Percent vs Percentage

My Dictionary says "Percentage" is the "result obtained by multiplying a quantity by a percent". So 10 **percent** of 50 apples is 5 apples: the 5 apples is the **percentage**.

But in practice people use both words the same way.

<u>Working with Decimals, Fractions and Percentages</u>

Decimals, Fractions and Percentages are just different ways of showing the same value:

A Half can be written...

As a fraction:	$^1/_2$
As a decimal:	0.5
As a percentage:	50%

A Quarter can be written...

As a fraction:	$^1/_4$
As a decimal:	0.25
As a percentage:	25%

Example Values

Here is a table of commonly occurring values shown in Percent, Decimal and Fraction form:

Percent	Decimal	Fraction
1%	0.01	$^1/_{100}$
5%	0.05	$^1/_{20}$
10%	0.1	$^1/_{10}$
12½%	0.125	$^1/_8$
20%	0.2	$^1/_5$
25%	0.25	$^1/_4$
$33^1/_3$%	0.333...	$^1/_3$
50%	0.5	$^1/_2$
75%	0.75	$^3/_4$
80%	0.8	$^4/_5$
90%	0.9	$^9/_{10}$
99%	0.99	$^{99}/_{100}$
100%	**1**	
125%	1.25	$^5/_4$
150%	1.5	$^3/_2$
200%	2	

Conversions

> **From Percent to Decimal**

To convert from percent to decimal: divide by 100, and remove the "%" sign.

The easiest way to divide by 100 is to **move the decimal point 2 places to the left**. So:

From Percent

To Decimal

75%   0.7 5.   0.75   move the decimal point **2 places to the left**, and remove the "%" sign.

2 Places

> **From Decimal to Percent**

To convert from decimal to percent: multiply by 100, and add a "%" sign.

The easiest way to multiply by 100 is to **move the decimal point 2 places to the right**. So:

From Decimal   To Percent

0.125   0.12.5   12.5%   move the decimal point **2 places to the right**, and add the "%" sign.

> ➢ **From Fraction to Decimal**

The easiest way to convert a fraction to a decimal is to divide the top number by the bottom number (divide the numerator by the denominator in mathematical language)

Example: Convert $^2/_5$ to a decimal

Divide 2 by 5: $2 \div 5 = 0.4$

Answer: $^2/_5 = 0.4$

> ➢ **From Decimal to Fraction**

To convert a decimal to a fraction needs a little more work.

Example: To convert 0.75 to a fraction

Steps	Example
First, write down the decimal "over" the number 1	$^{0.75}/_1$
Then multiply top and bottom by 10 for every number after the decimal point (10 for 1 number, 100 for 2 numbers, etc)	$^{0.75 \times 100}/_{1 \times 100}$
(This makes it a correctly formed fraction)	$= ^{75}/_{100}$
Then Simplify the fraction	$^3/_4$

> ➢ **From Fraction to Percentage**

The easiest way to convert a fraction to a percentage is to divide the top number by the bottom number. then multiply the result by 100, and add the "%" sign.

Example: Convert $^3/_8$ to a percentage

First divide 3 by 8: $3 \div 8 = 0.375$,
Then multiply by 100: $0.375 \times 100 = 37.5$
Add the "%" sign: 37.5%

Answer: $^3/_8 = 37.5\%$

➢ **From Percentage to Fraction**

To <u>convert a percentage to a fraction</u>, first convert to a decimal (divide by 100), then use the steps for converting decimal to fractions (like above).

Example: To convert 80% to a fraction

Steps	Example
Convert 80% to a decimal (=80/100):	0.8
Write down the decimal "over" the number 1	$^{0.8}/_1$
Then multiply top and bottom by 10 for every number after the decimal point (10 for 1 number, 100 for 2 numbers, etc)	$^{0.8 \times 10}/_{1 \times 10}$
(This makes it a correctly formed fraction)	$= ^8/_{10}$
Then simplify the fraction	$^4/_5$