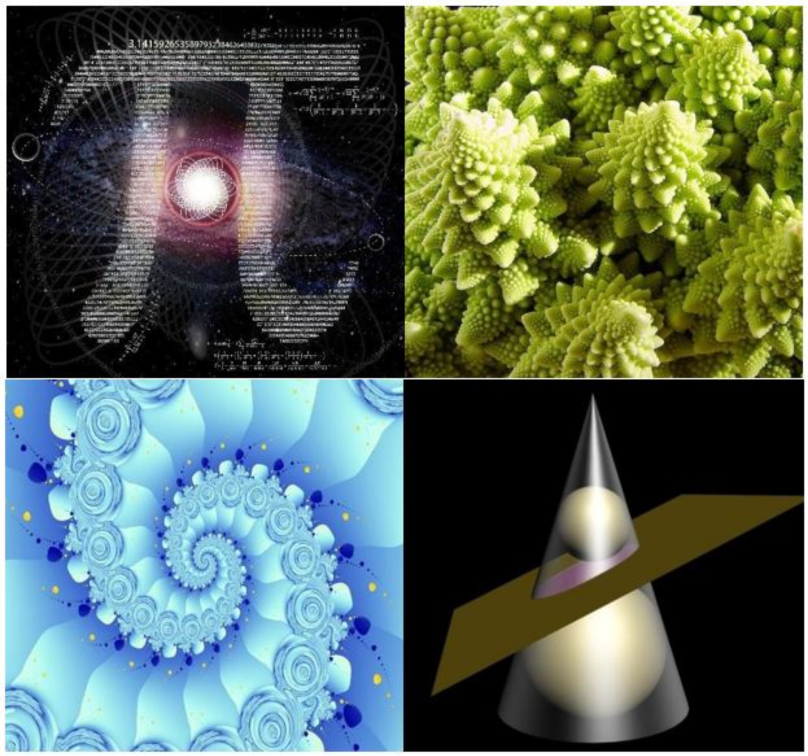
Washington State

Bridge to College Mathematics Course

Unit 6. Exponential Functions and an Introduction to Logarithms



Adapted from Math Ready
A Southern Regional Education Board Transition Course

Unit 6. Exponential Functions and an Introduction to Logarithms

Overview

PURPOSE

In this unit, students will experience exponential functions through a real-world lens of finance. Beginning with an overall look into financial decisions they will face as adults, students study the mathematics involved in purchasing a car, planning for retirement and even deciding on a job. They will extend their understanding of inverse functions as they explore logarithms as inverses to exponential functions and use logarithms to solve problems involving exponential growth and decay.

For Bridge to College course overview, refer to traditional Bridge to College course material.

Special Statement

The resources that follow were developed, adapted or compiled by more than 80 Washington Bridge to College Math teachers to support continuous learning during long term school closure determined necessary because of the COVID-19 pandemic. They are identified as Electronic or Printable resources to provide the most equitable access possible for students with varying levels of technology and internet capability. All possible steps were taken to compile meaningful and accurate resources and to attribute them appropriately, but if, in order to offer them in a timely fashion for immediate use, any errors exist, please notify the Mathematics Office at OSPI by email to mathematics@k12.wa.us.

We recognize that the collaborative problem solving structure developed in Bridge classrooms all year may not be possible at this time, but encourage teachers to seek ways to promote discussion and mutual supports between students using technology, email or phone calls.

This document currently contains Lesson 1, Unit 6 with a recommendation that this learning extends for approximately one week. Lesson 2 will be added within that week and the remaining Lessons 3 through 6 will follow shortly thereafter. Lesson 7, as a Capstone, will be left the discretion of Bridge to College teachers for the most suitable format and content for their students.

LESSON PROGRESSION OVERVIEW

Lesson Big Idea	Lesson Details	Content Standards	Standards for Mathematical Practice
Lesson 2: Properties of Exponents	This lesson is intended to help you assess how well students are able to interpret exponential and linear functions. Students will translate between descriptive, algebraic and tabular data, and graphical representation of the functions and will also be asked to recognize how, and why, a quantity changes per unit interval. To achieve these goals students work on simple and compound interest problems.	A-SSE.1 A-SSE.1a A-SSE.1b A-SSE.2 A-SSE.3 A-SSE.3a A-SSE.3b A-SSE.3c A-SSE.4 F-LE.1 F-LE.1a F-LE.1b F-LE.1c	MP 2 MP 3 MP 7

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Exponential Functions and an Introduction to Logarithms

Lesson 2 of 7

DESCRIPTION

In this lesson, students will explore two Big Ideas:

- Linear and exponential growth
- Properties of exponents

To explore and develop these ideas, students will:

- Dive deeper into linear and exponential growth through multiple representations (graphs, tables, similarities and differences)
- Develop a deeper understanding of properties of exponents

Lesson Key	
Blue Table	Electronic Resources
Orange Table	Printable (Paper) Resources

COMMON CORE STATE STANDARDS ADDRESSED

- A-SSE.1: Interpret expressions that represent a quantity in terms of its context.
  - A-SSE.1a: Interpret parts of an expression, such as terms, factors, and coefficients
  - A-SSE.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*
- A-SSE.2: Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$*, thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.
- A-SSE.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
  - A-SSE.3a: Factor a quadratic expression to reveal the zeros of the function it defines.

- A-SSE.3b: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
- A-SSE.3c: Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

- A-SSE.4: Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*[*]
- F-LE.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - F-LE.1a: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
  - F-LE.1b: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
  - F-LE.1c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MATHEMATICAL PRACTICE STANDARD(S) EMPHASIZED

- MP 2: Reason abstractly and quantitatively.
- MP 3: Construct viable arguments and critique the reasoning of others.
- MP 7: Look for and make use of structure.

Sequence of Instruction	Activities Checklist

LESSON

Electronic Resources

Activity Description	Links
Students will: • Be able to calculate simple and compound interest • Be able to identify interest rate, principal, and time via tables, graphs, verbal descriptions, and algebraically • Be able to differentiate between simple and compound interest problems given different problem representations (tabular, graphical, algebra, verbal description)	Unit 6 Lesson 2 Card Sort Desmos Version
Students will: • Be able to apply the rules of exponents: (e.g. multiplication, division, power of a power, power of a product, and power of a fraction exponent rules, and the negative and fractional exponent rules)	Task 5 Desmos version

Printable (Paper) Resources

Activity Description	Link
Students will: • Understand simple vs compound interest • Understand how to write a simple interest linear equation • Understand how to write compound interest linear equation • Find similarities between linear and exponential graphs tables and equations • Understand when two situations have the same growth rate • Understand how growth rates affect the tables and graphs	Printable Resource: Intro – Simple and Compound Interest Printable Resource: Sorting Activity with Answers and Reflection Questions
Students will: • Develop a deeper understanding of the properties of exponents	Printable Resource: Exponents & Task 5

ATTRIBUTION AND LICENSE INFORMATION

This work has been created in partnership with expert mathematics educators from across Washington state. We express our gratitude to all the contributors to this effort. Without their support and expertise, this resource would not be possible.

We also thank the following sources

Desmos | Desmos, Inc. *linked only*
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Mathematics Assessment Project | Mathematics Assessment Resource Service, University of Nottingham
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PRINTABLE RESOURCES

Simple and Compound Interest

Odd One Out?

Investment 1
$100
Simple Interest Rate: 5%

Investment 2
$400
Simple Interest Rate: 5%

Investment 3
$200
Simple Interest Rate: 10%

Projector Resources   Representing Linear and Exponential Growth   P-1

1. For the three investments above, which is the odd one out? Explain your reasoning.

2. Look at the other two investments. Can you decide why each of them could be the odd one out? Explain why.

3. Write each of the investments as an algebraic equation using ***A*** as the amount in the bank and ***n*** as the time in years. Assign ***r*** as the interest rate (remember to convert the % to a decimal!) and assigning ***P*** as the amount invested.

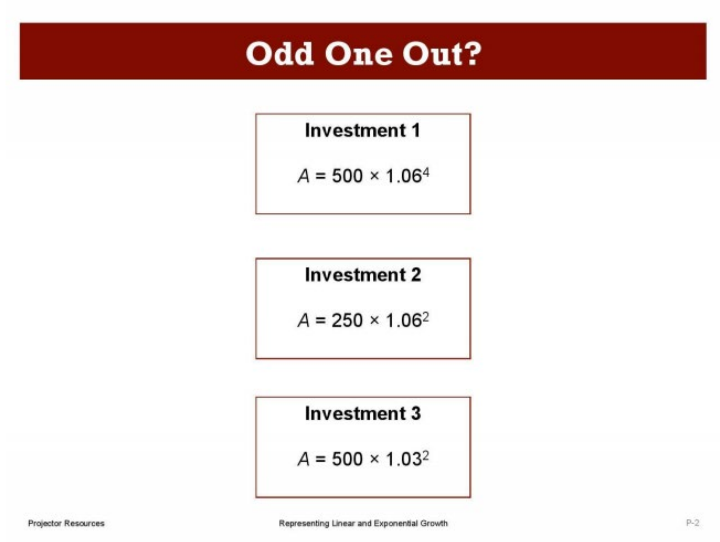
$$A = P + rn$$

Write your formulas in the table and use them to complete it.

	Investment 1	Investment 2	Investment 3
Algebraic Formula			
Value after 5 years			
Length of time to double*			
Verification of doubling^			

* To find the length of time to double—let ***A*** be equal to twice the amount invested and solve for ***n.***

^ To verify the time to double, show that the time slightly less than your answer is just less than double the investment (or that time just longer is a bit more than double).



4. These investments represent compound interest. How can you tell?

5. For each, identify the investment amount, the interest rate and the time.

	Investment amount	Interest Rate (as a %)	Time invested
1			
2			
3			

6. Which is the odd one out? Explain your reasoning.

7. Why might the other two investments be the odd one out? Explain why.

8. Write the general form for each investment (***n*** represents the time) in the table and use the formulas to complete it.

	Investment 1	Investment 2	Investment 3
Algebraic Formula			
Value after 5 years			
Length of time to double*			
Verification of doubling^			

9. What do you notice about compound interest versus simple interest?

Sorting Activity

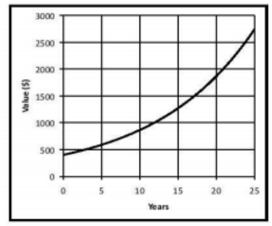
Card Set: Investment Plans and Formulas

P1 Investment: $400 Simple Interest Rate: 16%	P2 Investment: $400 Compound Interest Rate: 2%
P3 Investment: $400 Simple Interest Rate: 8%	P4 Investment: $200 Compound Interest Rate: 2%
P5 Investment: $400 Compound Interest Rate: 8%	P6 Investment: $400 Simple Interest Rate: 2%

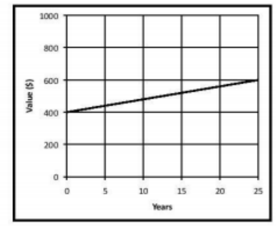
F1 $A = 400 \times 1.08^n$	F2 $A = 400 + 32n$
F3 $A = 400 \times 1.02^n$	F4 $A = 400 + 8n$
F5	F6

Card Set: Graphs

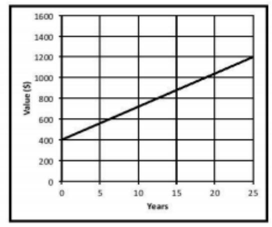
G1



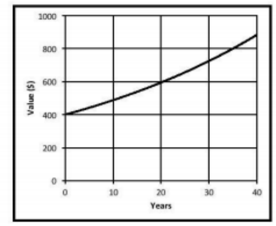
G2



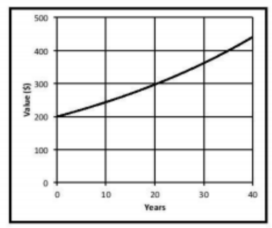
G3



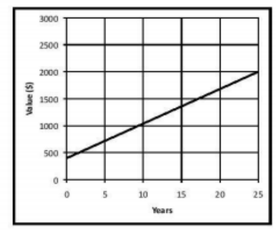
G4



G5



G6



Card Set: Tables

T1

Years	Value ($)
0	400.00
1	432.00
2	466.56
3	
4	544.20
5	587.73

T2

Years	Value ($)
0	200.00
1	204.00
2	
3	212.24
4	216.49
5	220.82

T3

Years	Value ($)
0	400.00
1	408.00
2	
3	424.00
4	432.00
5	440.00

T4

Years	Value ($)
0	400.00
1	408.00
2	
3	424.48
4	432.97
5	441.63

T5

Years	Value ($)
0	400.00
1	432.00
2	464.00
3	
4	528.00
5	560.00

T6

Years	Value ($)
0	400.00
1	464.00
2	
3	592.00
4	656.00
5	720.00

Card Set: Statements

S1 These two investments will take the same time to double your money.	S2 This investment will double your money in 12 years 6 months.
S3 This investment gives the worst return for your money over two years or more.	S4 This investment is the best one over 10 years.
S5 This investment is the best one over 20 years.	

1. What do you notice about the common characteristics of the simple interest graphs (G2, G3, and G6)?

2. What do you notice about the common characteristics of the compound interest graphs (G1, G4, and G5)?

3. What do you notice about the common characteristics of the simple interest equations (F2, F4, and F6)?

4. What do you notice about the common characteristics of the compound interest equations (F1, F3, and F5)?

Try the card sort matching a one card from each set (Plans, Formulas, Graphs, Tables, Statements) that show the same investment a different way . You can cut them out to match them up (like we do in class) or use a color code to show which go together. There is a key at the end of this document but give the sort a try and see how far you can get first.

5. What is the difference between the compound interest tables and the simple interest tables?

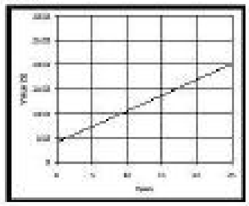
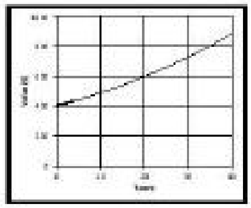
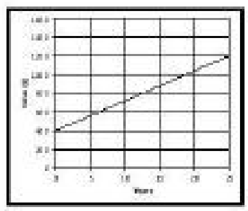
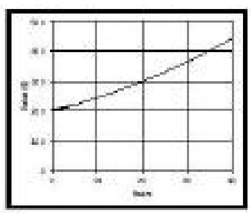
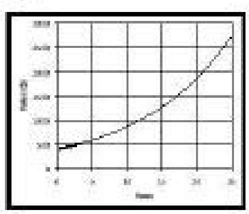
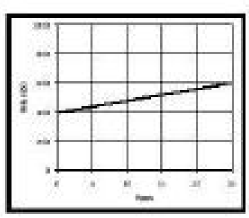
6. In row P1 where do you see the initial amount and interest rate in the equation, table, and graph?

7. In row P5 where do you see the initial amount and interest rate in the equation, table, and graph?

8. Why is P5 the best over 20 years? Justify your answer.

9. Will P2 and P4 double in the same amount of time? Justify your answer.

Answers for card sort. Did you try it on your own first? If you got stuck, take a hint here and try again!

<table>
<tr>
<td>P1
Investment: $400
Simple Interest
Rate: 16%</td>
<td>F6
$A = 400 + 64n$</td>
<td>G6
</td>
<td>T6
<table><tr><th>Years</th><th>Value</th></tr><tr><td>0</td><td>400.00</td></tr><tr><td>1</td><td>464.00</td></tr><tr><td>2</td><td>528.00</td></tr><tr><td>3</td><td>592.00</td></tr><tr><td>4</td><td>656.00</td></tr><tr><td>5</td><td>720.00</td></tr></table></td>
<td>S4
This investment is the best one over 10 years.</td>
</tr>
<tr>
<td>P2
Investment: $400
Compound
Interest Rate: 2%</td>
<td>F3
$A = 400 \times 1.02^n$</td>
<td>G4
</td>
<td>T4
<table><tr><th>Years</th><th>Value</th></tr><tr><td>0</td><td>400.00</td></tr><tr><td>1</td><td>408.00</td></tr><tr><td>2</td><td>416.16</td></tr><tr><td>3</td><td>424.48</td></tr><tr><td>4</td><td>432.97</td></tr><tr><td>5</td><td>441.63</td></tr></table></td>
<td>S1 (P2 and P4)
These two investments will take the same time to double your money.</td>
</tr>
<tr>
<td>P3
Investment: $400
Simple Interest
Rate: 8%</td>
<td>F2
$A = 400 + 32n$</td>
<td>G3
</td>
<td>T5
<table><tr><th>Years</th><th>Value</th></tr><tr><td>0</td><td>400.00</td></tr><tr><td>1</td><td>432.00</td></tr><tr><td>2</td><td>464.00</td></tr><tr><td>3</td><td>496.00</td></tr><tr><td>4</td><td>528.00</td></tr><tr><td>5</td><td>560.00</td></tr></table></td>
<td>S2
This investment will double your money in 12 years 6 months.</td>
</tr>
<tr>
<td>P4
Investment: $200
Compound
Interest Rate: 2%</td>
<td>F5
$A = 200 \times 1.02^n$</td>
<td>G5
</td>
<td>T2
<table><tr><th>Years</th><th>Value</th></tr><tr><td>0</td><td>200.00</td></tr><tr><td>1</td><td>204.00</td></tr><tr><td>2</td><td>208.08</td></tr><tr><td>3</td><td>212.24</td></tr><tr><td>4</td><td>216.49</td></tr><tr><td>5</td><td>220.82</td></tr></table></td>
<td>S1 (P2 and P4)
These two investments will take the same time to double your money.</td>
</tr>
<tr>
<td>P5
Investment: $400
Compound
Interest Rate: 8%</td>
<td>F1
$A = 400 \times 1.08^n$</td>
<td>G1
</td>
<td>T1
<table><tr><th>Years</th><th>Value</th></tr><tr><td>0</td><td>400.00</td></tr><tr><td>1</td><td>432.00</td></tr><tr><td>2</td><td>466.56</td></tr><tr><td>3</td><td>503.88</td></tr><tr><td>4</td><td>544.20</td></tr><tr><td>5</td><td>587.73</td></tr></table></td>
<td>S5
This investment is the best one over 20 years.</td>
</tr>
<tr>
<td>P6
Investment: $400
Simple Interest
Rate: 2%</td>
<td>F4
$A = 400 + 8n$</td>
<td>G2
</td>
<td>T3
<table><tr><th>Years</th><th>Value</th></tr><tr><td>0</td><td>400.00</td></tr><tr><td>1</td><td>408.00</td></tr><tr><td>2</td><td>416.00</td></tr><tr><td>3</td><td>424.00</td></tr><tr><td>4</td><td>432.00</td></tr><tr><td>5</td><td>440.00</td></tr></table></td>
<td>S3
This investment gives the worst return for your money over two years or more.</td>
</tr>
</table>

Exponents and Task 5

Rules and Properties of Exponents Reference

Expression	Algebraic Formula	Numerical Example
Product of Powers	$a^n \cdot a^m = a^{n+m}$	$2^3 \cdot 2^4 = 2^{3+4} = 128$
	$a^n \cdot b^n = (a \cdot b)^n$	$3^2 \cdot 4^2 = (3 \cdot 4)^2 = 144$
Quotient of Powers	$\frac{a^n}{a^m} = a^{n-m}$	$\frac{2^5}{2^3} = 2^{5-3} = 4$
	$\frac{a^n}{b^n} = (\frac{a}{b})^n$	$\frac{4^3}{2^3} = (4/2)^3 = 8$
Power of a Power	$(b^n)^m = b^{n.m}$	$(2^3)^2 = 2^{3.2} = 64$
	$b^{n^m} = b^{(n^m)}$	$2^{3^2} = 2^{(3^2)} = 512$
	$\sqrt[m]{b^n} = b^{\frac{n}{m}}$	$\sqrt[3]{2^6} = 2^{\frac{6}{3}} = 4$
	$b^{\frac{1}{n}} = \sqrt[n]{b}$	$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$
Negative exponents	$b^{-n} = \frac{1}{b^n}$	$8^{-1} = \frac{1}{8^1} = 0.125$
Zero as an exponent	$b^0 = 1$	$5^0 = 1$
	$0^n = 0$, for $n>0$	$0^5 = 0$
One as an exponent	$b^1 = b$	$5^1 = 5$
	$1^n = 1$	$1^5 = 1$

Review the Rules and Properties of Exponents Reference and use it to complete Task #5.

Task #5: Homework Mishap

Five students were checking their answers on their math homework. David said, "Oh no, we all got different answers on the first problem! Only one of us must be right!" Charisse said, "Actually, only one of is *wrong*! Four of these answers are equivalent to each other, and only one is different from the others."

The students' answers are shown below. Find which student's answer is different from the others, and show that the other four are equivalent to each other using properties of exponents.

Anna: $3^{2x} + 3^4$

Barry: $81 \cdot 9^x$

Charisse: 3^{2x+4}

David: 9^{x+2}

Enzo: $\frac{3^4 \cdot 3^x}{3^{-x}}$