Name ______________________________ Date __________ Class __________

LESSON 7-4

Reteach

Properties of Logarithms

Use properties of logarithms to simplify logarithms.

The Product Property uses addition instead of multiplication.

Product Property

The logarithm of a product can be written as the sum of the logarithm of the numbers.

$$\log_b mn = \log_b m + \log_b n$$

where m, n, and b are all positive numbers and $b \neq 1$

Simplify: $\log_8 4 + \log_8 16 = \log_8 (4 \cdot 16) = \log_8 64 = 2$

The bases must be the same for both logarithms.

Think: 8 to what power is equal to 64, or $8^{?} = 64$.

The Quotient Property uses subtraction instead of division.

Quotient Property

The logarithm of a quotient can be written as the logarithm of the numerator minus the logarithm of the denominator.

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

where m, n, and b are all positive numbers and $b \neq 1$

Simplify: $\log_3 243 - \log_3 9 = \log_3 \left(\frac{243}{9}\right) = \log_3 27 = 3$

The bases must be the same for both logarithms.

Think: 3 to what power is equal to 27, or $3^{?} = 27$.

Complete the steps to simplify each expression.

1. $\log_6 54 + \log_6 4$

$\log_6 (54 \cdot 4)$

$\log_6 216$

____

2. $\log_2 128 - \log_2 8$

$\log_2 \left(\frac{128}{8}\right)$

____

____

3. $\log_9 3 + \log_9 27$

____

____

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Name ________________________ Date ___________ Class ____________

LESSON **7-4**

Reteach

Properties of Logarithms (continued)

The Power Property uses multiplication instead of exponentiation.

Power Property

The logarithm of a power can be written as the product of the exponent and the logarithm of the base.

$$\log_b a^p = p \log_b a$$

for any real number p

where a and b are positive numbers and $b \neq 1$

Simplify: $\log_4 64^5 = 5 \log_4 64 = 5(3) = 15$

"Bring down" the exponent to multiply.

Think: 4 to what power is equal to 64, or $4^? = 64$.

Logarithms and exponents undo each other when their bases are the same.

Inverse Properties	
The logarithm of b^x to the base b is equal to x. $\log_b b^x = x$ ↑↑ The logarithm undoes the exponent when the bases are the same. Simplify: $\log_7 7^{4x} = 4x$ The base of the log is 7 and the base of the exponent is 7.	b raised to the logarithm of x to the base b is equal to x. $b^{\log_b x} = x$ ↑↑ The exponent undoes the logarithm when the bases are the same. Simplify: $3^{\log_3 64} = 64$ The base of the exponent is 3 and the base of the log is 3.

Simplify each expression.

4. $\log_5 125^2$

$2 \log_5 125$

____

5. $\log_2 16^4$

$4 \log_2 16$

____

6. $\log_9 81^3$

____

____

7. $\log_6 6^{5y}$

____

8. $4^{\log_4 75}$

____

9. $2^{\log_2 3x}$

____

LESSON 7-4

Practice A

Properties of Logarithms

Express as a single logarithm. Simplify, if possible.

1. $\log_3 9 + \log_3 27$

$\log_3 (9 \cdot 27) = \log_3 243$

$3^x = 243$, so $x =$ **4**

2. $\log_2 16 + \log_2 4$

$\log_2 (16 \cdot 4) = \log_2$ **64**

$2^x =$ **64**, so $x =$ **6**

3. $\log_5 125 + \log_5 25$

$\log_5 (125 \cdot 25) = \log_5$ **3125**

$5^x =$ **3125**, so $x =$ **5**

4. $\log_{10} 250 + \log_{10} 40$ **$\log_{10} 10{,}000 = 4$**

5. $\log_6 3 + \log_6 2$ **$\log_6 6 = 1$**

6. $\log_8 16 + \log_8 4$ **$\log_8 64 = 2$**

Express as a single logarithm. Simplify, if possible.

7. $\log_5 250 - \log_5 10$ **$\log_5 25 = 2$**

8. $\log_3 21 - \log_3 7$ **$\log_3 3 = 1$**

9. $\log_2 160 - \log_2 5$ **$\log_2 32 = 5$**

10. $\log_4 128 - \log_4 8$ **$\log_4 16 = 2$**

11. $\log_6 72 - \log_6 2$ **$\log_6 36 = 2$**

12. $\log_5 1000 - \log_5 8$ **$\log_5 125 = 3$**

Simplify, if possible.

13. $\log_6 36^2$

$2 \log_6 36$

$2 \cdot 2 =$ **4**

14. $\log_5 5^4$

$4 \log_5 5$

4

15. $\log_2 8^3$

$3 \log_2 8$

9

16. $\log_3 3^4$ **4**

17. $\log_4 64^4$ **12**

18. $\log_8 8^2$ **2**

Evaluate. Round to the nearest hundredth.

19. $\log_5 13$ **1.59**

20. $\log_3 7$ **1.77**

21. $\log_8 21$ **1.46**

Solve.

22. The Richter magnitude of an earthquake, M, is related to the energy released in ergs, E, by the formula $M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$. Find the energy released by an earthquake of magnitude 6.8. **10^{22} ergs**

LESSON 7-4

Practice B

Properties of Logarithms

Express as a single logarithm. Simplify, if possible.

1. $\log_3 9 + \log_3 27$ **$\log_3 243 = 5$**

2. $\log_2 8 + \log_2 16$ **$\log_2 128 = 7$**

3. $\log_{10} 80 + \log_{10} 125$ **$\log_{10} 10{,}000 = 4$**

4. $\log_6 8 + \log_6 27$ **$\log_6 216 = 3$**

5. $\log_3 6 + \log_3 13.5$ **$\log_3 81 = 4$**

6. $\log_4 32 + \log_4 128$ **$\log_4 4096 = 6$**

Express as a single logarithm. Simplify, if possible.

7. $\log_2 80 - \log_2 10$ **$\log_2 8 = 3$**

8. $\log_{10} 4000 - \log_{10} 40$ **$\log_{10} 100 = 2$**

9. $\log_4 384 - \log_4 6$ **$\log_4 64 = 3$**

10. $\log_2 1920 - \log_2 30$ **$\log_2 64 = 6$**

11. $\log_3 486 - \log_3 2$ **$\log_3 243 = 5$**

12. $\log_6 180 - \log_6 5$ **$\log_6 36 = 2$**

Simplify, if possible.

13. $\log_4 4^6$ **6**

14. $\log_5 5^{x-5}$ **$x - 5$**

15. $7^{\log_7 30}$ **30**

16. $12^{\log_{12} 1}$ **1**

17. $\log_8 8^5$ **5**

18. $\log_3 9^4$ **8**

Evaluate. Round to the nearest hundredth.

19. $\log_{12} 1$ **0**

20. $\log_3 30$ **3.10**

21. $\log_5 10$ **1.43**

Solve.

22. The Richter magnitude of an earthquake, M, is related to the energy released in ergs, E, by the formula $M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$. Find the energy released by an earthquake of magnitude 4.2. **$10^{18.1}$ ergs**

LESSON 7-4

Practice C

Properties of Logarithms

Express as a single logarithm. Simplify, if possible.

1. $\log_6 12 + \log_6 18$ **$\log_6 216 = 3$**

2. $\log_3 81 - \log_3 27$ **$\log_3 3 = 1$**

3. $\log_4 128 - \log_4 8$ **$\log_4 16 = 2$**

4. $\log_6 18 + \log_6 72$ **$\log_6 1296 = 4$**

5. $\log_5 3125 - \log_5 25$ **$\log_5 125 = 3$**

6. $\log_8 128 + \log_8 256$ **$\log_8 32{,}768 = 5$**

7. $\log_5 5 + \log_5 125$ **$\log_5 625 = 4$**

8. $\log_2 256 - \log_2 64$ **$\log_2 4 = 2$**

9. $\log_3 8019 - \log_3 99$ **$\log_3 81 = 4$**

10. $\log_8 80 + \log_8 51.2$ **$\log_8 4096 = 4$**

11. $\log_7 13.3 - \log_7 1.9$ **$\log_7 7 = 1$**

12. $\log_{10} 125 + \log_{10} 80$ **$\log_{10} 10{,}000 = 4$**

Evaluate. Round to the nearest hundredth.

13. $\log_8 8^6$ **6**

14. $2^{\log_2 8^x}$ **8^x**

15. $\log_2 16^5$ **20**

16. $\log_3 3^{(2x+1)}$ **$2x + 1$**

17. $\log_4 16^{(x-1)}$ **$2x - 2$**

18. $5^{\log_5 17}$ **17**

19. $\log_3 5^2$ **2.93**

20. $\log_5\left(\frac{1}{125}\right)^2$ **-6**

21. $\log_6\left(\frac{1}{6^4}\right)^3$ **-12**

22. $\log_4 20^2$ **4.32**

23. $\log_9 27^4$ **6**

24. $\log_2 10$ **3.32**

Solve.

25. Carmen has a painting presently valued at \$5000. An art dealer told her the painting would appreciate at a rate of 6% per year. In how many years will the painting be worth \$8,000?

a. Write a logarithmic expression. **$\log_{1.06} 1.6$**

b. Simplify your expression. **8 years**

LESSON 7-4

Reteach

Properties of Logarithms

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The Product Property uses addition instead of multiplication.

Product Property

The logarithm of a product can be written as the sum of the logarithm of the numbers.

$$\log_b mn = \log_b m + \log_b n$$

where m, n, and b are all positive numbers and $b \neq 1$

Simplify: $\log_8 4 + \log_8 16 = \log_8 (4 \cdot 16) = \log_8 64 = 2$

The bases must be the same for both logarithms.

Think: 8 to what power is equal to 64, or $8^? = 64$.

The Quotient Property uses subtraction instead of division.

Quotient Property

The logarithm of a quotient can be written as the logarithm of the numerator minus the logarithm of the denominator.

$$\log_b \frac{m}{n} = \log_b m - \log_b n$$

where m, n, and b are all positive numbers and $b \neq 1$

Simplify: $\log_3 243 - \log_3 9 = \log_3 \left(\frac{243}{9}\right) = \log_3 27 = 3$

The bases must be the same for both logarithms.

Think: 3 to what power is equal to 27, or $3^? = 27$.

Complete the steps to simplify each expression.

1. $\log_6 54 + \log_6 4$

$\log_6 (54 \cdot 4)$

$\log_6 216$

3

2. $\log_2 128 - \log_2 8$

$\log_2 \left(\frac{128}{8}\right)$

$\log_2 16$

4

3. $\log_9 3 + \log_9 27$

$\log_9 (3 \cdot 27)$

$\log_9 81$

2

LESSON 7-4

Reteach

Properties of Logarithms (continued)

The Power Property uses multiplication instead of exponentiation.

Power Property

The logarithm of a power can be written as the product of the exponent and the logarithm of the base.

$$\log_b a^p = p \log_b a$$

for any real number p

where a and b are positive numbers and $b \neq 1$

Simplify: $\log_4 64^5 = 5 \log_4 64 = 5(3) = 15$

"Bring down" the exponent to multiply.

Think: 4 to what power is equal to 64, or $4^? = 64$.

Logarithms and exponents undo each other when their bases are the same.

Inverse Properties	
The logarithm of b^x to the base b is equal to x.	b raised to the logarithm of x to the base b is equal to x.
$\log_b b^x = x$	$b^{\log_b x} = x$
↑↑	↑↑
The logarithm undoes the exponent when the bases are the same.	The exponent undoes the logarithm when the bases are the same.
Simplify: $\log_7 7^{4x} = 4x$	Simplify: $3^{\log_3 64} = 64$
The base of the log is 7 and the base of the exponent is 7.	The base of the exponent is 3 and the base of the log is 3.

Simplify each expression.

4. $\log_5 125^2$

$2 \log_5 125$

$2 \cdot 3 = 6$

5. $\log_2 16^4$

$4 \log_2 16$

$4 \cdot 4 = 16$

6. $\log_9 81^3$

$3 \log_9 81$

$3 \cdot 2 = 6$

7. $\log_6 6^{5y}$

$5y$

8. $4^{\log_4 75}$

75

9. $2^{\log_2 3x}$

$3x$

LESSON 7-4

Challenge

Some Other Properties of Logarithms

Logarithmic properties allow simplification of expressions that otherwise would be difficult to compute. One such property is

$$\log_b \sqrt[r]{x} = \frac{\log_b x}{r}$$

which shows that the logarithm of a root is equal to the logarithm of the radicand divided by the index of the radical.

1. Evaluate $\log_{10} \sqrt{1000}$ on your calculator and then evaluate $\frac{\log_{10} 1000}{2}$ without a calculator. How do your answers compare?

Both expressions equal $\frac{3}{2}$.

2. Evaluate $\log_2 \sqrt[4]{64}$ with your calculator and with the above formula. Which is easier to compute? (Hint: You will need the Change of Base Formula for one of the calculations.)

Result is $\frac{3}{2}$; formula is easier to compute.

3. Evaluate $\log_3 \sqrt[5]{729}$ with your calculator and with the above formula. Which is easier to compute?

Result is $\frac{6}{5}$; formula is easier to compute.

Another useful property of logarithms is called the Chain Rule for Logarithms.

$$\log_a b \cdot \log_b c = \log_a c$$

4. Prove this formula by changing all the logarithms to base 10.

$$\log_a b \cdot \log_b c = \log_a c$$

$$\log_a b \cdot \log_b c = \frac{\log b}{\log a} \cdot \frac{\log c}{\log b}$$

$$= \frac{\log c}{\log a} \cdot \frac{\log b}{\log b} = \frac{\log c}{\log a} = \log_a c$$

5. Evaluate $\log_2 3 \cdot \log_3 5 \cdot \log_5 8 \cdot \log_8 13$ using your calculator and the Change of Base Formula both with and without using the above formula.

$$\log_2 13 = \frac{\log 13}{\log 2} \approx 3.7$$

6. Evaluate $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdot \log_5 6 \cdot \cdots \cdot \log_{31} 32$ both with and without using the above formula. Which is easier to compute?

$\log_2 32 = 5$; possible answer: using the Chain Rule is much easier.

LESSON 7-4

Problem Solving

Properties of Logarithms

Trina and Willow are researching information on earthquakes. One of the largest earthquakes in the United States, centered at San Francisco, occurred in 1906 and registered 7.8 on the Richter scale. The Richter magnitude of an earthquake, M, is related to the energy released in ergs, E, by the formula $M = \frac{2}{3} \log \left(\frac{E}{10^{11.8}}\right)$.

1. Find the amount of energy released by the earthquake in 1906.

a. Substitute 7.8 for magnitude, M, in the equation. $7.8 = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$

b. Solve for the value of log E. $23.5 = \log E$

c. Willow says that E is equal to 10 to the power of the value of log E. Is she correct? What property or definition can be used to find the value of E? Explain.

Yes; by the definition of logarithm; $E = 10^{23.5}$

d. Trina says the energy of the 1906 earthquake was 3.16×10^{23} ergs. Willow says the energy was $10^{23.5}$ ergs. Who is correct? How do you know?

They are both correct; $10^{23.5} = 3.16 \times 10^{23}$.

Choose the letter for the best answer.

2. An earthquake in 1811 in Missouri measured 8.1 on the Richter scale. About how many times as much energy was released by this earthquake as by the California earthquake of 1906?

[✓ **A**] 2.8
B 3.0
C 3.6
D 5.7

3. Another large earthquake in California measured 7.9 on the Richter scale. Which statement is true?

F 0.1 times as much energy was released by the larger earthquake.
[✓ **G**] The difference in energy released is 1.3×10^{23} ergs.
H The energy released by the second earthquake was 3.26×10^{23} ergs.
J The total energy released by the two earthquakes is equal to the energy released by an 8.0 earthquake.

4. Larry wrote the following: $\log 10^{0.0038} = 3.8 \times 10^{-3}$. Which property of logarithms did he use?

A Product Property
B Quotient Property
[✓ **C**] Inverse Property
D Power Property

5. Vijay wants to change $\log_5 7$ to base 10. Which expression should he use?

[✓ **F**] $\frac{\log_{10} 7}{\log_{10} 5}$
H $\frac{\log_{10} 7}{\log_5 5}$
G $\frac{\log_{10} 5}{\log_{10} 7}$
J $\frac{\log_7 5}{\log_{10} 7}$

LESSON 7-4

Reading Strategy

Identify Relationships

The inverse relationship between logarithmic and exponential functions can be used to find all the properties of logarithms. For example, to find the logarithm of a product, add the logarithms of its factors. Remember, when the base of the logarithm is not given, it is assumed to be base 10.

	Property of Exponent	Property of Logarithm
Product Property	$b^x b^y = b^{x+y}$	$\log_b (xy) = \log_b x + \log_b y$
Quotient Property	$\frac{b^x}{b^y} = b^{x-y}$	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
Power Property	$(b^x)^y = b^{x \cdot y} = b^{xy}$	$\log_b x^y = y \log_b x$
Inverse Property	$b^{\log_b x} = x$	$\log_b b^x = x$

Write true or false for each. Explain your answer using a property.

1. $\log 5 + \log 2 = 1$

True; Product Property

2. $\log_2 40 - \log_2 5 = 3$

True; Quotient Property

3. $\log_2 25^2 = 3$

False; Power Property

4. $4^{\log_4 18} = 16$

False: Inverse Property

Simplify each expression. Tell which property or properties you used.

5. $\log x^5$

$5 \log x$; Power Property

6. $\log x^2 - \log x$

$\log x$; Quotient Property

7. $\log x + \log\left(\frac{10}{x}\right)$

1; Product Property

8. $\frac{1}{2}(x^{-2}x^3)(\log x^2)$

$x \log x$; Power Property

9. $\log x^3y^5 - \log xy^3$

$2 \log xy$; Quotient Property and Power Property

10. $7(\log 0.2 + \log 50)$

7; Product Property and Inverse Property

Solve.

11. $64^x = 2^{(x+5)}$

$x = 1$