math MAMMOTH

Grade 6-B Worktext

Light Blue Series

Factoring

Fractions

Integers

Geometry

Statistics

By Maria Miller

EDITION 3.0

Contents

Chapter 9: Geometry

Chapter 10: Statistics

Foreword

Math Mammoth Grade 6-A and Grade 6-B worktexts comprise a complete math curriculum for the sixth grade mathematics studies, aligned to the Common Core Standards.

This part B covers the latter half of the 6th grade. In chapter 6, we first review prime factorization, and then study the greatest common factor and the least common multiple.

Chapter 7 first presents a thorough review of fraction operations from 5th grade (addition, subtraction, and multiplication). The topic of focus in the chapter is fraction division. Students also do some problem solving with fractions.

In chapter 8, students are introduced to integers. They plot points in all four quadrants of the coordinate plane and learn to add and subtract with negative numbers. Multiplication and division of integers are studied in 7th grade.

In geometry (chapter 9), the focus is on area of polygons. This is studied in a logical progression: first, the area of right triangles, then the area of parallelograms, then, the area of triangles, and lastly the area of polygons. We also review some topics from earlier grades (quadrilaterals, perimeter, basic drawing). Other major topics in this chapter are calculating surface area of some solids using nets, and volume of rectangular prisms with fractional edge lengths.

Lastly, in chapter 10, students study statistics. The focus idea is that of a statistical distribution. Students learn about measures of center and measures of variability. They learn how to make dot plots, historgrams, boxplots, and stem-and-leaf plots. All of these lessons are necessary to summarize and analyze distributions.

Part 6-A covers the first half of 6th grade topics: a review of the four operations, beginning algebra, decimals, ratios, and percent.

I wish you success in teaching math!

Maria Miller, the author

Chapter 6: Prime Factorization, GCF, and LCM
Introduction

The topics of this chapter belong to a branch of mathematics known as *number theory*. Number theory has to do with the study of whole numbers and their special properties. In this chapter, we review prime factorization, and study the greatest common factor (GCF) and the least common multiple (LCM).

The main application of factoring and the greatest common factor in arithmetic is in simplifying fractions, so that is why I have included a lesson on that topic. However, it is not absolutely necessary to use the GCF when simplifying fractions, and the lesson emphasizes that fact.

The concepts of factoring and the GCF are important to understand because they will be carried over to algebra, where students will factor polynomials. In this chapter, we lay a groundwork for that by factoring simple sums, such as 27 + 45, using the GCF. When factored, the sum 27 + 45 becomes 9(3 + 5).

Similarly, the main use for the least common multiple in arithmetic is with fractions, this time with the addition of fractions, and we study that topic in this chapter in connection with the LCM.

Primes are fascinating "creatures", and you can let students read more about them using the Internet resources listed below. The really important, but far more advanced, application of prime numbers is in cryptography. Some students might be interested in reading additional material on that subject—please see the list below for Internet resources.

The Lessons in Chapter 6

Helpful Resources on the Internet

Primes

An Interactive Sieve of Eratosthenes
To find all prime numbers below a given number, use a sieve of Eratosthenes to "weed out" all the composite numbers.
http://www.cut-the-knot.org/Curriculum/Arithmetic/Eratosthenes.shtml

An Another Interactive Sieve of Eratosthenes
Click on a number to remove its multiples from the grid.
http://nlvm.usu.edu/en/nav/frames_asid_158_g_3_t_1.html?open=instructions

Primes, Factors and Divisibility - Explorer at CountOn.org
Lessons explaining divisibility tests, primes, and factors.
http://www.counton.org/explorer/primes/

Prime Number Calculator
This calculator tests if a number is a prime, and tells you its smallest divisor if it is not prime.
http://www.basic-mathematics.com/prime-number-calculator.html

Prime Numbers as Building Blocks - Euclid's Greatest Discovery
A short video about the fundamental theorem of arithmetic: that each composite number has a unique prime factorization.
http://www.youtube.com/watch?v=5kl28hmhin0

The Prime Pages
Learn more about primes on this site: the largest known primes, finding primes, how many are there, and more.
http://primes.utm.edu/

Primality of 1 from Wikipedia
Discussing whether 1 should or should not be counted as a prime number.
http://en.wikipedia.org/wiki/Prime_number#Primality_of_one

Arguments for and Against the Primality of 1
http://primefan.tripod.com/Prime1ProCon.html

Prime factorization

Factorization Forest
For each number you factorize, you'll get to grow a tree in your forest! You can choose between 6 different trees, also.
http://mrnussbaum.com/forest/

Factor Trees at Math Playground
Factor numbers to their prime factors using an interactive factor tree, or find the GCF and LCM of numbers.
http://www.mathplayground.com/factortrees.html

MathGoodies Interactive Factor Tree Game
Type in a missing number to the factor tree, and the program will find the other factor, and continue drawing the tree as needed.
http://www.mathgoodies.com/factors/prime_factors.html

Free Workheets for Prime Factorization
Generate free, printable worksheets for prime factorization or for finding all the factors of a given number. Customize the worksheets in various ways (difficulty level, spacing, font size, number of problems.)
http://www.homeschoolmath.net/worksheets/factoring.php

The Cryptoclub. Using Mathematics to Make and Break Secret Codes (book)
Cryptoclub kids strive to break the codes of secret messages, and at the same time learn more and more about encrypting and decrypting. The book contains problems to solve at the end of each chapter, little tips, and historical information how cryptography has been used over the centuries. By solving the problems you can actually learn to do all of it yourself.
http://www.amazon.com/gp/product/156881223X?tag=homeschoolmat-20

Greatest common factor and least common multiple

Fruit Shoot - Greatest Common Factor
Shoot the fruit that has the greatest common factor of two given numbers. Three levels and two different speeds.
http://www.sheppardsoftware.com/mathgames/fractions/GreatestCommonFactor.htm

Fruit Shoot - Least Common Multiple
Shoot the fruit that has the least common multiple of two given numbers. Three levels and two different speeds.
http://www.sheppardsoftware.com/mathgames/fractions/LeastCommonMultiple.htm

Factors and Multiples Jeopardy Game
A jeopardy game where the questions have to do with factors, multiples, prime factorization, GCF, and LCM.
http://www.math-play.com/Factors-and-Multiples-Jeopardy/Factors-and-Multiples-Jeopardy.html

Factors, LCM, and GCF - Activity from Math Playground
Choose 'Find the prime factorization of 2 numbers, GCF, and LCM'. First, you find the prime factorization of two different numbers, using the factor tree. Once that is done, the activity shows you a Venn diagram. Drag the factors of the two numbers in the correct areas, then figure out their GFC and LCM.
http://www.mathplayground.com/factortrees.html

Free Workheets for Greatest Common Factor and Least Common Multiple
Generate free, printable worksheets for GCF and LCM. Customize the worksheets in various ways (choose number range, font size, etc.)
http://www.homeschoolmath.net/worksheets/GCF_LCM.php

Least Common Multiple Tutorial
An animated tutorial and exercises for the Least common multiple from e-learning for Kids.
http://e-learningforkids.org/Courses/EN/M1105

Factors Millionaire Game
A millionaire game where the questions have to do with factors, prime numbers, and the greatest common factor.
http://www.math-play.com/Factors-Millionaire/Factors-Millionaire.html

Greatest Common Factor - That Quiz
10-question quiz, not timed, difficulty level 5 (medium). You can also change the parameters to your liking.
http://www.thatquiz.org/tq-r/?-j2-l5-p0

Least Common Multiple - That Quiz
10-question quiz, not timed, difficulty level 5 (medium). You can also change the parameters to your liking.
http://www.thatquiz.org/tq-r/?-j4-l5-p0

GCF and LCM Quiz
10-question quiz, not timed, difficulty level 5 (medium). You can also change the parameters to your liking.
http://www.thatquiz.org/tq-r/?-j4-l5-p0

Math Problems with LCM & GCF
A quiz of 10 word problems involving the usage of the greatest common factor and the least common multiple.
http://www.funtrivia.com/playquiz/quiz2715661f17598.html

Greatest Common Factor - Activity from Glencoe
First, the activity asks you to click on all the factors of two numbers (which represent how many apples and oranges there are to bag). Then you find the GFC of them. Next, it gives you practice problems for finding all factors of a number, finding common factors of two numbers, and finding the GCF of two numbers.
http://www.glencoe.com/sites/common_assets/mathematics/mc1/cim/chapter_04/M1_06/M1_06_dev_100.html

Snowball Fight! - Least Common Multiple (LCM)
Multiple-choice questions on the LCM of two numbers - when you click the right answer, a snowball flies into a kid.
http://www.fun4thebrain.com/beyondfacts/lcmsnowball.html

Pyramid Math
This includes games for GCF, LCM, exponents, and square roots. The question to solve appears in the right, under 'example'. Choose the triangular tile with the correct answer, and drag it to the solution vase. Includes easy and hard levels, timed and non-timed versions.
http://www.mathnook.com/math/pyramidmath.html

Factors - for review

Product game
For two players; each selects a factor, computer colors the product - who gets four in row wins.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=29

Factor Feeder
Eat factors of the given number, and avoid numbers that are not factors of the given number in this Pacman-style game. Use Arrow Keys to move.
http://hoodamath.com/games/factorfeeder.php

Sliding Tile Factorization Game
Slide a number over another to capture it, if it is a factor of the other. Number 1 is only supposed to be used to capture a prime number.
http://www.visualmathlearning.com/Games/sliding_factors.html

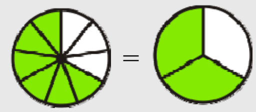
Snake
Eat factors, multiples, and prime numbers in this remake of the classic game.
http://www.spacetime.us/arcade/play.php?game=2

(This page intentionally left blank.)

Using Factoring When Simplifying Fractions

On this page, we will review simplifying fractions. Let your teacher decide if you can skip this page.

You are used to seeing the process of **simplifying fractions** like this: →



These slices have been joined together in threes.

$$\frac{6}{9} \underset{\div 3}{\overset{\div 3}{=}} \frac{2}{3}$$

In simplifying fractions, we divide both the numerator and the denominator by the same number. The fraction becomes *simpler*, which means that the numerator and the denominator are now *smaller* numbers than they were before.

However, this does NOT change the actual value of the fraction. It is the "same amount of pie" as it was before. It is just cut differently.

We can simplify a fraction only if its numerator and denominator are divisible by the same number:

We *can* simplify $\frac{25}{65}$ because both 25 and 65 are divisible by 5: →

$$\frac{25}{65} \underset{\div 5}{\overset{\div 5}{=}} \frac{5}{13}$$

We *cannot* simplify $\frac{11}{20}$ because 11 and 20 do not have any common divisors except 1.

You can simplify in multiple steps. Just start somewhere, using the divisibility tests. The goal is to simplify the fraction to the *lowest terms*. Then, the numerator and the denominator do not have any common factors.

$$\frac{42}{60} \underset{\div 2}{\overset{\div 2}{=}} \frac{21}{30} \underset{\div 3}{\overset{\div 3}{=}} \frac{7}{10}$$

$$\frac{180}{780} \underset{\div 10}{\overset{\div 10}{=}} \frac{18}{78} \underset{\div 2}{\overset{\div 2}{=}} \frac{9}{39} \underset{\div 3}{\overset{\div 3}{=}} \frac{3}{13}$$

1. Simplify the fractions to the lowest terms, if possible.

a. $\frac{12}{36}$	**b.** $\frac{45}{55}$	**c.** $\frac{15}{23}$	**d.** $\frac{13}{6}$
e. $\frac{15}{21}$	**f.** $\frac{19}{15}$	**g.** $\frac{17}{24}$	**h.** $\frac{24}{30}$

2. Simplify the fractions. Use your knowledge of divisibility.

a. $\frac{95}{100}$	**b.** $\frac{66}{82}$	**c.** $\frac{69}{99}$
d. $\frac{120}{600}$	**e.** $\frac{38}{52}$	**f.** $\frac{72}{84}$

A new notation

We will start using a new way to indicate simplifying fractions. When a numerator or a denominator gets simplified, we will *cross it out* **with a slash,** and write the *new* numerator or denominator next to it (either above or below it).

The number you divide by (the 4) does ***not*** get indicated in any way! You only think about it in your mind: "I divide 12 by 4, and get 3. I divide 20 by 4, and get 5."

You may not see any advantage over the "old" method yet, but this shortcut will come in handy soon.

$$\frac{\overset{3}{\cancel{12}}}{\underset{5}{\cancel{20}}} = \frac{3}{5}$$

$$\frac{\overset{7}{\cancel{35}}}{\underset{11}{\cancel{55}}} = \frac{7}{11}$$

3. Simplify the fractions. Write the simplified numerator and denominator above and below the old ones.

a. $\frac{14}{16}$	**b.** $\frac{33}{27}$	**c.** $\frac{12}{26}$	**d.** $\frac{9}{33}$	**e.** $\frac{42}{28}$

Using factoring when simplifying

Study carefully the example on the right where we factor 96/144.

$$\frac{96}{144} = \frac{\overset{2}{\cancel{8}} \times \overset{1}{\cancel{12}}}{\underset{3}{\cancel{12}} \times \underset{1}{\cancel{12}}} = \frac{2}{3}$$

- First, we factor (write) 96 as 8 × 12, and 144 as 12 × 12.
- Then we simplify in two steps:
  1. 8 and 12 are both divisible by 4, so they simplify into 2 and 3.
  2. 12 and 12 are divisible by 12 so they simplify into 1 and 1. Essentially, they cancel each other out.

For a comparison, the "old" way looks like this:

$$\frac{96}{144} \overset{\div 12}{=} \frac{8}{12} \overset{\div 4}{=} \frac{2}{3}$$

Let's study some more examples. (Remember that the number that you divide by is not shown.)

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \times \overset{2}{\cancel{6}}}{\underset{5}{\cancel{35}} \times \underset{1}{\cancel{3}}} = \frac{2}{5}$$

$$\frac{45}{150} = \frac{\overset{3}{\cancel{9}} \times \overset{1}{\cancel{5}}}{\underset{10}{\cancel{30}} \times \underset{1}{\cancel{5}}} = \frac{3}{10}$$

4. The numerator and the denominator have already been factored in some problems. Your task is to simplify.

a. $\frac{56}{84} = \frac{7 \times 8}{21 \times 4} =$	**b.** $\frac{54}{144} = \frac{6 \times 9}{12 \times 12} =$	**c.** $\frac{120}{72} = \frac{10 \times \boxed{}}{\boxed{} \times 9} =$
d. $\frac{80}{48} = \frac{\boxed{} \times 8}{\boxed{} \times 8} =$	**e.** $\frac{36}{90} = \frac{\qquad}{\qquad} =$	**f.** $\frac{28}{140} = \frac{\qquad}{\qquad} =$

Simplify "criss-cross"

These examples are from the previous page. This time the 45 in the numerator has been written as 5 × 9 instead of 9 × 5. We can cancel out the 5 from the numerator with the 5 from the denominator (we simplify criss-cross).

Also, we can simplify the 9 in the numerator and the 30 in the denominator criss-cross. The other example (simplifying 42/105) is similar.

This same concept is used before multiplying two fractions.

$$\frac{45}{150} = \frac{\overset{1}{\cancel{5}} \times \overset{3}{\cancel{9}}}{\underset{10}{\cancel{30}} \times \underset{1}{\cancel{5}}} = \frac{3}{10}$$

$$\frac{42}{105} = \frac{\overset{1}{\cancel{7}} \times \overset{2}{\cancel{6}}}{\underset{1}{\cancel{3}} \times \underset{5}{\cancel{35}}} = \frac{2}{5}$$

5. Simplify.

a. $\frac{14}{84} = \frac{2 \times 7}{21 \times 4} =$	**b.** $\frac{54}{150} = \frac{9 \times \square}{10 \times \square} =$	**c.** $\frac{138}{36} = \frac{2 \times \square}{\square \times 4} =$
d. $\frac{27}{20} \times \frac{10}{21} =$	**e.** $\frac{75}{90} = \frac{}{} =$	**f.** $\frac{48}{45} \times \frac{55}{64} =$

In this example, the simplification is done in two steps. In the first step, 12 and 2 are divided by 2, leaving 6 and 1. In the second step, 6 and 69 are divided by 3, leaving 2 and 23.

These two steps can also be done without rewriting the expression. The 6 and 69 are divided by 3 as before. This time we simply did not rewrite the expression in between but just continued on with the numbers 6 and 69 that were already written there.

If this looks too confusing, you do not have to write it in such a compact manner. You can rewrite the expression before simplifying it some more.

$$\frac{48}{138} = \frac{\overset{6}{\cancel{12}} \times 4}{\underset{1}{\cancel{2}} \times 69} = \frac{\overset{2}{\cancel{6}} \times 4}{1 \times \underset{23}{\cancel{69}}} = \frac{8}{23}$$

$$\frac{48}{138} = \frac{\overset{\overset{2}{\cancel{6}}}{\cancel{12}} \times 4}{\underset{1}{\cancel{2}} \times \underset{23}{\cancel{69}}} = \frac{8}{23}$$

6. Simplify the fractions to the lowest terms, or simplify before you multiply the fractions.

a. $\frac{88}{100}$	**b.** $\frac{84}{102}$	**c.** $\frac{85}{105}$
d. $\frac{8}{5} \times \frac{8}{20} =$	**e.** $\frac{72}{120}$	**f.** $\frac{104}{240}$
g. $\frac{35}{98}$	**h.** $\frac{5}{7} \times \frac{17}{15} =$	**i.** $\frac{72}{112}$

The Greatest Common Factor (GCF)

Let's say we have TWO whole numbers. We can then list all the *factors* of each number, and then find the factors that are common in both lists. Lastly, we can choose the greatest or largest among those "common factors." That is the **greatest common factor** of the two numbers. The term itself really tells you what it means!

Example 1. Find the greatest common factor of 18 and 30.

The factors of 18: 1, 2, 3, 6, 9, and 18.
The factors of 30: 1, 2, 3, 5, 6, 10, 15, and 30.

Their *common* factors are 1, 2, 3, and 6. The ***greatest* common factor** is 6.

Here is a **method** that can help you **to list all the factors of a given number**.

Example 2. Find the divisors (factors) of 36.

We check if 36 is divisible by 1, 2, 3, 4, 5, and so on. Each time we find a divisor, we write down *two* factors.

- 36 is divisible by 1. We write $36 = 1 \cdot 36$, and that equation gives us *two* factors of 36: both the smallest (**1**) and the largest (**36**).
- Next, 36 is also divisible by 2. We write $36 = 2 \cdot 18$, and that equation gives us two more factors of 36: the second smallest (**2**) and the second largest (**18**).
- Next, 36 is divisible by 3. We write $36 = 3 \cdot 12$, and now we have found the third smallest factor (**3**) and the third largest factor (**12**).
- Next, 36 is divisible by 4. We write $36 = 4 \cdot 9$, and we have found the fourth smallest factor (**4**) and the fourth largest factor (**9**).
- Finally, 36 is divisible by 6. We write $36 = 6 \cdot 6$, and we have found the fifth smallest factor (**6**) which is also the fifth largest factor.

We know that we are done because the list of factors from the "small" end (1, 2, 3, 4, 6) has **MET** the list of factors from the "large" end (36, 18, 12, 9, 6).

Therefore, all of the factors of 36 are: 1, 2, 3, 4, 6, 9, 12, 18, and 36.

1. List all of the factors of the given numbers.

a. 48	**b.** 60
c. 42	**d.** 99

2. Find the greatest common factor of the given numbers. Your work above will help!

a. 48 and 60	**b.** 42 and 48	**c.** 42 and 60	**d.** 99 and 60

3. List all of the factors of the given numbers.

a. 44	**b.** 66
c. 28	**d.** 56
e. 100	**f.** 45

4. Find the greatest common factor of the given numbers. Your work above will help!

a. 44 and 66	**b.** 100 and 28	**c.** 45 and 100	**d.** 45 and 66
e. 28 and 44	**f.** 56 and 28	**g.** 56 and 100	**h.** 45 and 28

Example 3. What is the greatest common factor useful for?

It can be used to simplify fractions. For example, let's say you know that the GCF of 66 and 84 is 6. Then, to simplify the fraction 66/84 to the lowest terms, you divide both the numerator and the denominator by 6. →

$$\frac{66}{84} \underset{\div 6}{\overset{\div 6}{=}} \frac{11}{14}$$

However, it is *not* necessary to use the GCF when simplifying fractions. You can always simplify in several steps. See the example at the right. →

$$\frac{66}{84} \underset{\div 2}{\overset{\div 2}{=}} \frac{33}{42} \underset{\div 3}{\overset{\div 3}{=}} \frac{11}{14}$$

Or, you can *simplify by factoring*, like we did in the previous lesson:

$$\frac{66}{84} = \frac{6 \cdot 11}{7 \cdot 6 \cdot 2} = \frac{11}{14}\ .$$

In fact, these other methods might be quicker than using the GCF.

5. Simplify these fractions, if possible. Your work in the previous exercises can help!

a. $\frac{48}{66}$ **b.** $\frac{42}{44}$ **c.** $\frac{42}{48}$ **d.** $\frac{99}{60}$

e. $\frac{48}{100}$ **f.** $\frac{100}{99}$ **g.** $\frac{56}{28}$ **h.** $\frac{44}{99}$

Using prime factorization to find the greatest common factor (optional)

Another, more efficient way to find the GCF of two or more numbers is to use the prime factorizations of the numbers to find *all* of the common prime factors. The product of those common prime factors forms the GCF.

Example 4. Find the GCF of 48 and 84.

The prime factorizations are: $48 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$ and $84 = 2 \cdot 2 \cdot 3 \cdot 7$.

We see that the common prime factors are 2 and 2 and 3. Therefore, the GCF is $2 \cdot 2 \cdot 3 = 12$.

Example 5. Find the GCF of 75, 105, and 125.

The prime factorizations are: $75 = 3 \cdot 5 \cdot 5$, $105 = 3 \cdot 5 \cdot 7$, and $150 = 2 \cdot 3 \cdot 5 \cdot 5$.

The common prime factors for all of them are 3 and 5. Therefore, the GCF of these three numbers is $3 \cdot 5 = 15$.

6. Find the greatest common factor of the numbers.

a. 120 and 66	**b.** 36 and 136
c. 98 and 76	**d.** 132 and 72
e. 45 and 76	**f.** 64 and 120

7. Find the greatest common factor of the given numbers.

a. 75, 25 and 90	**b.** 54, 36, and 40
c. 18, 24, and 36	**d.** 72, 60, and 48

Find the greatest common factor of 187 and 264.

Puzzle Corner

(This page intentionally left blank.)

Chapter 7: Fractions
Introduction

In this chapter, first of all, we *review* fraction arithmetic from fifth grade; namely addition, subtraction, simplification, and multiplication of fractions. Then, the focus is on the division of fractions.

In the first lesson on fraction division, we study reciprocal numbers, tying this in with the idea that "how many times does *this* number fit into *that* number" is always solved with division. For example, we can write a division from the question, "How many times does 1/3 fit into 1?" The answer is, obviously, 3 times. We can write the division 1 ÷ (1/3) = 3, and the multiplication 3 × (1/3) = 1. These two numbers, 1 and 1/3, are *reciprocal numbers* because their product is 1.

Students solve such questions using visual models, and writing division sentences that match them. The goal is the shortcut of fraction division: each division can be changed into a multiplication by the reciprocal of the divisor (also called flip and multiply rule). This is, in itself, just a shortcut. It *is* necessary to memorize it, but note that the shortcut does not help us make sense of fraction division—students also need to study the concept of division and the visual models to understand the processes.

In the next two lessons, students solve problems involving fractions or fractional parts. A lot of the problems in these lessons are review in the sense that they involve previously learned concepts and are similar to problems students have solved earlier, but some involve the division of fractions (the new concept of this chapter).

The Lessons in Chapter 7

Helpful Resources on the Internet

General

Fraction Models
Explore improper fractions, mixed numbers, decimals, and percentages. The activity includes several models: bar, area, pie, and set. Adjust numerators and denominators to see how they alter the representations of the fractions and the models.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=11

Visual Fractions
Great site for studying all aspects of fractions: identifying, renaming, comparing, addition, subtraction, multiplication, division. Each topic is illustrated by either a number line or a circle with a Java applet. Also a couple of games, for example: make cookies for Grampy.
http://www.visualfractions.com

Conceptua Math Fractions Tools
Free and interactive fraction tools for identifying fractions, adding and subtracting, estimating, comparing, equivalent fractions, multiplying, dividing, finding common denominators and more. Each activity uses several fraction models such as fraction circles, horizontal and vertical bars, number lines, etc. that allow students to develop a conceptual understanding of fractions. A free registration required.
http://www.conceptuamath.com/app/tool-library

Fraction Games at Sheppard Software
Games for addition and subtraction of fractions, simplifying fractions, equivalent fractions, and a fraction of a set.
http://www.sheppardsoftware.com/mathgames/menus/fractions.htm

Fraction Lessons at MathExpression.com
Tutorials, examples, and videos explaining all the basic fraction topics.
http://www.mathexpression.com/learning-fractions.html

Visual Math Learning
Free tutorials with some interactivity about all the fraction operations. Emphasizes visual models and lets student interact with those.
http://www.visualmathlearning.com/pre_algebra/chapter_9/chap_9.html

Online Fraction Calculator
Add, subtract, multiply, or divide fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction_calculator.php

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for the four operations with fractions and mixed numbers.
http://www.homeschoolmath.net/worksheets/fraction.php

Fraction Worksheets: Equivalent Fractions, Simplifying, Convert to Mixed Numbers
Create custom-made worksheets for some other fraction operations.
http://www.homeschoolmath.net/worksheets/fraction-b.php

Addition and Subtraction

Fraction Videos 1: Addition and Subtraction
My own videos that cover equivalent fractions, addition and subtraction of like and unlike fractions, and of mixed numbers.
http://www.mathmammoth.com/videos/fractions_1.php

MathSplat
Click on the right answer to addition problems (like fractions) or the bug splats on your windshield!
http://fen.com/studentactivities/MathSplat/mathsplat.htm

Adding Fractions
Illustrates how to find the common denominator when adding two unlike fractions using interactive pie models.
http://nlvm.usu.edu/en/nav/frames_asid_106_g_3_t_1.html

Adding and Subtracting Fractions with Uncommon Denominators Tool at Conceptua Fractions
A tool that links a visual model to the procedure of adding two unlike fractions. A free registration required.
https://www.conceptuamath.com/app/tool/adding-fractions-with-uncommon-denominators
https://www.conceptuamath.com/app/tool/subtracting-fractions-with-uncommon-denominators

Old Egyptian Fractions
Puzzles to solve: add fractions like a true Old Egyptian Math Cat!
http://www.mathcats.com/explore/oldegyptianfractions.html

Fraction Bars Blackjack
The computer gives you two fraction cards. You have the option of getting more or "holding". The object is to get as close as possible to 2, without going over, by adding the fractions on your cards.
http://fractionbars.com/Fraction_Bars_Black_Jack/

Multiplication and Division

Fraction Videos 2: Multiplication and Division
My own videos that cover multiplying and dividing fractions.
http://www.mathmammoth.com/videos/fractions_2.php

Multiply Fractions Jeopardy
Jeopardy-style game. Choose a question by clicking on the tile that shows the points you will win.
http://www.quia.com/cb/95583.html

Fractions Mystery Picture Game
Solve problems where you find a fractional part of a quantity, and uncover a picture.
http://www.dositey.com/2008/math/mistery2.html

Math Basketball - Dividing Fractions Game
First make a basket, and then you get to solve a fraction division problem with multiple choice answers.
http://www.math-play.com/math-basketball-dividing-fractions-game/math-basketball-dividing-fractions-game.html

Soccer Math - Dividing Fractions Game
In order to kick the ball and score points, you first have to answer math problems correctly.
http://www.math-play.com/soccer-math-dividing-fractions-game/soccer-math-dividing-fractions-game.html

Number Line Bars
Fraction bars that illustrate visually how many times a fraction “fits into” another fraction .
http://nlvm.usu.edu/en/NAV/frames_asid_265_g_2_t_1.html?open=activities&from=category_g_2_t_1.html

Fraction Worksheets: Addition, Subtraction, Multiplication, and Division
Create custom-made worksheets for fraction addition, subtraction, multiplication, and division.
http://www.homeschoolmath.net/worksheets/fraction.php

Comparing Fractions

Comparison Shoot Out
Choose level 2 or 3 to compare fractions and shoot the soccer ball to the goal.
http://www.fuelthebrain.com/Game/play.php?ID=47

Comparing Fractions—XP Math
Simple timed practice with comparing two fractions.
http://xpmath.com/forums/arcade.php?do=play&gameid=8

Comparing Fractions Tool at Conceptua Fractions
An interactive tool where students place numbers, visual models, and decimals on a number line.
http://www.conceptuamath.com/app/tool/comparing-fractions

Fractional Hi Lo
The computer has selected a fraction. You make guesses and it tells if your guess was too high or too low.
http://www.theproblemsite.com/games/hilo.asp

Comparing/Ordering Fractions Worksheets
Create free worksheets for comparing two fractions or ordering 3-8 fractions. Compare fractions with the same denominator, fractions with the same numerator, or you compare a fraction to 1/2, or to 1, and so on. You can also include images (fraction pies).
http://www.homeschoolmath.net/worksheets/comparing_fractions.php

(This page intentionally left blank.)

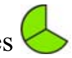
Dividing Fractions: Reciprocal Numbers

First, let's review a little.

How many times does one number go into another?

From this situation, you can <u>*always*</u> write a **division**.

Yes — EVEN if the numbers are fractions! Ask:
"How many times does the <u>*divisor*</u> go into the <u>*dividend*</u>?"

How many times does  go into ?

Three times. We write the division: $2 \div \frac{2}{3} = 3$.

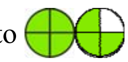
Then check the division: $3 \times \frac{2}{3} = \frac{6}{3} = 2$.

1. Solve. Write a division. Then write a multiplication that checks your division.

a. How many times does  go into ?

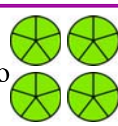
$3 \div \frac{1}{3} =$ _____

Check: _____ $\times \frac{1}{3} =$

b. How many times does  go into ?

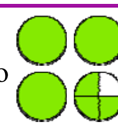
$\div$ ___ = _____

Check: _____ $\times$ ___ =

c. How many times does  go into ?

$\div$ ___ = _____

Check:

d. How many times does  go into ?

$\div$ ___ = _____

Check:

2. Solve. Think how many times the divisor goes into the dividend. Can you find a *pattern* or a *shortcut*?

a. $3 \div \frac{1}{6} =$	**b.** $4 \div \frac{1}{5} =$	**c.** $3 \div \frac{1}{10} =$	**d.** $5 \div \frac{1}{10} =$
e. $7 \div \frac{1}{4} =$	**f.** $4 \div \frac{1}{8} =$	**g.** $4 \div \frac{1}{10} =$	**h.** $9 \div \frac{1}{8} =$

The shortcut is this:

$$5 \div \frac{1}{4} \;\to\; 5 \times 4 = 20 \qquad 3 \div \frac{1}{8} \;\to\; 3 \times 8 = 24 \qquad 9 \div \frac{1}{7} \;\to\; 9 \times 7 = 63$$

That is, <u>multiply</u> the number by the <u>**reciprocal**</u> of the divisor. Notice that 1/4 upside down, or inverted, is 4/1 or just 4. We call 1/4 and 4 ***reciprocal numbers***, or just ***reciprocals***.

Does the shortcut make sense to you? For example, consider the problem $5 \div (1/4)$.
Since 1/4 goes into 1 exactly four times, it must go into 5 exactly $5 \times 4 = 20$ times.

Two numbers are reciprocal numbers (or reciprocals) of each other if, when multiplied, they make 1.	
$\frac{3}{4}$ is a reciprocal of $\frac{4}{3}$, because $\frac{3}{4} \times \frac{4}{3} = \frac{12}{12} = 1$.	$\frac{1}{7}$ is a reciprocal of 7, because $\frac{1}{7} \times 7 = \frac{7}{7} = 1$.

You can find the reciprocal of a fraction $\frac{m}{n}$ by flipping the numerator and denominator: $\frac{n}{m}$.

This works, because $\frac{m}{n} \times \frac{n}{m} = \frac{n \times m}{m \times n} = 1$.

To find the reciprocal of a mixed number, first write it as a fraction, then "flip" it.

Since $2\,\frac{3}{4} = \frac{11}{4}$, its reciprocal number is $\frac{4}{11}$.

3. Find the reciprocal numbers. Then, write a multiplication with the given number and its reciprocal.

a. $\frac{5}{8}$	**b.** $\frac{1}{9}$	**c.** $1\,\frac{7}{8}$	**d.** 32	**e.** $2\,\frac{1}{8}$
$\frac{5}{8} \times \frac{\square}{\square} = 1$	$\frac{\square}{\square} \times \frac{\square}{\square} = 1$	$\frac{\square}{\square} \times \frac{\square}{\square} = 1$	$32 \times \frac{\square}{\square} = 1$	$\frac{\square}{\square} \times \frac{\square}{\square} = 1$

4. Write a division sentence to match with each multiplication above.

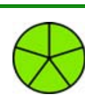
a. $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**b.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**c.** $1 \div \frac{\square}{\square} = \frac{\square}{\square}$	**d.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$	**e.** ___ $\div \frac{\square}{\square} = \frac{\square}{\square}$

Read, and try to understand. This is important!

Let's now try to **make sense** of the reciprocal numbers and division of fractions *visually*.

Thinking of the division problem **1 ÷ (2/5)**, we ask, first of all, **how many times does 2/5 fit into 1?**

Using pictures: How many times does  go into  ?

From the picture we can see that  goes into  two times, and then we have 1/5 left over.

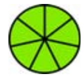
But, how many times does $\frac{2}{5}$ fit into the leftover piece, $\frac{1}{5}$? How many times does  go into  ?

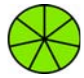
That is like trying to fit a TWO-part piece into a hole that holds just ONE part.
Only 1/2 of the two-part piece fits! So, 2/5 fits into 1/5 exactly half a time.

In total, we find that 2/5 fits into one exactly **2 1/2 times**. We can write the division $1 \div \frac{2}{5} = 2\,\frac{1}{2}$ or $\frac{5}{2}$.

Notice, we got $1 \div \frac{2}{5} = \frac{5}{2}$. Checking that with multiplication, we get $\frac{5}{2} \times \frac{2}{5} = 1$. **Reciprocals again!**

One more example. Thinking of the division problem **1 ÷ (5/7)**, we ask **how many times does 5/7 fit into 1?**

Using pictures: How many times does  go into  ?

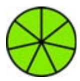
From the picture we can see that  goes into  just once, and then we have 2/7 left over.

But, how many times does $\frac{5}{7}$ fit into the leftover piece, $\frac{2}{7}$? How many times does  go into  ?

The FIVE-part piece fits into a hole that is only big enough for two parts just 2/5 of the way.

So in total, 5/7 fits into one exactly **1 2/5 times**. The division is $1 \div \frac{5}{7} = 1\,\frac{2}{5}$ or $1 \div \frac{5}{7} = \frac{7}{5}$.

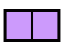
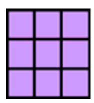
5. Solve. Think how many times the given fraction fits into one whole. Write a division.

a. How many times does  go into  ?

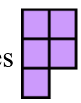
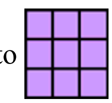
$1 \div \frac{\square}{\square} =$

b. How many times does  go into  ?

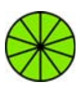
$1 \div \frac{\square}{\square} =$

c. How many times does  go into  ?

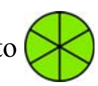
$1 \div \frac{\square}{\square} =$

d. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

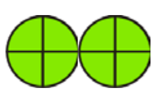
e. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

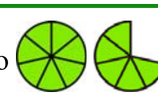
f. How many times does  go into  ?

$1 \div \frac{\square}{\square} =$

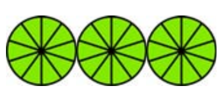
6. Solve. Think how many times the given fraction fits into the other number. Write a division.

a. How many times does  go into  ?

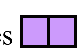
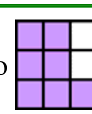
$2 \div \frac{\square}{\square} =$

b. How many times does  go into  ?

$\square\,\frac{\square}{\square} \div \frac{\square}{\square} =$

c. How many times does  go into  ?

$3 \div \frac{\square}{\square} =$

d. How many times does  go into  ?

$\frac{\square}{\square} \div \frac{\square}{\square} =$

SHORTCUT: instead of dividing, multiply by the reciprocal of the divisor.

Study the examples to see how this works.

How many times does  go into  ?

$$\frac{3}{4} \div \frac{1}{3}$$

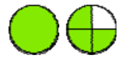
$$\downarrow \quad \downarrow$$

$$\frac{3}{4} \times 3 = \frac{9}{4} = 2\frac{1}{4}$$

Answer: 2 1/4 times.

Does it make sense?

Yes,  fits into  a little more than two times.

How many times does  go into  ?

$$\frac{7}{4} \div \frac{2}{5}$$

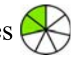
$$\downarrow \quad \downarrow$$

$$\frac{7}{4} \times \frac{5}{2} = \frac{35}{8} = 4\frac{3}{8}$$

Answer: 4 3/8 times.

Does it make sense?

Yes.  goes into 1 3/4 over four times.

How many times does  go into ?

$$\frac{2}{9} \div \frac{2}{7} =$$

$$\downarrow \quad \downarrow$$

$$\frac{\cancel{2}}{9} \times \frac{7}{\cancel{2}} = \frac{7}{9}$$

Answer: 7/9 of a time.

Does it make sense?

Yes, because  does not go into  even one full time!

Remember: There are **TWO** changes in each calculation:

1. You need to change the division into multiplication.
2. You need to use the reciprocal of the divisor.

7. Solve these division problems using the shortcut. **Remember to check if your answer makes sense.**

a. $\frac{3}{4} \div 5$

$$\downarrow \quad \downarrow$$

$$\frac{3}{4} \times \frac{1}{5} =$$

b. $\frac{2}{3} \div \frac{6}{7}$

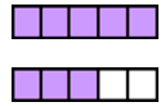
c. $\frac{4}{7} \div \frac{3}{7}$

d. $\frac{2}{3} \div \frac{3}{5}$

e. $4 \div \frac{2}{5}$

f. $\frac{13}{3} \div \frac{1}{5}$

8. **a.** Write a division to match the situation on the right.

How many times does  fit into  ?

We have 8/5, which is eight pieces, trying to fit into five pieces... so they fit 5/8 of the way.

b. Check your division by multiplication.

9. Fill in.

$2 \div \frac{3}{4} = ?$

Or, how many times does (3/4) go into (1) (1) ?

First, let's solve how many times (3/4) goes into (1).

Since $1 \div \frac{3}{4} = \frac{\square}{\square}$, it goes into one $\frac{\square}{\square}$ times.

If 3/4 fits into (1) _____ times, then it fits into (1) (1) **double that many times**, or _____ times.

We get the same answer by using the shortcut:

$2 \div \frac{3}{4}$

↓ ↓

$2 \times \frac{\square}{\square} =$

10. Fill in.

$\frac{\square}{\square} \div \frac{\square}{\square} = ?$

Or, how many times does (2/7) go into (5/6) ?

First, let's solve how many times (2/7) goes into (1).

Since $1 \div \frac{\square}{\square} = \frac{\square}{\square}$, it goes into one $\frac{\square}{\square}$ times.

If 2/7 fits into (1) _____ times, then it fits into (5/6) exactly 5/6 as many times as it fits into 1,

which is $\frac{5}{6} \times \frac{\square}{\square} = \frac{\square}{\square} =$

We get the same answer by using the shortcut:

$\frac{\square}{\square} \div \frac{\square}{\square}$

↓ ↓

$\frac{\square}{\square} \times \frac{\square}{\square} =$

(This page intentionally left blank.)

Chapter 8: Integers
Introduction

In chapter 8, students are introduced to integers, the coordinate plane in all four quadrants, and integer addition and subtraction. Multiplication and division of integers are studied in the 7th grade.

Integers are introduced using the number line to relate them to the concepts of temperature, elevation, and money. We also study briefly the ideas of absolute value (an integer's distance from zero) and the opposite of a number.

Next, students learn to locate points in all four quadrants and how the coordinates of a figure change when it is reflected across the x or y-axis. Students also move points according to given instructions, and find distances between points with the same first coordinate or the same second coordinate.

Adding and subtracting integers is presented through two main models: (1) movements along the number line and (2) positive and negative counters. With the help of these models, students should not only learn the shortcuts, or "rules," for adding and subtracting integers, but also understand *why* these shortcuts work.

A lesson about subtracting integers explains the shortcut for subtracting a negative integer using three different viewpoints (counters, number line movements, and as a distance or difference). There is also a roundup lesson for addition and subtraction of integers.

Lastly in this chapter, once again, we study graphing. Students plot points on the coordinate grid according to a given equation in two variables (such as $y = x + 2$), this time using also negative numbers. They see the patterns in the coordinates of the points and the pattern in the points drawn in the grid, and also work through some real-life problems.

The Lessons in Chapter 8

Helpful Resources on the Internet

Ordering integers

Number Balls Game
Click on the rotating number balls in the ascending order.
http://www.mathplayground.com/numberballs.html

Negative Numbers Hat Game
Put the hats with numbers on the people's heads in the right order.
http://www.primaryresources.co.uk/online/negnumorder.swf

Order Negative Numbers
Drag and drop the numbers in the right order onto the ladder (scroll down the page a bit to see the activity).
http://www.bbc.co.uk/bitesize/ks3/maths/number/negative_numbers/revision/2/

Addition and subtraction
The section for "all operations" below has more games for adding and subtracting.

Color Chips Addition
The user drags positive/negative chips to the working area, then combines them in pairs to see the sum.
http://nlvm.usu.edu/en/nav/frames_asid_161_g_2_t_1.html

Color Chips Subtraction
Drag positive/negative chips and zero pairs into working area as instructed, then subtract.
http://nlvm.usu.edu/en/nav/frames_asid_162_g_3_t_1.html

Integer Football
The football player advances either towards the positive or the negative yard lines.
http://www.mathgoodies.com/games/integer_game/football.html

Line Jumper
You see a number line and an addition or subtraction problem. Click the right answer on the number line.
http://www.funbrain.com/linejump

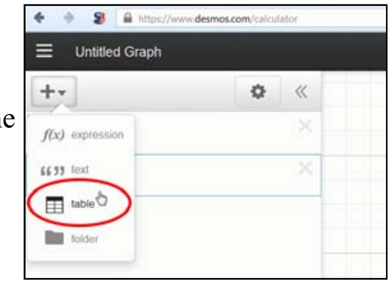
Space Coupe to the Rescue
By choosing a positive or negative number, the player controls the vertical position of a spaceship. If the spaceship reaches the same vertical position as a virus pod, the pod is destroyed.
http://pbskids.org/cyberchase/games/negativenumbers

Red and Black TripleMatch Game for Adding Integers
This is a fun card game with 2-5 people to practice adding integers.
http://mathmamawrites.blogspot.com/2010/07/black-and-red-triplematch-card-game-for.html

Adding and Subtracting Integers Gizmos from Explorelearning.com
Interactive simulations that illustrate adding and subtracting integers on a number line or with chips. Includes an exploration guide and assessment questions. You can get a 5-minute access for free, or a free 30-day trial account.
http://www.explorelearning.com/index.cfm?method=cResource.dspResourcesForCourse&CourseID=211

Graphing

Desmos Graphing Calculator
A versatile, easy-to-use, and free graphing calculator. To practice plotting points and lines as learned in this chapter, add an item from the + button and choose 'table'. Fill in *x* and *y* values, and Desmos will plot the points. You can then type the equation of the line in the form y = (something), such as y = 2x, and check if the line goes through your points.
https://www.desmos.com/calculator



Meta-Calculator 2.0
Choose Graphing Calculator. You can enter an equation to be graphed, or choose "plot points" from top menu to enter individual points.
http://www.meta-calculator.com/online/

Graph Mole
A fun game about plotting points in coordinate plane. Plot points before the mole eats the vegetables.
http://funbasedlearning.com/algebra/graphing/default.htm

Catch the Fly
Wait for the fly to land on the coordinate grid, then type its coordinates, and a frog will eat it.
http://hotmath.com/hotmath_help/games/ctf/ctf_hotmath.swf

Looking for the Top Quark Game
Each player receives six quarks that they hide on a grid. The players use coordinates to find their opponent's hidden quarks.
http://education.jlab.org/topquarkgame

Coordinate Grid Quiz from ThatQuiz.org
This quiz has 10 questions and asks to either plot a point or give the coordinates of a given point. You can also modify the quiz parameters to your liking.
http://www.thatquiz.org/tq-7/?-j8-l5-m2kc0-na-p0

Co-ordinate Game
You will see a red circle on the grid. Enter its co-ordinates and click check.
http://www.bgfl.org/bgfl/custom/resources_ftp/client_ftp/ks3/maths/coordinate_game/game1.htm

All operations / General

Arithmetic Four (Connect the Four game)
Practice any or all of the four operations with integers. First you answer a math problem, then you can do your move in a connect-the-four game. Choose addition and subtraction to practice the operations learned in this chapter.
http://www.shodor.org/interactivate/activities/ArithmeticFour

Flashcards with Negative Numbers
Interactive flashcards at AplusMath.com for integer addition, subtraction, multiplication, and division. Choose addition and subtraction to practice what students learn in this chapter.
http://www.aplusmath.com/Flashcards/sub-nflash.html

Create Integers Worksheets
Use the basic operations worksheet generator for integer worksheets by choosing the range of numbers to be from negative to positive.
http://www.homeschoolmath.net/worksheets/basic-operations-worksheets.php

How to Teach Integers
An article for the teacher about how to teach integer operations.
http://www.homeschoolmath.net/teaching/integers.php

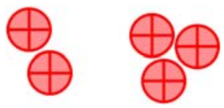
Free Downloadable Integer Fact Sheets
http://www.homeschoolmath.net/download/Add_Subtract_Integers_Fact_Sheet.pdf
http://www.homeschoolmath.net/download/Multiply_Divide_Integers_Fact_Sheet.pdf

The History of Negative Numbers
While they seem natural to us now, in the past negative numbers have spurred controversy and been called "fictitious" and other names.
http://nrich.maths.org/public/viewer.php?obj_id=5961
http://www.classzone.com/books/algebra_1/page_build.cfm?content=links_app3_ch2&ch=2

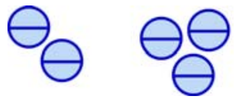
(This page intentionally left blank.)

Adding Integers: Counters

Addition of integers can be modeled using **counters.** We will use red counters with a "+" sign for positives and blue counters with a "–" sign for negatives.



Here we have the sum 2 + 3. There is a group of 2 positives and another of 3 positives.



This picture shows the sum (−2) + (−3). We *add* negatives and negatives. In total, there are five negatives, so the sum is −5.

$1 + (-1) = 0$

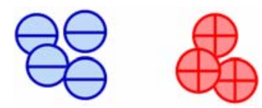
One positive counter and one negative counter *cancel* each other. In other words, their sum is zero!

$2 + (-2) = 0$

Two negatives and two positives also cancel each other. Their sum is zero.

$3 + (-1) = 2$

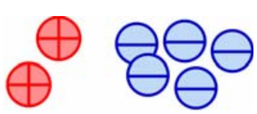
Here, one "positive-negative" pair is canceled (you can cross it out!). We are left with 2 positives.



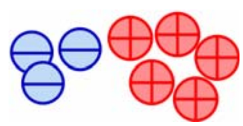
$(-4) + 3 = -1$

Now the negatives outweigh the positives. Pair up three negatives with three positives. Those cancel out. There is still one negative left.

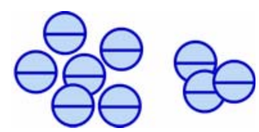
1. Refer to the pictures and add. Remember each "positive-negative" pair is canceled.



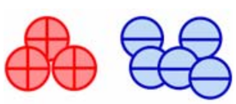
a. $2 + (-5) =$ _____



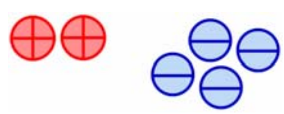
b. $(-3) + 5 =$ _____



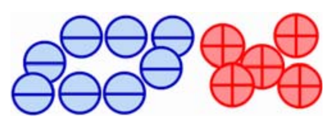
c. $(-6) + (-3) =$ _____



d. $3 + (-5) =$ _____

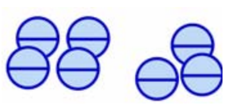


e. $2 + (-4) =$ _____

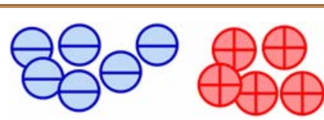


f. $(-8) + 5 =$ _____

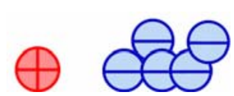
2. Write addition sentences (equations) to match the pictures.



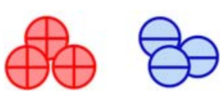
a.



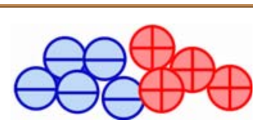
b.



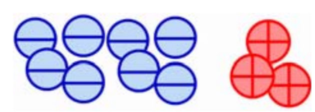
c.



d.

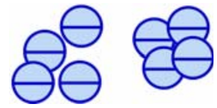


e.



f.

A note on notation



We can write an elevated minus sign to indicate a negative number: $^-4$.
Or we can write it with a minus sign and parentheses: (-4).
We can even write it without the parentheses if the meaning is clear: -4.

So $^-4 + {}^-4 = {}^-8$ is the same as $(-4) + (-4) = (-8)$, which is the same as $-4 + (-4) = -8$

You ***should*** write the parentheses if you have + and −, or two − signs, next to each other.

So, do <u>not</u> write "8 + − 4"; write "8 + (−4)." And do not write "3 − −3"; write "3 − (−3)."

3. Think of the counters. Add.

a. $7 + (-8) =$ $(-7) + 8 =$	**b.** $(-7) + (-8) =$ $7 + 8 =$	**c.** $5 + (-7) =$ $7 + (-5) =$	**d.** $50 + (-20) =$ $10 + (-40) =$
e. $^-2 + {}^-4 =$ $^-6 + 6 =$	**f.** $10 + {}^-1 =$ $^-10 + {}^-1 =$	**g.** $^-8 + 2 =$ $^-8 + {}^-2 =$	**h.** $^-9 + {}^-1 =$ $9 + {}^-1 =$

4. Rewrite these sentences using symbols, and solve the resulting sums.

a. The sum of seven positives and five negatives.

b. Add −3 and −11.

c. Positive 100 and negative 15 added together.

5. Write a sum for each situation, and solve.

a. Your checking account is overdrawn by $50. (This means your account is negative). Then you earn $60. What is the balance in your account now?

b. Hannah owes $20 to her mom. Then, she borrows $15 more from her mom. What is Hannah's "balance" now?

6. Consider the four expressions $2 + 6$, $(-2) + (-6)$, $(-2) + 6$, and $2 + (-6)$. Write these expressions in order from the one with **least** value to the one with **greatest** value.

7. Find the number that is missing from the equations.

a. $-3 +$ _______ $= -7$	**b.** $-3 +$ _______ $= 3$	**c.** $3 +$ _______ $= (-7)$
d. ______ $+ (-15) = -22$	**e.** $2 +$ _______ $= -5$	**f.** _______ $+ (-5) = 0$

Comparing number line jumps and counters

We can think of $-5 + (-3)$ as five negatives and three negatives, totaling 8 negatives or -8. We also know that $-5 - 3$ is like starting at -5 and jumping three steps towards the left on the number line, ending at -8.

Since both have the same answer, the two expressions $-5 + (-3)$ and $-5 - 3$ are equal:

$$-5 + (-3) = -5 - 3$$

It is as if the "+ −" in the middle is changed into a single − sign. This, indeed, is a *shortcut*!

Similarly, $2 + (-7)$ is the same as $2 - 7$. Either think of having 2 positive and 7 negative counters, totaling 5 negatives. Or, think of being at 2 and taking 7 steps to the left, ending at -5.

When solving integer problems, you may think of number line jumps or of counters, whichever is easier.

8. Compare how $-7 + 4$ is modeled on the number line and with counters.

a. On the number line, $-7 + 4$ is like starting at _____, and moving _____ steps to the _____________, ending at _____.

b. With counters, $-7 + 4$ is like _____ negatives and _____ positives added together. We can form _____ negative-positive pairs that cancel, and what is left is ____ negatives.

9. Add.

a. $4 + (-10) =$	**b.** $-8 + (-8) =$	**c.** $-5 + (-7) =$	**d.** $11 + (-2) =$
$-6 + 8 =$	$7 + (-8) =$	$12 + (-5) =$	$-10 + 20 =$

10. **a.** Find the value of the expression $x + (-4)$ for four different values of x. You can choose the values.

b. For which value of x does the expression $x + (-4)$ have the value 0?

11. Solve the problems, and observe the patterns.

a. $3 - 2 =$	**b.** $^{-}7 - 0 =$	**c.** $^{-}5 + 0 =$	**d.** $^{-}6 + 6 =$
$3 - 3 =$	$^{-}7 - 1 =$	$^{-}5 + 1 =$	$^{-}6 + 7 =$
$3 - 4 =$	$^{-}7 - 2 =$	$^{-}5 + 2 =$	$^{-}6 + 8 =$
$3 - 5 =$	$^{-}7 - 3 =$	$^{-}5 + 3 =$	$^{-}6 + 9 =$
$3 - 6 =$	$^{-}7 - 4 =$	$^{-}5 + 4 =$	$^{-}6 + 10 =$

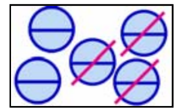
Subtracting a Negative Integer

We have already looked at such subtractions as 3 − 5 or −2 − 8, which you can think of as number line jumps. But what about **subtracting a negative integer?** What is 5 − (−4)? Or (−5) − (−3)?

Let’s look at this kind of expression with a “double negative” in several different ways.

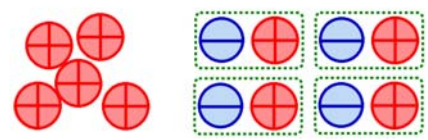
1. Subtraction as “taking away”:

We can model subtracting a negative number using counters. (−5) − (−3) means we start with 5 negative counters, and then we *take away* 3 negative counters. That leaves 2 negatives, or −2.



5 − (−4) cannot easily be modeled that way, because it is hard to take away 4 negative counters when we do not have any negative counters to start with. But you *could* do it this way:

Start out with 5 positives. Then *add* four positive-negative pairs, which is just adding zero! Now you can take away four negatives. You are left with nine positives.



Start out with 5.

Add four positive-negative pairs, which amount to zero.

Lastly, cross out four negatives. You are left with nine positives.

2. Subtracting a negative number as a number line jump:

5 − (−4) is like standing at 5 on the number line, and getting ready to subtract, or go to the left. But, since there is a minus sign in front of the 4, it “turns you around” to face the positive direction (to the right), and you take 4 steps to the right instead. So, 5 − (−4) = 5 + 4 = 9.

(−5) − (−3) is like standing at −5, ready to go to the left, but the minus sign in front of 3 turns you “about face,” and you take 3 steps to the right instead. You end up at −2.

3. Subtraction as a difference/distance:

To find the difference between 76 and 329, you subtract 329 − 76 = 253 (the smaller-valued number from the bigger-valued one). If you subtract the numbers the other way, 76 − 329, the answer is −253.

By the same analogy, we can think of 5 − (−4) as meaning the difference (distance) between 5 and −4. From the number line we can see the distance is **9**.

(−5) − (−3) *could* be the distance between −5 and −3, except it has the larger number, −3, subtracted from the smaller number, −5.

If we turn them around, (−3) − (−5) would give us the distance (difference) between those two numbers, which is 2. Then, (−5) − (−3) would be the opposite of that, or −2.

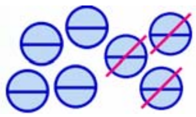
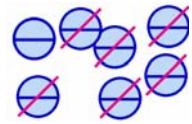
Two negatives make a positive!

You have probably already noticed that, any way you look at it, we can, in effect, replace those two minuses in the middle with a + sign.
In other words, 5 − (−4) has the same answer as 5 + 4.
And (−5) − (−3) has the same answer as −5 + 3.
It may look a bit strange, but it works out really well.

5 − (−4)
5 + 4 = 9

(−5) − (−3)
(−5) + 3 = −2

1. Write a subtraction sentence to match the pictures.

a. 	**b.** 

2. Write an addition or subtraction sentence to match the number line movements.

a. You are at −2. You jump 6 steps to the left.

b. You are at −2. You get ready to jump 6 steps to the left,
but turn around at the last minute and jump 6 steps to the right instead.

3. Find the distance between the two numbers. Then, write a matching subtraction sentence. *Remember* to subtract the smaller number from the bigger number.

a. The distance between 3 and −7 is _______. Subtraction: _______ − _______ = _______	**b.** The distance between −3 and −9 is _______. Subtraction: _______ − _______ = _______
c. The distance between −2 and 10 is _______. Subtraction: _______ − _______ = _______	**d.** The distance between −11 and −20 is _______. Subtraction: _______ − _______ = _______

4. Solve. Remember the shortcut: you can change each double minus "− −" into a plus sign.

a. $-8 - (-4) =$	**b.** $-1 - (-5) =$	**c.** $12 - (-15) =$
$8 - (-4) =$	$1 - (-5) =$	$-12 + 15 =$
$-8 + (-4) =$	$-1 - 5 =$	$-12 - 15 =$
$8 + (-4) =$	$1 - 5 =$	$12 + (-15) =$

5. Connect with a line the expressions that are equal (have the same value).

a.		**b.**	
$10 - (-3)$	$10 - 3$	$-9 + 2$	$-9 + (-2)$
$10 + (-3)$	$10 + 3$	$-9 - 2$	$-9 - (-2)$

6. Write an integer addition or subtraction to describe the situations.

a. A roller coaster begins at 90 ft above ground level.
Then it descends 105 feet.

b. Matt has $25. He wants to buy a bicycle from his friend that costs $40.
How much will he owe his friend?

Puzzle Corner

Solve $-1 + (-2) - (-3) - 4$.

(This page intentionally left blank.)

Chapter 9: Geometry
Introduction

The main topics in this chapter include

- the area of triangles
- the area of polygons
- nets and the surface area of prisms and pyramids
- conversions between units of area (both metric and customary)
- the volume of prisms with fractional side lengths

We start out by reviewing quadrilaterals and the basic drawing of shapes. Students need to use a ruler and a protractor, and measure the side lengths and angles to do the drawing problems.

Next, we get to the focus of the chapter, which is the area of polygons. First, we study the area of a right triangle, which is very easy, as a right triangle is always half of a rectangle. Building on from there, the area of a parallelogram is the same as the area of a certain rectangle, thus we arrive at the usual formula for the area of a parallelogram as being the product of its height and its base.

Then, the area of any triangle is half of the area of a certain parallelogram. Lastly, the area of polygons can be determined by dividing them into triangles, finding the areas of those, and adding them.

Students also practice their new skills in the context of a coordinate grid. They draw polygons in the coordinate plane and find their side lengths, perimeters, and areas.

Nets and surface area is the next major topic. Students draw nets and determine the surface area of prisms and pyramids using nets. They learn how to convert between different area units, not using conversion factors or formulas, but using logical reasoning where they learn to determine those conversion factors themselves.

Lastly, we study the volume of rectangular prisms, this time with fractional edge lengths. (Students have already studied this topic using whole-number edge lengths in the 5th grade.) The basic idea is to prove that the volume of a rectangular prism *can* be calculated by multiplying its edge lengths even when the edges have fractional lengths. To that end, students need to think how many little cubes with edges 1/2 or 1/3 unit go into a larger prism. Once we have established the formula for volume, students solve some problems concerning the volume of rectangular prisms.

The Lessons in Chapter 9

Helpful Resources on the Internet

Use these online resources as you see fit to supplement the main text.

Angles

Working with Angles
An interactive lesson with explanations and quiz from Absorb Mathematics course written by Kadie Armstrong, a mathematician.
http://www.absorblearning.com/mathematics/demo/units/KCA003.html

Angles in Polygons
An interactive lesson with explanations and quiz from Absorb Mathematics course written by Kadie Armstrong, a mathematician.
http://www.absorblearning.com/mathematics/demo/units/KCA004.html

Properties of Parallelograms
Investigate the sides and interior angles of parallelograms using these two interactive tools.
http://www.keymath.com/x3331.xml

Interior Angles
A nice lesson and explanation about interior angles of polygons.
http:www.coolmath4kids.com/interior.html

Geometry Bridge
An interactive review lesson on types of angles, types of triangles, angle sum of a triangle, and the Pythagorean Theorem. You get to build a bridge!
http://mysite.verizon.net/vzex2lij/

Similar Triangles Quiz from ThatQuiz.org This quiz has 10 questions and asks to provide a missing side length when two similar triangles are shown. You can also modify the quiz parameters to your liking.
http:www.thatquiz.org/tq-A/?-jg-l1i-m2kc0-na-p0

Polygons

Triangle Explorer
Practice calculating the area of a triangle using this interactive tool.
http://www.shodor.org/interactivate/activities/TriangleExplorer/

Interactive Quadrilaterals
See all the different kinds of quadrilateral “in action”. You can drag the corners, see how the angles change, and observe what properties do not change.
http://www.mathsisfun.com/geometry/quadrilaterals-interactive.html

Looking at Polygons
An interactive lesson with explanations and quiz from Absorb Mathematics course written by Kadie Armstrong, a mathematician.
http://www.absorblearning.com/mathematics/demo/units/KCA007.html

Interactive Tangram Puzzle
Place the tangram pieces so they form the given shape.
http://nlvm.usu.edu/en/nav/frames_asid_112_g_2_t_1.html

Congruent Transformations

Primary Resources: Reflection
Color the squares and reflect the given pattern in a line.
http://www.primaryresources.co.uk/online/reflection.swf

Primary Resources: Rotation
From the arrow you can change the shape. Use the circular arrow buttons to rotate the shapes either 90 or 45 degrees.
http://www.primaryresources.co.uk/online/rotation.swf

Similarity and Congruence
An interactive lesson with explanations and quiz from Absorb Mathematics course written by Kadie Armstrong, a mathematician.
http://www.absorblearning.com/mathematics/demo/units/KCA035.html

National Library of Virtual Manipulatives for Interactive Mathematics: Geometry
Collection of interactive geometry activities: Congruent triangles, fractals, geoboard activities, Golden rectangle, Ladybug leaf, Ladybug mazes, platonic solids, tangrams, tessellations, transformations and more.
http://nlvm.usu.edu/en/nav/category_g_3_t_3.html

Interactivate! Tessellate
An online, interactive tool for creating your own tessellations. Choose a shape, then edit its corners or edges. The program automatically changes the shape so that it will tessellate (tile) the plane. Then push the tessellate button to see your creation!
http://www.shodor.org/interactivate/activities/Tessellate

Symmetry Game
Tell how many lines of symmetry a shape has.
http://www.innovationslearning.co.uk/subjects/maths/activities/year3/symmetry/shape_game.asp

Similar Figures

Length, Perimeter, Area and Volume of Similar Figures
Use this interactive figure to explore how the scale factor affects the size and the area of similar figures. The discussion provided helps the teacher, but a specific lesson plan would be more helpful.
http://standards.nctm.org/document/eexamples/chap6/6.3/index.htm

Ratio and Scale
An interactive lesson with explanations and quiz from Absorb Mathematics course written by Kadie Armstrong, a mathematician.
http://www.absorblearning.com/mathematics/demo/units/KCA024.html

Circle /Pi

Area of a Circle, Formula & Illustrated Lesson
The interactive tool shows you the area of the circle as the radius increases. The page also includes a short quiz.
http://www.mathwarehouse.com/geometry/circle/area-of-circle.php

Circle Tool
An applet that allows you to investigate how the area and circumference of a circle compare to its radius and diameter in the Intro and Investigation sections and then hone your skills in the Problems section.
You can drag the radius to various lengths, and then click the “Add to Table” button to record the data in the table. You can also examine the ratios of any two measures, and make a graph of the data
http://illuminations.nctm.org/ActivityDetail.aspx?ID=116

Radius, Diameter, and Circumference
A simple lesson with an interactive quiz about radius, diameter, and circumference of a circle.
http://www.mathgoodies.com/lessons/vol2/circumference.html

The Area of a Circle as a Limit
An animation that illustrates how the area of a circle is a limit of the sum of the areas of interior triangles as the number of triangles goes to infinity. This is the idea I explain in this book (Math Mammoth Geometry 2).
http://www.learnerstv.com/animation/animation.php?ani=96&cat=physics

Area of Circle
An activity where you measure a circle and the resulting figure when you cut it into wedges and tape them together to form a crude parallelogram.
http://www.learner.org/courses/learningmath/measurement/session7/part_b/index.html

Amazing History of Pi
A short and simple introduction to the history of pi.
http://ualr.edu/lasmoller/pi.html

Approximating Pi
How did Archimedes find the approximate value of pi? This interactive tool illustrates Archimedes' basic approach with inscribed or circumscribed polygons.
http://www.pbs.org/wgbh/nova/archimedes/pi.html

Pi Day Activities & Links
Two pi-related activities to do at home, plus a lot of fun links about pi. Originally meant for Pi Day (celebrated 3/14 each year).
http://www.exploratorium.edu/pi/pi_activities/index.html

5 Trillion Digits of Pi
As of August 2010, the world record for computing digits of pi was 5 trillion digits. This will keep changing, of course.
http://www.numberworld.org/digits/Pi/

Rolling circle illustrating Pi This is a short animation where a circle with diameter 1 rolls on a number line one complete roll. Of course having rolled once around its circumference, it now lands at 3.14 or Pi.
http://i.imgur.com/dsCw0.gif

Volume & Surface Area

Geometric Solids
Rotate various geometric solids by dragging with the mouse. Count the number of faces, edges, and vertices.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=70

Interactivate: Surface Area and Volume
Explore or calculate the surface area and volume of rectangular prisms and triangular prisms. You can change the base, height, and depth interactively.
http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/

Geometry Area/Perimeter Quiz from ThatQuiz.org
An online quiz, asking either the area of perimeter of rectangles, triangles, and circles. You can modify the quiz parameters to your liking, for example to omit the circle, or instead of solving for area, you solve for an unknown side when perimeter/area is given.
http://www.thatquiz.org/tq-4/?-j201v-lc-m2kc0-na-p0

Geometry Volume/Surface Area Quiz from ThatQuiz.org An online quiz, asking either the volume or surface area of cubes, prisms, spheres, cylinders, or cones. You can modify the quiz parameters to your liking, for example to omit some shapes, solve only for volume or surface area, or instead of solving for volume/surface area, you solve for an unknown dimension (side or radius) when the volume or surface area is given.
http://www.thatquiz.org/tq-4/?-j3vu0-lc-m2kc0-na-p0

General

Geometry - Math Warehouse
Detailed lessons about angles, triangles, quadrilaterals, circles, similar triangles, parallelograms, polygons, and trapezoids.
http://www.mathwarehouse.com/geometry/

Geometry course from Learning Math
This online geometry course includes readings, problems, videos, interactive activities, homework problems and solutions. It is meant for K-8 teachers but will work well for middle school students as well.
www.learner.org/courses/learningmath/geometry

Geometry Reference Sheet
Both online and printable versions; includes area and volume formulas for common shapes plus Pythagorean Theorem.
www.ecalc.com/math-help/worksheet/geometry

Geometry Tutorials
Simple tutorials on triangles and their properties, polygons, symmetry, angles and much more. Also includes lots of solved geometry questions and some interactive applets.
www.analyzemath.com/geometry.html#tutorials

Online Kaleidoscope
Create your own kaleidoscope creation with this interactive tool.
http://www.zefrank.com/dtoy_vs_byokal/

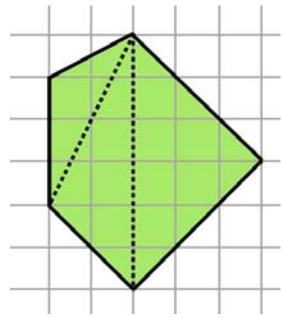
(This page intentionally left blank.)

Area of Polygons

To calculate the **area of polygons**, all you have to do is divide them into easy shapes, such as rectangles and triangles. Calculate the area separately of each easy shape, and add them to find the total area.

1. This figure is called a ______________________.

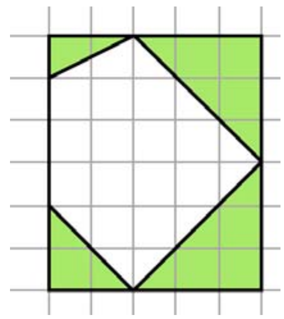
   Calculate its area using the three triangles.
   For each triangle, use the *vertical* side as the base.



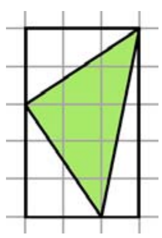
2. Here is another way of calculating the area of the same figure.

   1. Calculate the area of the rectangle that encloses the figure.
   2. Calculate the areas of the four shaded triangles.
   3. Subtract.

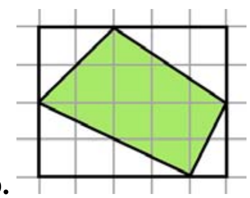
   Use this method and verify that you get the same result as above.



3. Find the areas of the shaded figures.

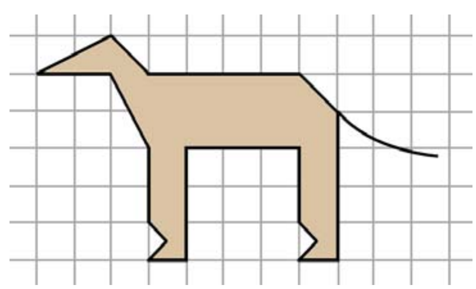


a.



b.

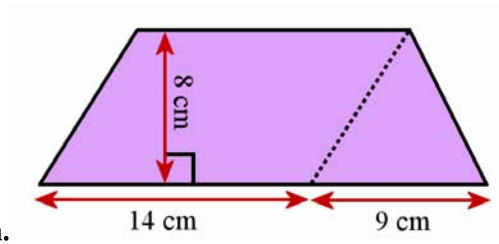
4. **a.** The side of each little square in the drawing on the right is 1 inch. Find the area of the polygon.



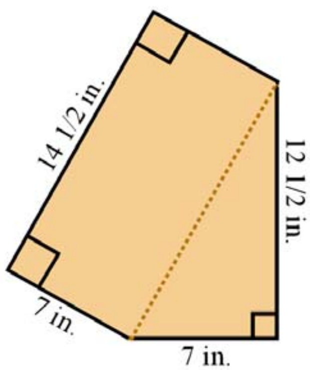
   b. Imagine that the side of each little square is 2 inches instead. What is the area now?

5. Calculate the total area of the figures.

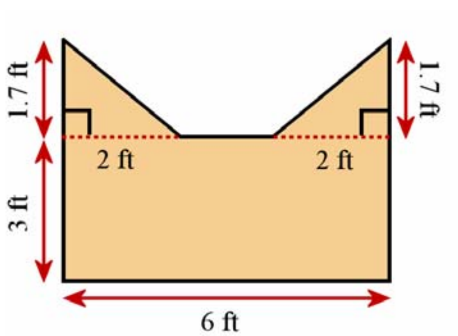
a.



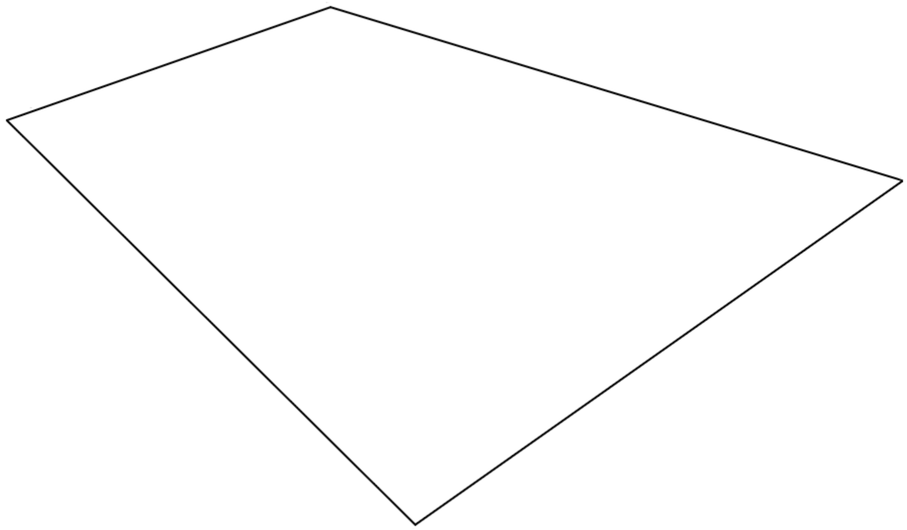
b.



c.

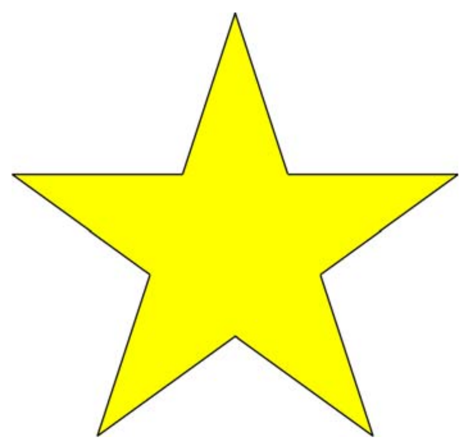


6. Divide this quadrilateral into two triangles, and then find its area in square centimeters. You may use a calculator.



Puzzle Corner

Measure what you need to from this star to find: (a) its perimeter in centimeters and (b) its area in square centimeters.



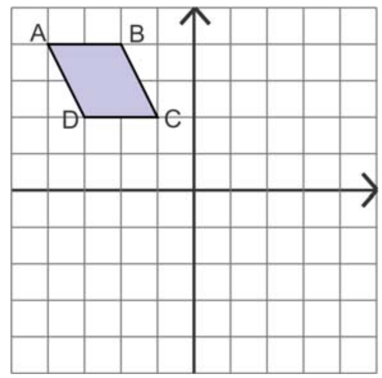
Polygons in the Coordinate Grid

1. What are these polygons called? Also, reflect or move them, and find their areas.

a. What is this polygon called?

b. Reflect it in the *x*-axis.

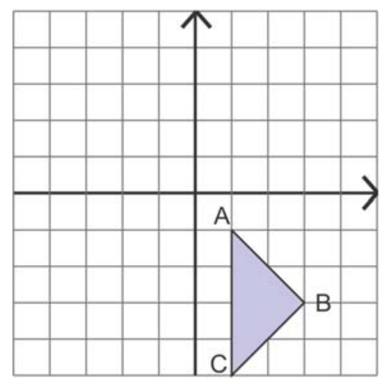
c. Find its area.



d. Classify this triangle according to its sides and angles.

e. Reflect it in the *y*-axis.

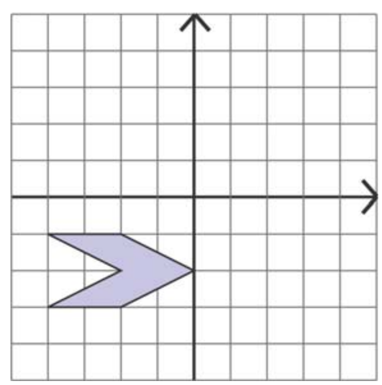
f. Find its area.



g. What is this polygon called?

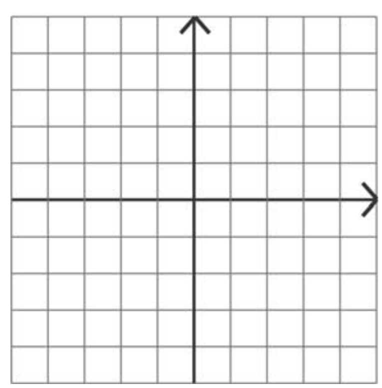
h. Move it 4 units up, and 2 units to the right.

i. Find its area.



j. Draw any pentagon using grid points as vertices.

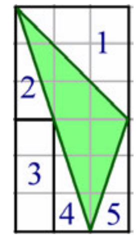
k. Find its area.



Here is a neat way to find **the area of any polygon whose vertices are points in the grid.**

Draw a rectangle around the polygon. Then, look at the area between the polygon and the rectangle, and divide it into triangles and rectangles. Now calculate those areas. Lastly, subtract those from the area of the large rectangle to find the area of your polygon.

Example. Find the area of the colored triangle. We draw a rectangle around it that is 3 units by 6 units. Then we find the areas marked with 1, 2, 3, 4, and 5:

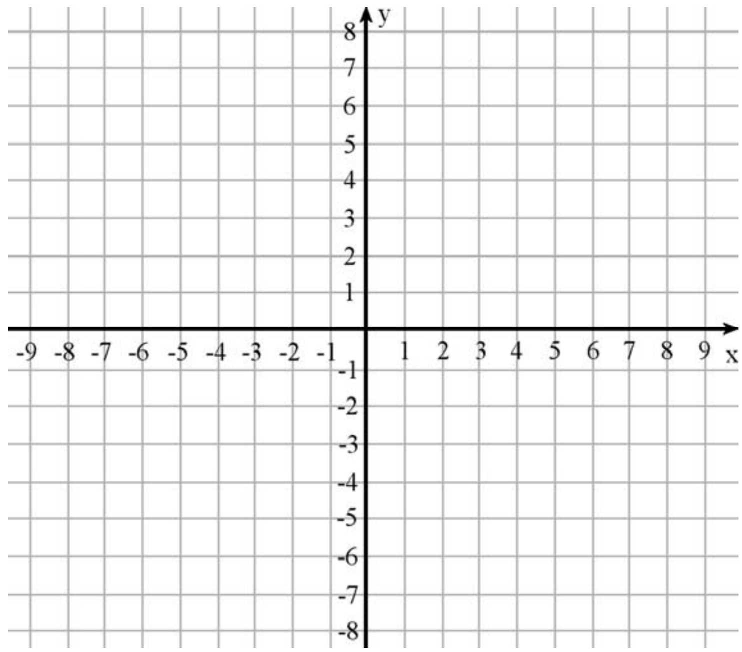


1: a triangle; $3 \times 3 \div 2 = 4.5$ square units
2: a triangle; $1 \times 3 \div 2 = 1.5$ square units
3: a rectangle; $1 \times 3 = 3$ square units
4: a triangle; $1 \times 3 \div 2 = 1.5$ square units
5: a triangle; $1 \times 3 \div 2 = 1.5$ square units
The total for the shapes 1, 2, 3, 4, and 5 is 12 square units.

Therefore, the area of the colored triangle is 18 sq. un. − 12 sq. un. = 6 square units.

2. Find the area of a triangle with given vertices.
   a. $(-8, 7)$, $(-5, 3)$, and $(4, 0)$.

   b. $(-7, -2)$, $(-2, -1)$, and $(-4, -7)$.



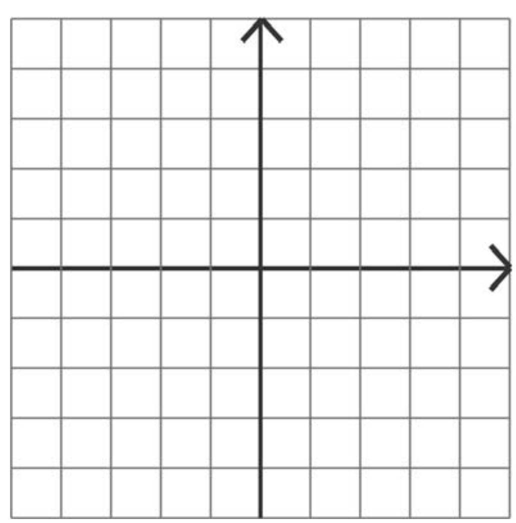
3. Draw a quadrilateral in the grid with vertices $(8, 5)$, $(3, 4)$, $(4, -5)$, $(7, -6)$

   Use the same technique to find its area.

4. The points $(1, 2.4)$, $(2.4, 1)$, $(2.4, -1)$, $(1, -2.4)$ are four vertices of a water fountain in the shape of a regular octagon. The other four points are found by reflecting these four in the *y*-axis.

   Find the length of one side of the fountain.

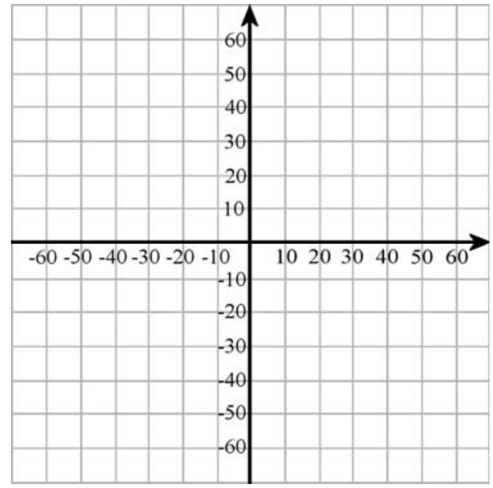
   Find its perimeter.



5. What polygon is formed when you join the following points in order with line segments?

   (−35, −40), (−35, 40), (−20, 40), (20, −15), (20, 40), (35, 40), (35, −40), (20, −40), (−20, 15), (−20, −40) and (−35, −40)

   Challenge: Find its area.



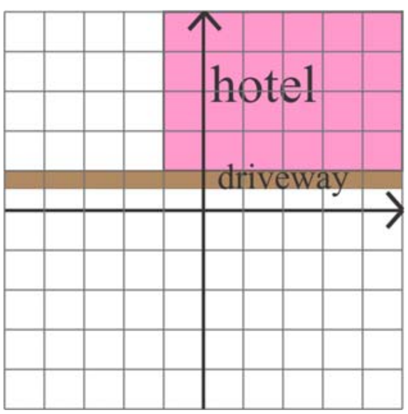
6. A hotel wants to build a swimming pool with a total area of 100 to 140 square meters. One of its sides has to be 12.5 meters.

   a. Suggest three different rectangular shapes and draw them in the grids below. Each unit in the grid is 5 m.

   b. For each pool, give the coordinates of the four corners of the pool.

   c. For each pool, calculate the pool's distance to the driveway.

Pool Design 1:

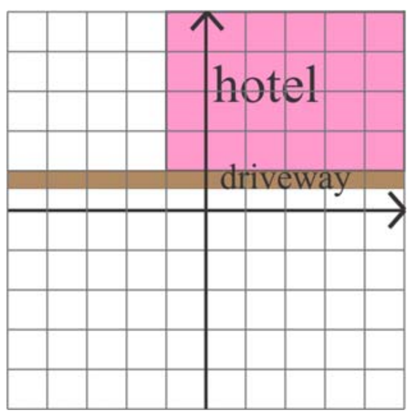


Coordinates of the corners:

Area:

Distance to the driveway:

Pool Design 2:

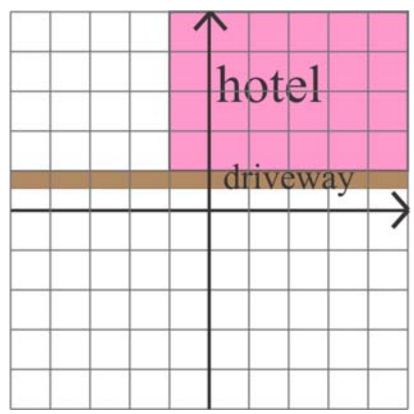


Coordinates of the corners:

Area:

Distance to the driveway:

Pool Design 3:



Coordinates of the corners:

Area:

Distance to the driveway:

(This page intentionally left blank.)

Chapter 10: Statistics
Introduction

The fundamental theme in our study of statistics is the concept of *distribution*. In the first lesson, students learn what a distribution is—basically, it is *how* the data is distributed. The distribution can be described by its center, spread, and overall shape. The shape is read from a graph, such as a dot plot or bar graph.

Two major concepts when summarizing and analyzing distributions are its center and its variability. We first study the center, in the lessons about mean, median, and mode. Students not only learn to calculate these values, but also relate the choice of measures of center to the shape of the data distribution and the type of data.

In the lesson *Measures of Variation* we study range, interquartile range, and mean absolute deviation. The last one takes many calculations, and the lesson gives instructions on how to calculate it using a spreadsheet program, such as Excel.

Then in the next lessons, students learn to make several different kinds of graphs: histograms, boxplots, and stem-and-leaf plots. In those lessons, students continue summarizing distributions by giving their shape, a measure of center, and a measure of variability.

The Lessons in Chapter 10

Helpful Resources on the Internet

Mean, Median, Mode, Range, etc.

Using and Handling Data
Simple explanations for finding mean, median, or mode.
http://www.mathsisfun.com/data/index.html#stats

Math Goodies Interactive Statistics Lessons
Clear lessons with examples and interactive quiz questions.
http://www.mathgoodies.com/lessons/vol8/range.html
http://www.mathgoodies.com/lessons/vol8/mean.html http://www.mathgoodies.com/lessons/vol8/median.html
http://www.mathgoodies.com/lessons/vol8/mode.html

Mean, Median, and Mode
Lesson on how to calculate mean, median, and mode for set of data given in different ways. Also has interactive exercises.
www.cimt.plymouth.ac.uk/projects/mepres/book8/bk8i5/bk8_5i2.htm

GCSE Bitesize Mean, Mode and Median Lessons
Explanations with simple examples.
www.bbc.co.uk/schools/gcsebitesize/maths/data/measuresofaveragerev1.shtml

Measures Activity
Enter your own data and the program will calculate mean, median, mode, range and some other statistical measures.
www.shodor.org/interactivate/activities/Measures

Landmark Shark Game
You're dealt five number cards, and using that as your data set you need to choose which of the range, median, or mode is the largest number.
http://media.emgames.com/emgames/demosite/playdemo.html?activity=M5A006&activitytype=dcr&level=3

Train Race Game
Calculate the median and range of travel times for four different trains, then choose a good train to take based on your results.
www.bbc.co.uk/education/mathsfile/shockwave/games/train.html

Graphing and Graphs

Bar Chart Virtual Manipulative
Build your bar chart online using this interactive tool.
http://nlvm.usu.edu/en/nav/frames_asid_190_g_1_t_1.html?from=category_g_1_t_1.html

An Interactive Bar Grapher
Graph data sets in bar graphs. The color, thickness and scale of the graph are adjustable. You can input your own data, or you can use or alter pre-made data sets.
http://illuminations.nctm.org/ActivityDetail.aspx?ID=63

Create a Graph
Kids can create bar graphs, line graphs, pie graphs, area graphs, and xyz graphs to view, print, and save.
http://nces.ed.gov/nceskids/createagraph/default.aspx

Circle Grapher
A tool to graph data sets in a circle graph. You can input your own data or alter a pre−made data set.
http://illuminations.nctm.org/activitydetail.aspx?id=60

Graphs Quiz from ThatQuiz.org
This quiz asks questions about different kinds of graphs (bar, line, circle graph, multi-bar, stem-and-leaf, boxplot, scattergraph). You can modify the quiz parameters to your liking, such as to plot the graph, answer different kinds of questions about the graph, or find mean, median, or mode based on the graph.
www.thatquiz.org/tq-5/math/graphs

Stem-and-Leaf
Enter the values and this web page creates your stem-and-leaf plot for you.
http://www.mrnussbaum.com/graph/sl.htm

Math Goodies Interactive Data & Graphs Lessons
Clear lessons with examples and interactive quiz questions, covering the concept and construction of line graphs, bar graphs, circle graphs, comparing graphs, and exercises.
http://www.mathgoodies.com/lessons/toc_vol11.html

Stem-and-Leaf Plots Quiz
This is from Glencoe mathematics, an online multiple-choice quiz that is created randomly. Refresh the page (or press F5) to get another quiz.
http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825200-8&chapter=12&lesson=1&&headerFile=4&state=na

Data Analysis and Probability Gizmos from Explorelearning.com
Interactive exploration activities online, with lesson plans. Topics include box-and-whisker plots, histograms, stem-and-leaf plots, lines of best fits using least squares, scatter plots; probability topics; and more. This is an excellent resource. The gizmos work for 5 minutes for free. You can also sign up for a free trial account.
www.explorelearning.com/index.cfm?method=cResource.dspChildrenForCourse&CourseID=129

Statistics Interactive Activities
(scroll down to Statistics and Probability concepts)
A set of interactive tools for exploring histograms, pie charts, boxplots, stem-leafs, and mean, median, variance, and standard deviation of data. Can enter your own data or explore the examples.
www.shodor.org/interactivate/activities/tools.html

Create a Boxplot
http://www.shodor.org/interactivate/activities/BoxPlot/

Statistics - Facts & Figures

GapMinder
Visualizing human development trends (such as poverty, health, gaps, income on a global scale) via stunning, interactive statistical graphs. This is an interactive, dynamic tool and not just static graphs. Download the software or the reports for free.
www.gapminder.org

WorldOdometers
World statistics updated in real time. Useful for general educational purposes - for some stunning facts.
www.worldometers.info

UN Data
The United Nations offers the ability to search across its statistical databases, including education, human development, population, trade and gender.
http://data.un.org

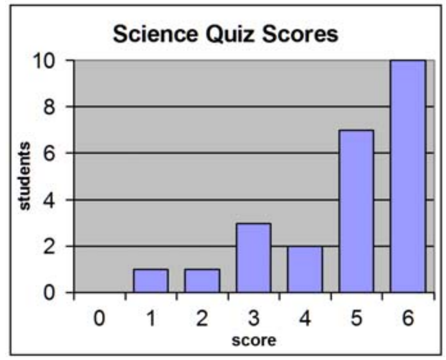
Graphs Quiz from ThatQuiz.org
This quiz asks questions about different kinds of graphs (bar, line, circle graph, multi-bar, stem-and-leaf, boxplot, scattergraph). You can modify the quiz parameters to your liking, such as to plot the graph, answer different kinds of questions about the graph, or find mean, median, or mode based on the graph.
http://www.thatquiz.org/tq-5/math/graphs

(This page intentionally left blank.)

Using Mean, Median, and Mode

Whether you use mean, median or mode depends both

- on the **type of data** *and*
- on the **shape of distribution.**



Example. This distribution of science quiz scores is heavily skewed to the right, and its "peak" is at 6. Which of the three measures of center would best describe this distribution?

Let us calculate the mean, median, and mode.

Mode: We can see from the graph that mode is 6.

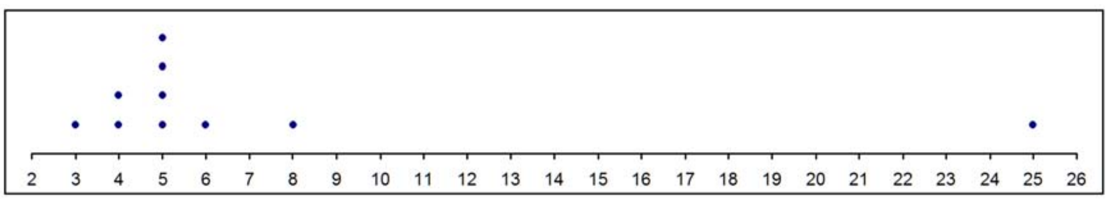
Median: There are 24 students. The students' actual scores are 1, 2, 3, 3, 3, 4, 4, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6.

Median is the average of the 12th and 13th score, which is 5.

Mean is $\dfrac{1 + 2 + 3 + 3 + 3 + 4 + 4 + 7 \times 5 + 10 \times 6}{24} = 4.79167 \approx 4.79.$

Notice, the mean is less than 5, whereas the two highest bars on the graph are at 5 and 6. In this case, the mean does not describe the peak of the distribution very well, because it actually falls outside the peak! Both median and mode do describe it well.

1. **a.** Find the mean, median, and mode of this data set: 3, 4, 4, 5, 5, 5, 5, 6, 8, 25.



mean _______ median _______ mode _________

b. Which of the three, mean, median, or mode, best describes the center of this data?

Clearly, either _____________ or _____________ , but *not* the _____________!

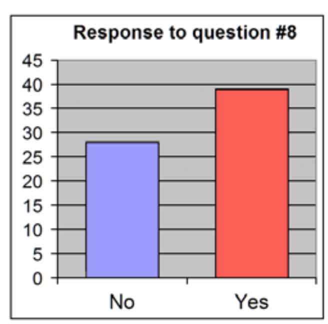
The _____________ is off from the central peak of the distribution.

The reason for this is that the data item 25 **throws it off**. This 25 is very different from the other data items in the set, and could even be a typing error! Such an item is called an **outlier**.

2. The graph shows the response to a certain question in a survey. It is measured as a *yes/no* question. Which of the below are possible to determine? (Mark with an "x").

____ mean ____ median ____ mode

Hint: Imagine what the original data that was used to create the graph looks like.



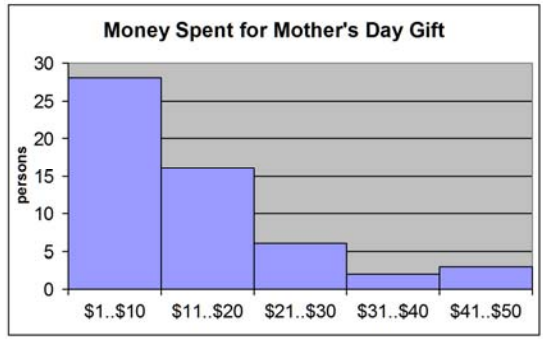
Guidelines for using mean, median, and mode

- *Mode* can be used with any type of data.
- *Median* can only be used if the data can be put in order.
- *Mean* can only be used if the data is numerical.

Sometimes, median and mean do not fall where the peak of the distribution is.

- The mean works best if the distribution is fairly close to a bell shape and does not have outliers.
- If the distribution is very skewed or has outliers, it is better to use median than mean.

3. Judith asked 55 teenagers about how much money they spent to purchase Mother's Day gifts.



a. Which of the numbers $11 and $9 is the mean? Which is the median?

b. Would mean or median better describe this data? Why?

c. *About* how many percent of these teenagers spent $10 or less on a Mother's Day gift?

4. - Name what is being studied (usually the *title* of the graph tells you this).
   - Describe how the data was measured and in what units. For example, perhaps the respondents gave numerical answers, in dollars. Perhaps they chose from yes/no as their answer.
   - Fill in the parts about mean, median, and mode. You do not have to find the mean, even when it is possible.

Hint: Think what kind of data was used to create the graph (the original data).

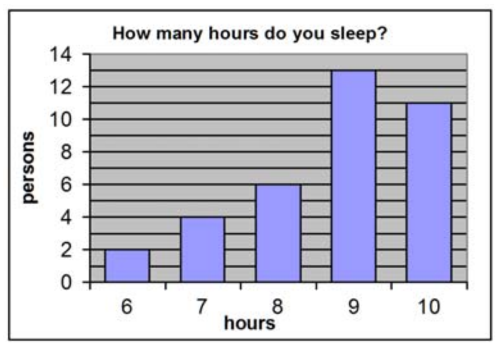
a. What is being measured or studied? ______________________

How is it measured?

Which are possible? (Mark with an "x").

____ mean ____ median ____ mode

Mode is: _____________ Median is: _____________



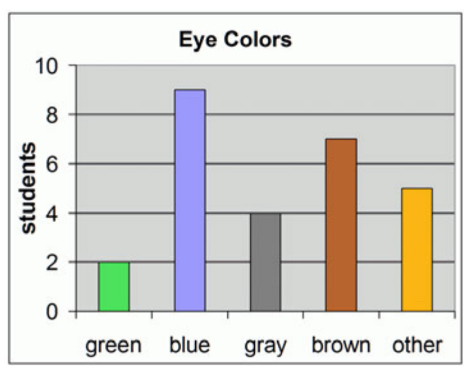
b. What is being measured or studied? ______________________

How is it measured?

Which are possible? (Mark with an "x").

____ mean ____ median ____ mode

Mode is: _____________ Median is: _____________



For the following data sets:

- Create a dot plot or a bar graph.
- Name your graph.
- Describe the shape of the distribution.
- Indicate how many observations there are.
- Choose measure(s) of center that describe the peak of the distribution, and give them.

5. **a.** word length of words on three pages in a certain children's story book:

7 5 6 8 3 6 6 2 4 2 2 3 3 4 4 3 5 5 4
5 4 3 2 5 2 1 4 4 7 5 4 8 3 3 3 3 3 5
5 3 4 2 3 1 6 2 5 4 4 3 4 3 2 8

Here is the same data sorted:

1 1 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 4 4 4
4 4 4 4 4 4 4 4 4 5 5 5 5 5 5 5 5 5 6 6 6 6 7 7 8 8 8

b. A restaurant asked its customers some questions regarding their service and food. The responses to the question, "How would you rate the meal you ate today?" are below. The customers had five choices for their answer: excellent, good, normal, not so good, and poor.

normal poor excellent good good excellent good
normal not so good excellent good good good
normal normal good excellent good good good
not so good not so good excellent good

Puzzle Corner

Can you find a quick, *mental math* method for calculating the mean for this data set? 102, 94, 99, 105, 96, 107, 101, 104
(weights of a litter of kittens at birth, in grams)