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Usefulness of High School Average and ACT Scores in Making College Admission Decisions

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Abstract

Ample correlational evidence indicates that high school GPA is usually better than admission test scores in predicting first-year college GPA, although test scores have incremental predictive validity. Many people conclude that this correlational evidence translates directly to usefulness in making admission decisions. The issue of usefulness is more complex than is implied by correlations or by other regression statistics, however.

This paper considers two common goals in college admission: maximizing academic success and accurately identifying potentially successful applicants. The usefulness of selection variables in achieving these goals depends not only on the predictive strength of the selection variables (such as measured by correlations), but also on other factors, including the distribution of the selection variables in the applicant population, institutions' selectivity, and their criteria for what constitutes success. This paper considers indicators of usefulness in achieving admission goals, and presents estimates of the indicators based on data from a large sample of four-year institutions.

The results suggest that high school GPA is more useful than admission test scores in situations involving low selectivity in admissions and minimal to average academic performance in college. In contrast, test scores are more useful than high school GPA in situations involving high selectivity and high academic performance. In nearly all contexts, test scores have incremental usefulness beyond high school GPA.

The paper also presents evidence for two other interesting phenomena: Students use their high school GPAs and test scores to select the institutions they want to attend, and this self-selection may be more important than institutions' selection in admissions. Moreover, high school GPA by test score interactions are important in predicting academic success.

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Usefulness of High School Average and ACT Scores in Making College Admission Decisions

Conventional wisdom supported by ample evidence holds that high school grades are usually better than college admission test scores in predicting first-year college GPA, but test scores have incremental predictive validity. For example, Morgan (1989) calculated correlations of high school rank, high school grades, and SAT scores with first-year college GPA in a study encompassing the academic years 1976 – 1985. Over this time span, multiple correlations for high school rank and grades ranged from .48 to .52. These correlations were .06 to .14 higher than the corresponding multiple correlations for SAT scores, but were .05 to .07 lower than the corresponding multiple correlations for the high school and test score variables jointly. More recently, Kobrin, Patterson, Shaw, Mattern, and Barbuti (2008) reported correlations of .36 for high school GPA, .35 for SAT scores, and .46 for high school GPA and SAT scores jointly. Evidence for ACT scores (1999, 2008c) is similar: For the academic years 1970-1971 through 2006-2007, multiple correlations of high school subject-area grade averages with first-year college GPA ranged from .48 to .51. The high school grade average correlations were .01 lower to .08 higher than the corresponding ACT score correlations, and were .04 to .09 lower than the corresponding correlations for ACT scores and high school grades jointly.

A plausible explanation of these results is that test publishers strive to make test scores direct measures of cognitive ability only. High school grades, in contrast, are composite measures of both cognitive ability and academically relevant behavior such as attendance, punctuality in turning in assignments, and participation in class (Stiggins, Frisbie, & Griswold, 1989; Brookhart, 1993; Cizek, Fitzgerald, & Rachor, 1995/1996; Noble, Roberts, & Sawyer, 2006). College grades are also composite measures of cognitive ability and academically relevant behavior (Allen, Robbins, Casillas, & Oh, 2008). For this reason, high school GPA should more strongly predict first-year college GPA than test scores do. Weighing in on the other side, test scores are standardized measures, but high school GPA is not; and test scores have higher reliability (Allen et al., 2008). On balance, high school GPA comes out somewhat higher than test scores in its correlation with first-year college GPA, but test scores have incremental predictive validity.

From this evidence, one might conclude that high school grades are more useful than test scores in making admission decisions, but that test scores have incremental usefulness. This conclusion, however, is oversimplified. As will be discussed below, the usefulness of a selection variable for admission to college does depend in large part on its predictive power, but it also depends on admission officers' goals. Correlations are related to the variance in an outcome variable that is explained by predictor variables. Admission officers, however, are not typically interested in explaining variance: They are interested in achieving their institutions' larger goals to educate students successfully. Usefulness also depends on other statistical issues, such as utility, applicant self-selection, and institution selectivity. In this paper, I show that the issue of usefulness is more complex and more interesting than is implied by correlations or other regression statistics. I show that in many cases, the conventional wisdom based on correlations does apply to usefulness, but that in some important respects, it does not.

This study is anchored in the current environment of admission, in which applicants to most four-year institutions must provide scores on admission tests. A broader analysis of the usefulness of test scores in admission would need to consider what would occur if test scores were not used. It is very likely that in an admission system without test scores, high school grades would be subject to inflationary pressure, thereby eroding their predictive power and, ultimately, their usefulness in achieving institutions' goals. This paper does not attempt to model the effects of inflation in high school grades in a hypothetical environment either without college admission tests or with tests optional. There is considerable evidence, however, that even in the current environment, high school grades are subject to inflation (Woodruff & Ziomek, 2004; Geisinger, 2009).

Admission Goals

To gauge the usefulness of a selection variable in achieving a goal, we need to specify the goal. Two common goals related to academic achievement are:

- To maximize academic success among enrolled students.
- To identify accurately those applicants who could benefit from attending the institution, and to enroll as many of them as possible.

These goals seem similar, but they are not identical. As will be described below, the first goal is related to the proportion of applicants who would succeed academically if they enrolled (success rate). The second goal is related to the proportion of applicants whom an institution correctly identifies as likely to succeed or likely to fail (accuracy rate). Both goals, however, pertain only to institutions with some degree of selectivity in their admission policies, rather than to institutions with open admission policies.

The admission selection strategy for accomplishing the first goal is relatively straightforward: So long as there is a positive relationship between the selection variable and the success criterion, an institution can increase the success rate of its enrolled students by admitting only applicants with the highest values of the selection variable. Highly selective institutions can easily pursue this goal because they typically have many more academically qualified applicants than they can admit. Less academically qualified students tend not to apply to these institutions to begin with; these potential applicants in effect select themselves out of the applicant pools. As a result, variance in the predictors is restricted at these institutions, typically resulting in smaller regression slopes and predictive correlations than at less selective institutions (see, for example, Kobrin & Patterson, 2010).

All institutions would ideally like to maximize academic success among their enrolled students. Publicly supported institutions, regional institutions, and other institutions that are moderately selective also consider the second goal to be important, however. These institutions' mission is to educate a broad portion of the population, not just the most academically able. A principal goal in their admission decision making, therefore, is to distinguish between applicants who are likely to be successful from those who are not likely to be successful. This goal is more difficult to achieve than the first, because it depends not only on maximizing academic success among enrolled students, but also on making sure that applicants who are denied admission would have had little chance of succeeding if they had enrolled. As will be shown later, it is possible that a selection variable can achieve the first goal (maximizing academic success), but not the second goal (accurately identifying potentially successful applicants).

Of course, institutions also use test scores in admission for reasons other than achieving these two goals. One reason is objectivity: to include in their decision making a component that can be interpreted the same way for all applicants.[1] Other potential reasons are to abet recruitment, to make course placement decisions, to support counseling and guidance services, to make scholarship decisions, and to assemble data for self-study or comparison to other institutions (Breland, Maxey, Gernand, Cumming, & Trapani, 2002). At less selective

[1] In principle, an institution could statistically adjust high school GPA for high school effects (such as in a hierarchical model), thereby making it more standardized. The practical benefits of doing this are limited, however, if admission test scores are also used (Willingham, 2005).

institutions, these other reasons (especially course placement) might be as important as admission selection.

Institutions' admission goals also relate to considerations other than future academic achievement (Camara, 2005). Examples of non-academic goals include assembling a student body with non-academic achievements, promoting cultural diversity, and promoting support from alumni. For this reason, test scores, high school course work, and high school grades are only part of institutions' admission decision making. Most institutions, in fact, make admission decisions using a holistic approach, rather than by an explicit formula (Breland et al., 2002). Laird (2005) further advocated a holistic individualized approach, in which institutions "make careful, individual decisions about each applicant based on the information at hand and the professional judgment of the admissions staff."

Student Self-Selection

This paper presents evidence that students use high school GPA and test scores in deciding on the institutions to which they apply for admission. Moreover, students' self-selection on these variables in applying to college is likely as important as institutions' reliance on these variables in making admission decisions. Therefore, high school grades and test scores contribute both directly and indirectly to attaining the goal of academic success.

Institutions' goals in making admission decisions overlap to some extent with those of prospective applicants. With respect to the first goal, for example, most students want to succeed academically. Therefore, the success rates associated with particular values of high school GPA and test scores are as relevant to students in their decision making as they are to institutions. Communicating usefulness in terms of success rates instead of correlations is likely to be more meaningful to students, just as it is to institutions.

Academic Success

In this paper, academic success is defined jointly by retention through the first year and by overall first-year college GPA (*FYGPA*). Other researchers (e.g., Saupe & Curs, 2008 and Bowen, Chingos, & McPherson, 2009) have studied long-term academic success (degree completion and cumulative GPA). Long-term success is clearly an important goal for all institutions. Attaining this goal through admission selection, however, is likely to be more feasible at highly selective institutions that attract and enroll only the most academically qualified applicants than at institutions whose mission is to educate a broad segment of students with diverse academic skills. At highly selective institutions, graduation rates and cumulative GPAs are typically very high, and many more highly qualified students apply than can be admitted. At less selective institutions, grades during the first year strongly mediate predictions of long-term success based on pre-enrollment measures (Allen et al., 2008). To achieve their goals, these institutions select applicants who have a reasonable chance of succeeding in the first year, given the interventions (e.g., counseling and course placement) that the institutions might provide.

For the analyses in this paper, students who complete the first year with a given level or higher of *FYGPA* are considered to be successful (S=1); otherwise, they are considered to be unsuccessful (S=0). Although dichotomizing a quasi-interval variable such as *FYGPA* degrades information in a statistical sense, dichotomies correspond more closely to admission officers' interpretations: Are a college student's grades high enough at least to get by, or has the student performed well? Moreover, as will become apparent, the usefulness of high school GPA and test scores depends strongly on which level of success one considers.

The analyses in this paper consider four levels of success:

* *S20*: Retention through first year, and 2.0 or higher FYGPA (minimal success)
* S30: Retention through first year, and 3.0 or higher FYGPA (typical level of success)
* S35: Retention through first year, and 3.5 or higher *FYGPA* (high level of success)
* *S37*: Retention through first year, and 3.7 or higher *FYGPA* (very high level of success)

By these criteria, students who either drop out or have a low *FYGPA* during their first year are unsuccessful. In the data on which this study is based, about 84% of students were at least minimally successful, about 52% were at least typically successful, about 27% were highly successful, and about 16% were very highly successful.

Target Population and Indicators of Usefulness

Admission selection rules are applied to applicants. Therefore, a common-sense indicator of the usefulness of particular selection rules is the estimated proportion of applicants for whom the admission goals would be achieved if they enrolled (Sawyer, 2007). For institutions whose goal is to select the applicants who are most likely to be successful, this proportion is the estimated success rate:

$$SR(c) \quad = \quad \frac{\sum_{i \geq cN} \hat{p}_{(i)}}{(1-c)N} \tag{1}$$

$$= \quad \text{estimated proportion of applicants who, if enrolled, would be successful.}$$

Here, $\hat{p}_{(1)} \leq \ \hat{p}_{(i)} \leq \hat{p}_{(N)}$ are the ordered estimated conditional probabilities of success of the N applicants, given one or more selection variable, and *1 - c* is equal to the proportion of applicants selected. I refer to the variable c as the "cutoff proportion"; it is equal to the cumulative relative frequency associated with a value of the selection variable. The variable *c* relates to an institution's selectivity in admission; the variable *SR(c)* is the estimated success rate among the students the institution does admit.

An institution serving a high-risk population might be concerned about minimizing the proportion of academic failure and near-failure (Fs and Ds) among its first-year students; it would consider *SR* for the *S20* criterion. An institution with few academic failures might instead be interested in maximizing the proportion of students who earn a B or higher average (*S30* criterion). A highly selective institution that expects most of its students to attain excellent academic achievement might consider *SR* for the *S35* or *S37* criteria. Alternatively, an institution using high school grades and test scores for merit scholarship selection, rather than admission, might also consider *SR* for the *S35* or *S37* criteria.

For institutions whose goal is accurately to identify potentially successful applicants, the relevant indicator is the estimated accuracy rate:

$$AR(c) \quad = \quad \frac{\sum_{i<cN}[1-\hat{p}_{(i)}] \quad + \quad \sum_{i\geq cN}\hat{p}_{(i)}}{N} \qquad (2)$$

$$= \quad \text{proportion of applicants for whom a correct admission decision is made.}$$

The *AR* indicator corresponds to an expected utility in which admitting an applicant who would be successful and denying admission to an applicant who would be unsuccessful are both weighted 1, and admitting an applicant who would be unsuccessful and denying admission to an applicant who would be successful are both weighted 0. The *AR* indicator can be generalized to other utilities by assigning different weights to the four outcomes (Sawyer, 1996). For example, if an institution believes that admitting students who do not succeed is less serious an error than denying admission to students who would have been successful, then it could assign a larger weight to the former outcome than to the latter.

An institution concerned about accurately identifying students who could benefit from attending the institution might consider *AR* for the *S30* criterion. A highly selective institution that expects excellent performance from its students, but that is concerned about denying admission to students who might perform at a high level, might consider *AR* for the *S35* or *S37* criteria.

Note that both indicators pertain to the target population of *applicants*, as a whole or in part, rather than only to enrolled students. The indicator *AR(c)* pertains to the entire applicant population for an institution, whereas *SR(c)* pertains to the subset of applicants who meet the institution's cutoff proportion. Predictive validity studies that only summarize correlations and other regression statistics based on data of enrolled students overlook this point. On the other hand, even though the indicators in Equations (1) and (2) pertain to all applicants, we must estimate their conditional probability of success component from the data of enrolled students: The reason is that we can obtain outcome data only from applicants who enroll at an institution.

Institutions are unlikely to forego using high school GPA and test scores in making admission decisions solely to do research.[2] We therefore need to make additional assumptions about the conditional probability of success components in the indicators. With the dichotomous success criterion S considered here, we assume that the conditional probability of success, given the selection variable(s) X, is the same for the non-enrolled applicants as for the enrolled students:

$$p(x) = \mathrm{P}[S{=}1 \mid X{=}x, \text{non-enrolled}] = \mathrm{P}[S{=}1 \mid X{=}x, \text{enrolled}]$$

[2] Although some non-open-enrollment institutions have test-score-optional admission policies, most of them continue to use test scores, at least for some applicants (Breland et al., 2002). Furthermore, it is doubtful that any institution disregards high school GPA (or its transformation to high school rank) in its admission decisions.

This assumption is analogous to that for the traditional adjustment of correlations for restriction of range, which requires that the applicant and enrolled student groups have the same conditional mean and variance functions (e.g., Lord & Novick, 1968). Although empirically testing the correctness of this assumption is not feasible, one could investigate the robustness of the indicators of usefulness to specified departures from the assumption.

With this assumption, calculating the indicators is straightforward: Simply score the entire applicant population with the fitted conditional probability of success function (e.g., using a logistic regression model), and then calculate the indicators from the ordered estimated conditional probabilities.

Cutoff proportion

Note that the indicators considered here assume that an institution admits the top proportion $1-c$ of the applicants, based on their conditional probability of success, and that it denies admission to the bottom proportion c. Unlike the correlation coefficient, which is a global measure, the indicators of usefulness described here depend explicitly on c. These indicators, as well as other considerations (such as capacity for the number of enrolled students), can inform an institution's choice of c.

Of course, institutions do not use simple cutoffs in making admission decisions. As is noted in the discussion following Table 4, institutions likely use high school grades and test scores as initial screens, but base their final admission decisions by considering additional variables. Equations (1) and (2) are mathematical idealizations that enable us to compare the properties of alternative selection variables. In this paper, cutoff proportions range from .01 (virtually all applicants admitted) to .99 (extreme selectivity).

Correlation Coefficient

According to (1) and (2), the indicators *SR(c)* and *AR(c)* are functions of the conditional probability of success *p(x)*, the distribution of *p(x)* in the applicant population, and the cutoff proportion *c*. If the selection variable *X* and the underlying outcome variable *Y* have a bivariate normal distribution in the applicant population, then we can more directly calculate *SR(c)* and *AR(c)* by integrating the bivariate normal density function over appropriate regions of *X* and *Y*:

$$SR(c) \quad = \quad \frac{\int_{z_S}^{\infty}\int_{z_c}^{\infty}\phi(x,y;\rho)\,dx\,dy}{1-c}$$

$$AR(c) \quad = \quad \int_{-\infty}^{z_S}\int_{-\infty}^{z_c}\phi(x,y;\rho)\,dx\,dy \quad + \quad \int_{z_S}^{\infty}\int_{z_c}^{\infty}\phi(x,y;\rho)\,dx\,dy\,, \qquad (3)$$

where z_c is the value of a standard normal variable corresponding to the cutoff proportion *c* on the selection variable *X*, $z_S = (y_S - \mu_Y)/\sigma_Y$ is the value of a standard normal variable corresponding to the success level y_S on the outcome variable *Y*, and ϕ is the bivariate standard normal density function with correlation parameter ρ .

A possible reason why researchers often interpret ρ as a measure of effectiveness in selection is that given a bivariate normal distribution, *SR* and *AR* depend on ρ . As is clear from (3), however, *SR* and *AR* also depend on the success variable *S* and the cutoff proportion *c*, as well as on ρ . Moreover, as we shall soon see, the assumption of bivariate normality is not tenable in the context of college admission selection. In particular, high school GPA has a severe negative skew and a pronounced ceiling.

Incremental Usefulness

A basic question that we should ask when evaluating the usefulness of a selection variable *X* is: Does using a variable *X* for selection increase the success rate and the accuracy rate over what would result if an institution did not use *X* (i.e., if it admitted all applicants or denied admission to all applicants)? Admitting all applicants amounts to setting $c=0$ in Equations (1) and (2); the resulting quantity for either indicator is the base success rate

$$BSR = SR(0) = AR(0) = \frac{\sum_{all\ i} \hat{p}_{(i)}}{N} \quad , \tag{4}$$

which is also equal to the overall (marginal) probability of success. Denying admission to all applicants amounts to setting $c=1$, and would result in an accuracy rate of *1-BSR*. I refer to admitting all applicants and denying admission to all applicants as "null decisions."

Note that *SR* depends on both the conditional probability of success function *p* and on the distribution of *p* (or, alternatively, of *X*) in the applicant population. Nevertheless, if *p* is an increasing function of *x* (the value of the selection variable *X*), then *SR* is also an increasing function of *x* (see Appendix), and therefore of *c*. Hence, if *p* is an increasing function of *X*, *SR(c)* exceeds *BSR*. Incremental usefulness in terms of success rate therefore corresponds to the traditional notion of nonzero regression slope or correlation.

As we shall see, however, the same need not be true of *AR*: Even though a variable is positively related to success, using it in selection might result in a decrease in classification accuracy. One can show by a straightforward differentiation argument (see Appendix) that *AR(c)* exceeds both *BSR* and *1-BSR* at some cutoff proportion if, and only if, the conditional probability of success function *p(x)* crosses 0.5. Moreover, *AR(c)* achieves its maximum value at the cutoff proportion c' associated with x' (and this maximum exceeds *BSR* and *1-BSR*) if, and only if, $p(x') = 0.5$.

Another important question involves comparing alternative selection variables *X* and *W*: Does using *X* and *W* jointly increase the success rate and accuracy rate over that which would occur if we used *X* only? This question pertains to incremental usefulness, and is analogous to the traditional notion of incremental predictive validity.

Because *SR(c)* and *AR(c)* depend on the cutoff proportion *c*, the answers to both questions can vary, depending on *c*. A selection variable might have incremental usefulness (either with respect to null decisions or with respect to another selection variable) at some cutoff proportions, but not at others.

Data

The analyses in this paper are based on data from 192 four-year postsecondary institutions that use ACT scores in their admission procedures (ACT, 2008a). The institutions provided outcome data either through their participation in ACT's predictive validity service or through participation in special research projects. The outcome data pertain to the following entering freshman class years: 2003 (1% of institutions), 2004 (31%), 2005 (68%), and 2006 (1%). For institutions that had data from more than one entering freshman class, I used the most recent data.

Institutional characteristics. Table 1 compares characteristics of the 192 institutions in the sample to those of all four-year institutions in the U.S. (ACT, 2008b). The sample institutions are broadly representative of four-year institutions in the U.S. with respect to their proportion of minority students and students' average ACT Composite scores. A majority of the institutions in this study are public, however, whereas only about 30% of all postsecondary institutions are public. Moreover, the institutions in this study tend to be much larger than four-year institutions generally.

TABLE 1

Summary of Institutions in Sample and of Four-Year Institutions in the U.S.

Characteristic	Sample (N=192)	Four-year institutions in the U.S. (N=2,044)
Affiliation: Proportion public	.57	.30
Self-reported admission selectivity: Proportion selective or highly selective	.27	.30
Median undergraduate enrollment	2,883	1,545
Median proportion minority	.21	.25
Median average ACT Composite score of enrolled students	21.8	22.0

When students register to take the ACT, they report their high school course work and grades. The analyses are based on *HSAvg*, the average of students' self-reported grades in standard college-preparatory courses, and on *ACT-C*, the Composite (average) of students' ACT scores in English, mathematics, reading, and science. The outcome success variables *S20*, *S30*, *S35*, and *S37*, are based on data reported by the postsecondary institutions.

Student characteristics (pooled sample). Table 2 contains the means and standard deviations of the pre-enrollment and outcome measures in the pooled sample of 120,338 students across all institutions.

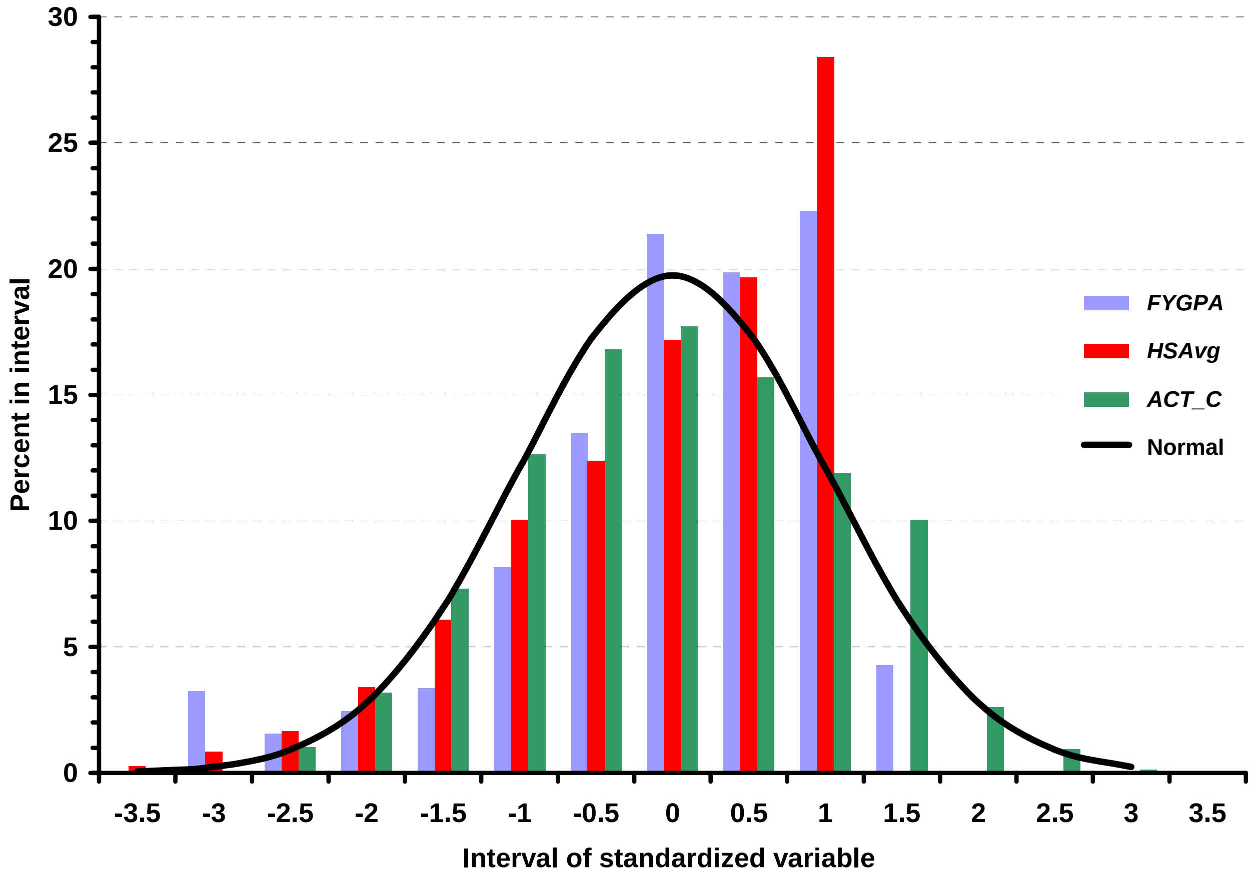
TABLE 2

Summary of Enrolled Student Characteristics
(Pooled Sample; N=120,338)

Characteristic	**Mean**	**SD**
Pre-enrollment measures		
HSAvg	3.42	0.50
ACT-C	22.6	4.3
Outcome measures		
S20	0.84	0.37
S30	0.52	0.50
S35	0.27	0.44
S37	0.16	0.37
FYGPA	2.78	0.95

As was noted previously, *SR* and *AR* are functions of the correlation ρ (and of other properties of the selection and outcome variables) when they have a bivariate normal distribution. Figure 1 shows histograms of *HSAvg*, *ACT-C*, and *FYGPA* standardized to z-scores with respect to the enrolled student population pooled over institutions. Figure 1 also shows a reference curve for the standard normal distribution.

FIGURE 1. Distributions of Standardized *FYGPA*, *HSAvg*, and *ACT-C*, Compared to the Standard Normal Distribution



Value on original scale							
FYGPA	0.0	0.9	1.8	2.8	3.7		
HSAvg		2.4	2.9	3.4	3.9		
ACT-C		14	18	22	27	31	35

As is clear from the marginal distributions shown in Figure 1, the assumption of bivariate normality is untenable. The distribution of *HSAvg* in our data has a pronounced negative skew (-0.9). The modal category of *HSAvg* is its maximum category, 0.75 to 1.25 standard deviations above the mean. *HSAvg* is also negatively skewed in broader populations: (-0.7) combined score sender / enrolled student population; (-0.7) all 2005 ACT-tested high school graduates; and (-0.4) a nationally representative sample of eleventh-grade students (Casillas, Robbins, Allen, Kuo, Hanson, & Schmeiser, 2010).

FYGPA is also negatively skewed (-1.1), and has a minor mode at its minimum category (2.75 to 3.25 standard deviations below the mean). The distribution of *ACT-C* is more nearly symmetric (skewness = 0.1). Furthermore, both the conditional mean of *FYGPA*, given *HSAvg*, and the conditional mean of *FYGPA*, given *ACT-C*, are slightly curvilinear (not shown in Figure 1). Therefore, one should be cautious in applying normal theory to calculate *SR* and *AR*.

Distribution of enrolled student characteristics among institutions. Table 3 summarizes the distribution of the means and correlations of the pre-enrollment and outcome variables among institutions.

TABLE 3

Summary of Enrolled Student Characteristics Among Institutions (N=192)

Characteristic	**Median**	**Min. – Max.**
Number of ACT-tested enrolled students	319	16 – 5,210
<u>Pre-enrollment measures</u>		
HSAvg mean	3.36	2.49 – 3.80
ACT-C mean	21.8	14.9 – 28.7
<u>Outcome measures</u>		
S20 mean	0.84	0.33 – 1.00
S30 mean	0.49	0.01 – 0.96
S35 mean	0.23	0.01 – 0.71
S37 mean	0.14	0.00 – 0.44
FYGPA mean	2.83	1.76 – 3.58
<u>Correlations</u>		
HSAvg / *ACT-C* correlation	0.44	-0.01 – 0.66
FYGPA / *HSAvg* correlation	0.48	-0.14 – 0.83
FYGPA / *ACT-C* correlation	0.41	-0.15 – 0.63
FYGPA / *HSAvg* & *ACT-C* multiple R	0.54	0.06 – 0.84

Pre-enrollment measures. The median mean HSAvg over the institutions in the sample (3.36) is similar to the mean HSAvg (3.32) of first-year college students nationally. The median mean *ACT-C* score over the institutions in the sample (21.8) is also similar to the mean score (22.1) of first-year college students nationally in 2008 (ACT, 2009a).

Outcome measures. At typical institutions in the sample, a huge majority (84%) of students completed the first year with at least a C average, and nearly half completed the first year with a B or higher average. Only a small proportion of students had *FYGPA*s of 3.7 or higher. The median average *FYGPA* of 2.83 reflects a minor mode in the *FYGPA* distribution at the value 0.0 (3% of students).

Among 1,634 institutions that recently completed a questionnaire administered by ACT, the average self-reported retention rate was 73% (ACT, 2009b). The 73% result was based on a different definition of retention (re-enrollment in the sophomore year) than that in this study (completion of the first year), but it suggests that the institutions in this study have higher retention rates than institutions generally.

Correlations. The median correlations at the bottom of Table 3 show the typical result: *HSAvg* is a better predictor of *FYGPA* than *ACT-C*, but *ACT-C* has incremental predictive validity. Of more interest is the huge variation among institutions in their correlations. At two institutions, *HSAvg* and *ACT-C* were both negatively correlated with *FYGPA*. None of the correlations were statistically significant ($p < .05$), even though neither institution's sample size was small (N=145 and 154). At the other extreme, *HSAvg* and *ACT-C* jointly accounted for nearly two-thirds of the variance in *FYGPA* (multiple R = .83 and .84) at two other institutions. Although not shown in Table 3, correlations for both predictor variables were higher at private institutions, at institutions with smaller percentages of minority students, and at institutions with larger standard deviations in the predictor variables. Correlations were lower at highly selective institutions.

Applicants and Score Senders

As was previously noted, postsecondary institutions make admission decisions about applicants; therefore, indicators of usefulness should be calculated for this target population. For several reasons, it is not feasible in a study involving many institutions to identify their applicants. I instead used score senders (students who sent their ACT scores to particular institutions) as a proxy for applicants. Score senders can be thought of as predecessors to applicants: Not all score senders decide to apply to an institution, but most applicants will have sent their scores.[3] The 192 institutions in the sample for this study had 483,451 non-enrolled score senders, in addition to their 120,338 enrolled students. Strictly speaking, therefore, the results reported here pertain to score senders, rather than to actual applicants.

Fifty-three of the 192 institutions represented in the study provided data on their actual applicants. By matching the applicant, score-sender, and enrolled student files of the 53 institutions, I determined whether an individual was a non-applicant, non-enrolled score sender, a non-enrolled applicant, or an enrolled student. Table 4 below compares the distributions of *HSAvg* and *ACT-C* among these three groups:

TABLE 4

Distribution of *HSAvg* and *ACT-C*, by Application and Enrollment Status at 53 Institutions

Application/enrollment status	N	***HSAvg*** Mean (SD)	***ACT-C*** Mean (SD)
1. Non-applicant, non-enrolled score senders	179,185	3.23 (0.59)	21.3 (4.4)
2. Non-enrolled applicants	17,305	3.40 (0.51)	22.3 (4.0)
3. Enrolled students	85,899	3.43 (0.48)	23.0 (4.1)

Note that the means of both variables increase from non-applicant score senders to non-enrolled applicants, and from non-enrolled applicants to enrolled students, but the standard deviations mostly decrease. Moreover, the non-applicant score-sender means and the non-enrolled applicant means differ more than the non-enrolled applicant means and the enrolled

[3] Some institutions permit applicants to submit test scores on their high school transcripts.

student means. This result suggests that with respect to *HSAvg* and *ACT-C*, applicant self-selection is at least as important as institutions' selection through admission decisions.

In terms of standard deviations, the difference between the mean *ACT*-C score of the non-enrolled applicants and the enrolled students is only moderately large (0.18 SD). The corresponding difference in mean *HSAvg* is quite small (0.06 SD). This result suggests either that institutions' admission decisions were based on other variables, in addition to *ACT-C* and *HSAvg*, or that admitted students' decisions to enroll were based on other variables, or (as seems likely) both. Although not addressed by these data, one plausible hypothesis is that students and institutions primarily use high school grades and test scores as initial screens, but base their final decisions substantially on other variables. Some of these other variables might include characteristics such as cost, location, personal goals, athletic talent, out-of-class accomplishments, and previous connections with the institution.

Method

Modeling Probability of Success

According to Equations (1) and (2), an essential component of the indicators *SR(c)* and *AR(c)* is $p(x) = \hat{P}[S = 1 \mid X = x]$, the estimated conditional probability of success, given the value of the selection variable *X*. I estimated the conditional probability of success using a hierarchical logistic regression model.

In logistic regression, we model the log-odds $\ln[p(x)/(1 - p(x))]$, rather than $p(x)$ directly; we can then calculate an estimated *p(x)* from the estimated log-odds. The intercept β_0 and the slope β_1 in the log-odds model are related to the point at which the probability-of-success curve crosses 0.5 and to its slope at this point: $p(-\beta_0 / \beta_1) = 0.5$, and

$p'(-\beta_0 / \beta_1) = \beta_1 / 4$. Additionally, in the hierarchical model, the intercept and slope coefficients for the linear predictor of the log-odds vary among institutions:

$$
\begin{aligned}
\ln\left(\frac{p_{ij}(x)}{1-p_{ij}(x)}\right) &= \beta_{0j} + \beta_{1j} x_{ij} \\
\beta_{0j} &= \gamma_0 + u_{0j} \\
\beta_{1j} &= \gamma_1 + u_{1j}
\end{aligned}
\tag{5}
$$

The intercept β_{0j} for institution j is the sum of a fixed effect γ_0 that is constant across institutions and a random effect u_{0j} that is specific to institution j. Similarly, the slope term β_{1j} for institution j is the sum of a fixed effect γ_1 that is constant across institutions and a random effect u_{1j} that is specific to institution j. The symbol x_{ij} refers to a selection variable (either *HSAvg* or *ACT-C*) for student i at institution j. To facilitate interpretation of the intercept, as well as computation, I centered x_{ij} about its mean across students (grand-mean centering).

The hierarchical model is more precise than a model based on data pooled across institutions, because it takes into account the dependence of observations within institutions (Snijders & Bosker, 1999). Furthermore, the hierarchical model is more parsimonious (has fewer parameters) than estimating a separate model for each of the 192 institutions. As a result, predictions based on the hierarchical model are likely to be more accurate at small institutions than predictions based on institution-specific models.

The concentration of *HSAvg* at its highest values suggests that we might improve prediction at the high end by a suitable transformation. One possibility is to replace *HSAvg* with its cumulative relative frequencies (in effect, causing it to have a more nearly uniform distribution). Another possibility is to transform *HSAvg* to have approximately a normal distribution. Although transformations like this can decrease skewness, they do not change the concentration of the distribution on particular values (e.g., 4.0).

I also estimated models based on *HSAvg* and *ACT-C* jointly. A standard way to model the relationship between probability of success and both variables is to include them as main effects x_{1ij} and x_{2ij} in the hierarchical model. It is possible, however, that the relationship between the log-odds and each selection variable depends on the value of the other selection variable; we can test this possibility with an interaction term $x_{1ij}x_{2ij}$ in the following model:

$$
\begin{aligned}
\ln\left(\frac{p_{ij}(x)}{1-p_{ij}(x)}\right) &= \beta_{0j} + \beta_{1j}x_{1ij} + \beta_{2j}x_{2ij} + \beta_{3j}x_{1ij}x_{2ij} \\
\beta_{0j} &= \gamma_0 + u_{0j} \\
\beta_{1j} &= \gamma_1 + u_{1j} \\
\beta_{2j} &= \gamma_2 + u_{2j} \\
\beta_{3j} &= \gamma_3
\end{aligned}
\qquad (6)
$$

For example, Equation (6) says that the slope of the log-odds on x_{1ij}, namely $\beta_{1j} + \beta_{3j}x_{2ij}$, depends on x_{2ij} [4]. As in Equation (5), I centered all the independent variables about their respective grand means.

Initially, I estimated the interaction Model (6) using data of all 120,338 students in the sample. Plots of the fixed effects in this initial model revealed that for values of *HSAvg* below 2.0, the estimated linear predictor was a weakly decreasing function of *ACT-C*. I therefore re-estimated the interaction model using data only for students with *HSAvg* above 2.0. For the 945 students whose *HSAvg* was less than 2.0, I set the probability of success equal to their overall base success rate.

---

[4] I would like to thank Professor Joseph Rodgers, University of Oklahoma, for his suggestion to estimate interaction models.

To simplify the extensive data processing required to estimate the hierarchical logistic models, and to calculate the indicators of usefulness *SR(c)* and *AR(c)*, I did both using the same software (SAS). I used PROC NLMIXED (SAS Institute, 2008) to estimate the hierarchical logistic models. With large data sets, NLMIXED requires gargantuan computer resources, and I could not use it to estimate a model with a random effect u_{3ij} in the interaction term coefficient β_{3j} and with level-2 means as additional fixed effects. I therefore used the simplified Model (6) to calculate success rates and accuracy rates.

More complex hierarchical models. To learn more about the variation in the coefficients among institutions, I estimated a more complex model with the HLM6 software (Raudenbush, Bryk, Cheong, & Congdon, 2004):

$$
\begin{aligned}
&\ln\left(\frac{p_{ij}(x)}{1-p_{ij}(x)}\right) = \beta_{0j} + \beta_{1j}x_{1ij} + \beta_{2j}x_{2ij} + \beta_{3j}x_{1ij}x_{2ij} \\
&\beta_{0j} \;=\; \gamma_{00} + \sum_s \gamma_{0s}W_{0sj} + u_{0j} \\
&\beta_{1j} \;=\; \gamma_{10} + \sum_s \gamma_{1s}W_{1sj} + u_{1j} \\
&\beta_{2j} \;=\; \gamma_{20} + \sum_s \gamma_{2s}W_{2sj} + u_{2j} \\
&\beta_{3j} \;=\; \gamma_{30} + \sum_s \gamma_{3s}W_{3sj} + u_{3j}
\end{aligned}
\tag{7}
$$

Potential level-2 predictors $W_{0sj}, W_{1sj}, W_{2sj}, W_{3sj}$ for $\beta_{0j}, \beta_{1j}, \beta_{2j}, \beta_{3j}$, respectively, were the institution means $\bar{x}_{1j}$ and $\bar{x}_{2j}$ of the centered selection variables x_{1ij} and x_{2ij}, the institutional mean of the centered interaction term, and the institutional characteristics affiliation (public/private), undergraduate enrollment, and percentage minority. I estimated models for which all the student-level fixed effects $\gamma_{00}, \gamma_{10}, \gamma_{20}, \gamma_{30}$ were statistically significant (p < .001), the fixed effects $\gamma_{0s}, \gamma_{1s}, \gamma_{2s}, \gamma_{3s}$ for the institution characteristics $W_{0sj}, W_{1sj}, W_{2sj}, W_{3sj}$ were statistically significant (p < .01), and the variances of the random effects $u_{0j}, u_{1j}, u_{2j}, u_{3j}$ were statistically significant (p < .01). Model (7) tells us whether the coefficients $\beta_{0j}, \beta_{1j}, \beta_{2j}, \beta_{3j}$ vary systematically by the institution characteristics $W_{0sj}, W_{1sj}, W_{2sj}, W_{3sj}$ and whether they vary randomly by institution. I did not use Model (7) to calculate success rates and accuracy rates.

Cross-classified models. Students are nested within high school, as well as within postsecondary institution. Therefore, the regression coefficients in Equations (5), (6), and (7) could vary among both high schools and postsecondary institutions, particularly when *HSAvg* is the selection variable. Estimating models with cross-classified random effects is difficult, especially in large data sets, and estimating nonlinear cross-classified models would be more complex still. I therefore deferred investigating them to a future study.

Indicators of Usefulness

From the estimated probabilities of success returned by NLMIXED for models (5) and (6), I calculated *SR(c)* and *AR(c)* using the cutoff proportions *c*= .01, .10, .20, .30, .40, .50, .60, .70, .80, .85, .90, .95, and .99 for each selection variable. These cutoff proportions correspond to increasing degrees of admission selectivity: The cutoff proportion .01 corresponds to admitting all but the bottom 1% of students, as ranked by their estimated probability of success; the cutoff proportion .99 corresponds to admitting only the top 1% of students.

Incremental success rate with respect to base rate. The incremental success rate associated with a selection variable is the difference between *SR(c)*, the success rate associated with admitting applicants at or above cutoff proportion *c*, and the base success rate *BSR*=*SR(*0*)*, the success rate associated with admitting all applicants. Recall that a selection variable has positive incremental success rate if its probability-of-success curve is increasing. As it turned out, the probability-of-success curves for all four success variables at all 192 institutions were increasing.[5] I calculated for each value of c the median incremental success rate across institutions:

$$Med.\,Inc.\,SR(c) \quad = \quad \underset{1 \le i \le 192}{median}\,\{SR_i(c) - BSR_i\}\,. \tag{8}$$

Incremental success rate of ACT-C with respect to HSAvg. As in evaluating correlations, it is important to determine whether a selection variable is incrementally useful with respect to another variable. To gauge the incremental usefulness of *ACT-C* with respect to *HSAvg* for maximizing the academic success of enrolled students, I calculated the median difference between the success rate for the model based on *HSAvg* and *ACT-C* jointly, and the success rate for the single-variable model based on *HSAvg* only:

$$Med.\,Inc.\,SR^{[ACT-C]}(c) \quad = \quad \underset{1 \le i \le 192}{median}\left\{SR_i^{[HSAvg\ and\ ACT-C]}(c) - SR_i^{[HSAvg]}(c)\right\}. \tag{9}$$

Incremental accuracy rate with respect to null decisions. As was noted earlier, a selection variable has incremental accuracy with respect to the base success rate (*BSR*) associated with accepting all applicants and the base failure rate (*1-BSR*) associated with denying admission to all applicants if, and only if, its probability-of-success curve crosses 0.5 somewhere. Otherwise, the institution would do better (in terms of accuracy) to choose either the null decision to accept all applicants or the null decision to reject all applicants. I therefore calculated:

Rel. Freq. Inc. Acc. = proportion of institutions whose probability-of-success curve crosses 0.5 somewhere. (10)

[5] Because hierarchical models shrink coefficient estimates toward the mean, all estimated slopes were positive, even at the two institutions with negative correlations.

This statistic is the proportion of institutions for which *AR(c)* exceeds max[*BSR*, 1-*BSR*] at *some* cutoff proportion; it is relevant for evaluating a selection variable globally across values of *c*.

Note that even if an institution's probability-of-success curve cross 0.5 at some cutoff proportion c', *AR(c)* need not exceed max[*BSR*, 1-*BSR*] at *all* cutoff proportions. I therefore calculated for each cutoff proportion *c* the relative frequency of incremental accuracy among institutions:

$$\textit{Rel. Freq. Inc. Acc}(c) = \text{proportion of institutions for which } AR_i(c) - \max[BSR_i, 1\text{-}BSR_i] > 0 \,. \tag{11}$$

I also calculated the median incremental accuracy rate among the institutions where it is positive:

$$Med.\ Inc.\ AR(c) \quad = \quad \underset{i:\, AR_i(c) - \max[BSR_i,\, 1-BSR_i] > 0.}{median} \left\{ AR_i(c) - \max[BSR_i, 1-BSR_i] \right\}, \tag{12}$$

This statistic shows the typical improvement in accuracy rate, above admitting or denying admission to everyone, at institutions where there is any such improvement.

Incremental accuracy rate of ACT-C with respect to HSAvg. To gauge the incremental usefulness of *ACT-C* for accurately identifying applicants who could benefit from attending an institution, I calculated the median difference between the accuracy rate for the model based on *HSAvg* and *ACT-C* jointly, and the accuracy rate for the single-variable model based on *HSAvg* only:

$$Med.\ Inc.\ AR^{[ACT-C]}(c) \quad = \quad \underset{i:\, AR_i^{[HSAvg\ and\ ACT-C]}(c) - \max[BSR_i,\, 1-BSR_i] > 0}{median} \left\{ AR_i^{[HSAvg\ and\ ACT-C]}(c) - AR_i^{[HSAvg]}(c) \right\}. \tag{13}$$

The median pertains to institutions for which the joint *HSAvg* and *ACT-C* model has incremental accuracy at or above cutoff proportion *c*.

Effects associated with extrapolation to score-sender population. Note that the success rate and accuracy rate statistics depend not only on the cutoff proportion *c* and the values of the estimated conditional probability of success function $\hat{p}$, but also on the distribution of $\hat{p}$ in the applicant population. Because applicant data were not available for most institutions in this study, I used data from score senders. From the previous discussion, we know that score senders have somewhat lower mean *HSAvg* and *ACT-C* than applicants, and that applicants have somewhat lower mean *HSAvg* and *ACT-C* than enrolled students. To obtain a rough notion of how using the score sender data affected the estimated success rates and accuracy rates, I recalculated these statistics using only the enrolled student data. If results calculated from the enrolled student data are similar to the results calculated from the combined group of non-enrolled score senders and enrolled students, then we can have more confidence that results calculated from the combined group of non-enrolled applicants and enrolled students would be similar.

Results

Hierarchical Models

Table 5 on the following page summarizes the simple hierarchical predictive models (Equations (5) and (6)) for each of the four success levels. Note that in both of the single-variable models (labeled A and B in Table 5), the fixed effects for the *HSAvg* and *ACT-C* slope coefficients are positive and statistically significant ($p < .001$). Moreover, the slope coefficients for *HSAvg* and *ACT-C* both increase with success level. For example, the *HSAvg* slope coefficient for the 2.0 success level is 1.596; for the 3.7 level, it is 3.759. This result suggests that *HSAvg* and *ACT-C* are more strongly related to high levels of success than they are to low levels of success.

TABLE 5

Simple Hierarchical Models for Predicting Success from *HSAvg* and *ACT-C*

Fixed effects		**Success level**							
		2.0 or higher		**3.0 or higher**		**3.5 or higher**		**3.7 or higher**	
Model	**Variable**	**Coefficient**	***p*** <	**Coefficient**	***p*** <	**Coefficient**	***p*** <	**Coefficient**	***p*** <
A	*Intercept*	2.019	.001	0.007	.877	-1.586	.001	-2.552	.001
	HSAvg	1.596	.001	2.283	.001	3.079	.001	3.759	.001
B	*Intercept*	1.974	.001	0.129	.004	-1.268	.001	-2.043	.001
	ACT-C	0.161	.001	0.232	.001	0.271	.001	0.295	.001
C	*Intercept*	2.196	.001	0.185	.001	-1.478	.001	-2.457	.001
	HSAvg	1.539	.001	1.931	.001	2.386	.001	2.825	.001
	ACT-C	0.105	.001	0.150	.001	0.163	.001	0.173	.001
	HSAvg X *ACT-C*	0.084	.001	0.097	.001	0.092	.001	0.095	.001

Random effects		**Success level**							
		2.0 or higher		**3.0 or higher**		**3.5 or higher**		**3.7 or higher**	
Model	**Variable**	**Std. dev.**	***p*** <	**Std. dev.**	***p*** <	**Std. dev.**	***p*** <	**Std. dev.**	***p*** <
A	*Intercept*	0.846	.001	0.642	.001	0.550	.001	0.690	.001
	HSAvg	0.332	.001	0.632	.001	0.866	.001	1.170	.001
B	*Intercept*	0.788	.001	0.579	.001	0.502	.001	0.562	.001
	ACT-C	0.051	.001	0.056	.001	0.059	.001	0.066	.001
C	*Intercept*	0.785	.001	0.636	.001	0.570	.001	0.710	.001
	HSAvg	0.252	.004	0.500	.001	0.705	.001	0.936	.001
	ACT-C	0.030	.004	0.033	.001	0.036	.001	0.041	.001

The variances of the *HSAvg* and *ACT-C* slope coefficients among institutions (lower half of Table 5) also increase with success level. The strength of these variables' relationships with higher levels of success varies more among institutions than does the strength of their relationships with lower levels of success.

The coefficients of variation for the *HSAvg* and *ACT-C* slopes (the standard deviation of the slope random effect divided by the slope fixed effect) are approximately 0.2 and 0.3, respectively. This result indicates that there is moderate variation among institutions in the slopes of the predictor variables. Nevertheless, the estimated slopes from the hierarchical model are positive at all institutions.

A typical way to compare the strength of predictor variables is to standardize their slope coefficients with respect to their standard deviations. On multiplying the fixed effects for the HSAvg and ACT-C slopes in Table 5 by the corresponding standard deviations in Table 1, we find that the standardized regression coefficients for HSAvg are uniformly larger than those for ACT-C. As was previously noted, however, the usefulness of selection variables depends on other properties, in addition to their regression coefficients. Examining the probability of success curves, rather than just the slope coefficients, lets us observe differences in strength of prediction across the entire ranges of predictor variables.

Figures 2 and 3 on the following pages show probabilities of success calculated from the fixed effects of *HSAvg* and *ACT-C*. These probability curves pertain to typical postsecondary institutions (i.e., those for which the random effects are 0). In both graphs, the horizontal axis is scaled in terms of both the values of the selection variables and their associated cutoff proportions (cumulative relative frequencies, *c*).

Figure 2. Probability of Success, Given *HSAvg*

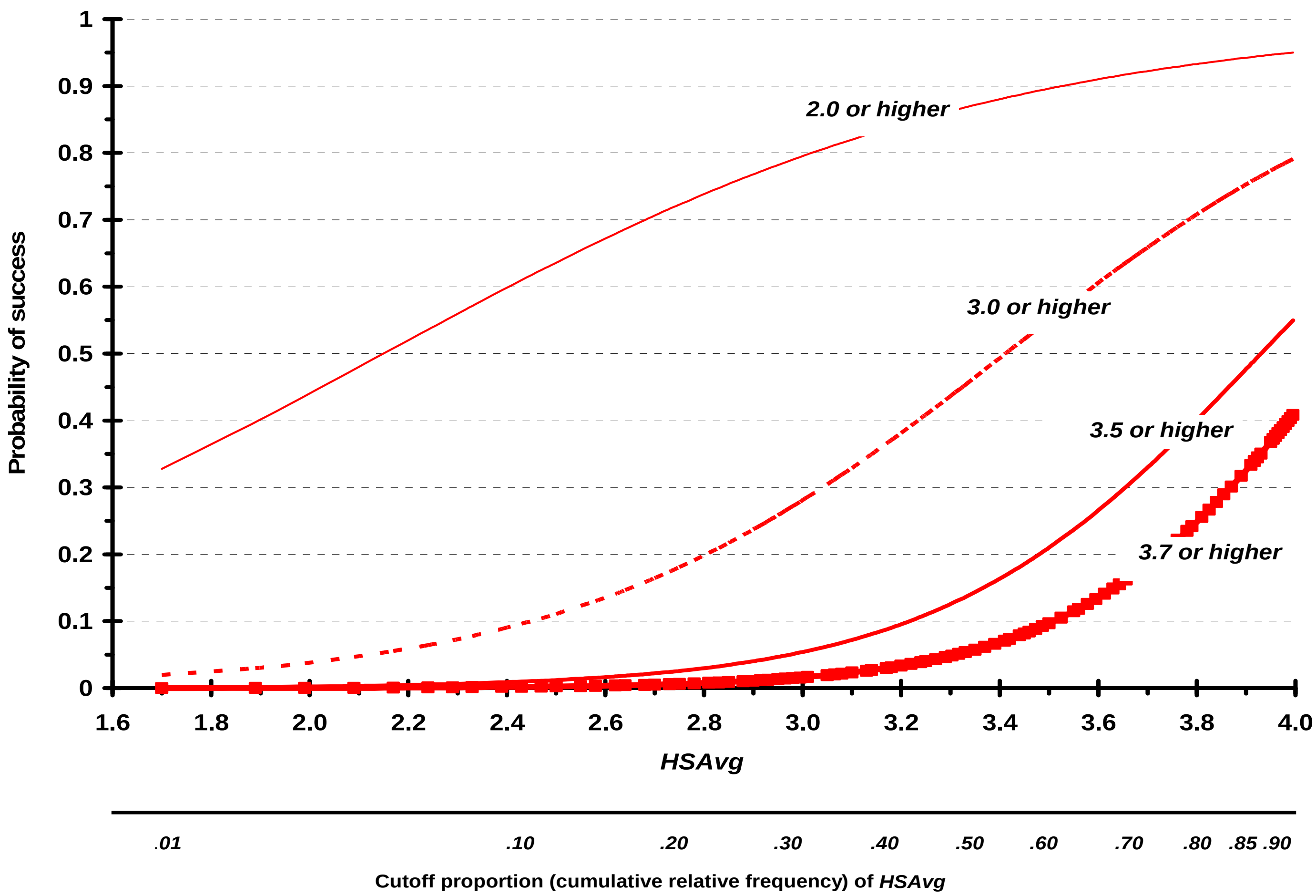
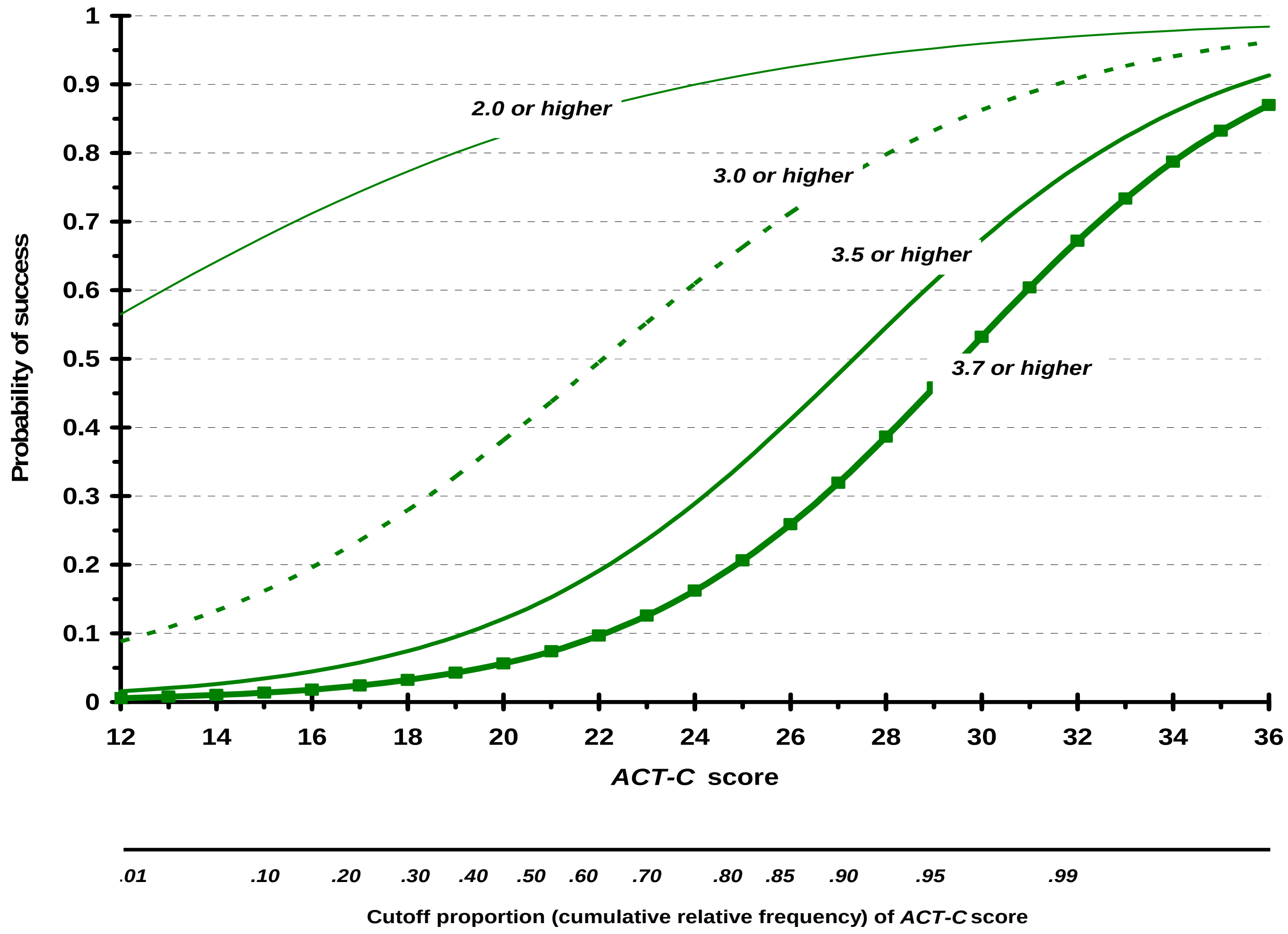


Figure 3. Probability of Success, Given *ACT-C* Score



To describe and compare the statistical relationships shown in Figures 2 and 3, I noted the following characteristics:

- What range of estimated probabilities is associated with the entire range of the selection variable? A broader range of estimated probabilities suggests better prediction.
- Does the probability curve cross 0.5? Recall, this property is required for incremental accuracy in selection.
- Over what values of the selection variable is the probability curve steepest? To answer this question, I noted the smallest interval of the selection variable associated with an increase of approximately 0.5 in the estimated probability. I also noted the range of cutoff proportions corresponding to the interval.

For predicting the 2.0 or higher success level, the *HSAvg* curve assumes values between .33 and .95. The *ACT-C* curve, in contrast, ranges between .56 and .98, indicating that at typical institutions, *ACT-C* does not have incremental accuracy in selection with respect to this success level. The steepest part of the *HSAvg* curve is associated with values of *HSAvg* between 1.70 and 3.17 (corresponding to cutoff proportions between .01 and .41).

For predicting the 3.0 or higher success level, the *HSAvg* curve assumes values between .02 and .79. The *ACT-C* curve has a broader range of estimated probabilities, .09 to .96, but both curves cross 0.5. The *HSAvg* curve is steepest over the values 2.95 to 3.91 (corresponding to cutoff proportions .29 to .86). The *ACT-C* curve is steepest over the scores 19 to 29 (corresponding to cutoff proportions .37 to .96). Thus, *ACT-C* has a wider spread of estimated probabilities than *HSAvg*, and is most predictive at higher cutoff proportions. In contrast, *HSAvg* is most predictive at a middle range of cutoff proportions.

For predicting the 3.5 or higher success level, the *HSAvg* curve assumes values between .00 and .55; the *ACT-C* curve, on the other hand, has a much broader range of estimated probabilities, .02 to .91. Both curves cross 0.5, though the *HSAvg* just barely does so. The *HSAvg* curve is steepest over the values 2.98 to 3.99 (corresponding to cutoff proportions .31 to .99). The *ACT-C* curve is steepest over the scores 23 to 31 (corresponding to cutoff proportions .70 to .99). Thus, *ACT-C* has a wider spread of estimated probabilities than *HSAvg*, and is most predictive at higher cutoff proportions.

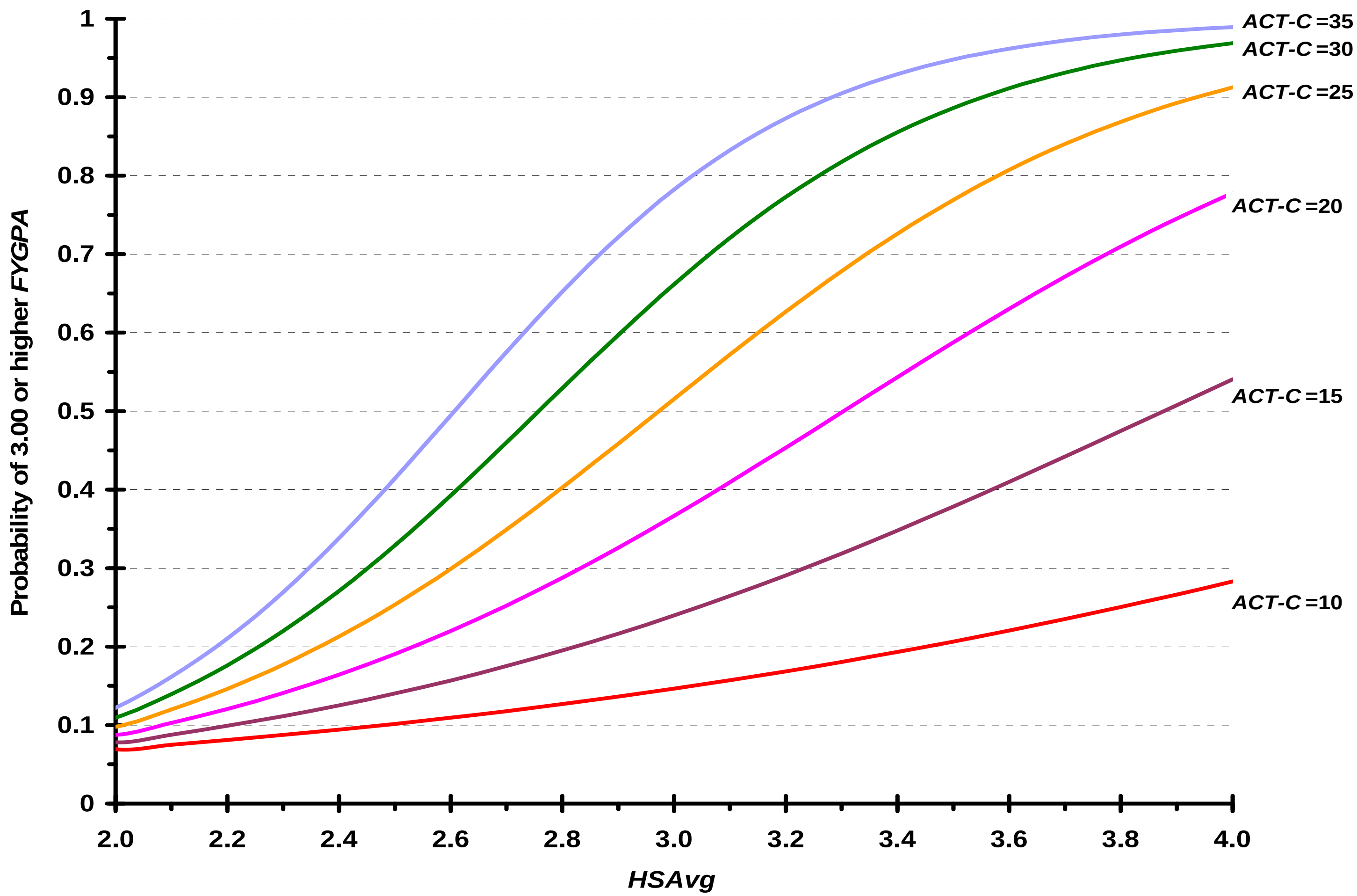
For predicting the 3.7 or higher success level, the *HSAvg* curve assumes values between .00 and .41, indicating that at typical institutions, it does not have incremental accuracy. The *ACT-C* curve, in contrast, has a very wide range of estimated probabilities, .01 to .87. The *ACT-C* curve is steepest over the values 22 to 31 (corresponding to cutoff proportions .62 to .99).

By these criteria, therefore, *HSAvg* is more predictive than *ACT-C* for the 2.0 or higher success criterion. On the other hand, *ACT-C* is somewhat more predictive than *HSAvg* for the 3.0 success level, and much more predictive than *HSAvg* for the 3.5 and 3.7 success levels. This result is consistent with that reported by Noble and Sawyer (2004).

In the joint models (labeled C in Table 5), the fixed effects for both the main effects and the interaction term are positive and statistically significant ($p < .001$). One interpretation of the interaction term is that *HSAvg* is more predictive for students with higher *ACT-C* scores than for students with lower *ACT-C* scores[6]. Figure 4 on the following page shows the probability of success (3.0 or higher), given different values of *HSAvg* and *ACT-C*. As *ACT-C* increases, the slope of the *HSAvg* probability-of-success curve increases markedly. Similar results occur for the other success levels.

[6] Alternatively, one could say that *ACT-C* is more predictive for students with high *HSAvg* than for students with low *HSAvg*.

Figure 4. Probability of 3.00 or Higher *FYGPA*, Given *HSAvg* and *ACT-C* Score



The coefficients of variation of the *HSAvg* and *ACT-C* slope coefficients in the joint models are similar to those in the single-variable models. They indicate moderate variation in the slopes, but are consistent with positive slopes at all institutions.

Table 6 on the following page summarizes the more complex hierarchical models (Equation (7)). These models include all the fixed and random effects in Model (6), as well as institution-level fixed effects for all terms and random effects for the interaction terms.

As one would expect, the student-level fixed effects (intercepts) in Table 6 are very similar to the corresponding fixed effects in Table 5. The only statistically significant institution-level fixed effects in Table 6 are mean *HSAvg* and mean *ACT-C*. The coefficients associated with mean *HSAvg* as a predictor of the *HSAvg* slope are positive, which indicates that the *HSAvg* probability-of-success curves tend to be steeper at institutions where applicants have higher mean *HSAvg* than at institutions where applicants have lower mean *HSAvg.* In contrast, the slope coefficients associated with mean *ACT-C* as a predictor of the *ACT-C* slope are negative. This result indicates that the *ACT-C* probability-of-success curves are steeper at institutions where applicants have lower mean *ACT-C.*

The other institution variables that I considered (affiliation, undergraduate enrollment, self-rated selectivity, and percent minority) did not meet the threshold of statistical significance ($p < .01$) required to enter the model after mean *HSAvg* and mean *ACT-C* had already been included. Apparently the effects of affiliation, percent minority, and undergraduate enrollment on the probability-of-success curves are not distinguishable from those associated with mean *HSAvg* and mean *ACT-C.*

TABLE 6

Complex Hierarchical Models for Predicting Success from HSAvg and ACT-C

Fixed effects			**Success level**							
			2.0		**3.0**		**3.5**		**3.7**	
Model	**Level 1 variable**	**Level 2 variable**	**Coeff.**	***p*** <	**Coeff.**	***p*** <	**Coeff.**	***p*** <	**Coeff.**	***p*** <
D	*Intercept*	*Intercept*	1.983	.001	-0.002	.971	-1.584	.001	-2.539	.001
		Mn_HSAvg	1.734	.001	. . .	. . .	. . .	. . .	-0.783	.004
	HSAvg	*Intercept*	1.582	.001	2.277	.001	3.043	.001	3.720	.001
		Mn_HSAvg	0.009	.001	0.010	.001	0.013	.001	0.019	.001
E	*Intercept*	*Intercept*	1.957	.001	0.127	.005	-1.252	.001	-2.015	.001
		Mn_ACT-C	0.158	.001	. . .	. . .	. . .	. . .	-0.076	.001
	ACT-C	*Intercept*	0.158	.001	0.231	.001	0.272	.001	0.300	.001
		Mn_ACT-C	. . .	. . .	. . .	. . .	-0.008	.001	-0.009	.002
F	*Intercept*	*Intercept*	2.205	.001	0.210	.001	-1.436	.001	-2.409	.001
		Mn_ACT-C	0.112	.001	. . .	. . .	. . .	. . .	-0.136	.001
	HSAvg	*Intercept*	1.535	.001	1.934	.001	2.389	.001	2.864	.001
	ACT-C	*Intercept*	0.105	.001	0.152	.001	0.169	.001	0.185	.001
		Mn_ACT-C	. . .	. . .	. . .	. . .	-0.008	.001	-0.007	.003
	HSAvg X ACT-C	*Intercept*	0.085	.001	0.098	.001	0.089	.001	0.081	.001

Random effects		**Success level**							
		2.0		**3.0**		**3.5**		**3.7**	
Model	**Level 2 residual**	**Std. dev.**	***p*** <	**Std. dev.**	***p*** <	**Std. dev.**	***p*** <	**Std. dev.**	***p*** <
D	*Intercept*	0.740	.001	0.635	.001	0.542	.001	0.650	.001
	HSAvg	0.258	.001	0.573	.001	0.795	.001	1.059	.001
E	*Intercept*	0.691	.001	0.574	.001	0.501	.001	0.537	.001
	ACT-C	0.050	.001	0.056	.001	0.056	.001	0.063	.001
F	*Intercept*	0.739	.001	0. 639	.001	0.573	.001	0.637	.001
	HSAvg	0.244	.001	0.518	.001	0.710	.001	0.918	.001
	ACT-C	0.048	.001	0.039	.001	0.038	.001	0.053	.001
	HSAvg X *ACT-C*	. . .	. . .	0.044	.001	0.058	.001	0.071	.002

Note: Terms that did not meet the required threshold of statistical significance are flagged by an ellipsis (**. . .**).

As one would also expect, including institutional characteristics in the *HSAvg* and *ACT-C* main-effects models (labeled D and E in Table 6) reduced the standard deviations of the random effects. In particular, including mean *HSAvg* in the models labeled D reduced the standard deviation of the *HSAvg* slope among institutions by 8% to 22%, depending on success level. Including mean *ACT-C* in the models labeled E reduced the standard deviation of the *ACT-C* slope by 5% or less.

There are also statistically significant random effects for the *HSAvg* by *ACT-C* interaction term (models labeled F), except at the 2.0 success level. Recall from Figure 3 that the interaction term indicates that the steepness of the *HSAv*g probability-of-success curve increases as *ACT-C* increases[7]. The random effects for the interaction term result indicate that the increase in steepness varies among institutions.

Incremental Success Rate with Respect to Base Rate

The incremental success rate for a selection variable at an institution is the difference between the success rate associated with a particular cutoff proportion and the base success rate associated with admitting all applicants. Table 7 on the following page shows the median incremental success rates (Equation (8)) associated with the four success levels and the three sets of selection variables.

The success rate at an institution is bounded from below by the base success rate and from above by one. Therefore, the incremental success rate is always less than one minus the base success rate. The last row of Table 7 shows a reference maximum, equal to one minus the median base success rate.

[7] Or, alternatively, the steepness of the *ACT-C* probability-of-success curve increases as *HSAvg* increases.

TABLE 7

Median Incremental Success Rate with Respect to Base Success Rate, by First-Year GPA Success Level, Cutoff Proportion, and Selection Variable (N = 192)

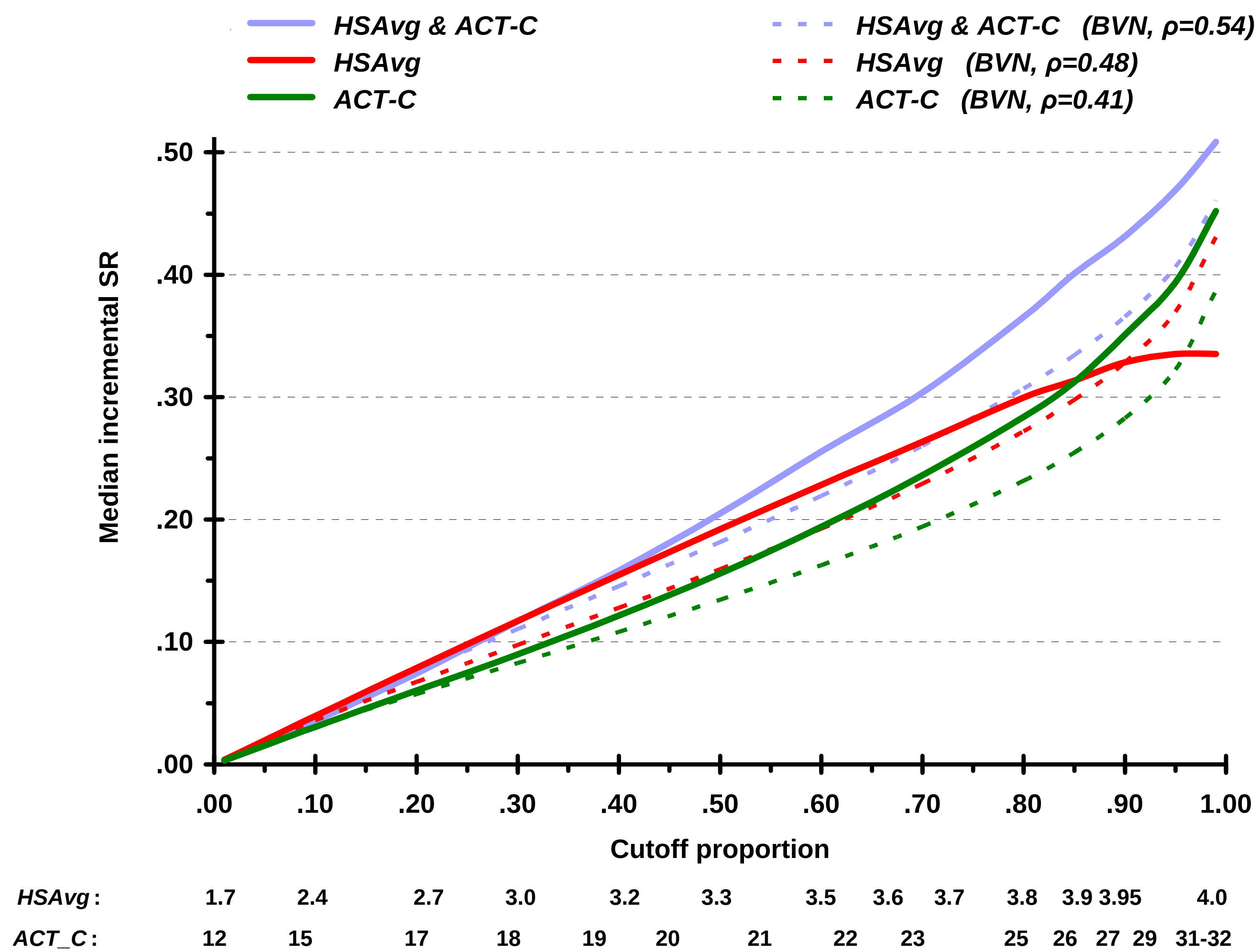
	Approx. value of		Success level											
			2.0			3.0			3.5			3.7		
Cutoff proportion	*HSAvg*	*ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*
.01	1.7	12	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00	.00
.10	2.4	15	.03	.02	.03	.04	.03	.04	.02	.02	.02	.01	.01	.01
.20	2.7	17	.06	.03	.05	.08	.06	.07	.04	.04	.04	.03	.02	.03
.30	3.0	18	.07	.05	.07	.12	.09	.12	.07	.06	.07	.05	.04	.04
.40	3.2	19	.09	.06	.09	.15	.12	.16	.11	.08	.10	.07	.06	.06
.50	3.3	20-21	.10	.07	.10	.19	.16	.20	.14	.12	.14	.10	.08	.09
.60	3.5	22	.11	.08	.12	.23	.19	.26	.19	.15	.19	.13	.10	.13
.70	3.7	23	.12	.09	.13	.26	.24	.30	.23	.20	.25	.17	.14	.18
.80	3.8	25	.13	.11	.15	.30	.28	.37	.28	.26	.34	.21	.20	.25
.85	3.9	26	.13	.11	.15	.31	.31	.40	.31	.30	.39	.24	.24	.30
.90	3.95	27	.13	.12	.16	.33	.35	.43	.32	.36	.45	.26	.30	.37
.95	4.0	29	.13	.13	.17	.34	.39	.47	.34	.43	.53	.28	.39	.46
.99	4.0	31-32	.13	.14	.18	.34	.45	.51	.34	.56	.63	.29	.54	.61
Reference maximum			.20			.57			.80			.88		

Scanning across the rows of Table 7, we see that incremental success rates increase markedly with success level up to 3.5, but then decrease slightly at 3.7. For example, selection based on *HSAvg* results in a maximum incremental success rate of .13 for 2.0 or higher *FYGPA*, .34 for 3.0 and 3.5 or higher, and .29 for 3.7 or higher. Relative to the reference maximums, on the other hand, the selection variables become relatively less effective as success level increases. With *HSAvg*, for example, .13/.20 > .34/.57 > .29/.88.

Some of the results in Table 7 are more apparent when displayed graphically. The solid curves in Figure 5 illustrate the following results for the 3.0 success level:

- *HSAvg* has higher incremental success rates than *ACT-C* at low to moderate cutoff proportions, but *ACT-C* does better than *HSAvg* at high cutoff proportions. At all success levels, the *HSAvg* incremental success rate curves flatten out at high cutoff proportions, but the *ACT-C* incremental success rate curves get steeper.
- At higher cutoff proportions, selection based on *ACT-C* and *HSAvg* jointly increases the incremental success rate over that for selection based on *HSAvg* or *ACT-C* only. For the 3.0 success level, for example, this occurs around the cutoff proportion .40 (*HSAvg*=3.2 or *ACT-C*=19).

Figure 5. Median Incremental Success Rate with Respect to Base Success Rate, by Prediction Model and Cutoff Proportion (3.0 or Higher *FYGPA*)



Equation (3) relates the success rate to the correlation coefficient, assuming that the selection and outcome variable have a bivariate normal distribution. The dashed curves in Figure 5 show success rates calculated from Equation (3) using correlation coefficients equal to the median correlations reported in Table 3. It is clear from Figure 5 that the success rates based on an assumption of bivariate normality differ substantially from those modeled from the data (Equations (1), (5), and (6)). The bivariate normal success rate for *HSAvg* is smaller than the modeled success rate when *HSAvg* is less than 3.9, and often substantially so. On the other hand, the bivariate normal success rate for *HSAvg* is much larger than the modeled success rate when *HSAvg* is greater than 3.9. In contrast, the bivariate normal assumption results in underestimated success rates for all values of *ACT-C* and of the joint *HSAvg & ACT-C* selection variable.

Incremental Success Rate of ACT-C with Respect to HSAvg

Table 8 shows that the median incremental success rate of *ACT-C* with respect to *HSAvg* (Equation (9)) depends on both success level and on cutoff proportion. For the 2.0 success level, *ACT-C* increases success rate only modestly above that attainable with *HSAvg*. As success level increases, the incremental success rate associated with *ACT-C* increases sharply at higher cutoff proportions.

TABLE 8

Median Incremental Success Rate of *ACT-C* with Respect to *HSAvg*, by First-Year GPA Success Level and Cutoff Proportion (N = 192)

Cutoff proportion	Approx. value of *HSAvg*	Success level			
		2.0	**3.0**	**3.5**	**3.7**
.01	1.7	.00	.00	.00	.00
.10	2.4	.00	.00	.00	.00
.20	2.7	.00	.00	.00	.00
.30	3.0	.00	.00	.00	.00
.40	3.2	.00	.00	.00	.00
.50	3.3	.00	.00	.00	.00
.60	3.5	.01	.01	.00	.00
.70	3.7	.01	.03	.01	.00
.80	3.8	.02	.05	.04	.03
.85	3.9	.02	.07	.07	.05
.90	3.95	.02	.09	.11	.09
.95	4.0	.03	.12	.17	.18
.99	4.0	.03	.16	.27	.30

The second column of Table 8 shows the approximate values of *HSAvg* associated with the cutoff proportions in the first column. This column suggests that for the 2.0 and 3.0 success levels, *ACT-C* typically has incremental usefulness with respect to *HSAvg* when *HSAvg* is 3.5 or greater. For the 3.5 and 3.7 success levels, *ACT-C* has incremental usefulness when *HSAvg* is at least 3.7 and 3.8, respectively.

Incremental Accuracy Rate with Respect to Null Decisions

Table 9 shows the percentage of institutions for which there is incremental accuracy in selection at particular cutoff proportions (Equation (11)). The results for the *HSAvg* and *ACT-C* models are displayed graphically in Figure 6.

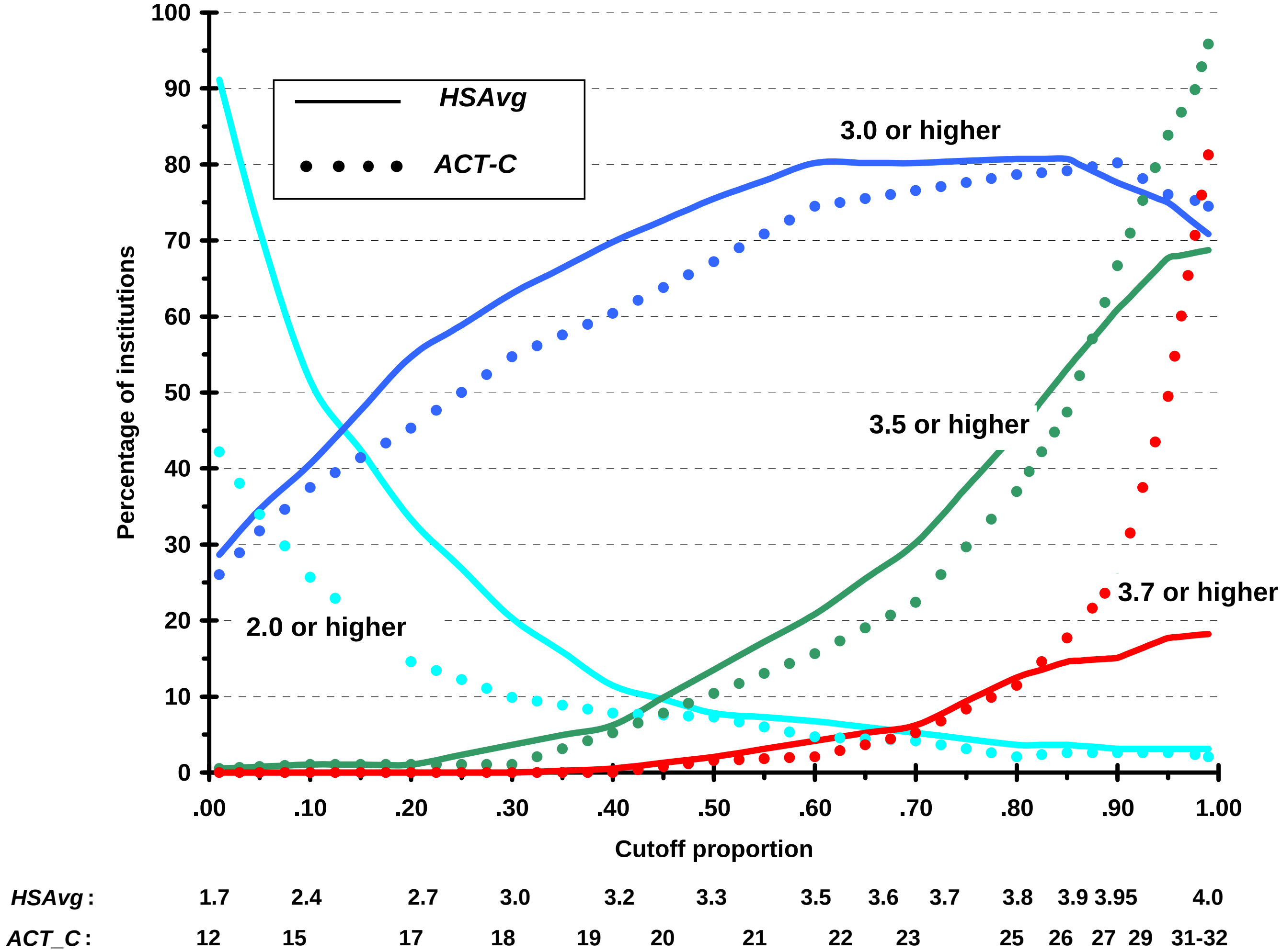
TABLE 9

Percentage of Institutions with Incremental Accuracy with Respect to Null Decisions, by First-Year GPA Success Level, Cutoff Proportion, and Selection Variable (N = 192)

			Success level											
			2.0			3.0			3.5			3.7		
Cutoff proportion	Approx. value of *HSAvg*	Approx. value of *ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*
.01	1.7	12	91	42	95	29	26	24	1	1	1	0	0	0
.10	2.4	15	52	26	41	41	38	37	1	1	1	0	0	0
.20	2.7	17	33	15	28	55	45	50	1	1	1	0	0	0
.30	3.0	18	20	10	21	63	55	60	4	1	3	0	0	0
.40	3.2	19	11	8	11	70	60	67	6	5	6	1	0	0
.50	3.3	20-21	8	7	8	76	67	78	14	10	15	2	2	3
.60	3.5	22	7	5	7	80	74	84	21	16	21	4	2	4
.70	3.7	23	5	4	6	80	77	84	30	22	37	6	5	10
.80	3.8	25	4	2	4	81	79	85	45	37	58	13	11	20
.85	3.9	26	4	3	3	81	79	83	53	47	72	15	18	26
.90	3.95	27	3	3	3	78	80	83	61	67	84	15	26	44
.95	4.0	29	3	3	3	75	76	81	68	84	92	18	49	74
.99	4.0	31-32	3	2	2	71	74	77	69	96	99	18	81	94
Any cutoff proportion			97	54	100	97	99	99	69	99	99	18	94	97

Note: The “null decisions” are admitting every student or denying admission to every student.

Figure 6. Percentage of Institutions for Which Selection Variables Have Incremental Accuracy with Respect to Null Decisions, by Cutoff Proportion



For the 2.0 success level, *HSAvg* and *HSAvg & ACT-C* jointly have incremental accuracy at a majority of institutions only for very low cutoff proportions. *ACT-C* by itself does not have incremental accuracy at most institutions for any cutoff proportion.

All the selection variables have incremental accuracy most frequently with respect to the 3.0 success level, which corresponds to typical achievement. For all three variables, the percentage of institutions with incremental accuracy increases with cutoff proportion until about .85 or .90, at which point it declines somewhat. The maximum proportion of institutions for which *HSAvg* has incremental accuracy (81%) is slightly greater than that for *ACT-C* (80%). *ACT-C* has incremental accuracy at *some* cutoff proportion, however, at slightly more institutions (99%) than does *HSAvg* (97%). *HSAvg & ACT-C* jointly have incremental accuracy at 85% of institutions at cutoff proportion .80.

For the 3.5 success level, all three sets of selection variables have incremental accuracy at a majority of institutions for high cutoff proportions. *HSAvg* does better than *ACT-C* at lower cutoff proportions, but *ACT-C* is better at higher cutoff proportions. The percentages for all three sets of selection variables increase with cutoff proportion.

For the 3.7 success level, *ACT-C* and the joint model have incremental accuracy at a majority of institutions for very high cutoff proportions. *HSAvg*, in contrast, does not have incremental accuracy at most institutions for any cutoff proportion. Again, the percentage of institutions with incremental accuracy increases with cutoff proportion.

The bottom row of Table 9 shows the percentage of institutions for which there is incremental accuracy at *any* cutoff proportion (Equation (10)). By this standard, *ACT-C* is useful at nearly all institutions for the 3.0, 3.5, and 3.7 success levels, but is useful at only slightly more than half of all institutions for the 2.0 success level. In contrast, *HSAvg* is useful at nearly all institutions for the 2.0 and 3.0 success levels, at about 69% of institutions for the 3.5 success level, and at only about 18% of institutions for the 3.7 success level. In contrast, the joint *HSAvg & ACT-C* selection variable is useful at nearly all institutions for all four success levels.

Table 10 on the following page shows the median incremental accuracy rate with respect to the null decisions of either admitting all applicants or denying admission to all applicants (Equation (12)). The medians in each cell of the table are based on only those institutions at which the incremental accuracy rate is positive (as summarized in Table 9).

Because the accuracy rate at an institution is bounded by 1.0, the maximum possible value of the incremental accuracy rate is always less than both the base success rate (*BSR*) and its complement (1-*BSR*). The last row of Table 10 shows the reference maximum, equal to the median over institutions of min(*BSR*, 1-*BSR*).[8]

For both the minimal level of success (2.0 or higher) and the very high level of success (3.7 or higher), the median incremental accuracy rate is often small (under .05). This result is a consequence of the relatively small reference maximums for these two success levels. As proportions of their reference maximums, however, the incremental accuracy rates are fairly large.

For the 3.0 and the 3.5 success levels, median incremental accuracy rates are often larger than .05. For example, the joint *HSAvg & ACT-C* selection variable has maximum incremental accuracy near .15 for the 3.0 success level, and near .25 for the 3.5 success level.

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[8] Strictly speaking, the reference maximum varies with cutoff proportion, as well as with success level. In the interest of simplicity, I have reported reference maximums by success level only.

TABLE 10

Median Incremental Accuracy Rate with Respect to Null Decisions Among Institutions At Which It is Positive, by First-Year GPA Success Level, Cutoff Proportion, and Selection Variable

			Success level											
			2.0			**3.0**			**3.5**			**3.7**		
Cutoff proportion	*Approx. value of* *HSAvg*	*ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*	*HSAvg*	*ACT-C*	*HSAvg & ACT-C*
.01	1.7	12	.01	.00	.00	.01	.01	.01	.01	.01	.01	.	.	.
.10	2.4	15	.03	.01	.02	.07	.04	.06	.06	.05	.08	.	.	.
.20	2.7	17	.05	.02	.04	.11	.07	.09	.15	.09	.17	.	.	.
.30	3.0	18	.08	.04	.06	.13	.09	.12	.23	.11	.25	.	.	.
.40	3.2	19	.08	.05	.07	.15	.09	.14	.09	.04	.08	.	.	.
.50	3.3	20-21	.07	.04	.07	.14	.10	.15	.07	.04	.08	.03	.03	.05
.60	3.5	22	.06	.04	.07	.12	.09	.14	.07	.05	.08	.05	.04	.08
.70	3.7	23	.07	.02	.07	.11	.09	.14	.05	.05	.09	.05	.03	.09
.80	3.8	25	.08	.06	.09	.09	.08	.11	.04	.04	.06	.02	.03	.05
.85	3.9	26	.04	.04	.07	.07	.07	.09	.03	.03	.05	.02	.03	.05
.90	3.95	27	.03	.03	.05	.05	.05	.07	.02	.02	.04	.01	.02	.04
.95	4.0	29	.02	.02	.03	.03	.03	.04	.01	.02	.03	.01	.01	.02
.99	4.0	31-32	.00	.01	.01	.01	.01	.01	.00	.01	.01	.00	.00	.01
Reference maximum				.20			.40			.20			.12	

Note: The "null decisions" are admitting every student or denying admission to every student. Empty cells indicate that no institutions had incremental accuracy with respect to null decisions.

Comparing different cells in Table 10 to each other is difficult, because the medians in each cell are based on different institutions. One can more easily interpret the median incremental *AR*s (Table 10) in connection with the percentages of institutions where incremental *AR* is positive (Table 9). Figures 7 - 10 on pages 51 and 53 show the median incremental *AR*s in Table 10 plotted against the percentages in Table 9, by cutoff proportion. Points farther from the origin are more desirable than points closer to the origin. The plot area in each figure is divided into three regions:

- The red-shaded region corresponds to cutoff proportions for which 50% or fewer institutions have positive incremental *AR*. The points plotted in the red-shaded region show the typical (median) incremental accuracy at the minority of institutions for which there is incremental accuracy at these cutoff proportions.
- The green-shaded region corresponds to cutoff proportions for which at least 50% of institutions have positive incremental *AR* and for which the median incremental *AR* is .05 or higher. The points plotted in the green-shaded region indicate that a selection variable is useful at most institutions for increasing *AR* beyond that achievable from the null decisions, and that the typical incremental accuracy is at least .05.
- The yellow-shaded region corresponds to cutoff proportions for which at least 50% of institutions have positive incremental *AR* but for which the median incremental *AR* is less than .05. The points plotted in the yellow-shaded region indicate that a selection variable is useful at most institutions for increasing *AR* beyond that from null decisions, but the typical incremental accuracy is less than .05.

Each point in the figures corresponds to a particular cutoff proportion (*c*). The plotted points for the same cutoff proportion are connected by light lines.

Figure 7 shows results for the 2.0 success level. Except at very low cutoff proportions

(c = .01 and .10), *HSAvg* and *HSAvg & ACT-C* jointly do not have incremental accuracy at a majority of institutions. At cutoff proportions above .50, *HSAvg* and *HSAvg & ACT-C* jointly have incremental accuracy at less than 10% of institutions, but at those institutions where they do have incremental accuracy, it is sometimes greater than .05. *ACT-C* by itself does not have incremental accuracy for the 2.0 success level at a majority of institutions at any cutoff proportion; and at the institutions where *ACT-C* does have incremental accuracy, it is typically less than .05. For the 2.0 success level, therefore, *ACT-C* has little or no benefit over *HSAvg* by itself for increasing accuracy rate (see also discussion under Table 11).

The story is quite different for the 3.0 success level (Figure 8). *HSAvg* has incremental accuracy at a majority of institutions for cutoff proportions .20 and higher, and *ACT-C* has incremental accuracy at a majority of institutions for cutoff proportions .30 and higher. The typical incremental accuracy at these institutions is greater than .05 for cutoff proportions between .30 and .85. According to the joint criteria of percentage of institutions with incremental accuracy and median incremental accuracy, *HSAvg* is more effective than *ACT-C* for cutoff proportions below .85, but *ACT-C* is more effective than *HSAvg* for cutoff proportions above .85.

Note also that *HSAvg & ACT-C* jointly is better than *HSAvg* alone for cutoff proportions above .50. In other words, *ACT-C* has incremental usefulness for the 3.0 success level at these cutoff proportions.

Figure 7. Median Incremental Accuracy Rate Where It is Positive, by Percentage of Institutions, Selection Variables, and Cutoff Proportion (2.0 or Higher *FYGPA*)

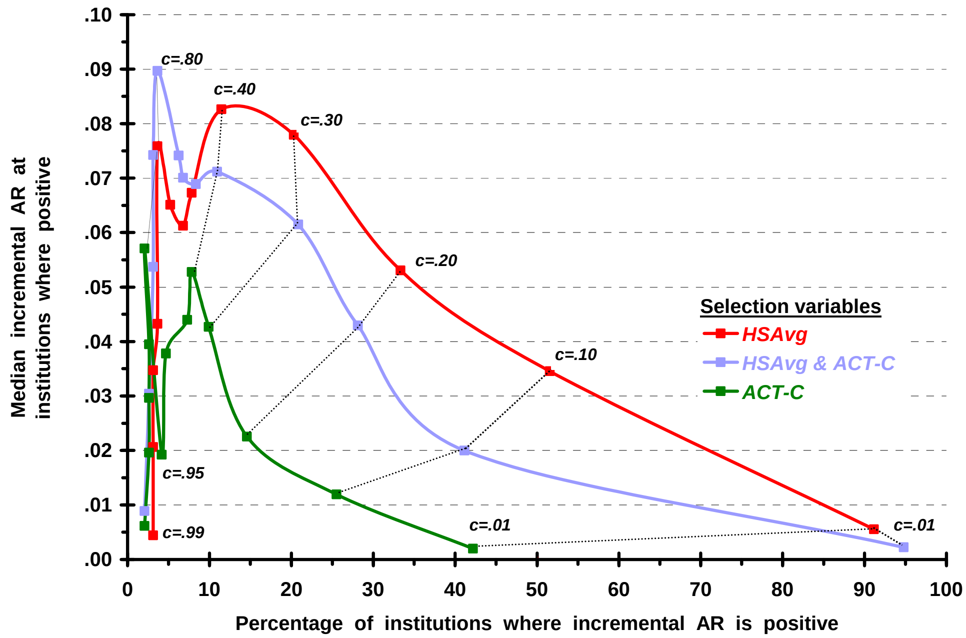
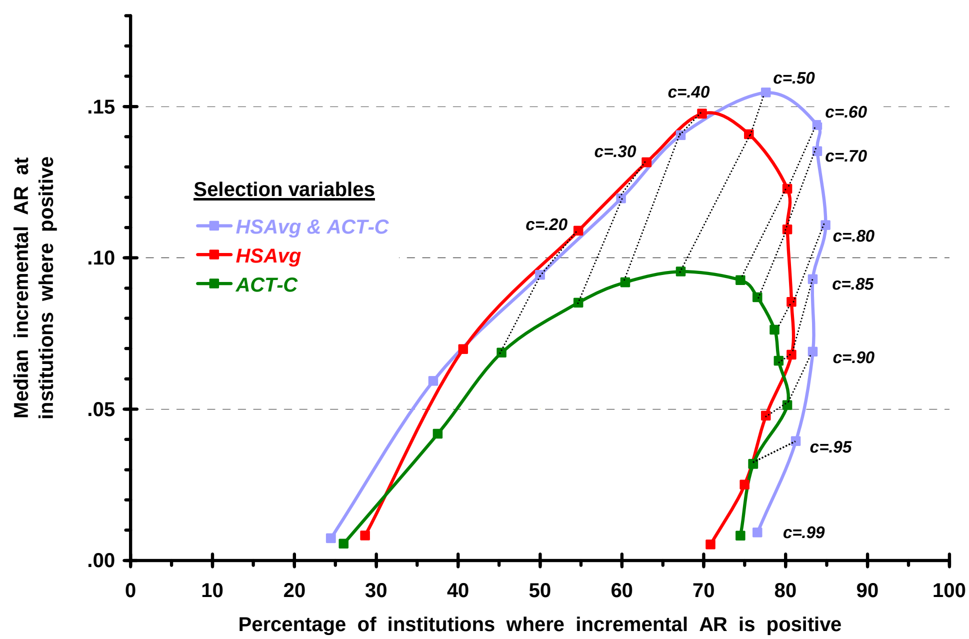


Figure 8. Median Incremental Accuracy Rate Where It is Positive, by Percentage of Institutions, Selection Variables, and Cutoff Proportion (3.0 or Higher *FYGPA*)



For the 3.5 success level (see Figure 9 on the following page), *HSAvg* has incremental accuracy at a majority of institutions for cutoff proportions .85 and higher, and *ACT-C* has incremental accuracy at a majority of institutions for cutoff proportions .90 and higher. *ACT-C* is more effective, in terms of median incremental accuracy, than *HSAvg* at these cutoff proportions; the median incremental accuracy for both selection variables, however, is below .05. Interestingly, at a small proportion of institutions, *HSAvg* and *HSAvg & ACT-C* jointly have very large incremental accuracy (about .25) near cutoff proportion .30.

According to Figure 9, *HSAvg & ACT-C* jointly is better than *HSAvg* alone for cutoff proportions above .40. Therefore, *ACT-C* has incremental benefit over *HSAvg* alone for increasing accuracy rate for these cutoff proportions.

At the 3.7 success level (see Figure 10 on following page), *HSAvg* does not have incremental accuracy at more than 20% of institutions for any cutoff proportion. Among the institutions where it does have incremental accuracy, the median incremental accuracy is typically less than .05. In contrast, *ACT-C* and the joint *HSAvg & ACT-C* selection variables have incremental accuracy at a majority of institutions at cutoff proportions .95 and higher. The median incremental accuracy at these cutoff proportions is also less than .05, however. At cutoff proportions .70 to .90, the joint *HSAvg & ACT-C* selection variable has incremental accuracy at fewer institutions, but its median incremental accuracy is larger than .05 at these institutions.

According to Figure 10, *HSAvg & ACT-C* jointly is better than *HSAvg* alone for all cutoff proportions. In other words, *ACT-C* has incremental benefit over *HSAvg* alone at all cutoff proportions for the 3.7 success level.

Figure 9. Median Incremental Accuracy Rate Where It is Positive, by Percentage of Institutions, Selection Variables, and Cutoff Proportion (3.5 or Higher *FYGPA*)

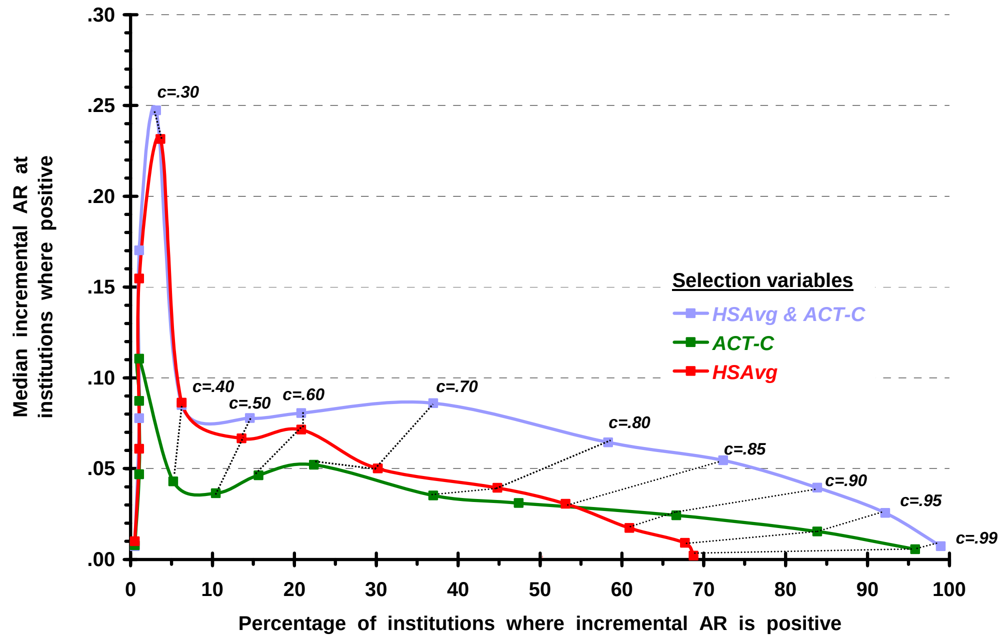
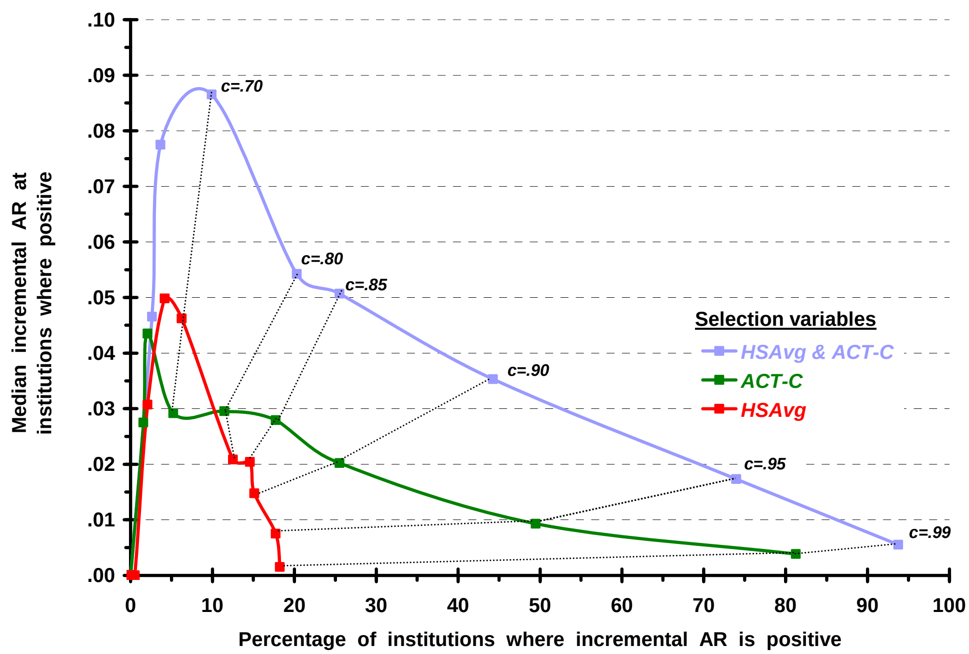


Figure 10. Median Incremental Accuracy Rate Where It is Positive, by Percentage of Institutions, Selection Variables, and Cutoff Proportion (3.7 or Higher *FYGPA*)



Incremental accuracy rate of ACT-C with respect to HSAvg

As shown in Table 11, the incremental accuracy rate of *ACT-C* with respect to *HSAvg* (Equation (13)) is similar to the incremental success rate of *ACT-C* with respect to *HSAvg*: *ACT-C* has little incremental accuracy with respect to *HSAvg* for the 2.0 success level, but it increases with success level. The maximum increase in accuracy rate associated with adding *ACT-C* to a selection rule based on *HSAvg* is about .04.

TABLE 11

Median Incremental Accuracy Rate of *ACT-C* with Respect to *HSAvg*,
by First-Year GPA Success Level and Cutoff Proportion

Cutoff proportion	Approx. value of *HSAvg*	Success level			
		2.0	3.0	3.5	3.7
.01	1.7	.00	.00	.00	.00
.10	2.4	.00	.00	.00	.00
.20	2.7	.00	.00	.00	.00
.30	3.0	.00	.00	.00	.00
.40	3.2	.00	.00	.00	.00
.50	3.3	.00	.01	.02	.01
.60	3.5	.01	.02	.03	.02
.70	3.7	.00	.03	.03	.03
.80	3.8	.00	.03	.04	.04
.85	3.9	.01	.03	.04	.04
.90	3.95	.01	.03	.03	.04
.95	4.0	.00	.02	.02	.03
.99	4.0	.00	.01	.01	.01

The second column of Table 11 suggests that for the 3.0, 3.5, and 3.7 success levels, *ACT-C* typically has incremental accuracy with respect to *HSAvg* when *HSAvg* is 3.3 or greater.

Score Senders vs. Enrolled Students

The selection rates and accuracy rates obtained from the enrolled students only (results available from author) were very similar to those obtained from the combined group of non-enrolled score senders and enrolled students. Because applicants are midway between score senders and enrolled students in their mean *HSAvg* and *ACT-C*, we have some assurance that the results for score senders also apply to applicants.

Summary and Discussion

The usefulness of selection variables depends on the goals intended by decision makers. In college admission, one plausible goal is to maximize the success rate (*SR*) of enrolled students with respect to some outcome variable (e.g., first-year GPA). Another plausible goal is to maximize the accuracy rate (*AR*) of an institution's decisions to admit or reject applicants. When the selection variable and outcome variable have a bivariate normal distribution with correlation coefficient ρ, the *SR* and *AR* indicators of usefulness are functions of the success level, the cutoff proportion (proportion of applicants not admitted), and ρ. It is perhaps for this reason that people often interpret ρ as an indicator of usefulness. The bivariate normal assumption is not realistic in college admission, however, because *HSAvg* has a pronounced skew. This study suggests that *SR* and *AR* can depart substantially from values calculated from ρ.

Both *HSAvg* and *ACT-C* predict academic success in the first year of college. As shown in Figures 2 and 3, however, their probability-of-success curves vary with different levels of success and with different cutoff proportions. Both variables have steeper slopes for the higher levels of success (*S30*, *S35*, and *S37*) than for the minimal level of success (*S20*). *HSAvg* is a stronger predictor than *ACT-C* for *S20*, but *ACT-C* is much stronger than *HSAvg* for predicting *S35* and *S37*. *HSAvg* does better at lower cutoff proportions, and *ACT-C* does better at higher cutoff proportions.

The statistical relationship of *HSAvg* and *ACT-C* with any level of success also depends on the joint values of both predictors (Figure 4). *HSAvg* is a much stronger predictor among students with high *ACT-C* scores than among students with low *ACT-C* scores. Correspondingly, *ACT-C* is a much stronger predictor among students with high values of *HSAvg* than among students with low values of *HSAvg*.

There is moderate variation among institutions in all the intercept and slope coefficients defining the conditional probability of success functions. Typically, the standard deviations of the coefficients among institutions are about two-tenths to three-tenths of the corresponding mean values. This variation indicates that institutions would benefit from doing their own local predictive validity studies, rather than relying on global studies.

Institutions can use either *HSAvg* or *ACT-C* to increase their success rates beyond the base success rate, no matter which success level they choose. *HSAvg* is more effective than *ACT-C* for increasing success rates at low to moderate cutoff proportions, but *ACT-C* is more effective at higher cutoff proportions. Using both selection variables, however, is more beneficial for improving success rates than using either variable by itself for cutoffs above *HSAvg*=3.5 or *ACT-C*=22.

Increasing accuracy rates, beyond that associated with the null decisions of either admitting all applicants or denying admission to all applicants, is more difficult than increasing success rates. The reason is that accurate classification requires both the success of admitted applicants and the failure of non-admitted applicants. The effectiveness of both *HSAvg* and *ACT-C* for increasing accuracy rates depends strongly on the success level:

- For the minimal 2.0 success level, neither *HSAvg* nor *ACT-C* increase accuracy rate at a majority of institutions, except at very low cutoff proportions. A possible reason for this result is that institutions do not admit students whose *HSAvg* or *ACT-C* score suggests that they are unlikely to earn at least a 2.0 *FYGPA*, and that lack of success at this level is due to other reasons.

- For the 3.0 success level (representing typical performance), *HSAvg* and *ACT-C* individually increase accuracy rate at most institutions for a broad range of cutoff proportions. *HSAvg* is effective at somewhat more institutions than *ACT-C* at cutoff proportions below .85, but ACT-*C* is effective at somewhat more institutions than *HSAvg* at higher cutoff proportions.
- For the 3.5 success level, both *HSAvg* and *ACT-C* individually increase accuracy rate at most institutions, but only at high cutoff proportions.
- For the 3.7 success level, *HSAvg* does not increase accuracy rate at most institutions for any cutoff proportion. *ACT-C*, however, increases accuracy at most institutions for high cutoff proportions

For all success levels, *ACT-C* has incremental accuracy beyond *HSAvg* at most institutions for cutoffs above *HSAvg*=3.3 or *ACT-C*=20 to 21.

These results are based on the assumption that institutions use strict cutoff proportions (*c*) in making admission decisions. This assumption is rarely if ever true, but it is necessary for this research, given the absence of quantifiable rules and data on institutions' actual admission policies. Institutions use other variables, in addition to high school course work, grades, and test scores, in making admission decisions. One should therefore interpret the results of this study with this in mind.

On the other hand, self-selection by potential applicants, based on their high school grades and test scores, might be as important as institutions' actual admission decisions in determining the colleges in which students enroll (see discussion following Table 4). From this perspective, high school grades and test scores contribute indirectly to attaining institutions' goals. This study captures some of the indirect contribution through its consideration of data from score senders. High school GPA and test scores likely also contribute, however, to the decisions by a broader pool of students to become score senders. Although the current study does not address the extent of this additional indirect contribution, one could presumably model it by making assumptions about the composition of the broader pool of all graduating high school students.

With these limitations in mind, we can conclude that the conventional wisdom based on correlations is correct in many, but not all respects. *HSAvg* by itself is better than *ACT-C* by itself for some, but not for all, degrees of selectivity and definitions of success. In some situations (for example, where an institution is interested in high levels of success), *ACT-C* is more useful. This study affirms the other aspect of the conventional wisdom, however: In most scenarios, using both high school grades and test scores jointly is better than using either by itself. In using both variables, moreover, it is important to take into account the *HSAvg* by *ACT-C* interaction effect, as well as the main effects.

Finally, it is worthwhile to keep in mind that although increasing *SR* and *AR* in making admission decisions is a plausible goal at many institutions, it is not their only goal. Other goals, such as objectivity and uniformity in making admission decisions, and providing data for counseling, placement, and institutional self-study are also important, and both high school grades and test scores contribute in different ways to these goals.

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Appendix

Relationship of Conditional Probability of Success and Its Marginal Distribution with Success Rate and Accuracy Rate

Proposition 1:

Let $p(x) = P[\text{Success}|\mathrm{X} = x]$ be strictly increasing in x

$f(x)$=be the probability density function for X.

Assume that f is non-zero everywhere.

Let $SR(k) = \int_k^\infty p(x) f(x) dx \Big/ \int_k^\infty f(x) dx$ be the success rate for a cutoff score k, and let $BSR = SR(-\infty)$. Then:

1. SR is strictly increasing in x.
2. $SR(x) > BSR$.
3. $SR(x) > p(x)$.

Proof:

Let $SR(x) = \dfrac{\int_x^\infty p(t) f(t) dt}{\int_x^\infty f(t) dt} = \dfrac{A}{B}$.

Then
$$SR'(x) = \frac{A'B - AB'}{B^2}$$
$$= \frac{-f(x)p(x)B + Af(x)}{B^2}$$
$$= \frac{f(x)[A - Bp(x)]}{B^2}.$$

Since p is increasing, $A = \int_x^\infty p(t) f(t) dt > p(x) \int_x^\infty f(t) dt$

$$= p(x)B,$$

and therefore, $SR'(x) > 0$. Thus, SR is strictly increasing in x, and $SR(x) > SR(-\infty) = BSR$.

Because $\dfrac{f(x)[A - Bp(x)]}{B^2} > 0$ for all x, $p(x)\int_x^\infty f(t)dt < \int_x^\infty p(t) f(t) dt$ for all x, and so $SR(x) > p(x)$ for all x.

Proposition 2:
Given the previous assumptions, let $AR(k) = \int_{-\infty}^{k} [1 - p(x)] f(x) dx + \int_{k}^{\infty} p(x) f(x) dx$ be the accuracy rate associated with cutoff score k. Then, AR has a local maximum at k_0 if, and only if, $p(k_0) = 1/2$.

Proof:

$$AR'(k) = [1 - p(k)] f(k) - p(k) f(k)$$
$$= f(k) - 2p(k) f(k).$$

$AR'(k_0) = 0$ if, and only if, $-2p(k_0) f(k_0) = f(k_0)$.

Since $f(k_0) > 0$, $AR'(k_0) = 0$ if, and only if, $p(k_0) = 1/2$.

Note that $AR''(K) = f'(k) - 2[p'(k) f(k) + p(k) f'(k)]$.

Therefore $AR''(K_0) = f'(k_0) - 2p'(k_0) f(k_0) - f'(k_0)$
$= -2p'(k_0) f(k_0)$, since $p(k_0) = 1/2$.

Because $p'(k_0) > 0$ and $f(k_0) > 0$, $AR''(k_0) < 0$.

Therefore, *AR* has a local maximum at k_0 if, and only if, $p(k_0) = 1/2$.

Proposition 3:
Let $BSR = \int_{-\infty}^{\infty} p(x) f(x) dx$. Then max *AR* exceeds *BSR* and 1-*BSR* if, and only if, *p* crosses 1/2.

Proof:

Suppose $AR(k) > BSR$ for some *k*. Proof by contradiction:

Suppose $p(x) > 1/2$ for all x. Then $\int_{-\infty}^{k} [1 - p(x)] f(x) dx < \int_{-\infty}^{k} p(x) f(x) dx$.

And so, $AR(k) < \int_{-\infty}^{k} p(x) f(x) dx + \int_{k}^{\infty} p(x) f(x) dx$
$=BSR$ *[contradiction]*.

If $p(x) < 1/2$ for all x, then $\int_{k}^{\infty} p(x) f(x) dx < \int_{k}^{\infty} [1 - p(x)] f(x) dx$.

And so, $AR(k) < \int_{-\infty}^{k} [1 - p(x)] f(x) dx + \int_{k}^{\infty} [1 - p(x)] f(x) dx$

$$= \int_{-\infty}^{\infty} [1 - p(x)] f(x) dx$$

$$= 1 - BSR \quad \textit{[contradiction]}.$$

Therefore $p(k_0) = 1/2$ for some k_0.

Since *p* is monotonic increasing, *p* must cross $1/2$ at k_0.

Now, suppose *p* crosses $1/2$. Let $p(k_0) = 1/2$ and $p(x) < 1/2$, for $x < k_0$, and $p(x) > 1/2$, for $x > k_0$.

Then $AR(k_0) = \int_{-\infty}^{k_0} [1 - p(x)] f(x) dx + \int_{k_0}^{\infty} p(x) f(x) dx$

$$> \int_{-\infty}^{k_0} p(x) f(x) dx + \int_{k_0}^{\infty} p(x) f(x) dx$$

$$= BSR.$$

Also, $AR(k_0) = \int_{-\infty}^{k_0} [1 - p(x)] f(x) dx + \int_{k_0}^{\infty} p(x) f(x) dx$

$$> \int_{-\infty}^{k_0} [1 - p(x)] f(x) dx + \int_{k_0}^{\infty} [1 - p(x)] f(x) dx$$

$$= 1 - BSR.$$