To Pre-Announce or Not: New Product Development in a Competitive Duopoly Market[1]

by

Ted Klastorin (tedk@uw.edu)

Hamed Mamani (hmamani@uw.edu)

Yong-Pin Zhou (yongpin@uw.edu)

ISOM Department
Michael G Foster School of Business
Box 353226
University of Washington
Seattle, WA 98195-3226

April, 2012; Revised, September, 2013

Abstract

In this paper, we consider the development and introduction of a new product in a durable goods duopoly market with profit maximizing firms. The first firm is an innovator who initially begins developing the product; the second firm is an imitator that begins developing a competing product as soon as it becomes aware of the innovator's product. We assume that consumers purchase at most one unit of the product when they have maximum positive utility surplus that is determined by the characteristics of the product, the consumer's marginal utility, and the consumer's discounted utility for future expected products and prices. The innovator firm can release information about its product when it begins developing the product or can guard information about its product until it introduces the product into the market. Our analysis shows that, contrary to conventional wisdom, conditions may exist when an innovator's profits increase by releasing information about its product prior to introduction. We discuss these conditions and their implications for new product development efforts.

***Keywords*:** New Product Development; Product Design; New Product Introduction

---

[1] *This is a working paper only and should not be reproduced or quoted in any way without the express written consent of the authors. Comments on the paper are welcome. The first author gratefully acknowledges the support of the Burlington Northern/Burlington Resources Foundation, and the third author gratefully acknowledges the support of a McCabe Fellowship. The authors thank Professor Jeff Schulman for his helpful comments on an earlier draft of this paper.*

To Pre-Announce or Not: New Product Development in a Competitive Duopoly Market

1. Introduction

The introduction of many durable goods follows a common pattern: an innovator firm initially develops a new product and is then followed by one or more imitator firms who subsequently produce competing products after learning of the innovator firm's product design. Many well-known technology products have followed this pattern; for example, consider the ubiquitous digital audio player (DAP). In 1997, Saehan Information Systems began development of the first mass-produced MP3 digital audio player (the *MPMan*) that it introduced to the US market in the summer of 1998. At some time after Saehan began developing the *MPMan*, Diamond Multimedia began developing a competing product, the *Rio PMP300*, that was introduced a few months after the *MPMan* (in September, 1998). Both players had similar technical characteristics (*e.g.*, they both used 32MB flash memory) and the market for both products lasted for approximately three years.

The history of digital audio players (DAPs) illustrates numerous factors that are common in the development and introduction of many durable goods. Saehan (the innovator firm) introduced their DAP into the US market first and enjoyed a brief monopoly period before the *Rio PMP300* was available. While it is unclear when Diamond Multimedia began to develop their DAP, it is possible that they started development after learning of the efforts by Saehan but prior to the introduction of the *MPMan*. Furthermore, although Saehan had a (brief) monopoly on DAPs during the summer of 1998, potential consumers during this time were undoubtedly aware of the impending introduction of the *Rio PMP300* later that year—knowledge that may have affected their purchase decisions. Finally, neither firm significantly changed the design of their respective player during the life span of these DAPs.

In this paper, we analyze the process of introducing a new durable good in a duopoly market with an innovator firm and an imitator firm. There is a finite set of potential consumers who purchase at most one unit. We assume that the product has a fixed life span and represents an incremental improvement of an existing product so that the information diffusion process can be considered exogenously and the potential demand is constant at any time during the product life cycle (Klastorin and Tsai 2004, Bayus *et al*. 1997, Cohen *et al*. 1996). Assuming that the firms are homogeneous with respect to development and production functions and want to maximize their discounted profits over the life of the product, we analyze this market with respect to both firms' optimal design and pricing decisions. We specifically consider the issue of whether an innovator firm has any incentive to release information about its development efforts prior to its product introduction. We find that conditions do exist when an innovator firm can increase its profits by pre-announcing its product, even though such an action provides information to the competitor and appears to contradict conventional wisdom that information about new product development efforts should always be guarded as long as possible (*The Economist* 2011).

We assume that consumers are rationally expectant or forward-looking (Coarse 1972, Dhebar 1994); that is, consumers' purchase decisions are determined by the utility surplus of current products as well as their (discounted) utility surplus of expected future products and prices. We assume that consumers initially enter the market when information about the innovator's product becomes known (although a product may not be available). For example, Apple's third quarter 2011 sales of the iPhone were below expectations when many customers postponed their purchase in anticipation of the rumored introduction of the iPhone 5, even though Apple never intentionally released such product development information (in fact, Apple introduced the iPhone 4S instead). The assumption of forward-looking (strategic) consumers is well supported both by empirical and theoretical research and is an integral part of our models.

We formulate the basic model as a Stackelberg game where the innovator firm initially sets the quality/design of the product[2]. At that time, it can either release information about this product or wait until it introduces the product into the market. When the imitator firm learns about the innovator firm's development effort, it immediately begins developing a competing product. This implies that product development efforts between the two firms proceed concurrently when firm X pre-announces its product but serially when firm X does not pre-announce.

In our basic model, we assume that all potential consumers enter the market when information about the new product becomes available. Following Moorthy (1988) and others, we assume that

---

[2] We refer to the design and quality of the product interchangeably but use these terms to refer to the number of features, durability, materials, and other attributes of the product.

the firms are homogeneous and the development time and variable cost of each product are functions of the characteristics of each product that can be measured by a single scalar. We also assume that marginal production cost (*i.e.*, cost of each unit) is a linear function of the design/quality level. With these assumptions, we show that a unique equilibrium exists and analytically derive characteristics of the market when both firms have positive duopoly sales. In addition to showing that an innovator firm may benefit by pre-announcing its development efforts, we also show that an innovator firm may choose to forgo a monopoly "opportunity" even though such action provides its competitor a significant advantage in the market.

To test the sensitivity of our results, we extended the basic model to include the cases when (1) production costs are a quadratic function of product design and quality, and (2) all potential consumers do not enter the market when the product is introduced; specifically, the number of potential consumers who enter at the beginning of the monopoly and duopoly periods are proportional to the length of the respective period. We show that most of the results derived from our analysis of the basic model hold in these cases as well, including the result that the innovator firm may want to pre-announce its development efforts under certain conditions.

1.1 Literature Review

Our work is related to several previous papers in the new product development literature. In an early paper, Dockner and Jorgensen (1988) generalized dynamic pricing strategies in a differential game model in dynamic oligopolies. Kouvelis and Mukhopadhyay (1999) studied pricing and product design quality in a competitive diffusion model setting. Klastorin and Tsai (2004) extended their work by including pricing and timing decisions in a duopoly market when the product lifespan is finite. Our work is also related to previous research on time-based competition. Cohen *et al*. (1996) analyzed the time-to-market decision for product replacement over a given entry period when demand was a function of product design level (they did not consider alternative pricing however). Bayus *et al*. (1997) studied the trade-off between time-to-market and product quality in a duopoly market over an infinite time horizon. They analyzed how one firm's decision affects the other firm's design and product entry-timing decisions. Bayus (1997) analyzed the optimal product quality and entry timing decisions for an imitator firm following the introduction of a new product under different cost, market and demand conditions.

Morgan *et al*. (2001) extended these previous papers by studying a multi-generation product with fixed cost of product development.

With respect to product and price competition, our work is related to Hotelling (1929), Moorthy (1988), and Schmidt and Porteus (2000). Moorthy (1988) studied a duopoly competition where two firms simultaneously determine their respective product designs and then subsequently set product prices. In his paper, Moorthy assumed that consumers were myopic and purchased the product that maximized their utility surplus at the time that they entered the market (a similar assumption was made by Klastorin and Tsai 2004). Schmidt and Porteus (2000) considered the case when a firm developed a new product that competes with an existing product in a market with linear reservation price curves. Finally, our work is related to the seminal paper by Shaked and Sutton (1982) which considered a market where competing firms sequentially decide to enter, set quality/design levels, and finally establish prices (after observing competitors' actions).

Moorthy and Png (1992) studied product positioning in a dynamic context (*i.e*., when should a supplier introduce a product sequentially?). Dhebar (1994) analyzed a two-period durable-good monopolist who introduces a new product in the first period and an upgraded version in the second period. In Dhebar's model, the firm determines the product prices in both periods as well as the upgraded product scope/design to maximize its expected net present profit over both periods. Consumers in Dhebar's model are forward-looking in the sense that they consider their expectation of the upgraded product when making a purchase decision in the first period. Kornish (2001) extended Dhebar's model to the case when the upgrade design/scope was exogenously determined. Ramachandran and Krishnan (2008) studied entry timing decisions in a monopoly market with a rapidly improving product design. Padmanabhan *et al*. (1997) analyzed a sequential product introduction problem and investigated the implications of consumer uncertainty regarding network externalities. In this work, the authors showed that it might be beneficial to the firm to provide "private" information to consumers.

A number of researchers have studied related problems in monopoly markets. For example, Fudenberg and Tirole (1998) studied the monopoly pricing of overlapping generations of a durable good under different market conditions. Dogan *et al*. (2006) considered a monopolist's software upgrade policies in a two period model when demand is random and impacted by word of mouth. Bala and Carr (2009) analyzed the upgrade pricing policies by characterizing the relationship between the product upgrade magnitudes and upgrade pricing structures along with the user upgrade costs. Yin *et al*. (2010) studied the impact of used goods on the product upgrade and pricing decisions in a monopoly market.

Other related papers include the work by Bhaskaran and Gilbert (2005) who studied the selling and leasing strategies of a durable goods manufacturer in the presence of a complementary product. In the context of the software industry, Mehra *et al*. (2010) considered the case when two firms compete by offering possible discounts to existing customers of the first-mover firm for upgrades.

1.2 Paper Contributions and Overview

To the best of our knowledge, this is the first study to analyze the issue of new product pre-announcements in a duopoly market with rationally expectant consumers. We model this market as a Stackelberg game and derive closed form solutions for the firms' design and price decision variables when firm X chooses (or not) to pre-announce its product. Using the results from our models, we show (contrary to conventional wisdom) that, under certain conditions, an innovator firm's profits may increase when information about its product is released as the firm begins its new product development process (even though such action benefits the innovator firm's competitor as well). The underlying insight can be used to explain other findings about the timing and pricing of new products in a duopoly market, including our finding that an innovator firm that enters the market first may knowingly decide to set a monopoly price that effectively discourages all monopoly sales. Our analysis shows that one of the most important considerations for an innovator firm is how the imitator firm reacts in terms of its design and pricing decisions. Specifically, we show that the imitator firm generally wants to differentiate its product from the innovator firm's product as much as possible; this increased differentiation generally increases the profits of both firms. We present a number of examples that illustrate how these factors interact to benefit both firms as model parameters vary.

The paper is organized as follows. In the second section, we define our basic model that assumes linear production costs and all potential consumers enter the market when the product is introduced. We analyze this model under the conditions that the innovator firm either pre-announces its development effort or waits until it introduces the product into the market. We show that a unique equilibrium exists in this Stackelberg game (assuming that both firms choose to compete in the duopoly market) and derive characteristics of both firms' products and prices. In the third section, we discuss the implications of the basic model and extend it to the case when production costs are quadratic and potential consumers enter the market at the beginning of the monopoly or duopoly periods (the number of consumers who enter each period is proportional to the length of those respective periods). We show that most of the results derived from the basic model continue to hold. Finally, we summarize our results and discuss several implications that our work offers for other fields (e.g., supply chain management).

2. Basic Model Defined

There are two competing firms; we will denote firm X as the innovator firm and firm Y as the imitator firm (throughout the remainder of this paper, we will use subscripts X and Y to indicate firm-specific variables). The firms are homogeneous with respect to cost and development time characteristics. Following Moorthy (1988), Cohen *et al*. (1996) and others, we assume that each firm develops a single durable product over a finite lifespan $[0, T]$ that is characterized by a scalar S_j, $j = X, Y$. The design level of the product reflects the number of features, the performance level, and the overall quality of the product. The time it takes to develop the product, $g(S)$, is a linear function of the product design level S: $g(S) = \beta S$ where $\beta > 0$ is the time to develop an additional unit of design/quality level. This assumption is based on empirical studies that demonstrated that the speed of product performance improvement can be represented by a Cobb-Douglas model (Cohen *et al*., 1996).

We assume that the marginal production cost (cost per unit sold) is a function of the design level, $c(S) = \alpha S^h \ (\text{for } h \geq 1)$ where α is the marginal production cost of a unit with design level S. In the basic model, we assume that $h = 1$ although we relax this assumption and consider quadratic costs ($h = 2$) in the following section. We assume that development costs are zero although these costs could be included by redefining the function $c(S)$.

We model both firms' product design and pricing decisions as a Stackelberg game. Firm X (the innovator firm) begins product development and sets a product design level S_X. For analytical tractability, we assume that S_X is an exogenously determined parameter; this is a reasonable assumption if the innovator product is the result of a technological breakthrough such that the value of S_X is largely determined by technological considerations. When firm Y (the imitator firm) learns of firm X's product development effort, it begins developing a competing product with design level S_Y. We assume that S_Y is a decision variable so that firm Y can differentiate its product from S_X if it wishes to do so. To avoid trivial cases, we assume that both firms make decisions that lead to positive sales during the product's finite life span [0, T].

We assume that the product's life span begins at time $t = 0$ when firm X introduces the product into the market. Consumers (and firm Y) become aware of firm X's product design S_X at time $t = -\beta S_X$ if firm X pre-announces its product, or time $t = 0$ if firm X does not pre-announce its product. If firm X does not pre-announce its product, it always enters the market first; if firm X pre-announces its product, the firm with the lower design level will enter the market first. The firm that enters the market first sells its product for a monopoly period that persists until the second firm introduces its product when both firms simultaneously set respective duopoly prices. In this paper, we focus our analysis on the case when firm X always enters the market first, regardless of whether or not it pre-announces its product. By focusing on this case, we can make a more accurate analysis of the trade-offs between pre-announcing and not pre-announcing a product development effort. The time lines indicating significant events during the product lifespan when firm X does or does not pre-announce its product are indicated in Figures *1a* and *1b*.

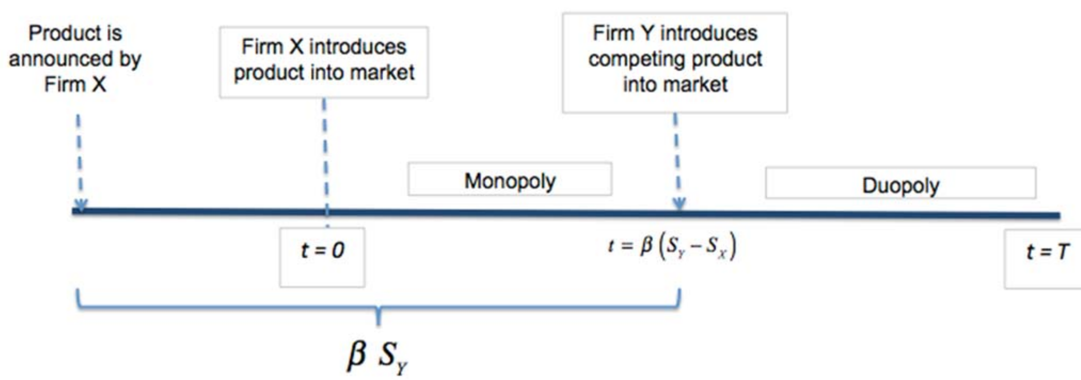

Figure 1a. Market Description: Firm X Pre-announces New Product

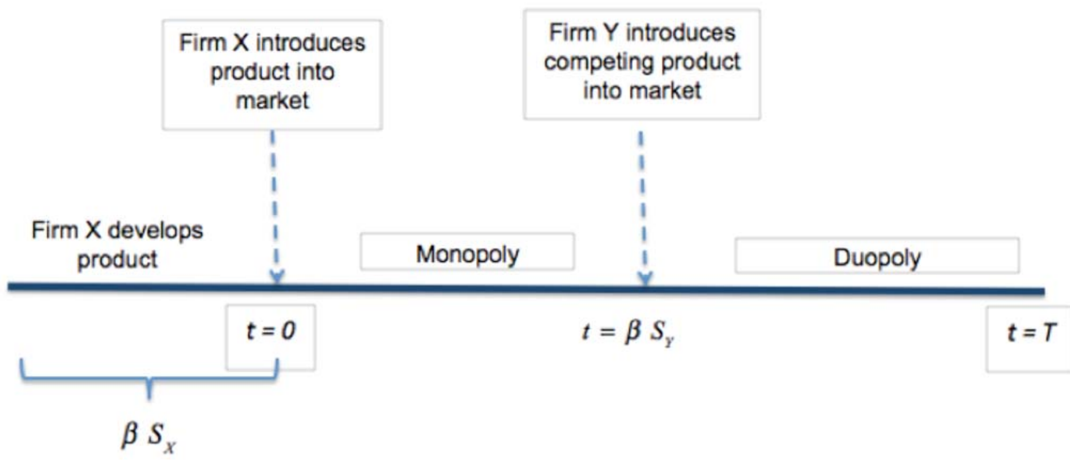

Figure 1b. Market Description: Firm X Does Not Pre-announce Product

We assume there are M potential consumers who enter the market at time $t = 0$ (we relax this assumption in the third section but show that our basic results still hold). Each consumer derives a utility $v\,S_j$ for firm j's product (j = X, Y), where v is a consumer's valuation of each unit of product design. Following previous studies (Kornish 2001; Klastorin and Tsai 2004), we assume $v \sim Uniform\left[0,b\right]$ for $b > 0$; without loss of generality, we normalize $b = 1$. A consumer's utility surplus for a product from firm j is $U_j = v\,S_j - p_j$ where p_j denotes the price set by firm j ($j = X$, Y) for its respective product. If consumers are myopic, they purchase the available product that provides the greater (positive) utility surplus when they enter the market. In our model, however, consumers may defer a current purchasing decision if their discounted utility surplus for an expected future product is greater than their utility surplus for a currently available product. Consumers who enter the market prior to any product introduction (*i.e.*, at the beginning of the monopoly period) remain in the market if their (discounted) expected utility surplus of future products is positive. Consumers purchase at most one copy of a product. Both consumers' utilities and firms' profits are discounted to time $t = 0$ at rate r. For mathematical tractability, we approximate the discount factor $\left(1+r\right)^{-t}$ by $\left(1-rt\right)$ based on a Taylor series first order approximation. For values of $r < 1$, this approximation introduces a relatively small error term.

Both firms seek to maximize their respective discounted profit in the Stackelberg game. We assume that $S_X < S_Y$ such that firm X always enters the market first. Given S_X, firm X determines the optimal price of its product in the monopoly period, p_{Xm}. As soon as firm Y becomes aware of firm X's product, it sets the design/quality level of its product, S_Y, and begins development. The introduction of firm Y's product into the market defines the beginning of the duopoly period when firms X and Y set their respective duopoly prices, p_{Xd} and p_{Yd}, in a simultaneous game. The notation we use in defining our models when firm X does not pre-announce its product is summarized below:

Decision variables (when firm X does not pre-announce product):

S_Y: design/quality level of firm *Y*'s product,
p_{Xm}: price of firm *X*'s product in the monopoly period,
p_{Xd} and p_{Yd}: respective prices of firm X and firm Y products in the duopoly period.

Parameters:

S_X: design/quality of firm X product,
$g(S) = \beta S$ where $\beta > 0$: time to develop a product of design level *S*,
$c(S) = \alpha S$ or αS^2 where $\alpha > 0$: marginal production cost for a product of design level *S*,
T: product lifespan,
v: consumer valuation of each unit of design where $v \sim Uniform[0,1]$,
M: total number of potential consumers in market, and
r: discount rate for firm profits and consumers' utility surplus.

When firm X pre-announces its product, we use $\hat{S}_Y$, $\hat{p}_{Xm}$, $\hat{p}_{Xd}$, and $\hat{p}_{Yd}$ to represent the respective decision variables. Throughout this paper, we assume that the following constraints hold in order to satisfy the assumption that a viable duopoly market exists and trivial cases are avoided:

$\beta\ S_Y < T$ (such that firm Y enters the market), and
$p_{jk} > c(S_j)$ for $j = X,Y;\ k = m,d$, and
$S_X < S_Y(1 - r\beta S_Y)$.

2.1. Firm X Does Not Pre-announce Product

All potential consumers enter the market at time $t = 0$ when firm X introduces its product and either purchase a product from firm X in the monopoly period or decide to wait for the duopoly period under the expectation that their (discounted) utility surplus in the duopoly period is greater than their current utility surplus. Some consumers may have negative utility surplus in both the monopoly and duopoly periods in which case they do not purchase a product at all.

Given the definition of consumers' utility surplus, a consumer would purchase a firm X product in the monopoly period if

$$vS_X - p_{Xm} > \left(1 - r\beta S_Y\right)\ \max\left(vS_X - p_{Xd}, vS_Y - p_{Yd}, 0\right) \tag{1}$$

where consumers' utility surplus in the duopoly period is discounted by the term $\left(1 - r\beta S_Y\right)$ given the length of the monopoly period, βS_Y. If a consumer does not purchase a product in the monopoly period, she will purchase a product from firm *k* (where *k* = *X*, *Y*) in the duopoly period if

$$vS_k - p_{kd} = \max_{j=X,Y}\left(vS_j - p_{jd}\right) > 0\ . \tag{2}$$

Equations (1) and (2) imply a partitioning of consumers into groups based on their values of *v*—those who purchase a firm X product in the monopoly period, those who purchase either a firm X or firm Y product in the duopoly period, or those who don't purchase either product.

There are two consumer partitions that firm X must consider. In the first, firm X has positive sales in both the monopoly and duopoly periods; in the second, firm X has no sales in the monopoly period and only sells its product in the duopoly period. We refer to the former as case A and the latter as case B.

Given $v \sim Uniform\left[0,1\right]$, consumer partitions in case A when firm X has positive sales in both the monopoly and duopoly periods are indicated in Figure *2a* (and analyzed in section 2.1.1). Case B occurs when $\frac{p_{Yd}\left(1 - r\beta S_Y\right) - p_{Xm}}{S_Y\left(1 - r\beta S_Y\right) - S_X} \leq \frac{p_{Xm} - p_{Xd}\left(1 - r\beta S_Y\right)}{r\beta S_Y S_X}$ and firm X has no sales in the monopoly period. In this case, a consumer is indifferent between the products of firm X and firm Y in the duopoly period when $v = \frac{p_{Yd} - p_{Xd}}{S_Y - S_X}$. We use this value to define the three consumer groups in case B (as indicated in Figure *2b* and analyzed in section 2.1.2).

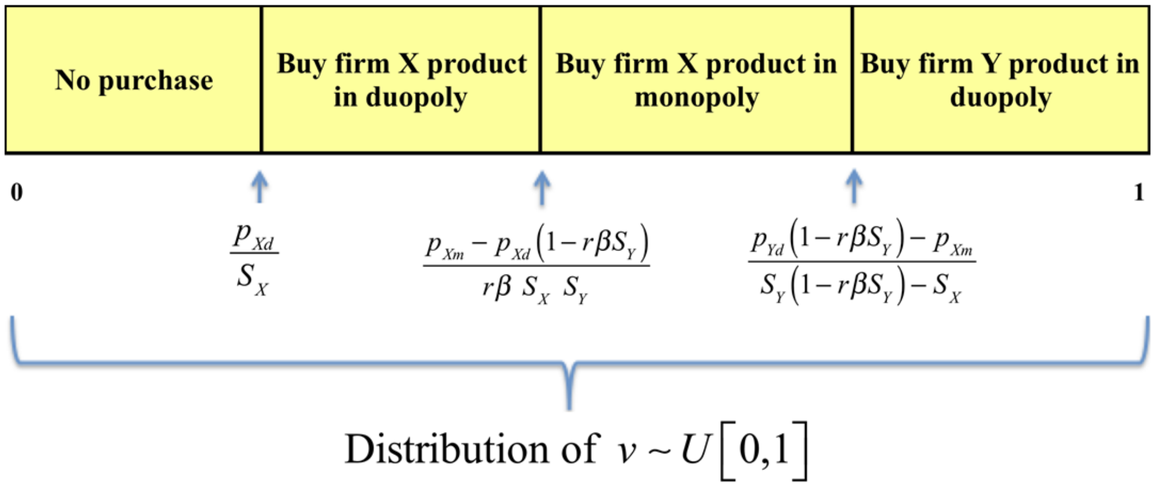

Figure 2a. *Consumer Partitions When Firm X Does Not Pre-announce Product Development and Has Sales in the Monopoly Period*

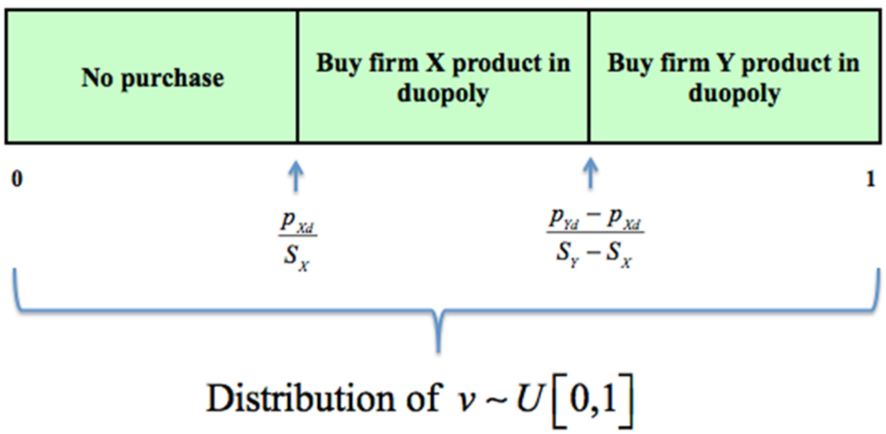

Figure 2b. *Consumer Partitions When Firm X Does Not Pre-announce Product Development and Has No Sales in the Monopoly Period*

2.1.1. Case A: Firm X has Positive Sales in the Monopoly Period

We use the superscript A to indicate the value of decision variables in case A. The discounted duopoly profit for firm X in this case is defined as follows:

$$\pi_{Xd}^{A} = \left(p_{Xd}^{A} - \alpha S_X\right)\left(1 - r\beta S_Y^{A}\right)\left[\frac{p_{Xm}^{A} - p_{Xd}^{A}}{r\beta S_X S_Y^{A}}\right]M\ . \tag{3}$$

Given (3), and using the first and second order conditions (FOC and SOC) wrt p_{Xd}^{A}, we find the optimal duopoly price for firm X, p_{Xd}^{A*}, to be:

$$p_{Xd}^{A*} = .5\left(p_{Xm}^{A} + \alpha S_X\right). \tag{4}$$

Since $p_{Xm}^{A} > \alpha\ S_X$, the optimal duopoly price defined by (4) indicates that $p_{Xd}^{A*} < p_{Xm}^{A*}$; *i.e.*, the competition in the duopoly period forces firm X to reduce its price. Substituting the definition of p_{Xd}^{A*} into (3), firm X's optimal discounted duopoly profit becomes:

$$\pi_{Xd}^{A} = \frac{\left(p_{Xm}^{A} - \alpha S_X\right)^2}{4r\beta S_X\ S_Y^A}\left(1 - r\beta\ S_Y^A\right)\ M.$$

Likewise, the discounted duopoly profit for firm Y is defined as follows:

$$\pi_{Yd}^{A} = \left(p_{Yd}^{A} - \alpha S_Y^A\right)\left(1 - r\beta S_Y^A\right)\left[1 - \frac{p_{Yd}^{A}\left(1 - r\beta S_Y^A\right) - p_{Xm}^{A}}{S_Y^A\left(1 - r\beta S_Y^A\right) - S_X}\right]M. \tag{5}$$

Using the FOC and SOC of (5), we find

$$p_{Yd}^{A*} = .5\left[S_Y^A\ \left(1+\alpha\right) - \frac{S_X - p_{Xm}^{A}}{\left(1 - r\beta\ S_Y^A\right)}\right], \tag{6}$$

indicating that firm Y can calculate its optimal duopoly price once the design and monopoly price of firm X's product is known. Using the results in (6), firm Y can find the optimal design/quality of its product, S_Y^{A*}; the following proposition indicates that there is a unique value of S_Y^{A*} that maximizes (5) subject to the constraints that $\pi_{Yd}^{A} > 0$ and the market segments indicated in Figure 2a.

<u>Proposition 1A</u>: For given values of p_{Xm}^{A} and S_X, the unique value of S_Y^{A*} that maximizes (5) in case A subject to the constraints that $S_Y^A\left(1-r\beta\, S_Y^A\right)>S_X$ and $0\le\frac{p_{Xd}^A}{S_X}\le\frac{p_{Xm}^A-p_{Xd}^A\left(1-r\beta S_Y^A\right)}{r\beta\, S_Y^A\, S_X}\le\frac{p_{Yd}^A\left(1-r\beta S_Y^A\right)-p_{Xm}^A}{S_Y^A\left(1-r\beta S_Y^A\right)-S_X}\le 1$ is given by $S_Y^{A*}=\frac{1}{2r\beta}$.

Proof: See Appendix B

Proposition 1A indicates that firm Y sets its product's design/quality level independent of firm X's design level when firm X has sales in the monopoly period. Given the optimal design value S_Y^{A*} defined by Proposition *1A*, the discount factor $\left(1-r\beta S_Y\right)$ is equal to 0.5. Substituting the value of S_Y^{A*} into (6), the optimal duopoly price set by firm Y can be written as:

$$p_{Yd}^{A*}=\frac{1+\alpha}{4r\beta}-S_X+p_{Xm}^A\ . \tag{7}$$

Under the conditions in Proposition *1A*, there is a unique equilibrium solution for this Stackelberg game. Given values of S_X and p_{Xm}^A, firm X knows firm Y's optimal response function. Using the results of Proposition 1A and the optimal duopoly price for firm Y defined by (7), the total discounted profit for firm X can be written as a function of S_X and the monopoly price, p_{Xm}^A:

$$\pi_X^A=\left\{\left(p_{Xm}^A-\alpha S_X\right)\left[\frac{\left(1+\alpha\right)\left(1-4r\beta S_X\right)-3\left(\dfrac{p_{Xm}^A}{S_X}\right)+\alpha+8r\beta\, p_{Xm}^A}{2\left(1-4r\beta\, S_X\right)}\right]+\frac{\left(p_{Xm}^A-\alpha S_X\right)^2}{4S_X}\right\}M\ . \tag{8}$$

For a given product design S_X, the monopoly price, p_{Xm}^A, that maximizes (8) can be calculated from the FOC of (8):

$$p_{Xm}^{A*}=\frac{S_X\left[1-4r\beta S_X+4\alpha\left(1-2r\beta S_X\right)\right]}{5-12r\beta\, S_X}\ . \tag{9}$$

SOCs confirm that (9) maximizes firm X's total profit defined by (9) and satisfies the constraints that define the Stackelberg game.

Using (4), (6), (9), and the results in Proposition 1, we can characterize the Stackelberg game completely when firm X conceals its development effort and does not pre-announce its product—assuming that firm X has positive sales in the monopoly period (*i.e.*, case A). Our results are summarized in Appendix A. In case B, when firm X sells only in the duopoly period (but still conceals its development effort), the Stackelberg game is analyzed in the following section.

2.1.2. Case B: Firm X has No Sales in the Monopoly Period

When $\frac{p_{Xm} - p_{Xd}\left(1 - r\beta S_Y\right)}{r\beta S_Y S_X} \geq \frac{p_{YD}\left(1 - r\beta S_Y\right) - p_{Xm}}{S_Y\left(1 - r\beta S_Y\right) - S_X}$ in Figure *2a*, firm X has set a design level and prices such that sales only occur in the duopoly period. In this case, the consumer partitions indicated in Figure *2b* are relevant and the discounted profit for firm X in the duopoly period is defined as:

$$\pi^B_{Xd} = \left(p^B_{Xd} - \alpha S_X\right)\left(1 - r\beta S^B_Y\right)\left[\frac{p^B_{Yd} S_X - p^B_{Xd} S^B_Y}{\left(S^B_Y - S_X\right) S^B_Y}\right] M\,. \tag{10}$$

Taking the FOC of (10), the duopoly price for firm X that maximizes (10) is:

$$p^{B*}_{Xd} = .5\left(\frac{p^B_{Yd} S_X}{S^B_Y} + \alpha S_X\right) \tag{11}$$

such that the discounted duopoly profit for firm X can be rewritten as

$$\pi^B_{Xd} = \frac{\left(1 - r\beta S^B_Y\right) S_X M}{4\left(S^B_Y - S_X\right) S^B_Y}\left(p^B_{Yd} - \alpha S^B_Y\right)^2.$$

Similarly, the discounted duopoly profit for firm Y is

$$\pi^B_{Yd} = \left(p^B_{Yd} - \alpha S^B_Y\right)\left(1 - r\beta S^B_Y\right)\left[\frac{S^B_Y - S_X - p^B_{Yd} + p^B_{Xd}}{S^B_Y - S_X}\right] M$$

such that the optimal duopoly price for firm Y becomes

$$p_{Yd}^{B*} = .5\left(S_Y^B - S_X + \alpha S_Y^B + p_{Xd}^B\right). \tag{12}$$

Using (11) and (12), we obtain the optimal duopoly prices for both firms in terms of S_X and S_Y^B:

$$p_{Yd}^{B*} = \frac{S_Y^B\left[2S_Y^B\left(1+\alpha\right) - S_X\left(2-\alpha\right)\right]}{4S_Y^B - S_X};\ \ p_{Xd}^{B*} = \frac{S_X\left(S_Y^B + 3\alpha S_Y^B - S_X\right)}{4S_Y^B - S_X}. \tag{13}$$

Given the optimal prices defined by (13) and FOC for S_Y, we can find the optimal design level for firm Y; this, leads to Proposition *1B*.

Proposition 1B: For a given value of $S_X > 0$, the unique value of S_Y^{B*} that maximizes firm Y's discounted duopoly profit subject to the constraints that $S_Y^{B*}\left(1 - r\beta\, S_Y^{B*}\right) > S_X$ and $\frac{p_{Xm}^B - p_{Xd}^B\left(1-r\beta S_Y^{B*}\right)}{r\beta S_Y^{B*} S_X} \geq \frac{p_{YD}^B\left(1-r\beta S_Y^{B*}\right) - p_{Xm}^B}{S_Y^{B*}\left(1-r\beta S_Y^{B*}\right) - S_X}$ and $0 \leq \frac{p_{Xd}^B}{S_X} \leq \frac{p_{Yd}^B - p_{Xd}^B}{S_Y^B - S_X} \leq 1$ is given by

$$S_Y^{B*} = \frac{2 + 3r\beta S_X + \sqrt{4 - 4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta}.$$

Proof: See Appendix B

In this case, we set $p_{Xm}^B = \frac{p_{Xd}^B\left[S_Y^{B*}\left(1-r\beta S_Y^{B*}\right) - S_X\right] + p_{Yd}^B r\beta S_X S_Y^{B*}}{S_Y^{B*} - S_X}$ such that $\frac{p_{Xm}^B - p_{Xd}^B\left(1-r\beta S_Y^B\right)}{r\beta\, S_Y^B\, S_X} = \frac{p_{Yd}^B\left(1-r\beta S_Y^B\right) - p_{Xm}^B}{S_Y^B\left(1-r\beta S_Y^B\right) - S_X}$ (the lower bound for which firm X has no monopoly sales). Any value greater than this lower bound would result in the same profit for both firms. Since $S_Y^{B*}\left(1-r\beta S_Y^{B*}\right) > S_X$, the monopoly price is greater than the duopoly price and greater than the monopoly price in case A. This, with the optimal prices defined by (13) and Proposition 1B, completely characterizes the Stackelberg game in case B. In the following section, we compare the results in cases A and B with no pre-announcement to the cases when firm X chooses to pre-announce its product and begins product development at time $t = -\beta S_X$.

2.2. Firm X Pre-announces Product

When firm X pre-announces its product, product development efforts by both firms now proceed concurrently, as indicated in Figure *1a*. While we assume that all potential consumers enter the market when the product is introduced, all consumers (and firm Y) become aware of the firm X product at time $t = -\beta S_X$. Our assumption that $\Delta S = \hat{S}_Y - S_X > 0$ implies that both firms have positive sales in the duopoly period, where $\hat{S}_Y$ denotes the design value of the firm Y product when firm X pre-announces its product. Similar to the "no pre-announcement" case, a consumer would purchase a product from firm X in the monopoly period if

$$v\ S_X - \hat{p}_{Xm} > \left(1 - r\beta\ \Delta S\right)\ \left[\max_{j=X,Y}\left(v\ S_j - \hat{p}_{jd}\right),\ 0\ \right]$$

and purchase a product from firm *k* (*k = X, Y*) in the duopoly period if condition (2) holds (given that the consumer did not purchase a product in the monopoly period). We continue to denote by case A the situation when firm X has positive sales in both the monopoly and duopoly periods, and by case B the situation where firm X has no sales in the monopoly period and only sells its product in the duopoly period.

2.2.1. Case A: Positive Monopoly Sales

We first consider case A. When firm X pre-announces its product and has positive monopoly sales, the consumer partitions are defined in Figure 3 and imply that $\hat{p}^A_{Xd} \le \hat{p}^A_{Xm} \le \hat{p}^A_{Yd}\left(1 - r\beta\ \Delta S\right)$.

Mathematically, case A with pre-announcement is similar to case A with no pre-announcement analyzed in section 2.1.1, with the exception that the discount factor is now $\left(1 - r\beta\ \Delta S\right)$. For example, the discounted duopoly profit for firm Y (that we denote by $\hat{\pi}^A_{Yd}$) is now defined as follows:

$$\hat{\pi}^A_{Yd} = \left(\hat{p}^A_{Yd} - \alpha \hat{S}^A_Y\right)\left(1 - r\beta\ \Delta S\right)\left[1 - \frac{\hat{p}^A_{Yd}\left(1 - r\beta\ \Delta S\right) - \hat{p}^A_{Yd}}{\hat{S}^A_Y\left(1 - r\beta\ \Delta S\right) - S_X}\right] M \qquad (14)$$

that compares to the firm Y duopoly profit defined by (5) for the "no pre-announcement" case. The analysis in this case proceeds in similar fashion and leads to the following proposition.

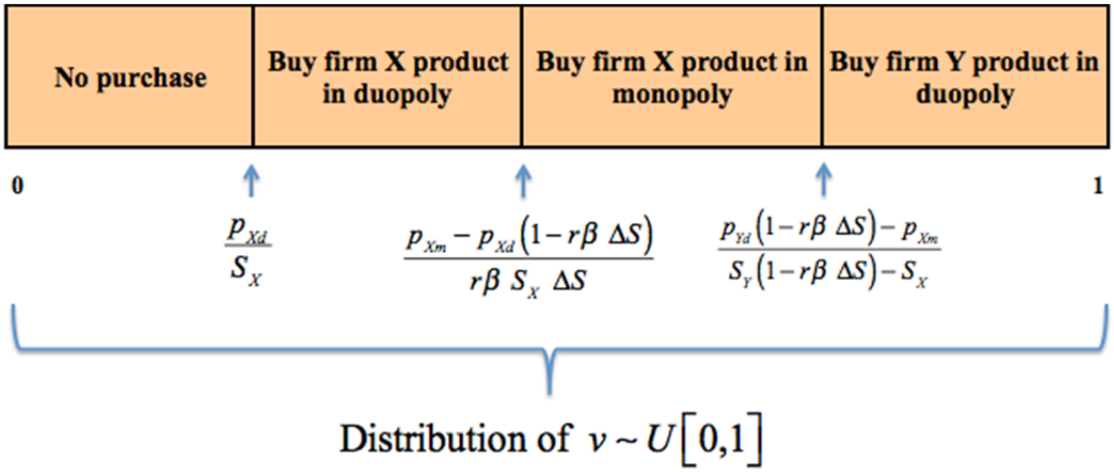

Figure 3. *Consumer Partitions When Firm X Pre-announces Product Development and Has Sales in the Monopoly Period*

Proposition 2A: For any values of $\hat{p}_{Xm}^{A}$ and S_X, the unique value of $\hat{S}_Y^{A*}$ that maximizes firm Y's discounted duopoly profit in case A defined by (14) subject to the constraints

$0 \le \frac{\hat{p}_{Xd}^{A}}{S_X} \le \frac{\hat{p}_{Xm}^{A} - \hat{p}_{Xd}^{A}(1-r\beta\ \Delta S)}{r\beta\ \Delta S\ S_X} \le \frac{\hat{p}_{Yd}^{A}(1-r\beta\ \Delta S) - \hat{p}_{Xm}^{A}}{\hat{S}_Y^{A}(1-r\beta\ \Delta S) - S_X} \le 1$ is given by $\hat{S}_Y^{A*} = 0.5\left(S_X + \frac{1}{r\beta}\right)$.

Proof: See Appendix B

The discount factor $(1-r\beta\ \Delta S)$ is now equal to $\left[0.5\left(1+r\beta\ S_X\right)\right]$ based on the optimal design value $\hat{S}_Y^{A*}$. Using $\hat{S}_Y^{A*} = 0.5\left(S_X + \frac{1}{r\beta}\right)$, we can rewrite the optimal price for firm Y's product as a function of the model parameters and $\hat{p}_{Xm}^{A}$ as follows:

$$\hat{p}_{Yd}^{A*} = .25\left[S_X + \frac{1}{r\beta}\right](1+\alpha) - \frac{S_X - \hat{p}_{Xm}^{A*}}{1 - S_X(1+r\beta)}.$$

The total discounted profit for firm X in this case is

$$
\hat{\pi}_X^A = \left(\hat{p}_{Xm}^A - \alpha S_X\right) M \left\{ \frac{.5\hat{p}_{Yd}^A\left(1+r\beta S_X\right) - \hat{p}_{Xm}^A}{.25\left[S_X + (r\beta)^{-1}\right]\left(1+r\beta S_X\right) - S_X} - \frac{\hat{p}_{Xm}^A - .25\left(\hat{p}_{Xm} + \alpha S_X\right)\left(1+r\beta S_X\right)}{.5r\beta S_X\left(1+r\beta S_X\right)} \right\} + \left(\hat{p}_{Xd}^A - \alpha S_X\right)\left[\frac{1+r\beta S_X}{2S_X}\right] M \ . \tag{15}
$$

Maximizing (15), we can obtain firm X's optimal prices for the monopoly and duopoly periods, respectively:

$$
\hat{p}_{Xm}^{A*} = \frac{S_X\left[r^2\left(1+3\alpha\right)\beta^2 S_X^4 - 2r\beta S_X^3\left(\alpha\beta r - \alpha + 1\right) + S_X^2\left(1+3\alpha+\alpha r^2\beta^2\right) + 2\alpha S_X - \alpha\right]}{1+4r^2\beta^2 S_X^3 + S_X^2\left(8 - r^2\beta^2\right) - 4S_X}.
$$

and $\hat{p}_{Xd}^{A*} = .5\left(p_{Xm}^A + \alpha S_X\right)$.

Note that the optimal duopoly price charged by firm X, $\hat{p}_{Xd}^{A*}$, is the same in both the pre-announcement and "no pre-announcement" cases; that is, $\hat{p}_{Xd}^{A*} = p_{Xd}^{A*} = .5\left(p_{Xm} + \alpha S_X\right)$ as defined by (4).

2.2.2. Case B: No Monopoly Sales for Firm X

The case when firm X has no monopoly sales (case B) is analogous to the case in section 2.1.2 with the difference that the discount factor is now equal to $\left(1 - r\beta\ \Delta S\right)$. This discount factor results in a changed optimal design $\hat{S}_Y^{B*}$ for firm Y; this value and its derivation is given in Proposition 2B.

<u>Proposition 2B</u>: For a given value of S_X, the unique value of $\hat{S}_Y^{B*}$ that maximizes firm Y's discounted duopoly profit subject to the constraints that $\hat{S}_Y^{B*}\left(1 - r\beta\left(\hat{S}_Y^{B*} - S_X\right)\right) > S_X$,

$$\frac{\hat{p}^B_{YD}\left(1-r\beta\left(\hat{S}^{B*}_Y-S_X\right)\right)-\hat{p}^B_{Xm}}{\hat{S}^{B*}_Y\left(1-r\beta\left(\hat{S}^{B*}_Y-S_X\right)\right)-S_X} \le \frac{\hat{p}^B_{Xm}-\hat{p}^B_{Xd}\left(1-r\beta\left(\hat{S}^{B*}_Y-S_X\right)\right)}{r\beta\left(\hat{S}^{B*}_Y-S_X\right)S_X}$$, and $0 \le \frac{\hat{p}^B_{Xd}}{S_X} \le \frac{\hat{p}^B_{Yd}-\hat{p}^B_{Xd}}{\hat{S}^B_Y-S_X} \le 1$ is given by $\hat{S}^{B*}_Y = \frac{2+5r\beta S_X+\sqrt{4+4r\beta S_X+\left(3r\beta S_X\right)^2}}{8r\beta}$.

Proof: See Appendix B

All other aspects of the model described in section 2.1.2 continue to hold; specific results are summarized in Appendix A.

3. Model and Managerial Implications

In the previous section, we were able to derive close-form expressions for all the key decisions – firm Y's design level and both firms' price decisions – as well as both firms' profit functions, in a variety of settings: when firm X pre-announces its product development effort or does not pre-announce; when firm X has sales in the monopoly period or not. These expressions are summarized in Appendix A; they not only allow us to understand the Stackelberg game dynamics and how the two firms strategically react to each other, but, more importantly, also allow us to evaluate firm X's profits under pre-announcement and no pre-announcement. We have used these expressions to carry out a range of numerical tests and will report the results below.

By comparing both firms' profits in the various settings, we find a number of significant implications for new product development managers. Two major findings emerge from our analysis. First, we found that the innovator firm X may, under some conditions, choose to forgo sales in the monopoly period. This result appears to be counter-intuitive. For example, consider case B (in section 2.1.2) when firm X does not pre-announce its product development effort and sells only in the duopoly period. In this case, it would seem that firm X could reduce its monopoly price p^B_{Xm} such that $\frac{p^B_{Xm}-p^B_{Xd}\left(1-r\beta S^B_Y\right)}{r\beta S^B_Y S_X} < \frac{p^B_{Yd}\left(1-r\beta S^B_Y\right)-p^B_{Xm}}{S^B_Y\left(1-r\beta S^B_Y\right)-S_X}$ and some consumers would then purchase firm X's product in the monopoly period. Since $p^B_{Xm} > p^B_{Xd}$, it would appear that firm X could do better by setting a monopoly price such that some monopoly sales occur.

Our analysis, however, suggests otherwise; namely, once firm X considers firm Y's reaction to its pricing and design decisions, we showed that firm X may lower its overall profits by reducing its monopoly price to stimulate monopoly sales. Specifically, when firm X sets a monopoly price that is sufficiently high to discourage any monopoly sales (case B), firm Y produces a product of higher quality/design than when firm X has monopoly sales (case A). This is supported by the following proposition that is based on previous results derived in section 2.

Proposition 3: For all $S_X > 0$, it holds that $S_Y^{B*} > S_Y^{A*}$ and $\hat{S}_Y^{B*} > \hat{S}_Y^{A*}$.

Proof: See Appendix B

Comparing (6) and (12), we can show that $p_{Yd}^{B} > p_{Yd}^{A}$ (that is, the optimal duopoly price for firm Y will increase when firm X has no monopoly sales). This increased product differentiation by firm Y increases the profitability of both firms X and Y as illustrated by the numerical example in Figure 4 where we set $S_X = 10$, $\alpha = .05$, $\beta = .15$, and $M = 1000$, varied the discount rate, r, from 0.01 to 0.10 in increments of 0.01, and calculated the discounted optimal profit for firm X in both cases A and B. As the discount rate (r) increases, firm Y decreases its design value S_Y; this decrease counteracts the increased rate in the discount factor $\left(1 - r\beta S_Y\right)$ and allows firm Y to enter the market earlier. If the discount factor increases, consumers are more willing to postpone their consumption decisions; hence, firm X will focus sales solely in the duopoly period. Furthermore, the products' similarity increases as firm Y reduces their design value. Our calculations confirm that the reduced product differentiation and increased discount factor results in reducing the profit of firm X (as indicated in Figure 4) and firm Y.

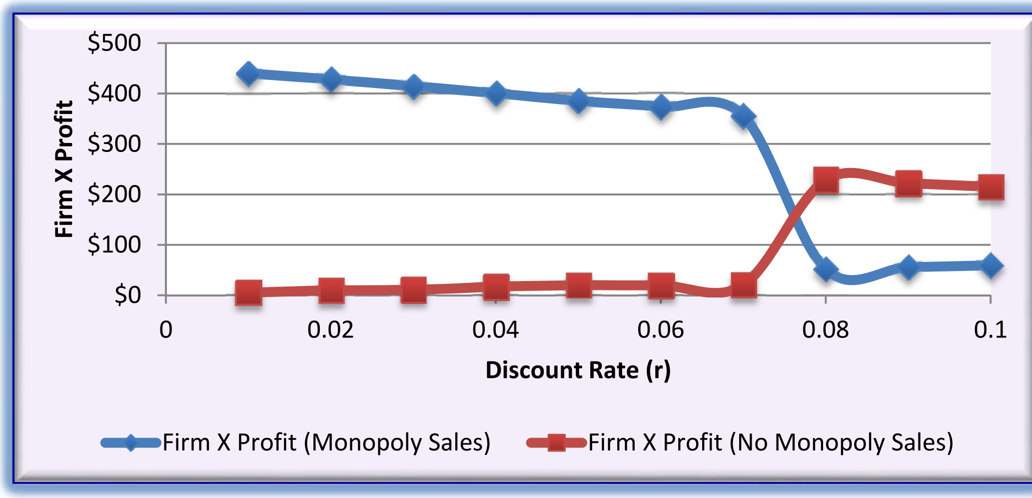

Figure 4. *Profit of Firm X with Monopoly and No Monopoly Sales as Function of Discount Rate, r*

The advantages (to both firms) of increased product differentiation is the main factor behind firm X's decision to pre-announce or not pre-announce its product development effort. A comparison of Propositions *1A* and *2A* indicate that firm Y's product will have a greater quality/design when firm X pre-announces its product; that is, $\hat{S}_Y^{A*} > S_Y^{A*}$ (assuming sales in the monopoly period). Again, the increased product differentiation by firm Y increases the profitability of both firms, thereby providing an incentive for firm X to pre-announce its product development effort (depending on the parameter values). The following numerical examples illustrate these concepts.

Normalizing T to 1, we set $\alpha = .05, \beta = .15, M = 1000, S_X = 10$, and varied the discount rate, r, from 0.01 to 0.10 in increments of 0.01. The results in Figure 5 indicate that firm X can increase its profit by pre-announcing its product development efforts as the discount rate, r, increases greater than (approximately) 0.075. Since consumers are forward looking, firm Y's product does not only compete with firm X's product in the duopoly period but also in the monopoly period. While firm Y's product has a higher design level that firm X's product, the increased discount rate diminishes the appeal of firm Y's product (compared to firm X's product in the monopoly period). Alternatively, the increased discount rate reduces the differences between the two products in the minds of monopoly consumers that, in turn, increases the competitive pressure on pricing. As a result, the profits of both firms are reduced. However, when firm X pre-announces its product development, firm Y begins its product development effort earlier. This has two effects: (1) firm X has a shorter monopoly period, and (2) firm Y now has an incentive to increase its product design level (that is, the longer development time is offset by the earlier development start). The first effect has a negative impact on firm X profits, but the second effect could increase firm X profits (since the products become more differentiated and the two firms can better segment the duopoly consumers). When the discount rate is high, the second effect tends to dominate; hence, firm X prefers to pre-announce its product development.

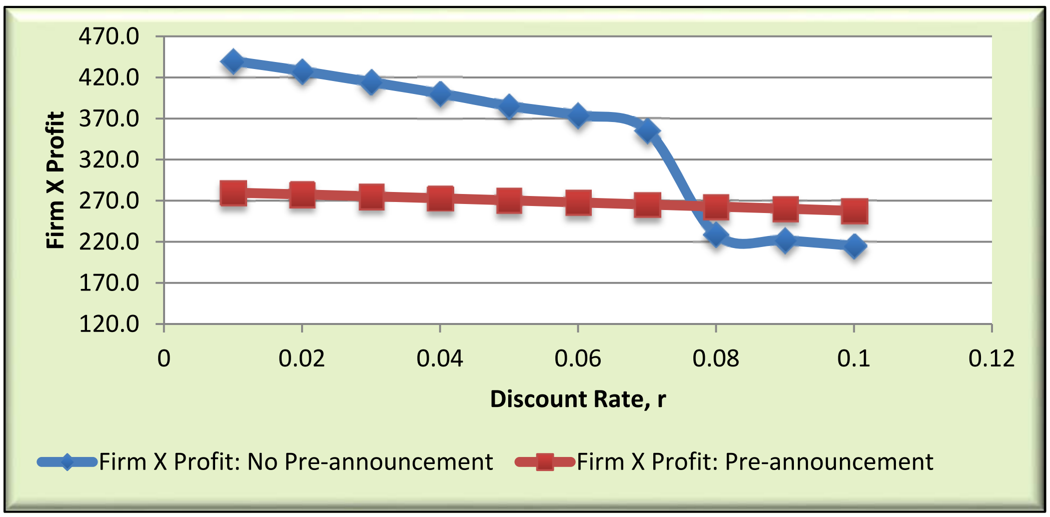

Figure 5. *Firm X Profits when Pre-announcing Product and Not Pre-announcing Product as Function of Discount Rate, r*

To investigate the impact of increasing marginal production cost (α), we normalized T = 1, set $r = .05$, $\beta = .15$, $S_X = 10$, $M = 1000$ and varied α from 0.01 to 0.10; results are indicated in Figure 6. For this example, firm Y sets its design level S_Y = 74.35 for all values of α when firm X pre-announces its product. However, when firm X does not pre-announce its product, firm Y increases the design level S_Y monotonically from 66.67 $(= 1/2r\beta)$ to 69.36 as α increases, reinforcing our finding that $\hat{S}_Y^{A*} > S_Y^{A*}$. For values of α less than 0.05, firm X has no sales in the monopoly period (case B); as the production cost increases greater then 0.05, firm X can increase its profits by reducing its monopoly price and beginning sales in the monopoly period. Our calculations indicate that both firm X and firm Y prices do not change monotonically with α; firm

X profits are maximized when α = 0.05 when firm X reduces its monopoly price to initiate monopoly sales. As α increases greater than 0.05, firm X increases its monopoly price (to offset the increase in costs) that reduces overall profits (but still retains monopoly sales). For values of α < 0.05, firm X can increase its profits by approximately 4.5 percent by pre-announcing its product development effort.

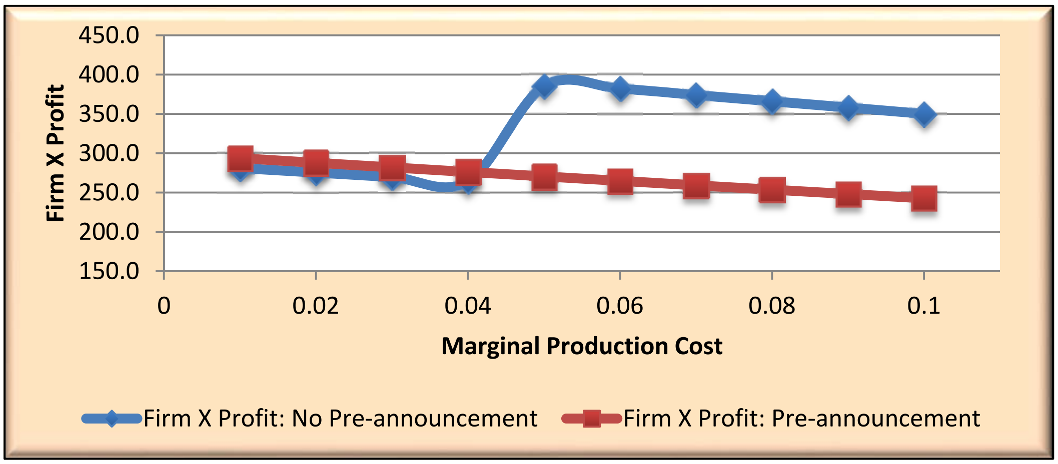

Figure 6. *Profit of Firm X when Pre-Announcing Product and Not Pre-announcing Product as Function of Marginal Production Cost,* α

To examine the impact of β (coefficient of development time), we normalized T = 1, set $r = .05$, $\alpha = .05$, $S_X = 10$, $M = 1000$ and varied β from 0.11 to 0.19 (see Figure 7). Our results in this case were similar to the case when we varied the marginal production cost; when β is relatively small (less than 0.14), firm X can increase its profits by pre-announcing its product and not selling any product in the monopoly period. When β > 0.14, firm X reduces its monopoly price p_{Xm} from \$7.88 to \$2.12, thereby generating monopoly sales and increasing its profit by approximately 53 percent.

In the basic model, S_X is treated as an exogenous parameter in order to maintain analytical tractability. Given a value of S_X, we can derive explicit expressions for the optimal decisions that maximize firm X's profits. While there is no analytical expression for the optimal value of S_X, we were able to use the results in section 2 to numerically investigate the trade-offs associated with varying values of S_X and analyze when firm X should or should not pre-announce its product development effort.

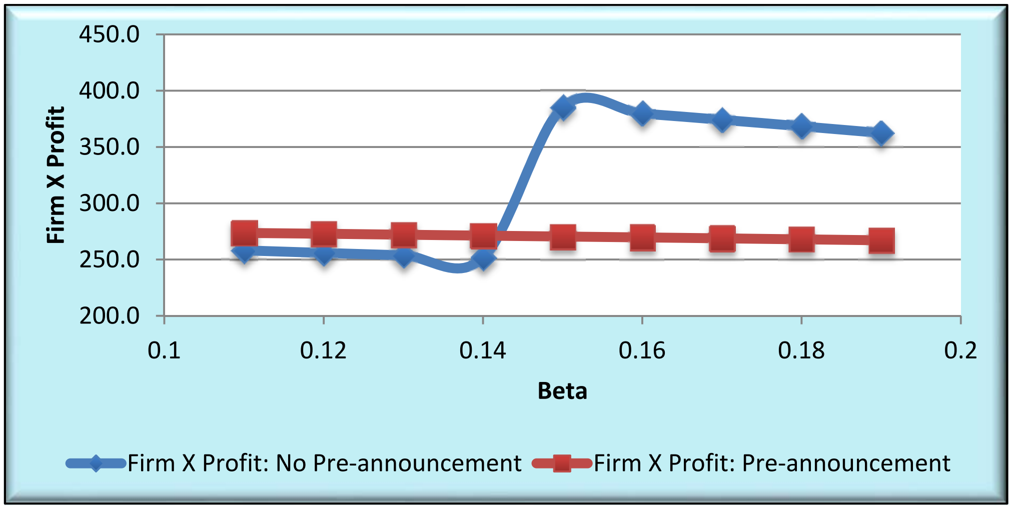

Figure 7. *Profit of Firm X when Pre-Announcing Product and Not Pre-announcing Product as Function of the Development Time Coefficient,* β

Following previous examples, we normalized T = 1, set r = .05, M = 1000, $\beta = .15$, $\alpha = 0.05$ and varied S_X from 5 to 20 in unit increments. Our results are indicated in Figure 8. In all cases, firm Y produced a product that has a significantly higher design level; optimal values of S_Y increased monotonically from 66.67 to 82.4. For any design level greater than 14, firm X would pre-announce their development effort; firm X's profit is maximized at S_X = 20 or higher. This is an important result as it indicates that firm X may choose to pre-announce its product development effort even when the S_X is endogenous.

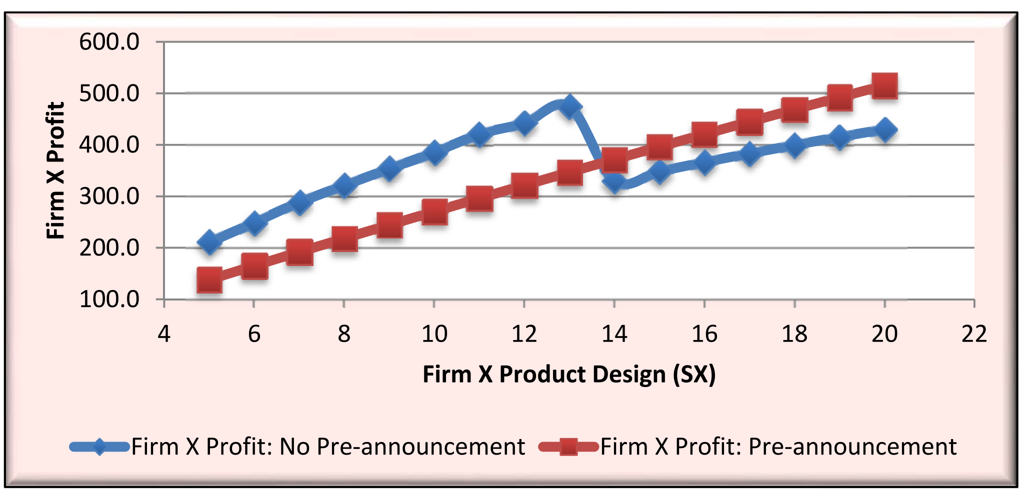

Figure 8. *Profit of Firm X when Pre-Announcing Product and Not Pre-announcing Product as Function of Product X Design,* S_X

To test the sensitivity of our results, we extended the basic model in two ways. First, we assumed that production costs were quadratic as indicated in some previous studies (Klastorin and Tsay, 2004); *i.e.*, we let $c(S) = \alpha S^2$ where $\alpha > 0$. Second, we relaxed the assumption that all potential consumers arrive when the product is initially introduced; in this case, we assumed that potential consumers arrive at the beginning of the monopoly and duopoly periods in numbers that are proportional to the length of each respective period. Specifically, when firm X does not pre-announce its product, the number of potential consumers who enter the market at time $t = 0$ is $\left(\frac{\beta S_Y}{T}\right) M$; the number of potential consumers who enter the market at the beginning of the duopoly period is $\left(1 - \frac{\beta S_Y}{T}\right) M$. The consumer partitions defined in Figures 2a and 2b continue to hold; thus, the total discounted profit for firm X (assuming sales in the monopoly period and quadratic costs) is defined as follows:

$$\pi_X = \left(p_{Xm} - \alpha S_X^2\right)\left[\frac{p_{Yd}\left(1 - r\beta S_Y\right) - p_{Xm}}{S_Y\left(1 - r\beta S_Y\right)} - \frac{p_{Xm} - p_{Xd}\left(1 - r\beta S_Y\right)}{r\beta S_X S_Y}\right]\left(\frac{\beta S_Y}{T}\right) M$$
$$+\left(p_{Xd} - \alpha S_X^2\right)\left(1 - r\beta S_Y\right) M \left\{\frac{p_{Xm} - p_{Xd}}{r\beta S_X S_Y}\left(\frac{\beta S_Y}{T}\right)^2 + \frac{p_{Yd} S_X - p_{Xd} S_Y}{S_X\left(S_Y - S_X\right)}\left(1 - \frac{\beta S_Y}{T}\right)\left(\frac{\beta S_Y}{T}\right)\right\}.$$

The analysis of this model is similar to the basic model described in section 2. For example, we can show that the optimal duopoly price for firm Y, p_{Yd}^*, is equal to

$$p_{Yd}^* = .5\left[S_Y\left(1 + \alpha S_Y\right) - \frac{S_X - p_{Xm}}{\left(1 - r\beta S_Y\right)}\right]$$

that is independent of firm X's sales in the monopoly period. However, in this case, Propositions 1 and 2 do not hold (that is, there is no unique equilibrium to the Stackelberg game); thus, to find S_Y^*, we solved the nonlinear programming problem:

$$\text{Max} \quad \pi_{Yd} = .25M\left(1-\frac{\beta S_Y}{T}\right)\frac{\left[\left(S_Y-\alpha S_Y^2\right)\left(1-r\beta S_Y\right)-S_X+p_{Xm}\right]^2}{\left[S_Y\left(1-r\beta S_Y\right)-S_X\right]}$$

subject to

$$\beta S_Y \leq T$$
$$S_Y\left(1-r\beta S_Y\right) \geq S_X$$

as an interpreted MATLAB application on a Dell OptiPlex 755 with 4GB of RAM running Windows 7 OS. Given S_Y^*, we then calculated p_{Xd}^* and p_{Xm}^* using a similar approach described in section 2 for the basic model.

Overall, we analyzed four models by varying the assumption about production cost (linear versus quadratic) and consumer arrival patterns (all arrive at time $t = 0$ or proportionately at the beginning of the monopoly and duopoly periods). When consumers arrived proportionately, we found that incentives for firm X to pre-announce its product were generally greater than the results we observed from the basic model. For example, when production costs are quadratic and consumers arrive proportionately at the beginning of each period, we varied S_X from 5 to 20 in unit increments and calculated the optimal decisions for firms X and Y (normalizing M = T = 1, and setting $r = .05$, $\beta = .15$, and $\alpha = 0.05$). The results are indicated in Figure 9; except for the cases when S_X = 15 and 16, firm X can increase its profit by pre-announcing its product development process.

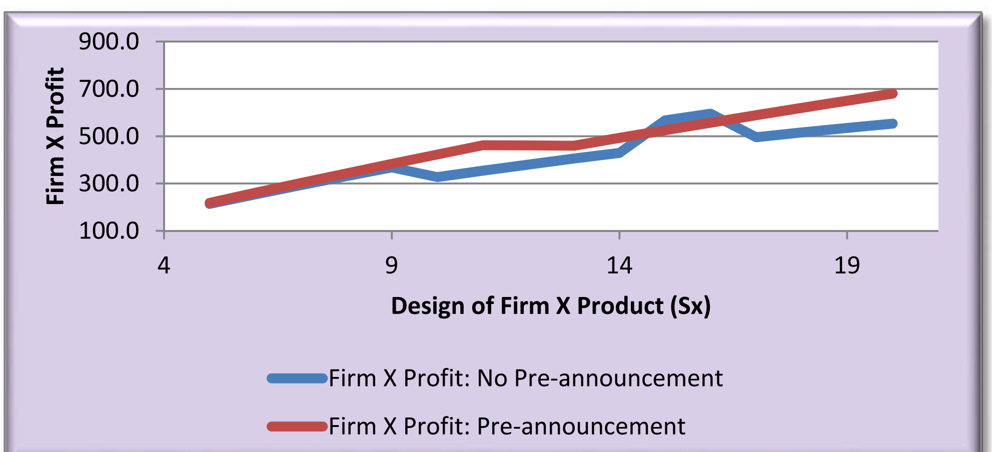

Figure 9. *Profit of Firm X when Pre-Announcing or Not Pre-announcing Product as Function of S_X, When Production Costs are Quadratic and Consumers Arrive at Beginning of each Period*

Generally, pre-announcement is a better strategy for firm X when consumers arrive proportionately over time. For the example in Figure 9 when all potential consumers arrive at time $t = 0$, firm X would only pre-announce its product for values of $S_X \geq 15$. Similarly, linear production costs tend to increase the benefits of pre-announcement; when we solved the example in Figure 9 with $c\left(S\right) = \alpha S$, firm X could increase profits by pre-announcing its product for all values of S_X. We found similar results as we reduced the values of α and β.

4. Conclusions and Extensions

In this paper, we studied a durable goods market with two competing homogeneous firms when one firm is an innovator who initiates development of a new product. The primary research question we addressed is whether the innovator firm should pre-announce its product development effort or not. Our work extends previous research in this area in several important ways. In addition to modeling a durable good market with a finite lifespan as a Stackelberg game, we modeled rationally expectant consumers who make decisions based on both the value of current goods and prices as well as their expectation of future products and prices. Using these models, we were able to show that a unique equilibrium exists for the Stackelberg game in some cases; in other cases, we were able to find optimal solutions numerically. We derived several implications for both firms and showed that conditions exist when the innovator firm benefits from pre-announcing its development project, even though releasing such information increases the profitability of its competitor and defies conventional wisdom.

This work suggests numerous directions for future research. In this paper, we only considered whether the innovator firm should pre-announce its new product when it begins development work (at time $t = -\beta S_X$) or wait until the product is introduced into the market (at time $t = 0$). Clearly, there may be some cases when the innovator firm should announce its product at some time between these values (as frequently observed in practice). We are also investigating measures of "competitive intensity" and its impact on pre-announcement decisions.[3]

---

[3] The authors gratefully acknowledge this suggestion by an anonymous referee.

In this case, we let λ denote a preference measure for firm Y's product such that the consumer utility for firm Y's product is $\lambda v S_Y - p_{Yd}$ (*i.e.*, consumers are indifferent between the two firms' products in the duopoly period if $\lambda v S_Y - p_{Yd} = v S_X - p_{Xd}$). A similar measure has been used in the study of gray markets (Ahmadi and Yang 2000; Xiao *et al*. 2011).

The models presented in this paper have implications for other areas in marketing and operations management. For example, a firm's product design decision impacts the resultant supply chain; a higher design level product is likely to require a more complex supply chain that may, in turn, increase both the marginal production cost and the development time (*i.e.*, the product development time $g(S)$ is no longer an exogenous function). In addition, firms can frequently compress the development time by increasing the level of resources allocated to product development (a modification of the time-cost trade-off problem discussed in the project management literature). This extension would allow a firm to study the impact of its development costs on resultant market share and profitability.

References

Ahmadi, R., B.R. Yang. 2000. "Parallel imports: Challenges from unauthorized distribution channels. *Marketing Science*. 18(3), 279-294.

Bala, R., S. Carr. 2009. Pricing software upgrades: The role of product improvement and user costs. *Production and Operations Management* **18**(5) 560-580.

Bhaskaran, S. R., S. M. Gilbert. 2005. Selling and leasing strategies for durable goods with complementary products. *Management Science* **51**(8) 1278-1290.

Bayus, B. L. 1997. Speed-to-market and new product performance trade-offs. *Journal of Product Innovation Management* **14** 485-497.

Bayus, B. L., S. Jain, A. G. Rao. 1997. Too little, too early: Introduction timing and new product performance in the personal digital assistant industry. *Journal of Marketing Research* **34**(1) 50-63.

Coase, R.H. 1972. Durability and monopoly. *Journal of Law and Economics*. **15**, 143-149.

Cohen, M., J. Eliashberg, T. H. Ho. 1996. New product development: The performance and time-to-market trade-off. *Management Science* **42**(2) 173-186.

Dogan, K., Y. Ji, V. Mookerjee, S. Radhakrishnan. 2011. "Managing the versions of a software product under variable and endogenous demand" *Information Systems Research*. 22 (1), 5-21.

Dhebar, A. 1994. Durable-goods monopolists, rational consumers, and improving products. *Marketing Science* **13**(1) 100-120.

Dockner, E., S. Jorgensen. 1988. Optimal pricing strategies for new products in dynamic oligopolies. *Marketing Science* **7**(4) 315-334.

Fudenberg, D., J. Tirole. 1998. Upgrades, trade-ins, and buy-backs. *The RAND Journal of Economics* **29**(2) 235-258.

Hotelling, H. 1929. Stability in competition. *The Economic Journal* **39** 41-57.

Klastorin, T., W. Tsai. 2004. New product introduction: Timing, design, and pricing. *Manufacturing & Service Operations Management* **6**(4) 302-320.

Kornish, L. J. 2001 Pricing for a durable-goods monopolist under rapid sequential innovation. *Management Science* **47**(11) 1552-1561.

Kouvelis, P., S. K. Mukhopadhyay. 1999. Modeling the design quality competition for durable products. *IIE Transactions* **31**(9) 865-880.

Mehra, A., R. Bala, R. Rankaranarayanan. "Competitive Behavior-based price discrimination for software upgrades." *Information Systems Research* 22(4).

Moorthy, S. K. 1988. Product and price competition in a duopoly. *Marketing Science* **7**(2) 141-168.

Moorthy, K. S., I. P. L. Png. 1992. Market segmentation, cannibalization, and the timing of the product introductions. *Management Science* **38**(3) 345-359.

Morgan,L. O., R. M. Morgan, W. L. Moore. 2001. Quality and time-to-market trade-offs when there are multiple product generations. *Manufacturing & Service Operations Management* **3**(2) 89-104.

Padmanabhan, V., S. Rajiv, K. Sirinivasan. 1997. New products, upgrades, and new releases: A rationale for sequential product introduction. *Journal of Marketing Research* **34**(4) 456-472.

Ramachandran, K., V. Krishnan. 2008. Design architecture and introduction timing for rapidly improving industrial products. *Manufacturing & Service Operations Management* **10**(1) 149-171.

Savin, S., C. Terwiesch. 2005. Optimal product launch times in a duopoly: Balancing life-cycle revenues with product cost. *Operations Research* **53**(1) 26-47.

Schmidt, G. L., E. Porteus. 2000. The impact of an integrated marketing and manufacturing innovation. *Manufacturing & Service Operations Management* **2**(4) 317-336.

Shaked, A. and J. Sutton. 1982. Relaxing Price Competition Through Product Differentiation. *The Review of Economic Studies*. 49 (1), pp. 3-13.

The Economist. February 26, 2011. The leaky corporation. The Economist Newspaper Limited, London, GB.

Yin, S., S. Ray, H. Gurnani, A. Animesh. 2010. Durable products with multiple used goods markets: Product upgrade and retail pricing implications. *Marketing Science* 29(3):540–560.

Xiao, Y. U. Palekar, Y. Liu. 2011. Shades of gray – The impact of gray markets on authorized distribution channels. *Quantitative Marketing and Economics*. 9, 155-178.

Appendix A. The optimal design level, prices, and profits.

No Preannouncement

Case A (positive monopoly market sales for firm X)

$$S_Y^{A*} = \frac{1}{2r\beta}$$

$$p_{Yd}^{A*} = \frac{15(1+\alpha) - 12r\beta S_X\left(7-\alpha\right) + (1-\alpha)r^2\beta^2 S_X^2}{12\left(5-12r\beta\ S_X\right)}$$

$$p_{Xd}^{A*} = \frac{\alpha S_X}{2} + \frac{\left(1+4\alpha-4r\beta S_X-8r\beta\alpha S_X\right)S_X}{2\left(5-12r\beta\ S_X\right)}$$

$$p_{Xm}^{A*} = \frac{S_X\left[1-4r\beta S_X+4\alpha\left(1-2r\beta S_X\right)\right]}{5-12r\beta\ S_X}$$

$$\pi_{Yd}^{A*} = \frac{\left(1-4r\beta S_X\right)\left(5-8r\beta S_X\right)^2\left(1-\alpha\right)^2}{16r\beta\left(5-12r\beta\ S_X\right)^2}$$

$$\pi_X^{A*} = \frac{\left(1-4r\beta S_X\right)\left(1-r\beta S_X\right)\left(1-\alpha\right)^2 S_X}{48\left(5-12r\beta\ S_X\right)}$$

No Preannouncement

Case B (no monopoly market sales for firm X)

$$S_Y^{B*} = \frac{2+3r\beta S_X+\sqrt{4-4r\beta S_X+\left(3r\beta S_X\right)^2}}{8r\beta}$$

$$p_{Yd}^{B*} = \frac{S_Y^{B*}\left(2S_Y^{B*}-2S_X+2\alpha S_Y^{B*}+\alpha S_X\right)}{4S_Y^{B*}-S_X}$$

$$p_{Xd}^{B*} = \frac{S_X\left(S_Y^{B*}-S_X+3\alpha S_Y^{B*}\right)}{4S_Y^{B*}-S_X}$$

$$p_{Xm}^{B*} \geq \frac{p_{Xd}^{B*}\left[S_Y^{B*}\left(1-r\beta S_Y^{B*}\right)-S_X\right]+p_{Yd}^{B*}r\beta S_X S_Y^{B*}}{S_Y^{B*}-S_X}$$

$$\pi_{Yd}^{B*} = \frac{4\left(S_Y^{B*}\right)^2\left(S_Y^{B*}-S_X\right)\left(1-r\beta S_Y^{B*}\right)\left(1-\alpha\right)^2}{\left(4S_Y^{B*}-S_X\right)^2}$$

$$\pi_X^{B*} = \frac{S_X S_Y^{B*}\left(S_Y^{B*}-S_X\right)\left(1-r\beta S_Y^{B*}\right)\left(1-\alpha\right)^2}{\left(4S_Y^{B*}-S_X\right)^2}$$

Preannouncement

Case A (positive monopoly market sales for firm X)

$$\hat{S}_Y^{A*} = 0.5\left(S_X + \frac{1}{r\beta}\right)$$

$$\hat{p}_{Yd}^{A*} = \frac{1}{\left(1+4r^2\beta^2S_X^3+\left(8-r^2\beta^2\right)S_X^2-S_X\right)r\beta\left(1+r\beta S_X\right)}\Big[4r^3\beta^3\left(r\left(\alpha+1\right)\beta+3\alpha+1\right)S_X^5 - \beta^2\left(r^2\left(\alpha+1\right)\beta^2+8r\beta-16\alpha\right)r^2S_X^4+2r\beta\left(r^2\left(\alpha+1\right)\beta^2+14\alpha-6\right)S_X^3 + 4\left(2r\beta+2\alpha+2\right)S_X^2-2\left(\alpha+1\right)\left(r\beta+2\right)S_X+1+\alpha\Big]$$

$$\hat{p}_{Xd}^{A*} = \frac{\alpha S_X}{2}+\frac{S_X\left[r^2\left(1+3\alpha\right)\beta^2S_X^4-2r\beta S_X^3\left(\alpha\beta r-\alpha+1\right)+S_X^2\left(1+3\alpha+\alpha r^2\beta^2\right)+2\alpha S_X-\alpha\right]}{2\left(1+4r^2\beta^2S_X^3+S_X^2\left(8-r^2\beta^2\right)-4S_X\right)}$$

$$\hat{p}_{Xm}^{A*} = \frac{S_X\left[r^2\left(1+3\alpha\right)\beta^2S_X^4-2r\beta S_X^3\left(\alpha\beta r-\alpha+1\right)+S_X^2\left(1+3\alpha+\alpha r^2\beta^2\right)+2\alpha S_X-\alpha\right]}{1+4r^2\beta^2S_X^3+S_X^2\left(8-r^2\beta^2\right)-4S_X}$$

$$\hat{\pi}_{Yd}^{A*} = \frac{1}{4r\beta\left(1+4r^2\beta^2S_X^3+\left(8-\beta^2r^2\right)S_X^2-4S_X\right)^2}\times\Big[\left(-\left(1+4\alpha\right)r^2\beta^2+4r\beta\left(1+5\alpha\right)+8-8\alpha\right)S_X^2 + 4r^2\beta^2\left(r\beta\left(\alpha-1\right)-3\alpha-1\right)S_X^4-r\beta\left(r^2\left(\alpha-1\right)\beta^2-\left(4+20\alpha\right)r\beta+12\alpha+4\right)S_X^3 - ((1+4\alpha)r\beta+4-4\alpha)S_X+1-\alpha)^2\Big]$$

$$\hat{\pi}_X^{A*} = \frac{S_X\left(r\beta S_X-1\right)\left(\left(1+3\alpha\right)r\beta S_X^3+\left(5\alpha-1-2\alpha r\beta\right)S_X^2+\left(2\alpha r\beta-6\alpha\right)S_X+2\alpha\right)}{4\left(4r^2\beta^2S_X^3-\beta^2r^2S_X^2+8S_X^2-4S_X+1\right)}\times$$

$$\begin{bmatrix}\left(1+3\alpha\right)\beta^4r^4S_X^6+2r^3\beta^3\left(\alpha\beta r+\alpha-1\right)S_X^5+4r^2\beta^2\left(1+5\alpha-r\left(4\alpha+2\right)\beta\right)S_X^4 \\ +8\alpha+2+2r\beta\left(r^2\beta^2\left(2\alpha+1\right)+2r\beta\left(2-7\alpha\right)-7-\alpha\right)S_X^3+ \\ \left(2\left(4\alpha-1\right)r^2\beta^2-8\left(4\alpha-1\right)r\beta+11+9\alpha\right)S_X^2+\left(2r\beta\left(6\alpha-1\right)-22\alpha-8\right)S_X\end{bmatrix}$$

Preannouncement

<u>Case B (no monopoly market sales for firm X)</u>

$$\hat{S}_Y^{B*} = \frac{2+5r\beta S_X+\sqrt{4+4r\beta S_X+\left(3r\beta S_X\right)^2}}{8r\beta}$$

$$\hat{p}_{Xm}^{B*} \ge \frac{\hat{p}_{Xd}^{B*}\left[\hat{S}_Y^{B*}\left(1-r\beta\left(\hat{S}_Y^{B*}-S_X\right)\right)-S_X\right]+\hat{p}_{Yd}^{B*}r\beta S_X\left(\hat{S}_Y^{B*}-S_X\right)}{\hat{S}_Y^{B*}-S_X}$$

$$\hat{p}_{Xd}^{B*} = \frac{S_X\left(\hat{S}_Y^{B*}-S_X+3\alpha\hat{S}_Y^{B*}\right)}{4\hat{S}_Y^{B*}-S_X}$$

$$\hat{p}_{Yd}^{B*} = \frac{2\hat{S}_Y^{B*}\left(1+\alpha\right)-\left(1+\alpha-r\beta(1+7\alpha)\hat{S}_Y^{B*}\right)S_X-r\beta\left(1+\alpha\right)S_X^2}{2r\beta\left(4\hat{S}_Y^{B*}-S_X\right)}$$

$$\hat{\pi}_X^{B*} = \left[\frac{S_X\left(1-\alpha\right)\left(1+r\beta S_X-r\beta\hat{S}_Y^{B*}\right)}{2r\beta\left(4\hat{S}_Y^{B*}-S_X\right)^2}\right]\times$$

$$\left[2\hat{S}_Y^{B*}\left(1+\alpha-r\beta\hat{S}_Y^{B*}\left(1+3\alpha\right)\right)-S_X\left(1+\alpha-r\beta\hat{S}_Y^{B*}\left(3+7\alpha\right)\right)-r\beta\left(1+\alpha\right)S_X^2\right]$$

$$\hat{\pi}_{Yd}^{B*} = \left[\frac{\left(2\left(1+\alpha\right)\hat{S}_Y^{B*}-8r\beta\alpha\left(\hat{S}_Y^{B*}\right)^2-\left(1+\alpha-\left(1+9\alpha\right)r\beta\hat{S}_Y^{B*}\right)S_X-r\beta\left(1+\alpha\right)S_X^2\right)}{4r^2\beta^2\left(\hat{S}_Y^{B*}-S_X\right)\left(4\hat{S}_Y^{B*}-S_X\right)^2}\right]\times$$

$$\left(1+r\beta S_X-r\beta\hat{S}_Y^{B*}\right)\left[-2\left(1+\alpha\right)\hat{S}_Y^{B*}+8r\beta\left(\hat{S}_Y^{B*}\right)^2+\left(1+\alpha-\left(9+\alpha\right)r\beta\hat{S}_Y^{B*}\right)S_X+r\beta\left(1+\alpha\right)S_X^2\right]$$

Appendix B. Proofs.

Proof of Proposition 1A:

In order to find the equilibrium of the game between firms X and Y, we adopt backward induction and start with the equilibrium of the duopoly game for a given p_{Xm}^{A} and S_X. The duopoly profits for firms X and Y are

$$\pi_{Xd}^{A}=\left(1-r\beta S_{Y}^{A}\right)\left(p_{Xd}^{A}-\alpha S_{X}\right)\left(\frac{p_{Xm}^{A}-p_{Xd}^{A}\left(1-r\beta S_{Y}^{A}\right)}{r\beta S_{X}S_{Y}^{A}}-\frac{p_{Xd}^{A}}{S_{X}}\right)M,$$

$$\pi_{Yd}^{A}=\left(1-r\beta S_{Y}^{A}\right)\left(p_{Yd}^{A}-\alpha S_{Y}^{A}\right)\left(1-\frac{p_{Xd}^{A}\left(1-r\beta S_{Y}^{A}\right)-p_{Xm}^{A}}{S_{Y}^{A}\left(1-r\beta S_{Y}^{A}\right)-S_{X}}\right)M$$

Finding the equilibrium solution of the duopoly game is non-trivial. Therefore, we resort to the following change of variables. Let the adjusted duopoly prices $\overline{p}_{Xd}^{A},\overline{p}_{Yd}^{A}$ be

$$\overline{p}_{Xd}^{A}=\frac{p_{Xd}^{A}}{S_{X}},\ \overline{p}_{Yd}^{A}=\frac{p_{Xd}^{A}\left(1-r\beta S_{Y}^{A}\right)-p_{Xm}^{A}}{S_{Y}^{A}\left(1-r\beta S_{Y}^{A}\right)-S_{X}}.$$

Note that for a given S_X,S_Y^A,p_{Xm}^A, the mapping between p_{Xd}^A,p_{Yd}^A and $\overline{p}_{Xd}^{A},\overline{p}_{Yd}^{A}$ is unique. The duopoly profits using this change of variables are

$$\pi_{Xd}^{A}=\frac{\left(1-r\beta S_{Y}^{A}\right)\left(\overline{p}_{Xd}^{A}-\alpha\right)\left(p_{Xm}^{A}-\overline{p}_{Xd}^{A}S_{X}\right)}{r\beta S_{Y}^{A}}M,$$

$$\pi_{Yd}^{A}=\left(\overline{p}_{Yd}^{A}\left(S_{Y}^{A}\left(1-r\beta S_{Y}^{A}\right)-S_{X}\right)-\alpha S_{Y}^{A}\left(1-r\beta S_{Y}^{A}\right)+p_{Xm}^{A}\right)\left(1-\overline{p}_{Yd}^{A}\right)M$$

Therefore, the equilibrium of the duopoly game between the two firms is characterized by the solution of the following system of equations.

$$\begin{cases} \dfrac{\partial \pi_{Xd}^{A}}{\partial \overline{p}_{Xd}^{A}} = \dfrac{p_{Xm}^{A} + \left(\alpha - 2\overline{p}_{Xd}^{A}\right) S_X}{r\beta S_Y^{A}} M \\ \dfrac{\partial \pi_{Yd}^{A}}{\partial \overline{p}_{Yd}^{A}} = \left[-2\left(S_Y^{A}\left(1 - r\beta S_Y^{A}\right) - S_X\right)\overline{p}_{Yd}^{A} + \left(1+\alpha\right) S_Y^{A}\left(1 - r\beta S_Y^{A}\right) - p_{Xm}^{A} - S_X\right] M \\ \dfrac{\partial \pi_{Yd}^{A}}{\partial S_Y^{A}} = \left(1 - 2r\beta S_Y^{A}\right)\left(\overline{p}_{Yd}^{A} - \alpha\right)\left(1 - \overline{p}_{Yd}^{A}\right) M \end{cases}$$

The FOC for S_Y^A indicates that either one of the following relationships should hold:

$\overline{p}_{Yd}^{A} = 1$ or $\overline{p}_{Yd}^{A} = \alpha$, or $2r\beta S_Y^A = 1$. Note that only the third case is a valid equilibrium solution, as in the first case firm Y makes no duopoly profit and in the second case firm X makes no duopoly profit. Therefore, the optimal design level for firm Y is always

$S_Y^{A*} = \dfrac{1}{2r\beta}$. *QED*

Proof of Proposition 1B:

When there is no monopoly market for firm X, the discounted (duopoly) profits for firms X and Y are

$$\pi_{Xd}^{B} = \left(1 - r\beta S_Y^{B}\right)\left(p_{Xd}^{B} - \alpha S_X\right)\left(\frac{p_{Yd}^{B} - p_{Xd}^{B}}{S_Y^{B} - S_X} - \frac{p_{Xd}^{B}}{S_X}\right) M, \quad \pi_{Yd}^{B} = \left(1 - r\beta S_Y^{B}\right)\left(p_{Yd}^{B} - \alpha S_Y^{B}\right)\left(1 - \frac{p_{Yd}^{B} - p_{Xd}^{B}}{S_Y^{B} - S_X}\right) M$$

Therefore, the equilibrium of the duopoly game between the two firms is characterized by the solution of the following system of equations.

$$\begin{cases} \dfrac{\partial \pi_{Xd}^B}{\partial p_{Xd}^B} = \dfrac{\left(p_{Yd}^B + \alpha S_Y\right) S_X - 2 p_{Xd}^B S_Y^B}{\left(S_Y^B - S_X\right) S_X} M \\ \dfrac{\partial \pi_{Yd}^B}{\partial p_{Yd}^B} = \dfrac{\left(1 - r\beta S_Y^B\right)\left(S_Y^B - S_X - 2 p_{Yd}^B + p_{Xd}^B + \alpha S_Y^B\right)}{S_Y^B - S_X} M \\ \dfrac{\partial \pi_{Yd}^B}{\partial S_Y^B} = \dfrac{\left[S_X\left(-r\beta p_{Yd}^B + 2\alpha r\beta S_Y^B - \alpha\right)\left(\left(S_X - S_Y^B\right)^2 + S_X\left(p_{Yd}^B - p_{Xd}^B\right)\right) + \left(p_{Yd}^B - p_{Xd}^B\right)\left(p_{Yd}^B - \alpha r\beta\left(S_Y^B\right)^2\right)\right]}{\left(S_Y^B - S_X\right)^2} M \end{cases}$$

Solving the first two equations as a function of S_Y^B gives us

$$p_{Xd}^B = \frac{S_X\left(S_Y^B - S_X + 3\alpha S_Y^B\right)}{4S_Y^B - S_X} \quad , \quad p_{Yd}^B = \frac{S_Y^B\left(2\alpha S_Y^B + \alpha S_X - 2S_X + 2S_Y^B\right)}{4S_Y^B - S_X}$$

Plugging the optimal prices into the third equation above leads to

$$\frac{d\pi_{Yd}^B}{dS_Y^B} = \frac{-2S_Y^B(1-\alpha)^2\left(S_X - S_Y^B\right)^2\left(4r\beta\left(S_Y^B\right)^2 - (2 + 3r\beta S_X)S_Y^B + S_X\right)}{\left(4S_Y^B - S_X\right)^2} M$$

Therefore, the optimal S_Y^B is the solution of the following equation.

$$4r\beta\left(S_Y^B\right)^2 - (2 + 3r\beta S_X)S_Y^B + S_X = 0,$$

or

$$S_Y^{B*} = \frac{2 + 3r\beta S_X \pm \sqrt{4 - 4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta} \; .$$

It remains to show that the smaller root, $S_Y^B = \dfrac{2 + 3r\beta S_X - \sqrt{4 - 4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta}$, is not feasible for the range of parameters considered, or $S_Y^B\left(1 - r\beta S_Y^B\right) \le S_X$ for $S_Y^B = \dfrac{2 + 3r\beta S_X - \sqrt{4 - 4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta}$:

$$S_Y^B\left(1-r\beta S_Y^B\right)-S_X=\frac{2+3r\beta S_X-\sqrt{4-4r\beta S_X+\left(3r\beta S_X\right)^2}}{8r\beta}\left(1-\frac{2+3r\beta S_X-\sqrt{4-4r\beta S_X+\left(3r\beta S_X\right)^2}}{8}\right)S_X$$

$$=\frac{-\left(2-3r\beta S_X\right)\sqrt{4-4r\beta S_X+\left(3r\beta S_X\right)^2}+4-24r\beta S_X-\left(3r\beta S_X\right)^2}{32r\beta}$$

$$=\frac{-T_1+T_2}{32r\beta}$$

where $T_1=\left(2-3r\beta S_X\right)\sqrt{4-4r\beta S_X+\left(3r\beta S_X\right)^2}$, $T_2=4-24r\beta S_X-\left(3r\beta S_X\right)^2$. Note that $T_1\geq 0$ (as $4r\beta S_X<1$ so there exists a feasible S_Y for $S_Y\left(1-r\beta S_Y\right)>S_X$). For $r\beta S_X\geq(\sqrt{20}-4)/3$ then $T_2\leq 0$ and the feasibility condition is violated. Suppose that $r\beta S_X<(\sqrt{20}-4)/3$ and $T_2>0$, then $T_1^2-T_2^2=64r\beta S_X\left(2-6r\beta S_X-9\left(r\beta S_X\right)^2\right)$, which is positive for all $r\beta S_X<(\sqrt{20}-4)/3$. Therefore $T_1>T_2$, and the feasibility condition is again violated. Consequently the only solution for the optimal design level for firm Y is $S_Y^{B*}=\dfrac{2+3r\beta S_X+\sqrt{4-4r\beta S_X+\left(3r\beta S_X\right)^2}}{8r\beta}$. *QED*

Proof of Proposition 2A:

Similar to the proof of Proposition 1A, we adopt the following change of variables in order to find the equilibrium solution of the duopoly game. Let the adjusted duopoly prices $\tilde{p}_{Xd}^A,\tilde{p}_{Yd}^A$ be

$$\tilde{p}_{Xd}^A=\frac{\hat{p}_{Xd}}{S_X},\ \tilde{p}_{Yd}^A=\frac{\hat{p}_{Xd}\left(1-r\beta\hat{S}_Y\right)-\hat{p}_{Xm}}{\hat{S}_Y\left(1-r\beta\hat{S}_Y\right)-S_X}.$$

Note that for a given S_X,S_Y^A,p_{Xm}^A, the mapping between p_{Xd}^A,p_{Yd}^A and $\tilde{p}_{Xd}^A,\tilde{p}_{Yd}^A$ is unique. The duopoly profits using this change of variables are

$$\hat{\pi}_{Xd}^A=\frac{\left(1-r\beta\left(\hat{S}_Y-S_X\right)\right)\left(\tilde{p}_{Xd}-\alpha\right)\left(\hat{p}_{Xm}-\tilde{p}_{Xd}S_X\right)}{r\beta\hat{S}_Y}M,$$

$$\hat{\pi}_{Yd}^A=\left(\tilde{p}_{Yd}^A\left(\hat{S}_Y\left(1-r\beta\left(\hat{S}_Y-S_X\right)\right)-S_X\right)-\alpha\hat{S}_Y\left(1-r\beta\left(\hat{S}_Y-S_X\right)\right)+\hat{p}_{Xm}\right)\left(1-\tilde{p}_{Yd}\right)M$$

Therefore, the equilibrium of the duopoly game between the two firms is characterized by the solution of the following system of equations.

$$\begin{cases} \dfrac{\partial \hat{\pi}_{Xd}^{A}}{\partial \tilde{p}_{Xd}^{A}} = \dfrac{\hat{p}_{Xm}^{A} + \left(\alpha - 2\tilde{p}_{Xd}^{A}\right) S_X}{r\beta\left(\hat{S}_Y^A - S_X\right)} M \\ \dfrac{\partial \hat{\pi}_{Yd}^{A}}{\partial \tilde{p}_{Yd}^{A}} = \left[-2\left(\hat{S}_Y^A\left(1 - r\beta\left(\hat{S}_Y^A - S_X\right)\right) - S_X\right)\tilde{p}_{Yd}^{A} + \left(1+\alpha\right)\hat{S}_Y^A\left(1 - r\beta\left(\hat{S}_Y^A - S_X\right)\right) - \hat{p}_{Xm}^{A} - S_X\right] M \\ \dfrac{\partial \hat{\pi}_{Yd}}{\partial \hat{S}_Y^A} = \left(1 - 2r\beta\hat{S}_Y^A + r\beta S_X\right)\left(\tilde{p}_{Yd}^{A} - \alpha\right)\left(1 - \tilde{p}_{Yd}^{A}\right) M \end{cases}$$

The FOC for S_Y^A indicates that either one of the following relationships should hold:

$\tilde{p}_{Yd}^{A} = 1$ or $\tilde{p}_{Yd}^{A} = \alpha$, or $2r\beta S_Y^A = 1 + r\beta S_X$. Note that only the third case is a valid equilibrium solution, as in the first case firm Y makes no duopoly profit and in the second case firm X makes no duopoly profit. Therefore, the optimal design level for firm Y is always

$\hat{S}_Y^{A*} = \dfrac{1 + r\beta S_X}{2r\beta}$. *QED*

Proof of Proposition 2B:

When there is no monopoly market for firm X, the discounted (duopoly) profits for firms X and Y are

$$\hat{\pi}_{Xd}^{B} = \left(1 - r\beta\hat{S}_Y^B\right)\left(\hat{p}_{Xd}^{B} - \alpha S_X\right)\left(\frac{\hat{p}_{Yd}^{B} - \hat{p}_{Xd}^{B}}{\hat{S}_Y^B - S_X} - \frac{p_{Xd}^{B}}{S_X}\right) M,\ \hat{\pi}_{Yd}^{B} = \left(1 - r\beta\hat{S}_Y^B\right)\left(\hat{p}_{Yd}^{B} - \alpha\hat{S}_Y^B\right)\left(1 - \frac{\hat{p}_{Yd}^{B} - \hat{p}_{Xd}^{B}}{\hat{S}_Y^B - S_X}\right) M$$

Therefore, the equilibrium of the duopoly game between the two firms is characterized by the solution of the following system of equations.

$$\begin{cases} \dfrac{\partial \hat{\pi}_{Xd}^{B}}{\partial \hat{p}_{Xd}^{B}} = \dfrac{\left(\hat{p}_{Yd}^{B} + \alpha\hat{S}_Y\right) S_X - 2\hat{p}_{Xd}^{B}\hat{S}_Y^B}{\left(\hat{S}_Y^B - S_X\right) S_X} M \\ \dfrac{\partial \hat{\pi}_{Yd}^{B}}{\partial \hat{p}_{Yd}^{B}} = \dfrac{\left(1 - r\beta\hat{S}_Y^B\right)\left(\hat{S}_Y^B - S_X - 2\hat{p}_{Yd}^{B} + \hat{p}_{Xd}^{B} + \alpha\hat{S}_Y^B\right)}{\hat{S}_Y^B - S_X} M \end{cases}$$

Solving the two price equations as a function of S_Y^B gives us

$$\hat{p}_{Xd}^B = \frac{S_X\left(\hat{S}_Y^B - S_X + 3\alpha\hat{S}_Y^B\right)}{4\hat{S}_Y^B - S_X} \quad , \quad \hat{p}_{Yd}^B = \frac{\hat{S}_Y^B\left(2\alpha\hat{S}_Y^B + \alpha S_X - 2S_X + 2\hat{S}_Y^B\right)}{4\hat{S}_Y^B - S_X}$$

Plugging the optimal prices into the third equation above leads to

$$\frac{\partial\hat{\pi}_{Yd}^B}{\partial\hat{S}_Y^B} = \frac{-2\hat{S}_Y^B(1-\alpha)^2\left(4r\beta\left(\hat{S}_Y^B\right)^2 - (2+5r\beta S_X)\hat{S}_Y^B + r\beta S_X^2 + S_X\right)}{\left(4\hat{S}_Y^B - S_X\right)^2}M$$

Therefore, the optimal S_Y^B is the solution of the following equation.

$$4r\beta\left(\hat{S}_Y^B\right)^2 - (2+5r\beta S_X)\hat{S}_Y^B + r\beta S_X^2 + S_X = 0 \quad ,$$

or

$$\hat{S}_Y^{B*} = \frac{2+3r\beta S_X \pm\sqrt{4+4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta} \ .$$

It remains to show that the smaller root, $\hat{S}_Y^B = \dfrac{2+5r\beta S_X - \sqrt{4+4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta}$, is not feasible for the range of parameters considered. Note that,

$$\hat{S}_Y^B - S_X = \frac{2+5r\beta S_X - \sqrt{4+4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta} - S_X$$

$$= \frac{\left(2-3r\beta S_X\right) - \sqrt{4+4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta} \quad \leq \quad 0$$

Consequently the only solution for the optimal design level for firm Y is

$$\hat{S}_Y^{B*} = \frac{2+5r\beta S_X + \sqrt{4+4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta} . \quad \text{QED}$$

Proof of Proposition 3:

For all $S_X > 0$, it holds that $S_Y^{B*} > S_Y^{A*}$ and $\hat{S}_Y^{B*} > \hat{S}_Y^{A*}$.

No Pre-announcement: When firm X has positive monopoly sales, we showed that $S_Y^{A*} = \dfrac{1}{2r\beta}$ and $S_Y^{B*} = \dfrac{2+3r\beta S_X + \sqrt{4-4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta}$ when firm X has no monopoly sales. We know that $\sqrt{4-4r\beta S_X + \left(3r\beta S_X\right)^2} > \sqrt{4-12r\beta S_X + \left(3r\beta S_X\right)^2} = \left|2-3r\beta S_X\right|$ for $S_X > 0$. Thus, it follows that

$$S_Y^{B*} = \frac{2+3r\beta S_X + \sqrt{4-4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta} > \frac{2+3r\beta S_X + \left(2-3r\beta S_X\right)}{8r\beta} = \frac{1}{2r\beta} = S_Y^{A*} \quad \text{for all}$$

$S_X > 0$.

Pre-announcement: We showed that $\hat{S}_Y^{A*} = 0.5\left(S_X + \dfrac{1}{r\beta}\right)$ and

$\hat{S}_Y^{B*} = \dfrac{2+5r\beta S_X + \sqrt{4+4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta}$ when firm X has positive and zero monopoly sales, respectively. Since $\hat{S}_Y^{B*} = \dfrac{1}{4r\beta} + \dfrac{5}{8}S_X + \dfrac{\sqrt{4+4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta}$, it holds that $\hat{S}_Y^{B*} > \hat{S}_Y^{A*}$ since $\dfrac{\sqrt{4r\beta S_X + \left(3r\beta S_X\right)^2}}{8r\beta} > 0$ for any $S_X > 0$. *Q.E.D.*