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THE CURRICULUM MATRIX: TRANSCENDENCE AND MATHEMATICS

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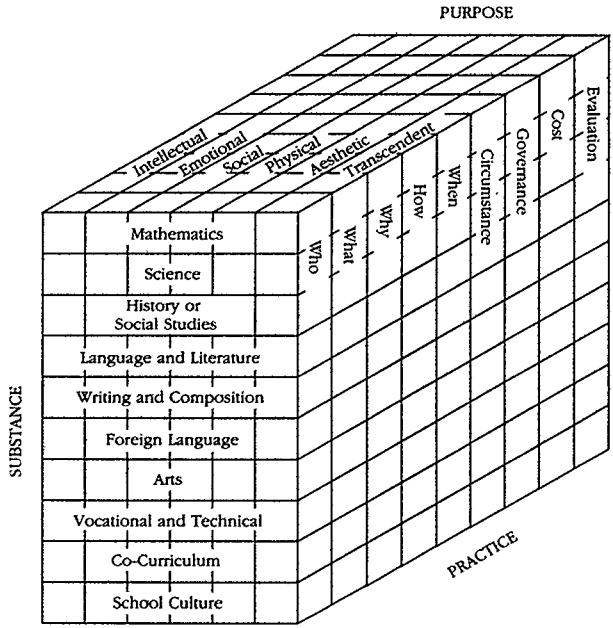
I am about to make a curriculum proposal. However, lest this proposal, like its many predecessors, be seen as yet another case of special pleading, let me put it into a context that at least seeks to be comprehensive. If the context is clear and adequate, it will be evident that this proposal is a segment of a much larger whole, with which it must interact.

Following the suggestion of Kuhn, who points out that any organized field of knowledge exists within a matrix, the parts of which are specified, I have constructed a curriculum matrix.[1] The matrix has three faces, which represent the principal dimensions of the curriculum field: *purpose, substance,* and *practice* (see Figure 1). That is, what we attempt in school seeks to accomplish something (it has a purpose), through the offering of organized experience (school subjects and other aspects of schooling), by means of the practices of teaching and learning. All the faces of the matrix interact, as do all the subparts, or specifications, of the three dimensions. I shall discuss one of the purposes here, leaving substance and practice for consideration on another occasion.

The first dimension of the curriculum field is its purpose, or its aims. The one continuing purpose of education, since ancient times, has been to bring people to as full a realization as possible of what it is to be a human being. Other statements of educational purpose have also been widely accepted: to develop the intellect, to serve social needs, to contribute to the economy, to create an effective work force, to prepare students for a job or career, to promote a particular social or political system. These purposes as offered are undesirably limited in scope, and in some instances they conflict with the broad purpose I have indicated; they imply a distorted human existence. The broader humanistic purpose includes all of them, and goes beyond them, for it seeks to encompass all the dimensions of human experience. What are these

[1]Thomas S. Kuhn, *The Structure of Scientific Revolutions, Volume II, Foundations of the Unity of Science,* 2nd ed., enlarged (Chicago: University of Chicago Press, 1970)

Figure 1. The Curriculum Matrix



dimensions? I think six may be identified, and they are presented in the matrix in the decreasing order of our knowledge about them. the intellectual, the emotional, the social, the physical, the aesthetic, and the transcendent or spiritual. Leave any one of these *wholly* out of an individual's development, and the person is grossly distorted, even destroyed.

The first of these dimensions is the intellectual. We know more about it than we do about any of the others. I need only mention the names of Piaget and Bloom to recall to educators the highly useful and influential work done by them and many others. Since ancient times, this dimension has dominated discussions of education, indeed, many believe that it is the only purpose to be served by organized education. Most of what we do in schools is intended to develop the intellect.

The second of these dimensions is the emotional, which we know much less about than we do the intellectual. However, this is the 20th century, not the 18th. We understand the place of emotions in human behavior, even if we know little about the nature of emotion or emotional development. Nevertheless, most of us would not define emotion the way Johnson defined passion, as a "disturbance of the reason."

The third dimension is the social. All human beings are social; so are many animals. Isolation breeds insanity. Included in this dimension are what is ordinarily called *character*, also morality, since these qualities are shown chiefly in social behavior. *Human relations* has become a popular object of attention in education, as well as in other human endeavors, and a good deal of knowledge has accumulated about it. However, the social development of children remains largely unexplored, despite Kohlberg's work on moral development.

The fourth of these dimensions is the physical. I do not mean the physician's view of the physical, but the psychologist's. Part of self-realization is the consciousness of one's self as a physical being. We continue to pay little attention to this dimension in the curriculum, relegating it to sports and interludes in the school day to "let off steam" and to rather casual instruction in personal health and hygiene.

The fifth, the aesthetic nature of human beings, is recognized by those who deign to attend to it, but we know little about it as educationists. The research literature on aesthetics in education continues to be scarce and generally of mediocre quality.

The sixth and most ignored of all the universal dimensions of human experience is the transcendent or spiritual. At least, it is ignored in education. We do not know what it is, we confuse it with the established institutions of religion; it is not rational in character and is therefore overlooked in educational institutions, where *Déesse Raison* ("The Goddess Reason") rules as much as she did during the 18th century.

To summarize, the curriculum field, as a whole, has three aspects: purpose, substance, and practice. Purpose is stated as the development of six dimensions of human experience, substance as ten aspects of the school experience; practice is specified as having nine subdivisions. All of this is represented as a curriculum matrix, in which, theoretically, all the subdivisions interact. The number of such theoretical interactions is $(6 \times 10 \times 9)^2/2 = 145{,}800$. Some of these are bizarre and would be ignored. The curriculum field is constantly in flux. All kinds of combinations of its components occur all the time. It is not static; it will not hold still. The matrix illustrates the enormous complexity of the field.

One way to test the usefulness of the matrix is to select for study the interaction that seems least likely to yield worthwhile results. I have therefore chosen to examine the relationship between the transcendent or spiritual experience and mathematics. I have been encouraged to make this attempt

because much of educators' response to this set of ideas has centered on the transcendent or spiritual dimension. What is it? Does it not threaten the intellectual purpose of education? I had no way to deal with these questions, though I realized they were related to experiences in my own life that had left me puzzled.

One such experience took place when I was with a busload of sightseers in Rome. We stopped at an elegant, small church to admire the building. At one side of the altar, and a bit to the rear of it, was Michelangelo's *Moses*. I was transfixed. The statue suddenly became my entire universe. I lost all sense of time and place, I was utterly absorbed in the wonder of the statue. Moses looked in rage at his people, the tablets fell from his hands. He was experiencing the shock of human frailty, having just come from God. For me, the experience transcended the place, the craft of the sculptor, the fact that, after all, I was looking at stone.

I don't know how long I stood there. When I "came to," the touring group had left and I was alone. The majesty, the awesome presence, the intensity of that moment, have remained with me to this day.

Now, this encounter of mine corresponds rather closely with what the psychologist Maslow calls the "peak experience." However, aside from a short study by Bloom and some further writing by Maslow and a few of his followers, I have not found psychologists pursuing it.[2] A recent book on intuition by Noddings and Shore comes close, but it does not deal with some important aspects of the experience as I have come to understand it.[3] The studies of teaching by Gage and his students, including Rosenshine, do not deal with it.[4] Bloom does not see the experience as directly linked with the curriculum, but only with what he calls "conditions."[5] Kohlberg speculates about children's notions of the transcendent, but he does not deal with the experience itself.[6]

I finally found an extensive literature, under the term *spiritual*, written, of course, by theologians—whom the social and behavioral scientists have wholly ignored. What, according to theologians, is the nature of this experience? What does it signify? What is its source? I received some help from acquaintances who are theologians and began a fascinating excursion into

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[2]Benjamin H. Bloom, "Peak Learning Experience," in *All Our Children Learning*, ed. Benjamin H. Bloom (New York. McGraw-Hill, 1981), pp. 193–199; Abraham H. Maslow, *The Farther Reaches of Human Nature* (New York. Viking-Penguin, 1971), Abraham H. Maslow, *Toward a Psychology of Being*, 2nd ed. (New York: Van Nostrand, 1968).

[3]Nel Noddings and Paul J. Shore, *Awakening the Inner Eye. Intuition in Education* (New York: Teachers College Press, 1984).

[4]Nathaniel L. Gage, ed., *Handbook of Research on Teaching* (Chicago: Rand McNally, 1963).

[5]Benjamin H. Bloom, "Peak Learning Experience," in *All Our Children Learning*, ed. Benjamin H. Bloom (New York: McGraw-Hill, 1981), pp. 193–199.

[6]Lawrence Kohlberg, *The Philosophy of Moral Development, Volume 1. Essays on Moral Development* (San Francisco: Harper & Row, 1981).

their discipline.[7] I read this literature with my mind on just one thing: What is the transcendent or spiritual experience?

In essence, the theologians had found what Maslow and Bloom have reported, found it earlier, and found much more. According to them, the experience is universal and ancient. Mankind has always been overwhelmed by the wonder of the life process—birth, growth, death. Primitive societies have marveled at the enormous, often hostile, forces of nature and the mysteries of the sky above.

People have responded to this kind of awareness in many ways: through ritual, supplication, religion, art objects, the emphasis and subordination of habits and feelings (such as celebrations and taboos), often in search of ecstasy—transforming experiences. The literature of the world's great religions is full of testimony to this fact, and it has found repeated expression in secular literature.

The idea of the transcendent or spiritual begins with awe—fear. Nature is viewed as essentially hostile, as in Beowulf's encounter with Grendel. In both Norse and Greek myth, a capricious, hostile nature was personified as gods and spirits such as Thor, Apollo, Vulcan, and Loki, to be feared and placated. Otto calls this fear "daemonic dread."[8]

The fear was expressed in aesthetic statements in every form, from idols and edifices to drama and music, as the great religious traditions took shape. What had begun as fear became almost overwhelming wonder, astonishment, even amazement, at the nature of things. It came to influence mankind's sense of existence.

Existence was viewed as essentially dimensional. Man dwells on Earth; there is a space between Earth and Heaven, where dwell the demons; there is Heaven, which is beyond comprehension, where dwell the gods. Early man saw the general in the particular—all of terror in a lion—much as Blake and Wordsworth did in their time—the universe in a grain of sand or in a daffodil. As Smith says, "The terrestrial plane proceeds from and is explained by the intermediate, the intermediate by the celestial, and the celestial by the infinite."[9]

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[7]Rudolph Otto, *The Idea of the Holy* (London. Oxford University Press, 1923); Martin Buber, *I and Thou*, trans. Walter Kaufman (New York: Scribner, 1970); Paul Tillich, *Theology of Culture* (London: Oxford University Press, 1959); Philip H. Phenix, "Transcendence and the Curriculum," in *Conflicting Conceptions of Curriculum*, ed. Elliot W. Eisner and Elizabeth Vallance (Berkeley, CA: McCutchan, 1974), chap. 7; Philip Phenix, *Realms of Meaning* (New York: McGraw-Hill, 1964); Huston Smith, *Forgotten Truth* (New York: Harper, 1977); Huston Smith, *Beyond the Post-Modern* Mind (New York: Crossroad, 1982); Evelyn Underhill, *Worship* (New York: Crossroad, 1982); Arthur W. Foshay, "Intuition and Curriculum," in *Beyond the Scientific: A Comprehenstve View of Consciousness*, ed. Arthur W. Foshay and Irving Morrissett (Boulder, CO: Social Science Education Consortium, 1978), chap. 4.

[8]Rudolph Otto, *The Idea of the Holy* (London: Oxford University Press, 1923).

[9]Huston Smith, *Forgotten Truth* (New York: Harper, 1977), p. 42.

The dimensional, spherical shape of existence was made manifest in many ways. Marshall McLuhan once pointed out to me that the cathedral contained for medieval man a spherical experience—when he was inside the building, all the most significant aspects of his world came at him from all directions, simultaneously. The Earth-centered cosmology of that time was also spherical. The Earth was at the center of a series of spheres, the lesser contained in the greater, from which the lesser derived its meaning. The whole system was believed to be in perfect harmony.

I discern two basic elements of the transcendent or spiritual experience from this account. the experience of dread or awe or fearfulness (later, wonder) and the experience of connectedness with something greater than what immediately appears. The latter is ordinarily called *transcendence*, which is the term I have adopted for this experience.

Other terms have gathered around the idea. Otto calls it the "wholly other," Tillich the "ground of being" or the "unconditioned," in which one deals with matters of "ultimate concern." "Ritual," according to Jones, "derives its energy and focus from the transcendent present in the individual or community consciousness or belief."[10] Phenix says, "the term *transcendent* refers to the experience of limitless going beyond any given state or realization of being." He points to "cognate" terms. *spirit, infinitude, idealization.* He continues by indicating how the idea of transcendence corresponds with Dewey's notion of the continuous progressive reconstruction of experience.

Bloom conducted a study interviewing 80 graduate students and found a few who claimed to have had a peak experience in the classroom. First, these students reported a sense of total involvement, analogous to my experience with the Michelangelo. Second, the students reported a loss of a sense of time and place; "the learning experience became the figure, while the ground . . . disappeared." Third, there was a feeling that the experience was immediately true in a fundamental way and that "organizing, analytic, and application types of thinking were temporarily suspended."[11] Fourth, Bloom's students described the experience as "awe-inspiring" and "wonderful."

These students' accounts correspond with what the theologians say. The experience is *awesome*, it *goes beyond* ordinary reason, it is *fundamental*—a moment of immediate truth—or, as Phenix and many others would have it, *transcendent.*

The theologians claim that the transcendent experience is latent in everyone and that it can be awakened in us. It is a sudden awareness of the

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[10]Paul D. Jones, *Rediscovering Ritual* (New York: Newman Press, 1973)

[11]Benjamin H. Bloom, "Peak Learning Experience," in *All Our Children Learning*, ed. Benjamin H. Bloom (New York: McGraw-Hill, 1981), p. 195.

connection between what is immediately apparent and a vastly larger sphere of being. This awareness may be evoked—called out.

This flash of awareness corresponds with Maslow's reports about his clients. He calls it "an illumination, a revelation, an insight" that leads to "the cognition of being."[12] Unlike the theologians, Maslow distinguishes between the people he calls "self-actualizing" (i.e., fully mature and internally organized) and others. The self-actualized have frequent peak experiences; others have them rarely or not at all.

These sudden awakenings are sometimes seen in the theological tradition as mystic visions. Mystics have reported them since the earliest times. I conclude that mysticism arises from a self-immolating encounter with ultimate reality—not unlike the experience of one who climbs out of Plato's cave, sees the source of all light, and is temporarily blinded. It is expressed nonrationally in poetry, as in Blake's "Tyger," which not only expresses the awe and wonder of the experience but also seeks to awaken it in us.

The idea of the transcendent or spiritual thus seems to have a scope, from daemonic dread or awe, at one extreme, to mystic expression at the other. The term includes this entire range of meanings in our time. It has shed none. The only new term associated with it is Otto's *numinous*, which "can only be incited, induced, and aroused"[13] or, in Maslow's language, "triggered."

Neither the theologians nor Maslow attempt to develop a *concept* of the transcendent or spiritual or peak experience, but only to assert its reality. The idea seems to exist in a *cloud* of meaning, not as a concept. It is a "state of mind," "purely felt experience, only to be indicated symbolically, by 'ideograms,'" says Otto.[14] In this sense, it is nonrational, somewhat as music and sculpture are nonrational. The idea goes beyond reason, as Bloom's students implied.

So, what does one do to grasp a cloud? To use reason to examine the nonrational? One way to proceed is to examine the language people use to talk about it. I have done so and have found that these accounts of the transcendent or spiritual or peak experience fall into categories: (1) the intellectual (i.e., the experience goes beyond reason), (2) the transcendent, or "going beyond"; (3) the mystic. In addition, there are two other categories: (A) those that describe the impact of the experience as a whole and (B) those that describe the impact of the experience in more particular, or detailed, terms. I have grouped this language into these five categories. Here is a sampling of the language so grouped, with the total number of times I found the terms (many occur in several different accounts) in each category.

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[12]Abraham H. Maslow, *Toward a Psychology of Being*, 2nd ed. (New York: Van Nostrand, 1968), p. 169.

[13]Rudolph Otto, *The Idea of the Holy* (London: Oxford University Press, 1923), p. 60.

[14]Ibid., p. 59.

1. The intellectual (17 terms)
   - nonrational
   - stupefying
   - ultimate meaning
   - unimaginable
   - boundless
2. The transcendent, or "going beyond" (15 terms)
   - transcendent
   - being unlimited
   - tropism of the soul toward the universe
   - fundamental
   - fusion of inner and outer being
3. The mystic (6 terms)
   - mystic vision
   - sublime
   - reverence
   - exaltation

A. The impact of the experience as a whole (45 terms)
   - awesome
   - singular awareness
   - timeless, spaceless
   - overpowering
   - nonvoluntary
   - astonishing

B. The impact of the more detailed qualities of the experience (18 terms)
   - inner illumination
   - wonder
   - amazing
   - of great moment
   - loss of fear

As Kaufman says of Buber, "such writers seem less concerned with precise denotation than with rich connotations and associations."[15]

Of these categories, the first two—the intellectual and the transcendent—appear essentially cognitive in character. It is as if the transcendent experience begins in the mind. But it does not. It begins as a kind of assault on one's sense of existence. The assault is in some part intellectual, but in some part an assault on pure reason, in which, remember, "organizing, analytic application types

[15]Martin Buber, *I and Thou*, trans. Walter Kaufman (New York. Scribner, 1970), p. 140.

of thinking are temporarily suspended."[16] The experience is self-contained. It is not instrumental to anything else.

The aspect of human experience we examine here appears to defy objective analysis, if we are to believe those who have given it most thought. How, then, shall we approach it? Perhaps by adopting the methods of inquiry used by the theologians and by the clinician Maslow.

The method takes as its domain the *testimony* of those who have undergone the experience. Theologians gather this evidence from the history, literature, and artifacts of the principal world religions. Maslow has listened to the accounts of his "self-actualized" clients. Bloom and his colleagues have interviewed 80 students.

The truth-claiming of these scholars rests on verification by the reader. The theologians seem to say the following:

- Here is the testimony of the ages—that people have spiritual, transcendent experiences.
- Look in on yourself. Do you not resonate to these accounts?
- Here is a theory of the origin and significance of the experience as it bears on some classic theological questions.

Maslow and Bloom proceed in essentially the same way. They gather testimony from clients and students and leave it to us to find parallels in their own lives. They then apply what they have found to their real-world preoccupation with education.

The theologians I have consulted are unanimous in rejecting the classic scientific methods used in the social sciences, judging the methods to be inapplicable to their concerns. Even Maslow was moved to remark that his interest in the peak experience was "strange for a scientist" (he was at one time president of the American Psychological Association). Telling of a conversation with a scientist while he was teaching at the Massachusetts Institute of Technology, Smith writes:

> As so often happened in such circumstances, the conversation turned to the difference between science and the humanities. We were getting nowhere when suddenly he broke in on something I was saying with the authority of a man who has discovered Truth. "I have it!" he exclaimed. "The difference between us is that I count, and you don't."[17]

Concealed in the scientist's double entendre is a complete lack of understanding of what people like Smith do.

There is mounting discussion of the methods of inquiry appropriate to the social and behavioral sciences. Wilson questions the usefulness of statis-

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[16]Benjamin H. Bloom, "Peak Learning Experience," in *All Our Children Learning*, ed. Benjamin H. Bloom (New York: McGraw-Hill, 1981), p. 195.

[17]Huston Smith, *Forgotten Truth* (New York: Harper, 1977), p. 11.

tical methods using aggregate data to deal with "hard-to-measure factors that operate deep within a complex social structure."[18] Some contemporary mathematicians insist that we recognize the limits of mathematical analysis. To these may be added some theologians, such as Buber, who claims that the "It-world" (of science) fails to admit his "you" or "thou," or Tillich, who insists that different levels of reality require different approaches. Smith, who has devoted much attention to this matter, suggests a strategy:

> In place of the usual tendency to begin with the accepted world and add to it only what collected evidence requires, I am asking if it would harm us to conjure the most interesting world we can and then drop from it what reason erases. There is some resemblance to Anselm's *credo ut intelligam*—"I believe in order to understand" [or better,] "I get involved in order to understand."[19]

What, then, is left for those preoccupied with curriculum and teaching? I suggest great teachers find what is captivating or transcendent about their disciplines and teach from these insights to awaken the transcendent experience in their students. (This point is strongly implied by the accounts in Epstein's *Masters*.[20]) I propose, with Bloom, that we extend our inquiry into the possibility that the experience of transcendence can take place in school.

My own current efforts go in this direction. Taking the categories that appear in my examination of the language used by those who have given this experience their attention, I ask what aspects of the school offering, or the curriculum, might be (in relation to the possibility of the experience of transcendence) intellectual, transcendent, and mystic. I ask also in what ways the subject matter we offer corresponds with the transcendent experience as a whole, and in particular. In so doing, I depart somewhat from Bloom and the others. My curriculum proposal is that we seek opportunities for the experience of transcendence in the subject matter we teach. I ask that we try to find the experience as a property of the subject matter itself.

To see where this effort would lead (and to test the usefulness of the curriculum matrix), I have taken the least understood dimension of the purpose of education—the transcendent or spiritual—and the least likely candidate in the school offering—mathematics—and examined how one might appear in the other.

MATHEMATICS

Mathematics is, of course, a field fundamental to our age. The idea that it has transcendent value seems strange, since most of us have thought of math as consisting of a number of memorized paradigms. To solve a math problem,

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[18]James Q. Wilson, "Thinking about Crime," *Atlantic Monthly* (September 1983). 75.
[19]Huston Smith, *Beyond the Post-Modern Mind* (New York. Crossroad, 1982), p. 151.
[20]Joseph Epstein, ed., *Masters. Portraits of Great Teachers* (New York. Basic Books, 1981).

one need only to match the paradigm with the problem. This may lead to an "aha!" experience or a gestalt, but scarcely to transcendence.

In venturing to examine the possibility of a transcendent experience, we must remember, first, that such experience is a private affair. As teachers, we cannot know whether it has taken place unless a student tells us it has. What we can do, therefore, is limited to providing opportunities for it. We cannot ourselves bring it about directly.

Where can we find such opportunities in mathematics? Recent literature on mathematics as a humanity is helpful. From this literature, and from the history of the field (of which this recent literature makes full use), some opportunities arise.[21]

First, let us consider the nature of the field. According to most mathematicians, mathematics is rigorous thinking. That is what it is, and that is all it is. In offering this impression of their discipline, they are, indeed, true to its history since Euclid. They are also responsible for the widespread phenomenon they call "math anxiety," for the rigorous thinking of mathematics is full of failure for a great many learners.

An emerging view of the field says that it consists of a series of remarkable feats of human imagination—leaps in understanding that make it possible to go beyond what is immediately perceived into a long series of transforming theories about reality—in other words, to *transcend* immediate reality, to *go beyond* common sense. The discoveries by mathematicians since ancient times are all of this kind.

The main thing to realize about mathematics is that it is profoundly *human*. If it is marvelous, and of course it is, it is because human beings are marvelous. It follows that mathematics can awaken within students an awareness of their own humanity, especially that aspect of it involving imagination. According to Klein:

> The achievements of mathematics demonstrate the capacity of the human mind, and this exhibition of what human reason can accomplish has given man the courage and confidence to tackle additional seemingly impenetrable mysteries of the cosmos, to seek cures for fatal diseases, and to question and improve the economic and political systems under which people live.[22]

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[21] A. D. Alexsandrov, A. N. Kolmogrow, and M. A. Lavrent'ev, *Mathematics: Its Content, Methods, and Meaning, Part I, A General View of Mathematics Analysis*, trans. S H Gould (Providence, RI. American Mathematical Society, 1962); David Bergamini and the Editors of *Life, Mathematics* (New York. Time Incorporated, 1963), Howard Eves, *An Introduction to the History of Mathematics*, 5th ed. (New York. CBS College Publishers, Saunders College Publishing, 1983); Martin Gardner, *Aha! Gotcha. Paradoxes to Puzzle and Delight* (San Francisco: W H Freeman, 1982), Felix Kaufman, *The Infinite in Mathematics and Its Elimination* (Dordrecht, Holland: D Reidel, 1978); Cassius J. Keyser, *The Human Worth of Rigorous Thinking*, 2nd ed (New York: Columbia University Press, 1925); Walter Popp, *History of Mathematics. Topics for Schools*, trans Maxim Bruckheimer (London: Transworld, 1975).

[22] Morris Klein, ed., *Mathematics. An Introduction to Its Spirit and Use* (San Francisco: W H Freeman, 1978), p. 4.

Mathematics is also a *spiritual* enterprise. To say so is not, of course, to say that it is religious. However, like religion, borrowing Tillich's terms, it does address some ultimate questions. For example, in discussing the concept of infinity, Guillen closes by summarizing where mathematical thought has brought us:

> The physical no longer does contain us wholly, if it ever did. We are beings at once finite and infinite, in the sense that our physical selves are prisoners of a finite realm, but not so our imaginative selves. . . . Now we roam freely beyond the ordinary infinity of the ponderable universe.[23]

In a famous passage, Galileo thought he had found that mathematics is the language of the universe:

> That vast book which stands forever open before our eyes, the universe, cannot be read until we have learned the language and become familiar with the characters in which it is written. It is written in the mathematical language, without which means it is humanly impossible to comprehend a single word.[24]

Small wonder that a popular quip among some mathematicians is "God is a mathematician." In Pope's words, "A mighty maze! But not without a plan."[25] With Klein, mathematicians say, "Our world is to a large extent what mathematicians say it is."[26]

Another characteristic of mathematics is that it is *astonishing*. A former student of mine said to me, "When I first heard of negative numbers at age 12, it blew my mind." When I knew her years later as a doctoral advisee, her continued enthusiasm for mathematics led her to write an outstanding dissertation, on something else, but she "thought like a mathematician."

Or, again. a high school mathematics teacher reported that when she tried to explain the concept of a variable in an equation, a student suddenly exclaimed, "You mean I can put *any* number I want in there? Wow!"

Finally, mathematics is *majestic*, overwhelmingly *powerful*. Shafaravich, one of the world's leading researchers on algebraic geometry, concludes:

> A superficial glance at mathematics may give an impression that it is a result of separate individual efforts of many scientists scattered about in continents and in ages. However, the inner logic of its development reminds one much more of the work of a single intellect, developing its thought systematically and consistently using a variety of human individualities only as a means.[27]

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[23]Michael Guillen, *Bridges to Infinity. The Human Side of Mathematics* (Boston. Houghton Mifflin, 1983).

[24]Quoted in Harold L. Jacobs, *Mathematics. A Human Endeavor* (San Francisco. W H. Freeman, 1970), p. xii.

[25]Alexander Pope, "An Essay on Man, Epistle 1," *The Complete Poetical Works of Alexander Pope* (Boston: Houghton Mifflin, 1903).

[26]Morris Klein, ed., *Mathematics. An Introduction to Its Spirit and Use* (San Francisco: W. H. Freeman, 1978), p. 1.

[27]Quoted in Philip J. Davis and Reuben Hersh, *The Mathematical Experience* (Boston. Houghton Mifflin, 1981), pp. 52–54.

The evidence for this conclusion is that mathematicians have made several discoveries, essentially identical, independently at entirely different times.

It is as if mathematics arose from a primal need to deal with nature. The ideas from which all mathematics descends, it has been speculated, are the ideas of more and less, greater and smaller, one and more than one.

We turn from this consideration of the nature of the field to some ideas within it that have had a transforming effect on the way human beings think. I have identified seven of these. Each of them, it seems to me, is a candidate for awakening the experience of transcendence among students. In each case, these ideas serve as historical milestones that split mathematics history, in the sense that math was one way before each of them, and another afterwards.

1. *Counting and the development of symbols.* Primitive people probably counted things such as their fingers and toes, objects around them, and the like. The first mathematical symbol was probably a tally. Now, enumeration itself was an imaginative feat, but even greater was the feat of inventing a symbol for an object, and later a set. Consider the difference between ///// and the figure V, or 5. Later consider the difference between MCMLXXXIV and 1984. Mankind got beyond the animals, which have common sense and can count a little, when we first invented symbols to stand for a group of objects.

2. *Combining quantities.* It was a great leap of the imagination for ancient man to go beyond enumerating to combining quantities, as in addition and multiplication. For example, it is difficult to combine quantities using Roman numerals. It was even more difficult for the ancient Egyptians, who could only multiply by 2 (notice the similarity to our binary computers), or for the Babylonians, who used base 60 (like our clocks). They had to shed a lot of baggage—for example, that certain numbers have religious properties—before the elegant simplicity of the modern number system became possible

3. *The nature of proof.* Euclid's *Elements* split mathematics history into two parts: pre- and post-Euclid. His contribution was the idea of proof. Having taken part in the legalistic disputations in the Agora at Athens, he proceeded to demonstrate that from a few self-evident axioms, it was possible to construct a whole apparatus—plane geometry. This method of proof dominated mathematics for 2,000 years. There was an inexorable certainty to such logic, all the way from the simplest syllogism to the most elaborate construction. The force of this logic has led to many a transcendent experience for students of mathematics for centuries. With Plato,[28] they have come to believe that "the knowledge at which geometry aims is the knowledge of the eternal."[29]

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[28]Plato, *The Republic*, Book VII, trans. A. D. Lindsay, Everyman Edition (London: J M. Dent, 1935), p. 527-b.

[29]Translation from Robert Ulich, *3,000 Years of Educational Wisdom* (Cambridge, MA: Harvard University Press, 1954), p. 54.

4. *Zero and negative numbers.* The invention of zero, which took place at about the same time in China, India, and by the Mayans, was in Western civilization a part of the invention of Arabic numerals. It spread to Baghdad by A.D. 700, and then very slowly into Europe, not being universal until the end of the 15th century.

Once one has the concept of zero, it immediately becomes inviting to go up from it and down from it, into positive and negative numbers. To imagine zero, we had to get rid of the idea that counting always involves something, or some things. We have to toy with the idea (expounded much later by Cantor and others) that *nothing* is *something.* We must at least think of the distinction (again, made much later) between *nothing* and *nothingness.*

5. *Sets and infinity.* The idea of infinity remained untamed from the Greeks to modern times. It defied satisfactory mathematical definition, retaining vague doctrinal meanings. The idea was tamed during the 19th century by Cantor, who is well known as the inventor of set theory, now widely taught to young children. Not so widely known is his impact on the idea of infinity

Cantor began by considering sets—of numbers, chairs, orchestras—anything. If the elements of a one set could be paired numerically with the elements of another, then the sets were equivalent in size. Notice that this definition of *set* does not require that the populations of the sets be counted. Any known set can be considered a subset of a still larger set, and that of a still larger set, ad infinitum. Now, Cantor made the leap. Since an infinite set is still a set, it could be thought of as a subset of a still more infinite set, and that of yet another, and so on. All the transfinite numbers and all the subsequent sets go beyond infinity as we have thought of it—"ordinary" infinity, as Cantor called it. The sequence of transfinite numbers is as boundless as the cosmos We are faced with an infinity of infinities—a strictly mind-boggling concept. (If it were not mind-boggling, it would still be a subset!) Guillen likens the concept to the idea of infinity as expressed by Kant and St. Gregory, whom he quotes. "No matter how far our mind may have progressed in the contemplation of God, it does not attain to what He is but to what is beneath Him."[30]

Again. we may cut something in half forever without wholly disposing of it. It is like the asymptotic line, which forever slopes toward another line but never touches it, or like an airplane that is forever landing but never touches the ground—or like a society, or a person, forever seeking perfection but never achieving it.

6. *Space and time.* The notion of multiple dimensions, or an infinity of dimensions, is one we can say, but we cannot grasp. One of the dimensions,

[30]Michael Guillen, *Bridges to Infinity: The Human Side to Mathematics* (Boston Houghton Mifflin, 1983), p. 48.

the fourth, is time. Einstein's contributions gave us a new vision of the nature of the universe. It consists of complex surfaces such that moving objects change their size. They are subject to not three dimensions but four, including time. Space, it develops, must be thought of as curved: The geometry of Euclid, which deals with planes, does not describe the universe as it is. Einstein was not the first to challenge Euclid; indeed, he relied on his 19th-century predecessors, who had.

Non-Euclidean geometry is associated with two men: Lobachevsky and Riemann. Others had come upon essentially the same ideas before Lobachevsky—Bolyai and Gauss. Both demonstrated that a geometry could be constructed as robust as Euclid's, but different. There is an alternative to Euclid's logic, which had been the only logic since ancient times. Riemann's geometry conceives of the mathematical "world" as a sphere, in which a straight line is like the arc of a great circle. Straight lines do not stretch to infinity, as in Euclid. The ends of a straight line eventually meet.

These difficult and mind-boggling ideas have transformed not only mathematics but our sense of where we are in the scheme of things. They certainly are candidates for evoking the experience of transcendence.

7. *Uncertainty and faith.* The work of Gödel, which surfaced in 1931 and has withstood the most rigorous examination by the mathematics community, deals with the basic structures of mathematics.[31] It requires the introduction of *uncertainty* into a field that had above all been certain that it was dealing with incontrovertible truth. It defies the belief in pure logic that until then had been the glory of the field. Hilbert, early in this century in Germany, claimed that anything at all could be understood mathematically, given rigor and persistence. (We in education are reminded of the dictum attributed to Thorndike: "Anything that exists, exists in some degree, and can be measured.")

Gödel's genius is that he uses logic to demonstrate the limits of logic. "In any mathematical system rich enough to include number theory," he says, "there is an expression expressible within the system that is true, yet is not provable within the system."[32] The method is deceptively simple. Gödel considers a hypothesis such as this: *Using logic, this hypothesis cannot be proved true.* That hypothesis can be proved either way. Since it can, one must suppose it to be true. Tricky! Guillen calls such statements, of which many can be made, "unprovable verities." But if something is true but cannot be proven, one must accept it on faith. Faith! The word is anathema to formal mathematicians, such as the followers of Hilbert.

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[31]Ernest Nagel and James R. Newman, *Gödel's Proof* (New York. New York University Press, 1958)

[32]Sherman K. Stein, *Mathematics. The Man-Made Universe*, 2nd ed. (San Francisco· W H Freeman, 1969), p. 351.

To this discussion we must add the recent work on chaos, which deals with the strictly unpredictable, such as the weather over a long term or the behavior of a single drop of water in a waterfall. There is an ultimate, but strange, kind of order in such chaos.[33]

THE POSSIBILITY OF THE EXPERIENCE OF TRANSCENDENCE IN MATHEMATICS

These ideas and others of their power suggest that the study of mathematics can become exciting, unforgettable, even transforming, because it pushes us out of our restricted environment into unlimited freedom of thought and imagination. However, there are prerequisites, two of which are becoming apparent. They have to do with the practical questions mentioned early in these remarks.

The first of these prerequisites is *immersion.* Ideas of this sort do not burst on the uniformed. I brought a lifetime of experience in literature and the arts to my encounter with the *Moses.* As Noddings and Shore point out, in mathematics we begin with a puzzle of some kind, then carry on the necessary study and drill to get the tools for dealing with it firmly within our grasp, whereupon the solution may (or may not) come suddenly, we just *might* have the experience of transcendence.[34] Poincaré, the great mathematician of the turn of the century, offers anecdotes of this kind from his own experience You cannot have the experience of transcendence in a vacuum.

The second is *time to think.* We often deny this prerequisite in the classroom. An experiment with "wait time" found that most teachers expect immediate answers to their constant questions, if they would wait as little as five seconds (preferably more), the response would improve in quality dramatically. In my experience with the *Moses,* I was alone, and silent, about it. Poincaré's experience occurred on a lonely walk. One quality of the experience of transcendence is the loss of a sense of time and place—solitude

Now let us return to the curriculum matrix. I have tried here to explore the contents of a block representing the intersection of one school subject—mathematics—and one dimension of human experience—the transcendent I have explored at some length one question of practice—how shall we conceive of the subject matter?—and only slightly another question of practice—how shall the teaching and learning be carried on?

I have tried to indicate how a teacher might conceive of mathematics in a way that might invite the experience of transcendence in some students. In the degree that we teach from this point of view, I believe, we will be emulating

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[33]James Glick, *Chaos. Making a New Science* (New York: Penguin, 1987).

[34]Nel Noddings and Paul J. Shore, *Awakening the Inner Eye: Intuition in Education* (New York: Teachers College Press, 1984), chaps. 5 and 6.

the greatest teachers, and our students are unlikely to forget what we have offered.

Mathematics as an experience of transcendence would not stand by itself in the offering. It would be more a matter of emphasis than of direct instruction. We would seek to point out the larger implications, to allow time for insight, and to convey our own excitement about the larger meanings of mathematics. If we can talk about these possibilities in an intelligent way, perhaps some teachers may achieve great moments.

I am encouraged in this effort by the recent appearance of some multidisciplinary books, such as Hofstadter's *Gödel, Escher, Bach*, which has been a big hit in the United States, and Boorstin's *The Discoverers*.[35] Others, chiefly French, are equally integrative. Maybe the times favor this kind of work. I hope so.[36]

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Ellsworth, Elizabeth, and Mariamne H. Whatley, eds. *The Ideology of Images in Educational Media: Hidden Curriculum in the Classroom.* New York: Teachers College Press, 1990. 150 pp. $31.95/$15.95.

Seven scholars show how educational films, videos, and photographs convey sexist, elitist, racist, and other oppressive meanings via their ostensibly neutral media conventions. The contributors offer ways to counter this historical linkage of content with privilege and power in curricular materials.

[35]Douglas R. Hofstadter, *Gödel, Escher, Bach. An Eternal Golden Braid* (New York: Random House, 1979); Daniel Boorstin, *The Discoverers* (New York: Random House, 1981).

[36]A version of this article appeared in *Curriculum* (Journal of the Association for the Study of Curriculum, United Kingdom) 11 (Autumn 1990): 100–111, and is published here with the permission of the author. See also Arthur W. Foshay, "The Curriculum Matrix," *The Educational Forum* 51 (Summer 1987): 343–353, and Arthur W. Foshay, "The Curriculum Matrix—Further Thoughts," *Thresholds in Education* 14 (August 1988): 8–10.