ECON 1101:
Supplemental Notes

October 30, 2012

MRS = Opportunity Cost

These notes concern the intuition behind the marginal rate of substitution = opportunity cost condition for optimal consumption with a wide range of preferences. Specifically think of the "middle ground" case from class.

First, remember the two issues to consider when you purchasing products: 1) what can I afford and 2) what do I want. The *optimal consumption* tells us what makes me happiest (gives me highest utility) among the things I can afford. Hopefully, you, unlike me, consider affordability when making your own purchasing decisions in real life. From class we learned that *opportunity cost* (OC) is related to what can we afford the *marginal rate of substitution* (MRS) is related to what we want.

Marginal Rate of Substitution and Opportunity Cost

Let's put ourselves in the world from class in which we decide to purchase one of two goods—cheese or wine—and define what MRS and OC are in this context:

Define: Marginal rate of substitution (of cheese for wine, here) is the amount of wine I would be *just willing* to give up to receive one unit of cheese, or the amount of wine I need to receive to be *just willing* give up one unit of cheese

- I highlighed *just willing* to express the fact that the person is indifferent in the trade. Obviously, if you offer me a million bottles of wine to give up one cheese wheel, I would be very happy to do it and would likely be better off (think I would have higher utility from the trade). The MRS tells us what amount of wine you could offer that would not make me better off from the trade but still willing to make the trade.

- For example, suppose I have 4 wine and 3 cheese and have a utility of 20. You come along and offer me one more cheese; you just tell me that I have to give up the amount of wine that would keep my utility at 20, so I give you 2 wine for the 1 cheese. After the trade I have 2 wine and 4 cheese and my utility is 20, so I am just as happy with 4 wine and 3 cheese or 2 wine and 4 cheese. Here, the marginal rate of substitution is $2\frac{wine}{cheese}$.

- Finally, it is worth remembering that the (negative) slope of an indifference curve is the marginal rate of substitution of the good on the x-axis for the good on the y-axis (so cheese is on the x-axis in my example above). The marginal rate of substitution for wine would be $\frac{1}{2}$.

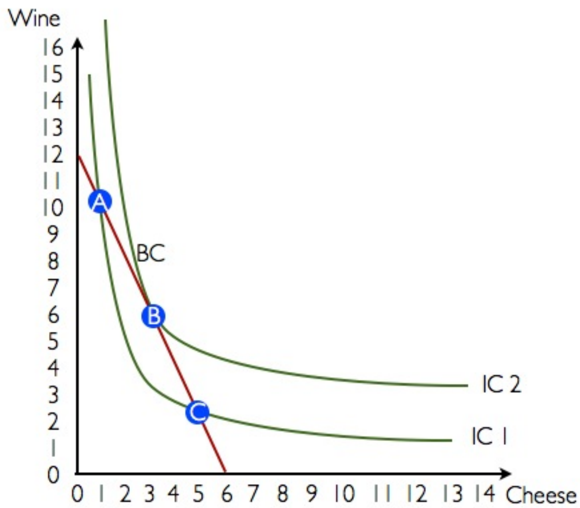
Define: Opportunity cost of cheese in terms of wine is the amount of wine I have to give up to receive one more cheese.

- Opportunity cost of cheese in terms of wine is also the ratio of price of cheese to price of wine.

- Opportunity costs change with a change in the ratio of prices.

MRS = Opportunity Cost

Now I want to provide intuition for why MRS = OC at the optimal point of consumption for "middle ground" preferences. The easiest way to explain this is by showing if 1) $MRS > OC$ or 2) $MRS < OC$, then I am able (can afford) another group of goods that would make me better off. We can also get graphical intuition, by looking at three candidate consumption points: A, B, and C labeled on the graph (notice all of them satisfy the condition that we are consuming a bundle on the budget constraint). At all of these points, the opportunity cost of cheese in terms of wine is 2 (from the slope of the budget constraint). From class we know that point B is the optimal consumption point. It is the affordable consumption bundle that gives me highest utility.



Case 1: $MRS > OC$

At point A, I am consuming something around 10 wine and 1 cheese. More importantly, the marginal rate of substitution of cheese for wine is higher than the opportunity cost of cheese in terms of wine (notice that the green line, my indifference curve, is steeper than the red line, my budget constraint). I do not know what the slope of my indifference curve is at point A, but it is something greater than 2. Suppose for this example it is 4, so my marginal rate of substitution for cheese in terms of wine at point A is 4.

$MRS = 4$ means that you could give me 1 cheese and take away 4 wine, and I would be just as happy as I was before. In other words I am just as happy (i.e. indifferent) between having 10 wine and 1 cheese, my original bundle, and 6 wine and 2 cheese, my bundle after the trade.

Suppose instead of taking away 4 wine when giving me 1 cheese, you only took away 2 wine. Then after this trade I would have 8 wine and 2 cheese versus 6 wine and 2 cheese with the old trade. Since I love more cheese and more wine, this second trade is a much better deal for me.

As luck would have it, that is exactly the trade the market is offering me! Through price ratios (and hence the opportunity cost of cheese in terms of wine) has told me that I can give up 2 wine and get 1 cheese. We just said that would make me better off (give me higher utility), and I can afford it! So there is no way that point A, where $MRS > OC$ can be an optimal consumption bundle.

In sum, if $MRS > OC$ I can improve my utility by buying more of the stuff on the x-axis with the money I get from selling some of the stuff I have on the $y-$axis. And compare optimal consumption point B to point A. Compared to A, with B I have more cheese (more $x-$axis) and less wine (less y-axis)

Case 2: $MRS < OC$

This case has a very similar argument.[1] At point C, I am consuming something around 5 cheese and 2 wine, and the marginal rate of substitution of cheese for wine is lower than the opportunity cost of cheese in terms of wine (check the slopes of the indifference curve versus the budget constraint). Since $OC = 2$, let's just say the $MRS = 1$ at point C.

[1] In fact, if you flip the x and y axis, you will notice that the exact argument for case 1 works since $MRS > OC$ now.

Since $MRS = 1$ you could take away 1 cheese and give me 1 wine, and I would be just as happy as before. So I am indifferent between 5 cheese and 2 wine or 4 cheese and 3 wine. Suppose instead of this trade, you offered to take away 1 cheese and give me 2 wine. With this new trade I would have 4 cheese and 4 wine. I have more wine and the same amount of cheese than with 4 cheese and 3 wine, so I must be happier (have higher utility) with this second trade.

As luck would have it again, this second trade is exactly what the market is offering me! Look again to the opportunity cost of cheese in terms of wine to see that the market is telling me I can give up 1 cheese and get 2 wine. This is a trade that would make me better off, as we just concluded, and I can afford it! So there is no way that point C, where $MRS < OC$ can be an optimal consumption bundle.

In sum, if $MRS < OC$ I can improve my utility by buying more of the stuff on the y-axis with the money I get from selling some stuff I have on the $x-$axis. And compare optimal consumption point B to point C. Compared to C, with B I have less cheese and more wine.

Final Comment

Hopefully this provides some intuition why at the optimal consumption bundle with middle-ground preferences, $MRS = OC$. When using the intutition above, just be careful to remember with your ratios. While on a graph this is easy to remember, in general keep constant which good is your "x-axis" good when comparing MRS to OC.