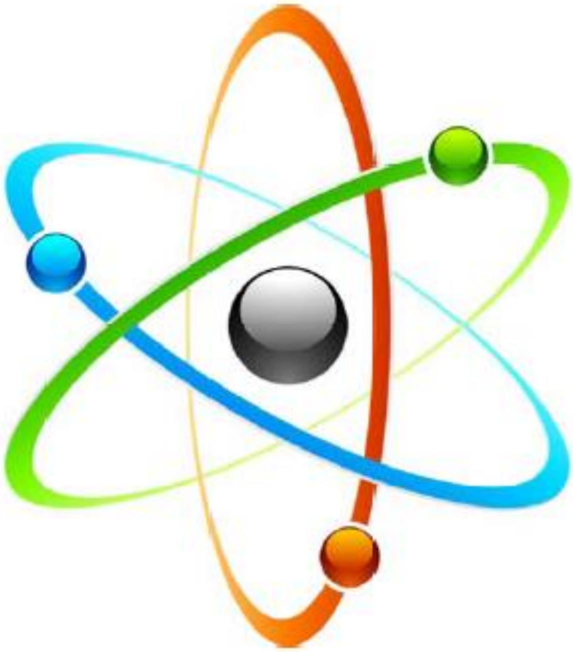
ADVANCED PHYSICS COURSE

CHAPTER 14:

REFRACTION

FOR HIGH SCHOOL PHYSICS CURRICULUM AND ALSO
THE PREPARATION OF ACT, DSST, AND AP EXAMS

This is a complete video-based high school physics course that includes videos, labs, and hands-on learning. You can use it as your core high school physics curriculum, or as a college-level test prep course. Either way, you'll find that this course will not only guide you through every step preparing for college and advanced placement exams in the field of physics, but also give you in hands-on lab practice so you have a full and complete education in physics. Includes text reading, exercises, lab worksheets, homework and answer keys.



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MATERIAL LIST

While you can do the entire course entirely on paper, it's not really recommended since physics is based in real-world observations and experiments! Here's the list of materials you need in order to complete all the experiments in this unit. *Please note: you do not have to do ALL the experiments in the course to have an outstanding science education. Simply pick and choose the ones you have the interest, time and budget for.*

- glass jar penny
- two glass containers, one of which MUST
- be Pyrex glass vegetable oil (cheap canola brand is what we used in the video)
- sink
- ruler
- gelatin (1 box)
- 1/2 cup sugar
- containers (2)
- hot (boiling) water with adult help
- knife with adult help
- fiber optics kit
- soldering iron with solder
- pliers
- wire strippers
- diagonal cutters
- optional: "helping hands" stand (makes it easier to solder components, but not required to build the project) shallow baking dish
- water
- old CD
- razor blades (2)
- cardboard tube hot glue gun
- washer, 3/8 inch inside diameter
- microscope slide
- petroleum jelly
- newsprint with a lot of type
- pipette, 1 mL
- old piece of wood
- single hair from your head
- aluminum foil
- clothespins (2-4)
- popsicle sticks (tongue-depressor size)
- scissors and a sharp razor
- meter sticks (2)
- bright light source
- biconvex plastic lens
- round balloon, white, 9 inches
- votive candle
- black marker
- matches
- metric ruler
- frog and dot printout
- scrap piece of cardboard
- Benham's disk printout
- string (about 3 feet)

INTRODUCTION

Imagine you have a thin rope attached to a thick rope, and you jerk the thin rope so it creates a pulse that travels down the rope. When it hits the boundary between the two ropes, the wave just doesn't stop and go away. Some of the energy from the wave is reflected back toward the source along the thin rope, and some of the energy is transmitted to the thicker (more dense) rope.

Since light is a wave, when it goes from a less dense to a more dense medium, some of the energy gets reflected back while some of it gets transmitted through. Aim a flashlight at a window and you'll find when the light goes from air to glass, it will both reflect back and transmit through the window.

When the light hits the glass, it not only reflects and transmits, it also changes speed and wavelength as it crosses the boundary AND it also changes directions. When it bends to change direction, it's called *refraction*.

REFRACTION AT A BOUNDARY

Imagine you have a thin rope attached to a thick rope, and you jerk the thin rope so it creates a pulse that travels down the rope. When it hits the boundary between the two ropes, the wave just doesn't stop and go away. Some of the energy from the wave is reflected back toward the source along the thin rope, and some of the energy is transmitted to the thicker (more dense) rope.

Since light is a wave, when it goes from a less dense to a more dense medium, some of the energy gets reflected back while some of it gets transmitted through. Aim a flashlight at a window and you'll find when the light goes from air to glass, it will both reflect back and transmit through the window.

When the light hits the glass, it not only reflects and transmits, it also changes speed and wavelength as it crosses the boundary AND it also changes directions. When it bends to change direction, it's called *refraction*.

BROKEN PENCIL

If you stick a pencil is a glass of water and look through the side of the glass, you'll notice that the pencil appears shifted. The speed of light is slower in the water (140,000 miles per second) than in the air (186,282 miles per second), called optical density, and the result is bent light beams and broken pencils.

You'll notice that the pencil doesn't always appear broken. Depending on where your eyeballs are, you can see an intact or broken pencil. When light enters a new substance (like going from air to water) perpendicular to the surface (looking straight on), refraction does not occur.

However, if you look at the glass at an angle, then depending on your sight angle, you'll see a different amount of shift in the pencil. Where do you need to look to see the greatest shift in the two halves of the pencil? (Hint: move the pencil back and forth slowly.)

Light Tricks

Overview: Today you get to see the science behind the illusion by learning how light striking an object affects how our eyes see it.

What to Learn: Light can be bent when it passes through materials. The amount that the light bends is called the *index of refraction.* How much light bends depends on the material it's passing through. This quality is measured for each individual material and is called the *optical density*. The more dense the substance, the slower the light travels through it, and the more the light bends.

Materials

- glass jar
- penny
- laser (optional)
- flashlight
- milk or flour

Light Tricks Data Table

When you do the pencil illusion trick, record your observations.

Water Level:	**Water Level:**	**Water Level:**

Experiment

1. Record your observations on the data sheet and in the exercises as you go.

2. Toss one coin into a water glass and fill with an inch of water. Hold the glass up and find where you need to look to see TWO coins.

3. Look through the top of the glass – how many coins are there now? What about when you look from the side?

4. Toss in a second coin – now how many are there?

5. Remove the coins and turn out the lights. Shine a flashlight beam through the glass onto a nearby wall. (Hint – if this doesn't work, try using a square clear container.) Stick a piece of paper on the wall where your light beam is and outline the beam with a pencil.

6. Shine the light at an angle up through the water so that it bounces off the surface of the water from underneath. Trace your new outline and compare… are they both the same shape?

7. Add a teaspoon of milk and stir gently. (No milk? Try sprinkling in a bit of white flour.) Now shine your flashlight through the container as you did in steps 4 and 5 and notice how the beam looks.

8. Use a round container instead of square… what's the difference?

Reading

Have you ever broken a pencil by sticking it into a glass of water? The pencil isn't really broken, but it sure looks like it! What's going on?

Light can be bent when it passes through materials. The amount that the light bends is called the *index of refraction.* How much light bends depends on the material it's passing through. This quality is measured for each individual material and is called the *opticaldensity*. The more dense the substance, the slower the light travels through it, and the more the light bends. To be exact, when a beam of light hits a different substance (like moving from air to water), the wavelength changes because the speed of the light changes.

If you're thinking that the speed of light is always constant, you're right… in outer space, light travels at 186,000 miles per second. But the Earth is covered with an atmosphere, and as soon as the light passes into this thick cloud of nitrogen and oxygen gas, it slows down a bit. The speed of light changes whenever it passes from one material to another, like when it moves from water to ice, or to sunglasses, smoke, fog, or windows. How much the light speed slows down depends on what the material is made of. Mineral oil and window glass will slow light down more than water, but not as much as diamonds do.

Exercises

1. When one coin is in the water, you can actually see two: Are the coins both the same size? Which one is the original coin?
2. In step 2 of the experiment: How many coins are there when viewed from the top of the glass? What about when you look from the side?
3. What happened when you tossed in a second coin?
4. How did your outlines compare?

Answers to Exercises: Light Tricks

1. When one coin is in the water, you can actually see two: Are the coins both the same size? Which one is the original coin? (the smaller coin is the reflection)

2. In step 2 of the experiment: How many coins are there when viewed from the top of the glass? What about when you look from the side? (one coin when looking from above, two when looking through the side)

3. What happened when you tossed in a second coin? (There were four.)

4. How did your outlines compare? (The first was a circle, the second was an oval.)

REFRACTIVE INDEX

The refractive index provides a measure of the relative speed of light in that particular medium which allows us to figure out speeds in other mediums as well as predict which way light will bend.

DISAPPEARING GLASS

When a beam of light hits a different substance (like glass), the speed of light changes. The color of the light (called the wavelength) can also change. In some cases, the change of wavelength turns into a change in the direction of the beam.

Depending on if the light is going from a lighter to an optically denser material (or vice versa), it will bend different amounts. Glass is optically denser than water, which is denser than air. Here's a chart:

Vacuum 1.0000
Air 1.0003
Ice 1.3100
Water 1.3333
Ethyl Alcohol 1.3600
Pyrex 1.4740
Karo Syrup 1.4740
Vegetable Oil 1.4740
Plexiglas 1.5100
Diamond 2.4170

This means if you place a Pyrex container inside a beaker of vegetable oil or Karo syrup, it will disappear (this also works for some mineral oils). Note however that the optical densities of liquids vary with temperature and concentration, and manufacturers are not perfectly consistent when they whip up a batch of this stuff, so some adjustments are needed.

Not only can you change the shape of objects by bending light (broken or whole), but you can also change the size. Magnifying lenses, telescopes, and microscopes use this idea to make objects appear different sizes.

WHY DOES LIGHT BEND?

But why does light bend? You can imagine a toy car going from a wood floor to carpeting. One wheel hits the carpet first and slows down before the other, causing the toy to turn. The direction of the wave changes in addition to the speed. The slower speed must also shorten its wavelength since the frequency of the wave doesn't change.

The bottom line is that bending is caused by the change in speed of light when it crosses a boundary. This is true everywhere, even in the vacuum of space if it's going from space to our atmosphere.

BENDING LIGHT RIGHT OR LEFT?

How does light know which way to bend? It depends on whether the wave is speeding up or slowing down when it moves across the boundary, which depends on the optical density of the mediums.

SPEAR FISHING

If you've ever tried to skewer something under the water from above it, you know that you can't aim directly at the object, because of the way light bends when it goes from a slower to a faster medium. Can you guess the one condition where light doesn't bend as it crosses a boundary?

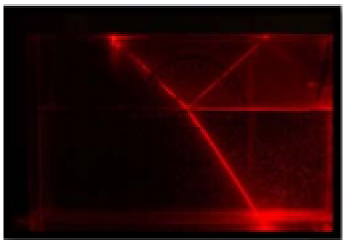
MATHEMATICS OF REFRACTION

How much incident light bends as it crosses a boundary can be calculated and measured if we know about the mediums, including information about the index of refraction. Snell's Law is a mathematical relationship between the refractive and incident angles of light and the optical density of the different mediums.

LASERS AND JELL-O USING SNELL'S LAW

If you're scratching your head during math class, wondering what you'll ever use this stuff like trigonometry and algebra for, here's a cool experiment that shows you how scientists use math to figure out the optical density of objects, called the "index of refraction".

Lasers, Jell-O and Trigonometry



Overview: If you're scratching your head during math class, wondering what you'll ever use this stuff for, here's a cool experiment that shows you how scientists use math to figure out the optical density of objects, called the "index of refraction".

Materials:

Paper
Laser
Pencil
Protractor
Ruler
Gelatin (1 box)
1/2 cup sugar
2 containers
Hot (boiling) water with adult help
Knife with adult help

Experiment:

1. Mix two packets of gelatin with one cup of boiling water and stir well.
2. To one of the containers, add 1/2 cup sugar. Label this one as "sugar" and put the lid on and store it in the fridge.
3. Label the other as "plain" and also store it in the fridge. It takes about 2 hours to solidify. Wait, and then:
4. Cut out a 3"x3" piece of gelatin from the plain container.
5. On your sheet of paper, mark a long line across the horizontal, and then another line across the vertical (the "**normal**" line) as shown in the video.
6. Mark the **angle of incidence** of 40o. This is the path your laser is going to travel on.
7. Lay down the gelatin so the bottom part is aligned with the horizontal line.
8. Shine your laser along the 40o angle of incidence. Make sure it intersects the origin.
9. Measure the **angle of refraction** as the angle between the bent light in the gelatin and the normal line. (It's 32o in the video.)
10. Use Snell's Law to determine the index of refraction of the gelatin: $n1 \sin q1 = n2 \sin q2$
11. Repeat steps 4-10 with the sugar gelatin. Did you expect the index of refraction to be greater or less than the plain version, and why?

What's going on? How much light bend as it goes through one medium to another depends on the index of refraction (refractive index) of the substances. There are lots of examples of devices that use the index of refraction, including fiber optics. Fiber optic cables are made out of a transparent material that has a higher index of refraction than the material around it (like air), so he waves stay trapped inside the cable and travel along it, bouncing internally along its length. Eye glasses use lenses that bend and distort the light to make images appear closer than they really are.

Questions to Ask:

1. Does reflection or refraction occur when light bounces off an object?
2. Does reflection or refraction occur when light is bent?
3. What type of material is used in a lens?
4. What would happen if light goes from air to clear oil?

SNELL'S LAW AND PRISMS

Incoming light refracts as it crosses two boundaries of a prism. Notice how prisms have non-parallel sides for a reason…

SNELL'S LAW AND THE INDEX OF REFRACTION

Now let's take a look at how to use Snell's Law to figure out the optical density of a medium by measuring how much the light bends when it goes from one medium to another.

TOTAL INTERNAL REFLECTION

The Law of Reflection states that when light reflects off the surface, the angle of incidence is equal to the angle of reflection. Snell's Law states that when light crosses into a new medium, the relationship between the angle of incidence (θ_i) and angle of refraction (θ_r) are related by the equation:

$$n_i \sin(\theta_i) = n_r \sin(\theta_r)$$

where *n* is the index of refraction.

There's actually a connection between light reflection and refraction, since they usually happen at the same time.

Total internal reflection happens at large incident angles and when light travels from a more optically dense medium to a lesser dense medium. Total refers to no loss in intensity (plane mirrors have a loss of about 4%).

For total internal reflection to occur, two things have to happen: light must be going from more optically dense to less dense mediums, and the angle of incidence is greater than the critical angle.

Total internal reflection happens when light travels from water to air, not from air to water. It also happens when light bends away from the normal at large angles of incidence. For water-to-air, it's greater than 48.6^o. Each set of mediums have their own critical angle.

TOTAL INTERNAL REFLECTION AND PRISMS

Let’s take a look at a glass prism and total internal reflection critical angles to determine the optical density of the glass.

TOTAL INTERNAL REFLECTION AND DIAMONDS

Let’s look at total internal reflection and diamonds.

FIBER OPTICS

Fiber Optics are one application of total internal reflection. Optical fibers are flexible, transparent fibers made from plastic or glass about the size of a human hair that can serve as a "light pipe" to transmit light from one location to another. The bundle of fibers is used in medical applications where doctors can see inside the body by attaching a small camera to one side of the cable. To make the project below, you can order this Fiber Optics kit.

Materials:

- Fiber Optics kit.
- Soldering iron with solder
- Pliers
- Wire strippers
- Diagonal cutters
- Optional: "helping hands" stand (makes it easier to solder components to the board, but not required to build the project)

Light wave communication over optical fiber networks are used today everywhere in fiber optic communications. These transmit over long distances at higher bandwidths than metal cables and don't have problems with electromagnetic interference or losses typical with copper wiring.

INTERESTING REFRACTION PHENOMENA

Dispersion is when visible light is separated into the colors that make up the light. We've already seen how optical density is a measure of how much a medium slows down light that travels through it. The index of refraction depends on the frequency of the light.

The index of refraction is 1.51 for red light and 1.53 for violet, which means that as light goes through glass, it slows down the violet light just a little it more than it does the red. Because of this, prisms can unmix light by dispersion because the prism has two (or more) boundaries where this effect adds to separate white light into its colors.

LIQUID PRISMS

In this experiment, water is our prism. A prism un-mixes light back into its original colors of red, green, and blue. You can make prisms out of glass, plastic, water, oil, or anything else you can think of that allows light to zip through.

Liquid Prism

Overview: A prism un-mixes light back into its original colors of red, green, and blue. In this experiment, water is our prism. You can make prisms out of glass, plastic, water, oil, or anything else you can think of that allows light to zip through.

What to Learn: Today you're going to play with splitting apart white light into its primary colors. The color of light striking an object affects how our eyes see it.

Materials

- mirror
- shallow baking dish
- sunlight
- index card

You'll also have one of the following:

- plain water
- baby oil or mineral oil
- water with one tablespoon of salt mixed in
- distilled white vinegar
- isopropyl rubbing alcohol
- clear liquid soap (do not mix with water)

Experiment

1. Set a tray of liquid in sunlight. If you're using water, then fill your tray with water. If you're using salt, mix a tablespoon of salt into the water and then set it in sunlight. If you're using anything else, fill it with your liquid and set it outside.
2. Lean a mirror against the inside edge of the tray and adjust it so that a rainbow appears.
3. Use the index card (or another white surface) so that you can clearly see the reflection from your prism.
4. You can also use a light bulb as an alternate light source by shining it through a slit in a flat cardboard surface. However, you'll find that sunlight is much more effective and will make a brighter, more complete rainbow.
5. Troubleshooting: This is one of the easiest experiments to do, and the most beautiful. The trouble is, you don't know where the water shadow will show up, so make sure you point the mirror to the sky and play with the angle of the mirror until you find the wavering rainbow. If you still have trouble, use a large sheet of white paper instead of the tiny index card.

Liquid Prism Data Table

Type of Liquid for the Prism	What did you observe?

Reading

What is a prism? Think of a beam of light. It zooms fast on a straight path, until it hits something, like a water drop. As the light goes through the water drop, it changes speed. This is called *refraction*, which we will discuss more in a future lesson. The speed change depends on the angle at which the light hits the water, and what the drop is made of. If it was a drop of mineral oil, the light would slow down a bit more because the fluid has more optical density. So when white light passes through a prism, like that water drop, it changes speed, which we can see with our eyes because it also turns colors.

Prisms un-mix light into its different wavelengths. When light hits the prism, most of it passes through, although a small bit of light does get reflected off the surface, but when it passes through it changes speed. Since the sunlight is made up of many different wavelengths (colors), each color gets bent by different amounts, and you see a rainbow out the other side. As long as the light travels at the same speed (like through the air), it’s white. But as soon as the light hits the water and bends at different angles, the wavelengths separate and spread out, making the rainbow you see.

Troubleshooting: This is one of the easiest experiments to do, and the most beautiful. The trouble is, you don't know where the water shadow will show up, so make sure you point the mirror to the sky and play with the angle of the mirror until you find the wavering rainbow. Because the shadow is constantly moving, you can snap a few pictures when you've got it so you can look over the finer details later. If this project still eludes you, take a large sheet and use it instead of the tiny index card.

Exercises

1. What serves as the prism in this experiment?

2. What property can help make something a good prism material?

3. What are some other items that could be used as prisms?

Answers to Exercises: Liquid Prism

1. What serves as the prism in this experiment? (water or other clear liquid)

2. What property can help make something a good prism material? (transparency: a material that allows light to pass through it)

3. What are some other items that could be used as prisms? (glass, oil, clear plastic)

WATER DROPS AND RAINBOWS

Ever notice how water has to be involved before you get a rainbow? Rainbows never happen on dry, clear days.

I remember how surprised I was when I saw a rainbow appear on a cloudless day while I was misting a soapy car with the garden hose. I was so amazed that the arc was larger than I realized that I climbed up a ladder before I realized that I could make the rainbow form in a cull circle!

Moonbows (also known as lunar rainbows) form from light reflected off the moon form in the atmosphere. Since they are formed from reflected sunlight, they tend to be very faint. If you want to find one, look in the opposite part of the sky from the moon. It will look like a white instead of the usual rainbow colors, but that's because the eye has a hard time seeing colors in the dark. If you take a long-exposure photograph, the colors will appear. Aristotle himself recorded observing moonbows on dark nights when the weather conditions were just right!

SPECTROMETER

Ever play with a prism? When sunlight strikes the prism, it gets split into a rainbow of colors. Prisms un-mix the light into its different wavelengths (which you see as different colors). Diffraction gratings are tiny prisms stacked together.

When light passes through a diffraction grating, it splits (diffracts) the light into several beams traveling at different directions. If you've ever seen the 'iridescence' of a soap bubble, an insect shell, or on a pearl, you've seen nature's diffraction gratings.

Spectrometers are used in chemistry and astronomy to measure light. In astronomy, we can find out about distant stars without ever traveling to them, because we can split the incoming light from the stars into their colors (or energies) and "read" what they are made up of (what gases they are burning) and thus determine their what they are made of. In this experiment, you'll make a simple cardboard spectrometer that will be able to detect all kinds of interesting things!

CDs are like a mirror with circular tracks that are very close together. The light is spread into a spectrum when it hits the tracks, and each color bends a little more than the last. To see the rainbow spectrum, adjust the CD and the position of your eye so the angles are perpendicular.

Spectrometers

Overview: Spectrometers (spectroscopes) are used in chemistry and astronomy to measure light. In astronomy, we can find out about distant stars without ever traveling to them, because we can split the incoming light from the stars into their colors (or energies) and "read" what they are made up of (what gases they are burning) and thus determine their what they are made of.

What to Learn: In this experiment, you'll make a simple cardboard spectrometer that will be able to detect all kinds of interesting things!

SPECIAL NOTE: This instrument is NOT for looking at the Sun. Do NOT look directly at the Sun. But you can point the tube at a sheet of paper that has the Sun's reflected light on it.

Materials:

Easy Spectrometer

- Old CD
- Razor
- Index card
- Cardboard tube at least 10 inches long

Advanced Spectrometer (Calibrated)

- Cardboard box (ours is 10″ x 5″ x 5″, but anything close to this will work fine)
- Diffraction grating
- 2 razor blades (with adult help)
- Masking tape
- Ruler
- Photocopy of a ruler (or sketch a line with 1 through 10 cm markings on it, about 4cm wide)

Easy Spectrometer:

1. A CD has a diffraction grating built into it. We're going to use a CD instead of a diffraction grating for this experiment.
2. Cut a clean slit less than 1 mm wide in an index card or spare piece of cardboard.
3. Tape it to one end of the tube.
4. Align your tube with the slit horizontally, and on the top of the tube at the far end cut a viewing slot about one inch long and ½ inch wide.
5. Cut a second slot into the tube at a 45-degree angle from the vertical away from the viewing slot.
6. Insert the CD into this slot so that it reflects light coming through the slit into your eye (viewing slot).
7. Aim the 1 mm slit at a light source such as a fluorescent light, neon sign, sunset, light bulb, computer screen, television, night light, candle, fireplace… any light source you can find EXCEPT THE SUN.
8. Look through the open hole at the light reflected off the compact disk (look for a rainbow in most cases) inside the cardboard tube.
9. Complete the data table.

Advanced Spectrometer (Calibrated)

1. Using a small box, measure 4.5 cm from the edge of the box. Starting here, cut a hole for the double-razor slit that is 1.5 cm wide 3 cm long.
2. From the other edge (on the same side), cut a hole to hold your scale that is 11 cm wide and 4 cm tall.
3. Print out the scale and attach it to the edge of the box.
4. Very carefully line up the two razors, edge-to-edge, to make a slit and secure into place with tape.
5. On the opposite side of the box, measure over 3 cm and cut a hole for the diffraction grating that is 4 cm wide and 3 cm tall.
6. Tape your diffraction grating over the hole.
7. Aim the razor slit at a light source such as a fluorescent light, neon sign, sunset, light bulb, computer screen, television, night light, candle, fireplace… any light source you can find. Put the diffraction grating up to your eye and look at the inner scale. Move the spectrometer around until you can get the rainbow to be on the scale inside the box.

How to Calibrate the Spectrometer with the Scale

8. Inside your box is a scale in centimeters. Point your slit to a fluorescent bulb, and you'll see three lines appear (a blue, a green, and a yellow-orange line). The lines you see in the fluorescent bulb are due to mercury superimposed on a rainbow continuous spectrum due to the coating. Each of the lines you see is due to a particular electron transition in the visible region of Hg (mercury).

    a. **blue line (435 nm)**

    b. **green line (546 nm)**,

    c. **yellow orange line (579 nm)**

    If you look at a sodium vapor street light you'll see a yellow line (actually 2 closely spaced) at 589 nm.

9. Line the razor slits along the length of the fluorescent tube to get the most intense lines. Move the box laterally (the lines will move due to parallax shift).
10. Take scale readings at the extreme of these movements and take the average for the scale reading. For instance, if the blue line averages to the 8.8 cm value, this corresponds to the 435 nm wavelength. Do this for the other 2 lines.
11. On graph paper, plot the cm (the ruler scale values) on the vertical axis and the wavelength (run this from 400-700 nm) on the horizontal axis.
12. Draw the best straight lines through the 3 points (4 lines if you use the Na (sodium) street lamp). You've just calibrated the spectrometer!
13. Line the razor slits up with another light source. Notice which lines appear and where they are on your scale. Find the value on your graph paper. For example, if you see a line appear at 5.5 cm, use your finger to follow along to the 5.5 cm until you hit the best-fit line, and then read the corresponding value on the wavelength axis. You now have the wavelength for the line you've just seen!
14.

Notes on Calibration and Construction: If you swap out different diffraction gratings, you will have to re-calibrate. If you make a new spectrometer, you will have to re-calibrate to the Hg (mercury) lines for each new spectrometer. If you do remake the box, use a scale that is translucent so you can see the numbers. If you use a clear plastic ruler, it may let in too much light from the outside making it difficult to read the emission line.

Spectrometer Data Table

Light Source	Draw what you see:	Wavelength
		For Advanced Spectrometers Only!

Reading

Diffraction gratings are found in insect (including butterfly) wings, bird feathers, and plant leaves. While I don't recommend using living things for this experiment, I do suggest using an old CD. That's how we're going to build the *Easy Spectrometer.*

CDs are like a mirror with circular tracks that are very close together. The light is spread into a spectrum when it hits the tracks, and each color bends a little more than the last. To see the rainbow spectrum, you've got to adjust the CD and the position of your eye so the angles line up correctly (actually, the angles are perpendicular).

You're looking for a spectrum (think of a rainbow). –Depending on what you look at (neon signs, chandeliers, incandescent bulbs, fluorescent bulbs, halogen lights, etc.), you'll see different colors of the rainbow.

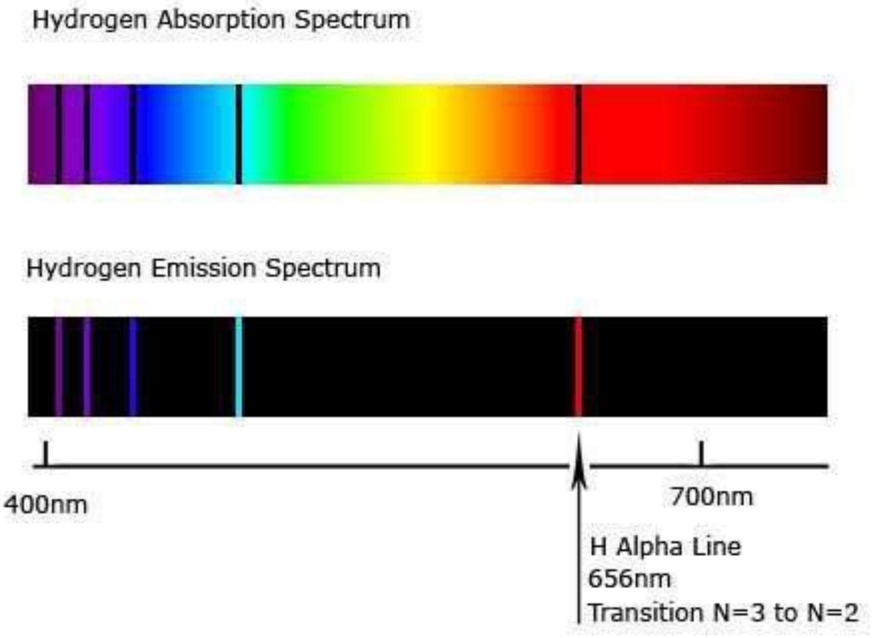
For the *Advanced Spectrometer,* we're actually going to calibrate it by plotting information on a graph and using a diffraction grating to make it more precise. It's much more like the instrument that scientists use in their labs.

Scientist use spectroscopes (spectrometers) to collect a small sample of light and test it to see what made the light. As the light passes through the diffraction grating, it gets split into different bands of light, and you'll see these as different wavelengths, or colors of light.

Scientists can figure out what fuel a star is burning, the age of the star, the composition of the star, how fast it's moving, and whether it's moving toward or away from Earth. For example, when hydrogen burns, it gives off light, but not in all the colors of the rainbow, only very specific colors in red and blue. It's like hydrogen's own personal fingerprint, or light signature.

While the spectrometers we're about to make aren't powerful enough to split starlight, they're perfect for using with the lights in your house, and even with an outdoor campfire. Next time you're out on the town after dark, bring this with you to peek different types of lights – you'll be amazed how different they really are.

SPECIAL NOTE: This instrument is NOT for looking at the Sun. Do NOT look directly at the Sun. But you can point the tube at a sheet of paper that has the Sun's reflected light on it.



How to Tell Which Elements are Burning

For example, if you were to view hydrogen burning with your spectroscope, you'd see the bottom appear in your spectrometer:

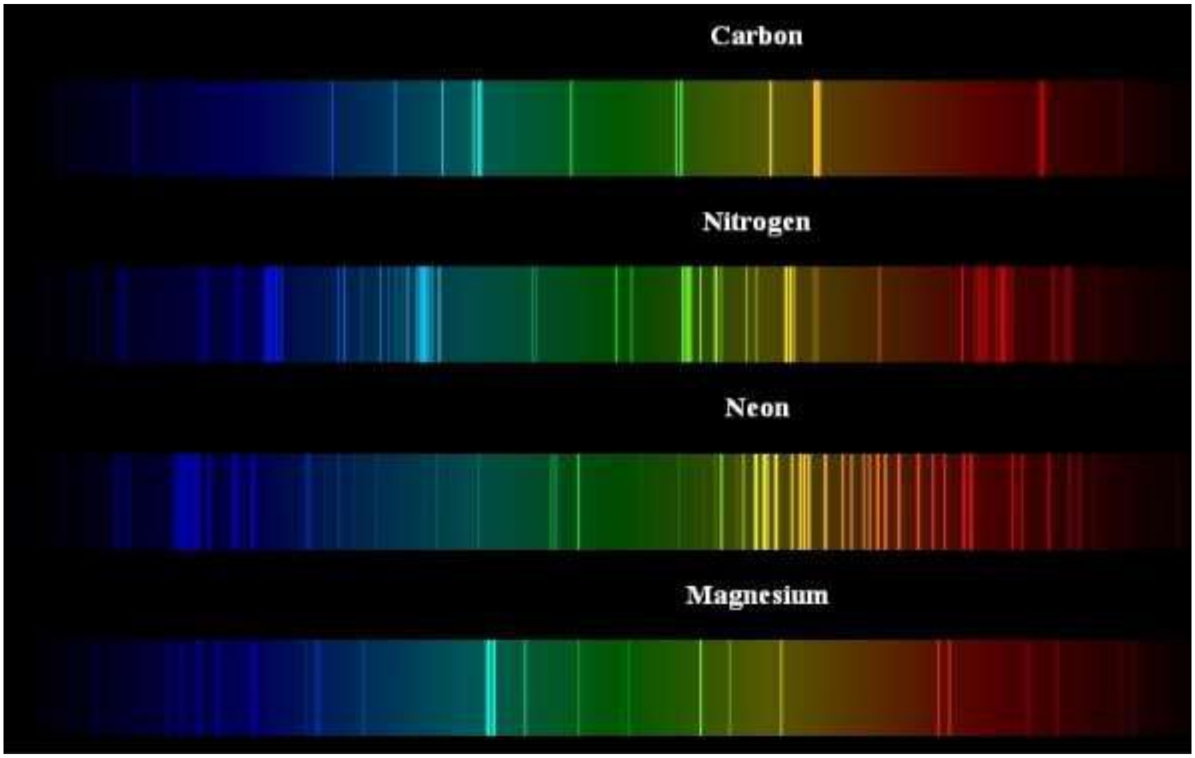
Notice how one fits into the other, like a puzzle. When you put the two together, you've got the entire spectrum.

What's the difference between the two? The upper picture (absorption spectrum of hydrogen) is what astronomers see when they use their spectrometers on distant stars when looking through the earth's atmosphere (a cloud of gas particles). The lower picture (emission spectrum of hydrogen) is what you'd see if you were looking directly at the source itself.

Note - Do NOT use your spectrometer to look at the Sun! When astronomers look at stars, they have computers look for them - they aren't putting their eye on the end of a tube.

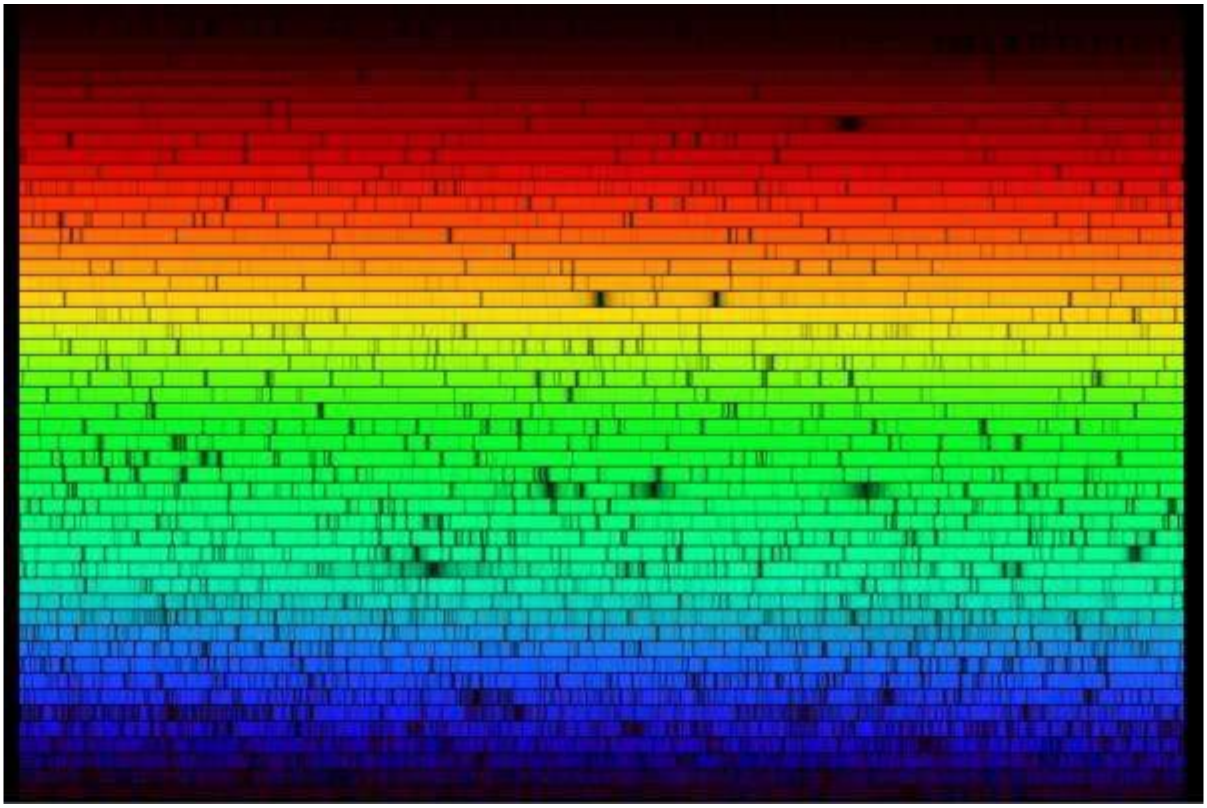
Each element has its own special 'signature', unique as a fingerprint, that it leaves behind when it burns. This is how we can tell what's on fire ***in*** a campfire.

For example, here's what you'd see for the following elements:



Just get the feel for how the signature changes depending on what you're looking at. For example, a green campfire is going to look a lot different from a regular campfire, as you're burning several elements in addition to just carbon. When you look at your campfire with your spectroscope, you're going to see ***all*** the signatures at the same time. Imagine superimposing all four sets of spectral lines above (carbon, neon, magnesium, and nitrogen) into one ***single*** spectrum... it's going to look like a mess! It takes a lot of hard work to untangle it and figure out which lines belong to which element. Thankfully, these days, computers are more than happy to chug away and figure most of it out for us.

Here's the giant rainbow of absorption lines astronomers see when they point their instruments at the Sun:



Do you see all the black lines? Those are called emission lines, and since astronomers have to look through a lot of atmosphere to view the Sun, there's a lot of the spectrum missing (shown by the black lines), especially corresponding to water vapor. The water absorbs certain wavelengths of light, which corresponds to the black lines.

Exercises

1. Name three more light sources that you think might work with your spectroscope.

2. Why is there a slit at the end of the tube instead of leaving it open?

Answers to Exercises: Spectrometers

1. Name three more light sources that you think might work with your spectroscope. (answers will vary)

2. Why is there a slit at the end of the tube instead of leaving it open? (The light that strikes the end of the tube gets mostly reflected away, and only a tiny amount of light gets inside the tube to the diffraction grating. If you had too much light, you wouldn't be able to see the spectrum.)

MIRAGES

Mirages happen on sunny days when the roads are heated by the sun to a point where it also heats the air above the road. Since hot air is less (optically) dense that cool air, the light refracts as it travels through it.

IMAGE FORMATION BY LENSES

Lenses are curved pieces of glass or plastic designed to bend (refract) light. A simple lens is just one piece, and a compound lens is like the lens of a camera – there's lots of them in there.

The first lenses were developed by nature – dewdrops on plant leaves are natural lenses. The light changes speed and bends when it hits the surface of the drop, and things under the drop appear larger. The earliest written records of lenses are found in the Greek archives and described as being glass globes filled with water.

Water Lens

Overview: Waves of light enter your eyes through the pupil, which is the small black dot right in the center of your colored iris. Your lens bends and focuses the light that enters your eye. In this experiment, we will study this process of bending light and we will look at the difference between concave and convex lenses.

Materials

- washer (3/8 inch inside diameter)
- microscope slide
- petroleum jelly (or lip balm)
- newsprint with small type
- pipette (1 mL) or eyedropper or spoon
- pen
- paper towel
- water

Experiment

1. Apply a little petroleum jelly on the washer's flat side. NOTE: Washers have flat and rounded sides, so be sure you are putting the petroleum jelly on the flat side of the washer.

2. Put the washer, petroleum jelly side down, on the middle of the microscope slide. Twist the washer a bit to seat it on the slide and make a seal. This should keep the water in place.

3. Put the washer and slide on the newsprint.

4. Fill the pipette with water.

5. Use the pipette to slowly place water in the washer. Fill the washer until the water makes a domed shape. You have just made a convex lens!

6. Find a letter *e* on the newspaper and put the lens over it. Draw a diagram of what the *e* looks like through the convex lens.

7. Now use the pipette to remove water from the washer (or you can absorb some with a paper towel). Your goal is to create a dip in the surface of the water. Now find the same *e* and place your new concave lens over the letter. Draw a picture of what the *e* looks like through the new lens.

Water Lens Data Table

Water Lens Type *(concave or convex)*	**Draw a Diagram of the Lens**	**Draw a Diagram of the Newsprint**

Reading

You can see that a convex lens bends outward and a concave lens bends inward. What does this do to light?

In a convex lens, the domed surface means that if light waves come in through the flat bottom surface, they will be spread out, or refracted, as they exit the curved portion of the lens. But since a concave lens dips inward it creates the opposite effect. When light waves exit the concave surface, they are brought together. This makes images appear smaller.

The lens does all the focusing work, but it is actually the shape of the eye that determines what you see. If you have a tall, oblong eye, you are far-sighted. And conversely, if your eyes are short and fat, you are near-sighted. In either case, the lenses are functioning properly but the actual shape of the eye needs a slight adjustment.

Exercises

1. What are the two main types of lenses?

2. How are the two main types of lenses shaped

3. How do the two main types of lenses work?

Answers to Exercises: Water Lens

1. What are the two main types of lenses? (convex and concave)
2. How are the two main types of lenses shaped? (convex bulges outward and concave dips inward)
3. How do the two main types of lenses work? (convex makes things appear larger, concave makes them appear smaller)

CONCAVE LENSES

Concave lenses are shaped like a 'cave' and curve inward like a spoon. Light that shines through a concave lens bends to a point (converging beam). Ever notice how when you peep through the hole in a door (especially in a hotel), you can see the entire person standing on the doorstep? There's a concave lens in there making the person appear smaller.

You'll also find these types of lenses in 'shoplifting mirrors'. Store owners post these mirrors around help them see a larger area than a flat mirror shows, although the images tend to be a lot smaller.

If you have a pair of near-sighted glasses, chances are that the lenses are concave. Near-sighted folks need help seeing things that are far away, and the concave lenses increase the focal point to the right spot on their retina.

Concave lenses work to make things look smaller, so there not as widely used as convex lenses. You'll find concave lenses inside camera lenses and binoculars to help clear weird optical problems that happen around the edges of a convex lens (called aberration).

Optical Bench

Overview: Mirrors and filters and lenses, oh my! In this lesson, we'll learn a lot more about each of these items and how you can use them together to make an optical bench. An optical table gives you a solid surface to work on and nails down your parts so they don't move. Scientists use optical benches when they design microscopes, telescopes, and other optical equipment. We're going to make a quick and easy optical lab bench to work with your lenses.

Well, technically our setup is called an optical rail, and the neat thing about it is that it comes with a handy measuring device so you can see where the focal points are for your lenses.

What to Learn: Lenses work to bend light in a certain direction, called refraction. Concave lenses work to make objects smaller and convex lenses make them larger. Light interacts with matter by transmission (including refraction), absorption, or scattering (including reflection).

Materials

- lenses (glass or plastic - magnifying lenses work also)
- two razor blades (new)
- index cards (about four)
- razor
- old piece of wood
- single hair from your head
- tape
- small binder clips
- aluminum foil
- clothespins (2-4)
- laser pointer
- popsicle sticks (tongue-depressor size)
- hot glue gun
- scissors and a sharp razor
- meter sticks (2)
- large candle (with adult supervision)
- bright light source (ideas for this are on the video)

Experiment

1. Use masking tape to fasten together the two meter sticks so that they're on top of each other. Be sure that you'll still be able to insert popsicle sticks between the two meter sticks in order to mount your lenses and filters.
2. Run a bead of glue along the tape you just added and attach a popsicle stick on each end of the now-attached meter sticks. These popsicle sticks will be your base, so be sure the numbers are at the top edge and easy to read.

3. To make the screen, grab a popsicle stick and a white index card. Place some glue along half of one long edge of the stick and attach it to the middle of the card. You can insert this new screen into the rail (between the two meter sticks) at one end.
4. Using a sharp knife, carefully cut out a small rectangle out of an index card. Remove the rectangle from the card.
5. Carefully place the two new razor blades side by side on the index card, facing each other atop the hole you just cut. They should be parallel and as close as you can get them without the razors actually touching. Secure them in place with tape.
6. Use a small bead of hot glue to attach another popsicle stick to the back of this index card which you've just taped the razors to. Insert the end of the stick into the optical bench. Tape around the meter sticks if it doesn't stand up straight.
7. To make an anti-slit, cut another rectangle in an index card with your razor blade.
8. Take a tiny piece of hair and tape it below the rectangular hole you just made. Stretch it across the middle of the hole and tape down the hair on the other side.
9. Flip over the card and glue a popsicle stick on the other side. Stand this up between the meter sticks in your optical rail (it doesn't really matter what order you put them in yet).
10. Place some hot glue on the side of another popsicle stick and attach this to a clothespin. Be sure the pin can still open and close. Press the stick down on the pin firmly until the glue dries.
11. To help with friction inside the clothespin, place a bead of hot glue inside on each jaw of the clothespin and hold it open until it dries. You can prop a popsicle stick inside and set aside while drying. This makes a higher friction surface to hold the lenses in place more securely. Alternately, you can wrap a rubber band around each jaw of the clothespin.
12. Insert a lens into the dry (or rubber banded) clothespin and insert the popsicle stick into the optical bench.
13. When making more clothespins for additional lenses, you can help prop them upright on the optical bench with small binder clips.
14. Now you've made your bench. Make sure all your items are about at the same height so that the light will hit everything evenly.
15. Turn out the lights and use a candle (with adult supervision) as your light source. Put it at one end of the optical bench. At the other end will be your screen. These items should be as close to the ends of the bench as possible so that your measurements are accurate.
16. Use the magnifying glass (a convex lens) in the middle, moving it back and forth until the image is focused on the screen.
17. Note how far away the magnifying glass is from the focused image. This is your focal point. Record the focal length on your data sheet.
18. Next, use two magnifying glasses. Move one at a time between the candle (or other light source) and your screen. Note where each focuses and record this data. Chances are they will not have the same focal point.
19. Then, put them close together and see where the focal point is when the magnifying glasses are held together like this. Note if your image size changes when both magnifying glasses are used. Also note if the image is more blurry or crisper.
20. Put your lenses in the optical bench and find the focal length for each individual one. Record this data on your table. You may want to record this on the edge of your lenses, or you can number them and put the number on the edge so that you can readily identity each lens.
21. *f* number is a ratio of focal length over diameter. Measure each individual lens diameter. Take the focal length data you recorded in the previous step and find the *f* number by dividing focal length by the diameter.
22. Next, mount your laser on the optical beam. Mount a slit opposite the beam.

23. Shine the beam through the slit toward a wall that's 6-10 feet away. You should see an interference pattern on the wall.

24. Complete the table:

Optical Bench Data Table

Lens Type	**Diameter** (inches or cm)	**Focal Length** *(inches or cm)*	***f* number** *f* = focal length ÷ diameter

Reading

Concave lenses are shaped like a "cave" and curve inward like a spoon. Light that shines through a concave lens bends to a point (converging beam). Ever notice how when you peep through the hole in a door (especially in a hotel), you can see the entire person standing on the doorstep? There's a concave lens in there making the person appear smaller.

You'll also find these types of lenses in "shoplifting mirrors." Store owners post these mirrors around help them see a larger area than a flat mirror shows, although the images tend to be a lot smaller.

If you have a pair of near-sighted glasses, chances are that the lenses are concave. Near-sighted folks need help seeing things that are far away, and the concave lenses increase the focal point to the right spot on their retina.

Concave lenses work to make things look smaller, so they're not as widely used as convex lenses. You'll find concave lenses inside camera lenses and binoculars to help clear weird optical problems that happen around the edges of a convex lens (called aberration).

Convex lenses bulge outward, bending the light out in a spray (diverging beam). A hand-held magnifying glass is a single concave lens with a handle. These lenses have been used as 'burning glasses' for hundreds of years by placing a small piece of paper at its focal point and using the sun as a light source, you can focus the light energy so intensely that you reach the flash point of the paper (the paper auto-ignites around 450°F).

When you stack a large convex lens above a solar panel, the magnification effect makes it so you can get away with using a smaller photovoltaic cell to get the same amount of energy from the sun. You'll find convex lenses in telescopes, microscopes, binoculars, eyeglasses, and more.

Mirrors: What if you coat one side of the lens with a reflecting silver coating? You get a mirror!

In the video, you'll see me stick wooden skewers into a piece of foam to simulate how the light rays reflect off the surface of the mirror. Note that when the mirror (foam) is straight, the light rays are straight (which is what you see when you look in the bathroom mirror). The light bounces off the straight mirror and zips right back at you, remaining parallel. Now arch the foam. Notice how the light rays (skewers) come to a point (focal point). After the focal point, the rays invert, so the top skewer is now at the bottom and the bottom is now at the top. This is your flipped (inverted) image. This is what you'd see when you look into a concave mirror, like the inside of a metal spoon. You can see your face, but it's upside-down.

Slits A slit allows light from only one source to enter. If you have light from other sources, your light beam is more scattered and your images and lines become blurry. Thin slits can be easily made by placing the edges of two razor blades very close together and securing into place. We're going to **use an anti-slit using a piece of hair, but you can** substitute a thin needle.

Filters: There are hundreds to thousands of different types of filters that are used in photography, astronomy, and sunglasses. A filter can change the amount and type of light allowed through it. For example, if you put on red-tinted glasses, suddenly everything takes on a reddish hue. The red filter blocks the rest of the incoming wavelengths (colors) and only allows the red colors to get to your eyeball.

There are color filters for every wavelength, even IR (infra-red) and UV (ultra-violet). UV filters reduce the haziness in our atmosphere, and are used on most high-end camera lenses, while IR filters are heat-absorbing filters used with hot light sources (like near incandescent bulbs or in overhead projectors).

A neutral density (ND) filter is a grayish-colored filter that reduces the intensity of all colors equally.

Photographers use these filters to get motion blur effects with slow shutter speeds, like a softened waterfall.

Exercises

1. Using only the shape, how can you tell the difference between a convex and a concave lens?

2. Which type of lens makes objects viewed through it appear smaller?

3. Which type of lens makes the objects viewed through it appear larger?

4. How do you get the *f* number?

Answers to Exercises: Optical Bench

1. How can you tell the difference between a convex and a concave lens? (Concave lenses are shaped like a “cave” and curve inward. Convex lenses curve outward.)
2. Which type of lens makes objects viewed through it appear smaller? (concave)
3. Which type of lens makes the objects viewed through it appear larger? (convex)
4. How do you get the f number? (It’s the ratio of focal length over diameter.)

CONVEX LENSES

Convex lenses bulge outwards, bending the light out in a spray (diverging beam). A hand-held magnifying glass is a single concave lens with a handle. These lenses have been used as ‘burning glasses’ for hundreds of years – by placing a small piece of paper at its focal point and using the sun as a light source, you can focus the light energy so intensely that you reach the flash point of the paper (the paper auto-ignites around 450^{o}F).

When you stack a large convex lens above a solar panel, the magnification effect makes it so you can get away with using a smaller photovoltaic cell to get the same amount of energy from the sun. You’ll find convex lenses in telescopes, microscopes, binoculars, eyeglasses, and more.

THIN LENSES

Thin lenses are either diverging or converging lenses that aren't very thick in the middle. We can simplify our ray tracing diagrams and our math equations by assuming a lens is thin.

CONVERGING LENSES

Converging lenses take incoming light and focus it down to a point before diverging out again. You can have single or double convex lenses, depending on the shape of the lens.

But what does the image look like? Remember the line of sight principle where in order to see an object, you have to be able to sight along a line at that object? This idea is how images are formed and what they will look like.

DIVERGING LENSES

The rays spread out when passing through a diverging lens. You can have single or double concave lenses.

HOW IMAGES CHANGE WITH DISTANCE

What happens if you bring an object from far away up close to a lens? How does the image change? The answer is that it depends on what type of lens and the distance it is from the lens.

THE LENS MAKER EQUATION

Imagine you were designing a pair of eyeglasses. How would you know what kind of lens to make? How curved would it be? What would the magnification be? Here's how you use the lens maker's equation to figure out the critical information about a lens.

THE EYE

The eye is a complex structure that detects and focuses light. Light first enters the eye through the cornea, a clear protective layer on the outside of the eye. The pupil, a black opening in the eye, lets light in. In dark rooms, the pupil will become larger, or dilate, in order to let in more light. If the room suddenly becomes bright, the pupil will become smaller. The pupil is surrounded by the brown, blue, grey, or green iris.

After passing through the pupil, light goes to the lens which, like a hand lens, is a clear curved structure that helps focus light on the retina, in the back of the eye. The retina is where the rods and cones are found.

EYE BALLOON

Now we are going to make an eyeball model using a balloon. This experiment should give you a better idea of how your eyes work. The way your brain actually sees things is still a mystery, but using the balloon we can get a good working model of how light gets to your brain.

Eye Balloon

Overview: In this lab, we are going to make an eyeball model using a balloon. This experiment should give you a better idea of how your eyes work. The way your brain actually sees things is still a mystery, but using the balloon we can get a good working model of how light gets to your brain.

What to Learn: We see objects when light traveling from an object enters our eye.

Materials

- biconvex plastic lens
- round balloon, white, 9 inches
- assistant
- votive candle
- black marker
- book of matches
- ruler

Experiment

1. Blow up the balloon until it is about the size of a grapefruit. If it's difficult to inflate, stretch the material a few times or ask an adult to help you.
2. You will need an extra set of hands for this portion. Ask your partner to hold the neck of the balloon closed to keep the air in while you insert the lens into the opening. The lens will need to be inserted perpendicular to the balloon's neck. It will prevent any air from escaping once it's in place. Like your eye, light will enter through the lens and travel toward the back of the balloon.
3. Hold the balloon so that the lens is pointing toward you. Take the lens between your thumb and index finger. Look into the lens into the balloon. You should have a clear view of the inside. Start to twist the balloon a little and notice that the neck gets smaller like your pupils do when exposed to light. Practice opening and closing the balloon's "pupil."
4. Have an adult help you put the candle on the table and light it. Turn out the lights.
5. Put the balloon about 20 to 30 centimeters away from the candle with the lens pointed toward it. The balloon should be between you and the candle. You should see a projection of the candle's flame on the back of the balloon's surface. Move the balloon back and forth in order to better focus the image on the back of the balloon and then proceed with data collection.
6. Describe the image you see on the back of the balloon. How is it different from the flame you see with your eyes? Draw a picture of how the flame looks.
7. The focal length is the distance from the flame to the image on the balloon. Measure this distance and record it.
8. What happens if you lightly push down on the top of the balloon? Does this affect the image? You are experimenting with the affect caused by near-sightedness.
9. To approximate a farsighted eye, gently push in the front and back of the balloon to make it taller. How does this change what you see?

Eye Balloon Data Table

Draw a picture of how the flame looks to you.	
Record the focal length from flame to the image.	
What happens to the image when you push down on the top of the balloon?	
What do you see when you push on the front and back of the balloon to make it taller?	

Reading

First, we'll discuss the parts of the balloon that relate to parts of your eye. The white portion of the balloon represents your sclera, which you may have already guessed is also the white part of your eye. It is actually a coating made of protein that covers the various muscles in your eye and holds everything together.

Of course, the lens you inserted represents the actual lens in your eye. The muscles surrounding the lens are called ciliary muscles and they are represented by the rubber neck of your balloon. The ciliary muscles help to control the amount of light entering your eyes. The retina is in the back of your eye, which is represented by the inside back of your balloon. The retina supports your rods and cones. They collect information about light and color and send it to your brain.

There are no light receptors in the area of your eye where the optic nerve attaches to your eyeball. This is your blind spot and if an image is in this spot, the light reflected off of it doesn't get perceived by your eye.

This is a fun experiment to play with using different ages of people. As folks get older, the shape of the eye changes and the blind spot can actually change. Ask a few fellow adults to help you demonstrate the lab and measure the distance for the blind spot based on how old the adults are. You can also test to see if different people with different vision have different blind spots. For example, is the blind spot the same for someone with 20/20 versus 20/40? Or with or without eyeglasses? Have fun with the different variations!

Exercises

1. How does your eye work like a camera?

2. How can you tell if a lens is double convex?

3. What is the difference between convex and concave?

4. Can you give an example of an everyday object that has both a convex and a concave side?

5. How can you change the balloon to make it like a near-sighted eye?

6. How can you change the balloon to make it like a far-sighted eye?

Answers to Exercises: Eye Balloon

1. How does your eye work like a camera? (Both have lenses, both produce images with lots of components working together.)

2. How can you tell if a lens is double convex? (When you run your fingers across it, you feel two bumps on each side.)

3. What is the difference between convex and concave? (A concave surface curves inward, while a convex surface bulges out.)

4. Can you give an example of an everyday object that has both a convex and a concave side? (spoon)

5. How can you change the balloon to make it near-sighted? (lightly push down on the top)

6. How can you change the balloon to make it far-sighted? (gently push in front and back of the balloon to make it taller)

NEARSIGHTEDNESS AND FARSIGHTEDNESS

The lens in the eye changes shape to bring objects into focus. Myopia (nearsightedness) is the lens' inability to bring objects that are far away into focus. The light gets brought into focus in front of the retina, so the eyeglasses needed to correct for this have diverging lenses.

Hyperopia (farsightedness) is when the image is focused behind the retina, which happens to people later in life. The way to correct it is with a pair of converging lens eyeglasses.

BLIND SPOTS

Your optic nerve can be thought of as a data cord that is plugged in to each eye and connects them to your brain. The area where the nerve connects to the back of your eye creates a blind spot. There are no receptors in this area at all and if something is in that area, you won't be able to see it. This experiment locates your blind spot.

There are no light receptors in the area of your eye where the optic nerve attaches to your eyeball. This is your blind spot and if an image is in this spot, the light reflected off of it doesn't get perceived by your eye. So you don't see it!

Disappearing Frog

Overview: Your optic nerve can be thought of as a data cord that is plugged in to each eye and connects them to your brain. The area where the nerve connects to the back of your eye creates a blind spot. There are no receptors in this area at all and if something is in that area, you won't be able to see it. This experiment locates your blind spot.

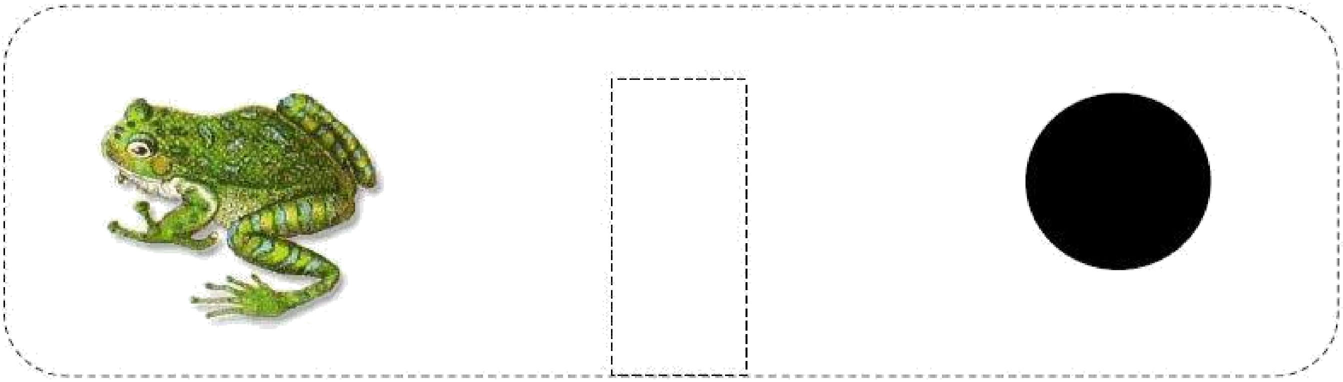
What to Learn: Cones and rods turn the light that enters the eye into images that are transmitted to the brain. Our eyes have a blind spot where the optic nerve connects to the back of the eye because there are no light receptors there.

Materials

- frog and dot printout
- meter stick
- scrap piece of cardboard

Experiment

1. Print out the frog and dot and remove the dotted portion. Attach it to the piece of cardboard, which should have a matching portion removed. You can place the paper and cardboard on the meter stick at the notched area.
2. Now to locate blind spots. First, close your left eye. Look at the frog with your right eye. Can you see the dot and the frog? You should be able to see both at this point, but concentrate on the frog. Now *slowly* move the stick toward you so that the frog is coming toward your eye. Pay attention and stop when the dot disappears from your peripheral vision. At this point, the light hitting the dot and reflecting back toward your eye is hitting the blind spot at the back of your right eyeball, so you can't see it. Record how far your eye is from the card for your right eye.
3. Continue to move the stick toward your face, and at some point you will notice that you are able to see the dot again. Keep moving the stick forward and back. What happens to the dot?
4. Repeat steps 2 and 3 with your left eye, keeping your right eye closed. This time, stare at the dot and watch for the frog to disappear. Move the paper on the stick back and forth *slowly* until you notice the frog disappears. You have found the blind spot for your left eye. Be sure to note the distance the paper is from your eye.



Disappearing Frog Data Table

Student Name	Right or Left Eye?	Distance from Eye to Frog

Exercises

1. What did you notice about the vision of the student and the blind spot that you measured?

2. Why do you think it's important to know where your blind spot is?

Answers to Exercises: Disappearing Frog

1. What did you notice about the vision of the student and the blind spot that you measured? (answers vary)
2. Why do you think it’s important to know where your blind spot is? (so you can expect it and work around it if you need to)

THE LENS OF THE EYE

Do you remember the eye balloon that you made earlier? The white portion of the balloon represents your sclera, which you may have already guessed is also the white part of your eye. It is actually a coating made of protein that covers the various muscle in your eye and holds everything together.

Of course, the lens you inserted represents the actual lens in your eye. The muscles surrounding the lens are called ciliary muscles and they are represented by the rubber neck of your balloon. The ciliary muscles help to control the amount of light entering your eyes.

The retina is in the back of your eye, which is represented by the inside back of your balloon. The retina supports your rods and cones. They collect information about light and color and send it to your brain.

Charles Benhamho (1895) created a toy top painted with the pattern (images on next page). When you spin the disk, arcs of color (called "pattern induced flicker colors") show up around the disk. And different people see different colors!

We can't really say why this happens, but there are a few interesting theories. Your eyeball has two different ways of seeing light: cones and rods. Cones are used for color vision and for seeing bright light, and there are three types of cones (red, green, and blue). Rods are important for seeing in low light.

One possibility is how the human eye is tuned for different colors. Your eyeballs respond at different rates to red, green, and blue colors. The spinning disk triggers different parts of the retina. This alternating response may cause some type of interaction within the nervous system that generates colors.

Another theory is that certain cones take longer react, and thus stay active, for longer amounts of time (though we're still talking milliseconds, here). To put another way, the white color activates all three cones, but then the black deactivates them in a certain sequence, causing your brain to get mixed and unbalanced signals. Your brain does the best it can to figure it out the information it's getting, and "creates" the colors you see in order to make sense of it all.

Neither of these theories explains the colors of Benham's disk completely and the reason behind the illusion remains unsolved. Can you help out these baffled scientists?

Yay! You completed this section! Now it's time for you to solve physics problems on your own:

Benham’s Disk

Overview: Charles Benham (1895) created a toy top painted with a specific pattern. When you spin the black and white pattern, surprising arcs of color (called “pattern induced flicker colors”) show up; and here’s the odd part: Different people see different colors!

What to Learn: The color of light striking an object affects how our eyes see it. The cones and rods inside our eyes collect images that are transmitted to the brain.

Materials

- Benham’s Disk sheet
  string (about 3 feet)
  8 index cards
- glue stick

Experiment

1. Cut out the Benham’s Disks.
2. Glue the disks to index cards for stability. Regular paper tends to flop around when you spin it quickly.
3. Label each disk with a number from 1 to 6 on the back side so you can record your observations in the data table later.
4. Spin the disks using the method you choose. Play with this a bit before you take data so you can get comfortable with how to do the experiment.
5. When you are ready, record your observations in the data table.

Reading

We can’t really say why this effect with Benham’s Disk happens, but there are a few interesting theories. The retina at the back of your eye has a bunch of light-sensitive cells called cones and rods. Your eyeball has two different ways of seeing light: cones and rods. Cones are used for color vision and for seeing bright light, and there are three types of cones (red, green, and blue). Rods are important for seeing in low light and they sense black, white, and gray shades. Together, they turn the light that enters your eye into an image.

Benham's Disk Data Table

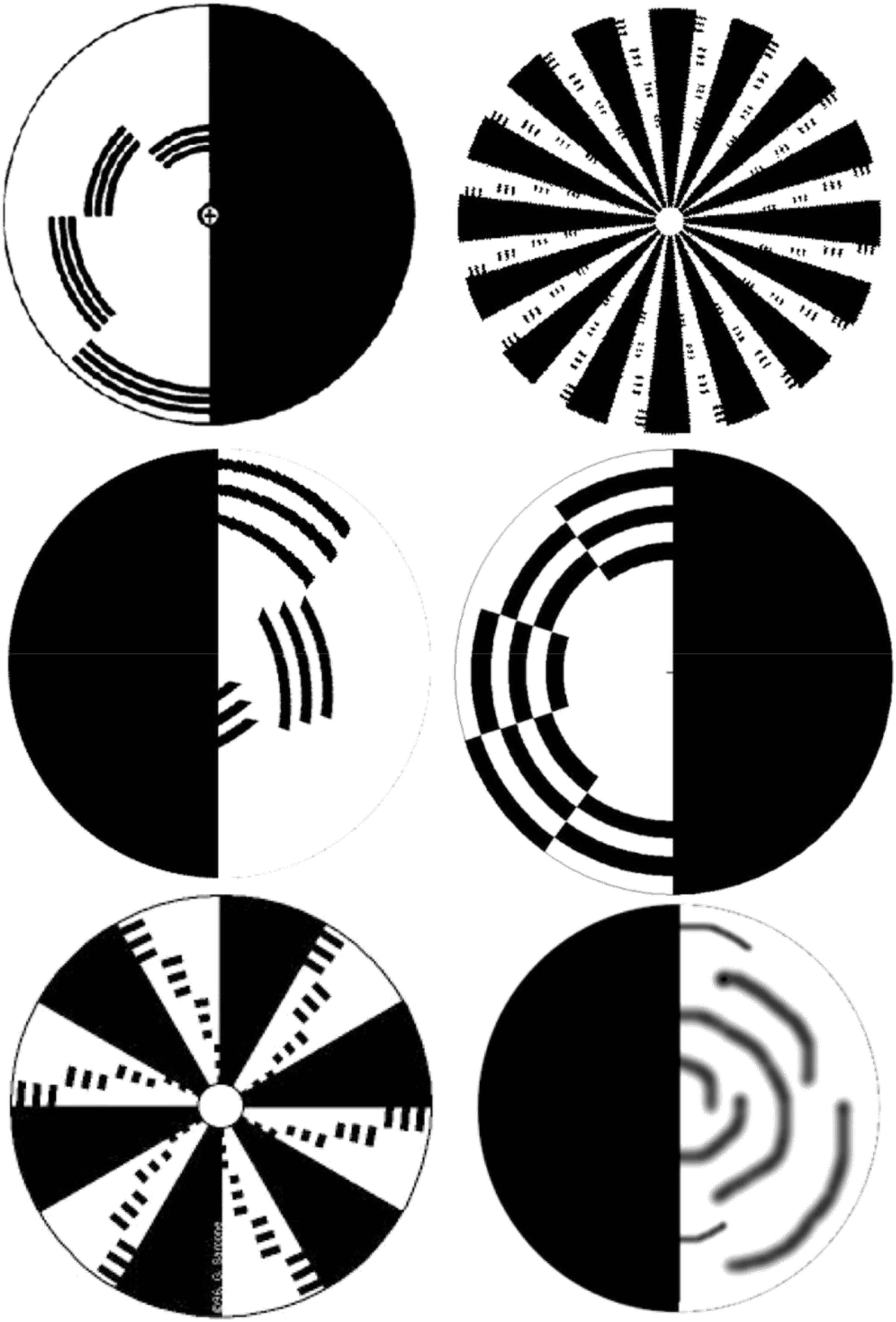
Disk#	**What Color Did You See ?**
1	
2	
3	
4	
5	
6	
Your Own Design #1	
Your Own Design #2	

Exercises

1. What colors were you able to see when the disks were spinning?

2. How did the different patterns look when they were spun?

3. How did speed and direction affect what you saw?



Benham's Disks

Answers to Exercises: Benham's Disk

1. What colors were you able to see when the disks were spinning? (answers vary)
2. How did the different patterns look when they were spun? (answers vary)
3. How did speed and direction affect what you saw? (answers vary)

HOMEWORK PROBLEMS WITH SOLUTIONS

On the following pages is the homework assignment for this unit. When you've completed all the videos from this unit, turn to the next page for the homework assignment. Do your best to work through as many problems as you can. When you finish, grade your own assignment so you can see how much you've learned and feel confident and proud of your achievement!

If there are any holes in your understanding, go back and watch the videos again to make sure you're comfortable with the content before moving onto the next unit. Don't worry too much about mistakes at this point. Just work through the problems again and be totally amazed at how much you're learning.

If you're scoring or keeping a grade-type of record for homework assignments, here's my personal philosophy on using such a scoring mechanism for a course like this:

It's more advantageous to assign a "pass" or "incomplete" score to yourself when scoring your homework assignment instead of a grade or "percent correct" score (like a 85%, or B) simply because students learn faster and more effectively when they build on their successes instead of focusing on their failures.

While working through the course, ask a friend or parent to point to three questions you solved correctly and ask you why or how you solved it.

Any problems you didn't solve correctly simply mean that you'll need to go back and work on them until you feel confident you could handle them when they pop up again in the future.

Student Worksheet for Refraction

After you've worked through the sample problems in the videos, you can work out the problems below to practice doing this yourself. Answers are given on the last page.

Equations:

$M = -d_i/d$

$n_i\sin(\Theta_i) = n_r\sin(\Theta_r)$

$1/f = 1/d_0 + 1/d_i$

$\Theta_{crit} = \sin^{-1}(n_1/n_2) \quad n_1 < n_2$

Practice Problems:

1. You want to throw a toy torpedo to your friend who is in a pool, underwater. Where would you aim in order to get the toy directly to your friend?

2. A ray of light strikes a block of flint glass (n=1.61) at an angle of 40^0 from normal, what is its angle of refraction?

3. A physics student shines a flashlight into a pool (n=1.33) and the angle of refraction is 40^0 from normal, at what angle did the light hit the surface of the water?

4. Light travels from a diamond (n=2.24) to an unknown material at an angle of 45^0 from normal and refracts at an angle of 66^0 from normal. What is the index of refraction of the unknown material?

5. Light travels from air to a cube of unknown material. The light hits the cube at an angle 58^0 from normal and refracts at an angle 25^0 from normal. What is the index of refraction of the unknown material?

6. Calculate the critical angle for an air-water boundary ($n_{air} = 1.00$ $n_{water} = 1.33$).

7. Calculate the critical angle for a Gallium phosphide (n=3.5) and Plexiglas (n=1.51) boundary.

8. An 11.2cm tall candle is placed 39.4cm from a double convex lens with a focal length of 14.3cm. Determine the image distance and image height.

9. Determine the focal length of a double concave lens that produces an image 17.3cm in front of the lens when the object is 29.4cm in front of the lens.

10. What is the focal length of a double convex lens that produces an image that is 3 times the height of the original when the original is 33.0cm from the lens?

Student Worksheet for Refraction

After you've worked through the sample problems in the videos, you can work out the problems below to practice doing this yourself. Answers are given on the last page.

Equations:

$$v = \frac{\omega}{\kappa} = \frac{\lambda}{T} = \lambda \nu$$

$$\nu = \frac{\omega}{2\pi} = \frac{1}{T}$$

$$v = \sqrt{\frac{\tau}{\mu}}$$

Practice Problems:

1. You want to throw a toy torpedo to your friend who is in a pool, underwater. Where would you aim in order to get the toy directly to your friend?

aim below where your friend
appears to be.

2. A ray of light strikes a block of flint glass (n=1.61) at an angle of 40^0 from normal, what is its angle of refraction?

$n_r = 1.61$
$\theta_i = 40°$
$n_i = 1.00$

$n_i \sin\theta_i = n_r \sin\theta_r$

$\theta_r = \sin^{-1}\left(\frac{n_i}{n_r}\sin\theta_i\right) = \sin^{-1}\left(\frac{1}{1.61}\sin(40°)\right)$

$\boxed{\theta_r = 23.53°}$

3. A physics student shines a flashlight into a pool (n=1.33) and the angle of refraction is ~~50~~^{0} from normal, at what angle did the light hit the surface of the water?
40°

$\theta_r = 40°$
$n_r = 1.33$
$n_i = 1.00$

$n_i \sin\theta_i = n_r \sin\theta_r$

$\theta_i = \sin^{-1}\left(\frac{n_r}{n_i}\sin\theta_r\right) = \sin^{-1}\left(1.33\sin(40°)\right)$

$\boxed{\theta_i = 58.75°}$

4. Light travels from a diamond (n=2.24) to an unknown material at an angle of 45^0 from normal and refracts at an angle of 66^0 from normal. What is the index of refraction of the unknown material?

$n_i = 2.24$

$\theta_i = 45°$

$\theta_r = 66°$

$$n_i \sin\theta_i = n_r \sin\theta_r$$

$$n_r = \frac{n_i \sin\theta_i}{\sin\theta_r} = \frac{2.24(\sin 45)}{\sin(66)}$$

$$\boxed{n_r = 1.73}$$

5. Light travels from air to a cube of unknown material. The light hits the cube at an angle ~~25^0~~ $58°$ from normal and refracts at an angle ~~58^0~~ $25°$ from normal. What is the index of refraction of the unknown material?

$n_i = 1.00$

$\theta_r = 25°$

$\theta_i = 58°$

$$n_i \sin\theta_i = n_r \sin\theta_r$$

$$n_r = n_i \frac{\sin\theta_i}{\sin\theta_r} = 1.00 \frac{\sin(58°)}{\sin(25°)}$$

$$\boxed{n_r = 2.01}$$

6. Calculate the critical angle for an air-water boundary ($n_{air} = 1.00$ $n_{water} = 1.33$).

$n_1 = 1.00$

$n_2 = 1.33$

$n_1 < n_2$

$$\theta_{crit} = \sin^{-1}\left(\frac{1.0}{1.33}\right)$$

$$\boxed{\theta_{crit} = 48.75°}$$

7. Calculate the critical angle for a Gallium phosphide (n=3.5) and Plexiglas (n=1.51) boundary.

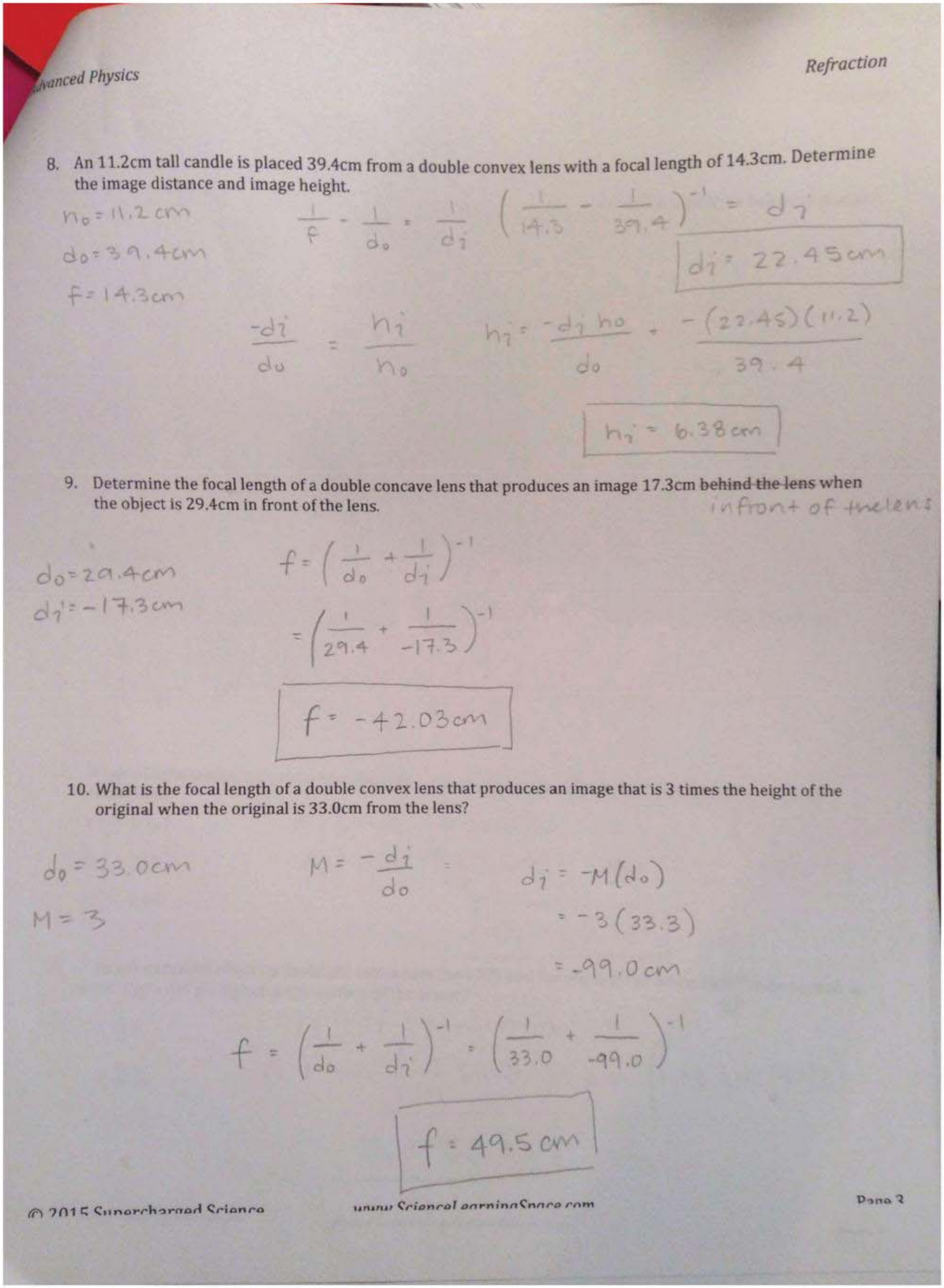
$n_1 = 1.51$

$n_2 = 3.5$

$n_1 < n_2$

$$\theta_{crit} = \sin^{-1}\left(\frac{1.51}{3.5}\right)$$

$$\boxed{\theta_{crit} = 25.56°}$$



8. An 11.2cm tall candle is placed 39.4cm from a double convex lens with a focal length of 14.3cm. Determine the image distance and image height.

$h_o = 11.2\text{ cm}$

$d_o = 39.4\text{cm}$

$f = 14.3\text{cm}$

$$\frac{1}{f} - \frac{1}{d_o} = \frac{1}{d_i} \qquad \left(\frac{1}{14.3} - \frac{1}{39.4}\right)^{-1} = d_i$$

$$\boxed{d_i = 22.45\text{ cm}}$$

$$\frac{-d_i}{d_o} = \frac{h_i}{h_o} \qquad h_i = \frac{-d_i h_o}{d_o} = \frac{-(22.45)(11.2)}{39.4}$$

$$\boxed{h_i = 6.38\text{ cm}}$$

9. Determine the focal length of a double concave lens that produces an image 17.3cm ~~behind the lens~~ in front of the lens when the object is 29.4cm in front of the lens.

$d_o = 29.4\text{cm}$

$d_i = -17.3\text{cm}$

$$f = \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1}$$

$$= \left(\frac{1}{29.4} + \frac{1}{-17.3}\right)^{-1}$$

$$\boxed{f = -42.03\text{cm}}$$

10. What is the focal length of a double convex lens that produces an image that is 3 times the height of the original when the original is 33.0cm from the lens?

$d_o = 33.0\text{cm}$

$M = 3$

$$M = -\frac{d_i}{d_o} \qquad d_i = -M(d_o)$$

$$= -3(33.3)$$

$$= -99.0\text{ cm}$$

$$f = \left(\frac{1}{d_o} + \frac{1}{d_i}\right)^{-1} = \left(\frac{1}{33.0} + \frac{1}{-99.0}\right)^{-1}$$

$$\boxed{f = 49.5\text{ cm}}$$

ADVANCED PLACEMENT PHYSICS 1 EQUATIONS, EFFECTIVE 2015

CONSTANTS AND CONVERSION FACTORS			
Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	Coulomb's law constant,	$k = 1/4\pi\varepsilon_0 = 9.0 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2$
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Universal gravitational constant,	$G = 6.67 \times 10^{-11}\ \text{m}^3/\text{kg}\cdot\text{s}^2$
Speed of light,	$c = 3.00 \times 10^8$ m/s	Acceleration due to gravity at Earth's surface,	$g = 9.8\ \text{m/s}^2$

UNIT SYMBOLS								
	meter,	m	kelvin,	K	watt,	W	degree Celsius,	°C
	kilogram,	kg	hertz,	Hz	coulomb,	C		
	second,	s	newton,	N	volt,	V		
	ampere,	A	joule,	J	ohm,	Ω		

PREFIXES

Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.
II. Assume air resistance is negligible unless otherwise stated.
III. In all situations, positive work is defined as work done on a system.
IV. The direction of current is conventional current: the direction in which positive charge would drift.
V. Assume all batteries and meters are ideal unless otherwise stated.

ADVANCED PLACEMENT PHYSICS 1 EQUATIONS, EFFECTIVE 2015

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x (x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_n\right|$$

$$a_c = \frac{v^2}{r}$$

$$\vec{p} = m\vec{v}$$

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

$$K = \frac{1}{2} m v^2$$

$$\Delta E = W = F_{\parallel} d = F d \cos\theta$$

$$P = \frac{\Delta E}{\Delta t}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$x = A \cos(2\pi f t)$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$\tau = r_{\perp} F = r F \sin\theta$$

$$L = I\omega$$

$$\Delta L = \tau\,\Delta t$$

$$K = \frac{1}{2} I \omega^2$$

$$\left|\vec{F}_s\right| = k\left|\vec{x}\right|$$

$$U_s = \frac{1}{2} k x^2$$

$$\rho = \frac{m}{V}$$

a = acceleration
A = amplitude
d = distance
E = energy
f = frequency
F = force
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
V = volume
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
ρ = density
τ = torque
ω = angular speed

$$\Delta U_g = m g \,\Delta y$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_p = 2\pi \sqrt{\frac{\ell}{g}}$$

$$\left|\vec{F}_g\right| = G \frac{m_1 m_2}{r^2}$$

$$\vec{g} = \frac{\vec{F}_g}{m}$$

$$U_G = -\frac{G m_1 m_2}{r}$$

ELECTRICITY

$$\left|\vec{F}_E\right| = k \left|\frac{q_1 q_2}{r^2}\right|$$

$$I = \frac{\Delta q}{\Delta t}$$

$$R = \frac{\rho \ell}{A}$$

$$I = \frac{\Delta V}{R}$$

$$P = I\,\Delta V$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

A = area
F = force
I = current
ℓ = length
P = power
q = charge
R = resistance
r = separation
t = time
V = electric potential
ρ = resistivity

WAVES

$$\lambda = \frac{v}{f}$$

f = frequency
v = speed
λ = wavelength

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2} bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

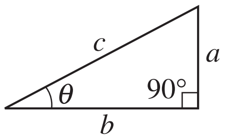
Rectangular solid
$V = \ell w h$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r \ell + 2\pi r^2$

Sphere
$V = \frac{4}{3} \pi r^3$
$S = 4\pi r^2$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle
$c^2 = a^2 + b^2$
$\sin\theta = \frac{a}{c}$
$\cos\theta = \frac{b}{c}$
$\tan\theta = \frac{a}{b}$



ADVANCED PLACEMENT PHYSICS 2 EQUATIONS, EFFECTIVE 2015

CONSTANTS AND CONVERSION FACTORS

Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ m^3/kg$\cdot$s^2
Universal gas constant,	$R = 8.31$ J/(mol$\cdot$K)	Acceleration due to gravity at Earth's surface,	$g = 9.8$ m/s^2
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K		

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\text{ J}\cdot\text{s} = 4.14 \times 10^{-15}\text{ eV}\cdot\text{s}$
	$hc = 1.99 \times 10^{-25}\text{ J}\cdot\text{m} = 1.24 \times 10^{3}\text{ eV}\cdot\text{nm}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2$
Coulomb's law constant,	$k = 1/4\pi\varepsilon_0 = 9.0 \times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\text{ (T}\cdot\text{m)/A}$
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}\text{ (T}\cdot\text{m)/A}$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\text{ N/m}^2 = 1.0 \times 10^{5}\text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. The frame of reference of any problem is assumed to be inertial unless otherwise stated.
II. In all situations, positive work is defined as work done on a system.
III. The direction of current is conventional current: the direction in which positive charge would drift.
IV. Assume all batteries and meters are ideal unless otherwise stated.
V. Assume edge effects for the electric field of a parallel plate capacitor unless otherwise stated.
VI. For any isolated electrically charged object, the electric potential is defined as zero at infinite distance from the charged object.

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS, EFFECTIVE 2015

MECHANICS

$$v_x = v_{x0} + a_x t$$

$$x = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$$

$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$

$$\left|\vec{F}_f\right| \le \mu \left|\vec{F}_n\right|$$

$$a_c = \frac{v^2}{r}$$

$$\vec{p} = m\vec{v}$$

$$\Delta\vec{p} = \vec{F}\,\Delta t$$

$$K = \frac{1}{2} m v^2$$

$$\Delta E = W = F_{\parallel} d = F d \cos\theta$$

$$P = \frac{\Delta E}{\Delta t}$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$x = A\cos(\omega t) = A\cos(2\pi f t)$$

$$x_{cm} = \frac{\sum m_i x_i}{\sum m_i}$$

$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I} = \frac{\vec{\tau}_{net}}{I}$$

$$\tau = r_{\perp} F = r F \sin\theta$$

$$L = I\omega$$

$$\Delta L = \tau\,\Delta t$$

$$K = \frac{1}{2} I \omega^2$$

$$\left|\vec{F}_s\right| = k\left|\vec{x}\right|$$

a = acceleration
A = amplitude
d = distance
E = energy
F = force
f = frequency
I = rotational inertia
K = kinetic energy
k = spring constant
L = angular momentum
ℓ = length
m = mass
P = power
p = momentum
r = radius or separation
T = period
t = time
U = potential energy
v = speed
W = work done on a system
x = position
y = height
α = angular acceleration
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed

$$U_s = \frac{1}{2} k x^2$$

$$\Delta U_g = m g \,\Delta y$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\left|\vec{F}_g\right| = G\frac{m_1 m_2}{r^2}$$

$$\vec{g} = \frac{\vec{F}_g}{m}$$

$$U_G = -\frac{G m_1 m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\frac{\left|q_1 q_2\right|}{r^2}$$

$$\vec{E} = \frac{\vec{F}_E}{q}$$

$$\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2}$$

$$\Delta U_E = q\,\Delta V$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\left|\vec{E}\right| = \left|\frac{\Delta V}{\Delta r}\right|$$

$$\Delta V = \frac{Q}{C}$$

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

$$E = \frac{Q}{\varepsilon_0 A}$$

$$U_C = \frac{1}{2} Q\Delta V = \frac{1}{2} C(\Delta V)^2$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$R = \frac{\rho\ell}{A}$$

$$P = I\,\Delta V$$

$$I = \frac{\Delta V}{R}$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
ℓ = length
P = power
Q = charge
q = point charge
R = resistance
r = separation
t = time
U = potential (stored) energy
V = electric potential
v = speed
κ = dielectric constant
ρ = resistivity
θ = angle
Φ = flux

$$\vec{F}_M = q\vec{v} \times \vec{B}$$

$$\left|\vec{F}_M\right| = \left|q\vec{v}\right|\left|\sin\theta\right|\left|\vec{B}\right|$$

$$\vec{F}_M = I\vec{\ell} \times \vec{B}$$

$$\left|\vec{F}_M\right| = \left|I\vec{\ell}\right|\left|\sin\theta\right|\left|\vec{B}\right|$$

$$\Phi_B = \vec{B} \bullet \vec{A}$$

$$\Phi_B = \left|\vec{B}\right|\cos\theta\left|\vec{A}\right|$$

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t}$$

$$\mathcal{E} = B\ell v$$

ADVANCED PLACEMENT PHYSICS 2 EQUATIONS, EFFECTIVE 2015

FLUID MECHANICS AND THERMAL PHYSICS

$$\rho = \frac{m}{V}$$

$$P = \frac{F}{A}$$

$$P = P_0 + \rho g h$$

$$F_b = \rho V g$$

$$A_1 v_1 = A_2 v_2$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\frac{Q}{\Delta t} = \frac{kA\,\Delta T}{L}$$

$$PV = nRT = Nk_B T$$

$$K = \frac{3}{2}k_B T$$

$$W = -P\,\Delta V$$

$$\Delta U = Q + W$$

A = area
F = force
h = depth
k = thermal conductivity
K = kinetic energy
L = thickness
m = mass
n = number of moles
N = number of molecules
P = pressure
Q = energy transferred to a system by heating
T = temperature
t = time
U = internal energy
V = volume
v = speed
W = work done on a system
y = height
ρ = density

MODERN PHYSICS

$$E = hf$$

$$K_{\max} = hf - \phi$$

$$\lambda = \frac{h}{p}$$

$$E = mc^2$$

E = energy
f = frequency
K = kinetic energy
m = mass
p = momentum
λ = wavelength
ϕ = work function

WAVES AND OPTICS

$$\lambda = \frac{v}{f}$$

$$n = \frac{c}{v}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f}$$

$$|M| = \left|\frac{h_i}{h_o}\right| = \left|\frac{s_i}{s_o}\right|$$

$$\Delta L = m\lambda$$

$$d\sin\theta = m\lambda$$

d = separation
f = frequency or focal length
h = height
L = distance
M = magnification
m = an integer
n = index of refraction
s = distance
v = speed
λ = wavelength
θ = angle

GEOMETRY AND TRIGONOMETRY

Rectangle

$$A = bh$$

Triangle

$$A = \frac{1}{2}bh$$

Circle

$$A = \pi r^2$$
$$C = 2\pi r$$

Rectangular solid

$$V = \ell w h$$

Cylinder

$$V = \pi r^2 \ell$$
$$S = 2\pi r \ell + 2\pi r^2$$

Sphere

$$V = \frac{4}{3}\pi r^3$$
$$S = 4\pi r^2$$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

Right triangle

$$c^2 = a^2 + b^2$$

$$\sin\theta = \frac{a}{c}$$

$$\cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{a}{b}$$