Section 2.4: Applications of Systems

Objective: Solve application problems by setting up a system of equations.

One application of system of equations are known as value problems. Value problems are ones where each variable has a value attached to it. For example, if our variable is the number of nickels in a person's pocket, those nickels would have a value of five cents each. We will use a table to help us set up and solve value problems. The basic structure of the table is shown below.

	Number	Value	Total
Item 1			
Item 2			
Total			

The first column in the table is used for the number of things we have. Quite often these will be our variables. The second column is used for the value each item has. The third column is used for the total value which we calculate by multiplying the number by the value. For example, if we have 7 dimes, each with a value of 10 cents, the total value is $(7)(10) = 70$ cents. The last row of the table is for totals. We only will use the third row (also marked total) for the totals that are given to use. This means sometimes this row may have some blanks in it.

Once the table is filled in we can easily make equations by adding each column, setting it equal to the total at the bottom of the column. This is shown in the following example.

Example 1.
In a child's bank there are 11 coins that have a value of $1.85. The coins are either quarters or dimes. How many coins of each type does the child have?

	Number	Value	Total
Quarter	q	25	
Dime	d	10	
Total			

Using value table; use q for quarters. d for dimes
Each quarter's value is 25 cents, each dime's is 10 cents

	Number	Value	Total
Quarter	q	25	$25q$
Dime	d	10	$10d$
Total			

Multiply number by value to get totals

	Number	Value	Total
Quarter	q	25	$25q$

We have 11 coins total. This is the number total.
We have 1.85 for the final total,

83

Dime	d	10	$10d$	
Total	11		185	

Write final total in cents (185) because 25 and 10 are written using cents

$$q + d = 11$$
$$25q + 10d = 185$$

First and last columns are our equations by adding

Solve by either addition or substitution.
Here we will use the addition method.

$$-10(q + d) = (11)(-10)$$
$$-10q - 10d = -110$$

Multiply first equation by -10

$$\begin{aligned} -10q - 10d &= -110 \\ 25q + 10d &= 185 \\ \hline 15q &= 75 \end{aligned}$$

Add together equations

$$\frac{15q}{15} = \frac{75}{15}$$

Divide both sides by 15

$$q = 5$$

We have our q, number of quarters is 5

$$(5) + d = 11$$

Plug into one of original equations

$$\begin{aligned} -5 -5 \\ \hline d = 6 \end{aligned}$$

Subtract 5 from both sides

$$d = 6$$

We have our d, number of dimes is 6

5 quarters and 6 dimes

Our Solution

World View Note: American coins are the only coins that do not state the value of the coin in numeric form. On the back of the dime it says "one dime" (not 10 cents). On the back of the quarter it says "one quarter" (not 25 cents). On the penny it says "one cent" (not 1 cent). The rest of the world (Euros, Yen, Pesos, etc) all write the value as a number so people who don't speak the language can easily use the coins.

Ticket sales also have a value. Often different types of tickets sell for different prices (values). These problems can be solved in much the same way.

Example 2.
There were 41 tickets sold for an event. Tickets for children cost $1.50 and tickets for adults cost $2.00. Total receipts for the event were $73.50. How many of each type of ticket were sold?

	Number	Value	Total
Child	c	1.5	
Adult	a	2	
Total			

Using our value table, c for child, a for adult
Child tickets have value 1.50; adult value is 2.00
(we can drop the zeros after the decimal point)

	Number	Value	Total
Child	c	1.5	$1.5c$
Adult	a	2	$2a$
Total			

Multiply number by value to get totals

	Number	Value	Total
Child	c	1.5	$1.5c$
Adult	a	2	$2a$
Total	41		73.5

We have 41 tickets sold. This is our number total
The final total was 73.50
Write in dollars as 1.5 and 2 are also dollars

$$c + a = 41$$
$$1.5c + 2a = 73.5$$

First and last columns are our equations by adding
We can solve by either addition or substitution

$$c + a = 41$$
$$\underline{-c \quad\quad -c}$$
$$a = 41 - c$$
$$1.5c + 2(41 - c) = 73.5$$
$$1.5c + 82 - 2c = 73.5$$
$$-0.5c + 82 = 73.5$$
$$\underline{-82 \quad -82}$$
$$\frac{-0.5c = -8.5}{-0.5 \quad\; -0.5}$$

We will solve by substitution.
Solve for a by subtracting c

Substitute into untouched equation
Distribute
Combine like terms
Subtract 82 from both sides
Divide both sides by -0.5

$$c = 17$$
$$a = 41 - (17)$$
$$a = 24$$
$$\text{17 child tickets and 24 adult tickets}$$

We have c, number of child tickets is 17
Plug into $a =$ equation to find a
We have our a, number of adult tickets is 24
Our Solution

Some problems will not give us the total number of items we have. Instead they will give a relationship between the items. Here we will have statements such as "There are twice as many dimes as nickels". While it is clear that we need to multiply one variable by 2, it may not be clear which variable gets multiplied by 2.

Generally the equations are backwards from the English sentence. If there are twice as many nickels as dimes, than we multiply the other variable (dimes) by two. So, the equation would be $n = 2d$. This type of problem is in the next example.

Example 3.

A man has a collection of stamps made up of 5¢ stamps and 8¢ stamps. There are three times as many 8¢ stamps as 5¢ stamps. The total value of all the stamps is $3.48. How many of each stamp does he have?

	Number	Value	Total
Five	f	5	
Eight	e	8	
Total			

Use value table, f for five cent stamp, and e for eight
Also list value of each stamp under value column

	Number	Value	Total
Five	f	5	$5f$
Eight	e	8	$8e$
Total			

Multiply number by value to get total

	Number	Value	Total
Five	f	5	$5f$
Eight	e	8	$8e$
Total			348

The final total was 348 (written in cents)
We do not know the total number, this is left blank.

$$e = 3f$$ **3 times as many 8¢ stamps as 5¢ stamps**

$$5f + 8e = 348$$ Total column gives second equation

$$5f + 8(3f) = 348$$ Substitution, substitute first equation in second

$$5f + 24f = 348$$ Multiply first

$$\frac{29f}{29} = \frac{348}{29}$$ Combine like terms
Divide both sides by 39

$$f = 12$$ We have f. There are 12 five cent stamps

$$e = 3(12)$$ Plug into first equation

$$e = 36$$ We have e, There are 36 eight cent stamps

12 five cent, 36 eight cent stamps   Our Solution

The same process for solving value problems can be applied to solving interest problems. Our table titles will be adjusted slightly as we do so.

	Invest	Rate	Interest
Account 1			
Account 2			
Total			

Our first column is for the amount invested in each account. The second column is the interest rate earned (written as a decimal - move decimal point twice left), and the last column is for the amount of interest earned. Just as before, we multiply the investment amount by the rate to find the final column, the interest earned. This is shown in the following example.

Example 4.

A woman invests $4000 in two accounts, one at 6% interest, the other at 9% interest for one year. At the end of the year she had earned $270 in interest. How much did she have invested in each account?

	Invest	Rate	Interest
Account 1	x	0.06	
Account 2	y	0.09	
Total			

Use our investment table, x and y for accounts
Fill in interest rates as decimals
Note: 6% = 0.06 and 9% = 0.09

	Invest	Rate	Interest
Account 1	x	0.06	$0.06x$
Account 2	y	0.09	$0.09y$
Total			

Multiply across to find interest earned.

	Invest	Rate	Interest
Account 1	x	0.06	$0.06x$
Account 2	y	0.09	$0.09y$
Total	4000		270

Total investment is 4000,
Total interest was 270

$$x + y = 4000$$
$$0.06x + 0.09y = 270$$

First and last column give our two equations
Solve by either substitution or addition

$$(-0.06)(x + y) = (4000)(-0.06)$$
$$-0.06x - 0.06y = -240$$

Using addition, multiply the first equation by -0.06

$$-0.06x - 0.06y = -240$$
$$\underline{0.06x + 0.09y = 270}$$

Add equations together

$$\frac{0.03y}{0.03} = \frac{30}{0.03}$$

Divide both sides by 0.03

$$y = 1000$$ We have $y = \$1000$ invested at 9%

$$x + 1000 = 4000$$ Plug into original equation

$$\underline{-1000 \quad -1000}$$ Subtract 1000 from both sides

$$x = 3000$$ We have $x = \$3000$ invested at 6%

$\$1000$ at 9% and $\$3000$ at 6% Our Solution

The same process can be used to find an unknown interest rate.

Example 5.

John invests $5000 in one account and $8000 in an account paying 4% more in interest. He earned $1230 in interest after one year. At what rates did he invest?

	Invest	Rate	Interest
Account 1	5000	x	
Account 2	8000	$x+0.04$	
Total			

Use our investment table. Use x for first rate
The second rate is 4% higher, or $x+0.04$
Be sure to write this rate as a decimal!

	Invest	Rate	Interest
Account 1	5000	x	$5000x$
Account 2	8000	$x+0.04$	$8000(x+0.04)$
Total			

Multiply to fill in interest column.
Be sure to distribute $8000(x+0.04)$

	Invest	Rate	Interest
Account 1	5000	x	$5000x$
Account 2	8000	$x+0.04$	$8000x+320$
Total			1230

Total interest was 1230.

$$5000x + 8000x + 320 = 1230$$ Last column gives our equation
$$13000x + 320 = 1230$$ Combine like terms
$$\underline{-320 \quad -320}$$ Subtract 320 from both sides
$$\frac{13000x}{13000} = \frac{910}{13000}$$ Divide both sides by 13000

$$x = 0.07$$ We have our x, 0.07 or 7% interest
$$(0.07) + 0.04$$ Second account is 4% higher
$$0.11$$ The account with \$8000 is at 0.11 or 11% interest
$$\$5000 \text{ at } 7\%, \text{ and } \$8000 \text{ at } 11\%$$ Our Solution

2.4 Practice

Solve.

1) A collection of dimes and quarters is worth $15.25. There are 103 coins in all. How many of each are there?

2) A collection of half dollars and nickels is worth $13.40. There are 34 coins in all. How many of each are there?

3) The attendance at a school concert was 578. Admission was $2.00 for adults and $1.50 for children. The total receipts were $985.00. How many adults and how many children attended?

4) A purse contains $3.90 made up of dimes and quarters. If there are 21 coins in all, how many dimes and how many quarters are there?

5) A boy has $2.25 in nickels and dimes. If there are twice as many dimes as nickels, how many of each kind does he have?

6) $3.75 is made up of quarters and half dollars. If the number of quarters exceeds the number of half dollars by 3, how many coins of each denomination are there?

7) A collection of 27 coins consisting of nickels and dimes amounts to $2.25. How many coins of each kind are there?

8) $3.25 in dimes and nickels were distributed among 45 boys. If each received one coin, how many received a dime and how many received a nickel?

9) There were 429 people at a play. Admission was $1 each for adults and 75 cents each for children. The receipts were $372.50. How many children and how many adults attended?

10) There were 200 tickets sold for a women's basketball game. Tickets for students were 50 cents each and for adults 75 cents each. The total amount of money collected was $132.50. How many of each type of ticket was sold?

11) There were 203 tickets sold for a volleyball game. For activity-card holders, the price was $1.25 each and for non-card holders the price was S2 each. The total amount of money collected was $310. How many of each type of ticket was sold?

12) At a local ball game the hotdogs sold for $2.50 each and the hamburgers sold for $2.75 each. There were 131 total sandwiches sold for a total value of $342. How many of each sandwich was sold?

13) At a recent Vikings game $445 in admission tickets was taken in. The cost of a student ticket was $1.50 and the cost of a non-student ticket was $2.50. A total of 232 tickets were sold. How many students and how many nonstudents attended the game?

14) A bank contains 27 coins in dimes and quarters. The coins have a total value of $4.95. Find the number of dimes and quarters in the bank.

15) A coin purse contains 18 coins in nickels and dimes. The coins have a total value of $1.15. Find the number of nickels and dimes in the coin purse.

16) A business executive bought 40 stamps for $9.60. The purchase included 25¢ stamps and 20¢ stamps. How many of each type of stamp were bought?

17) A postal clerk sold some 15¢ stamps and some 25¢ stamps. Altogether, 15 stamps were sold for a total cost of $3.15. How many of each type of stamp were sold?

18) A drawer contains 15¢ stamps and 18¢ stamps. The number of 15¢ stamps is four less than three times the number of 18¢ stamps. The total value of all the stamps is $1.29. How many 15¢ stamps are in the drawer?

19) The total value of dimes and quarters in a bank is $6.05. There are six more quarters than dimes. Find the number of each type of coin in the bank.

20) A child's piggy bank contains 44 coins in quarters and dimes. The coins have a total value of $8.60. Find the number of quarters in the bank.

21) A coin bank contains nickels and dimes. The number of dimes is 10 less than twice the number of nickels. The total value of all the coins is $2.75. Find the number of each type of coin in the bank.

22) A total of 26 bills are in a cash box. Some of the bills are one dollar bills, and the rest are five dollar bills. The total amount of cash in the box is $50. Find the number of each type of bill in the cash box.

23) A bank teller cashed a check for $200 using twenty dollar bills and ten dollar bills. In all, twelve bills were handed to the customer. Find the number of twenty dollar bills and the number of ten dollar bills.

24) A collection of stamps consists of 22¢ stamps and 40¢ stamps. The number of 22¢ stamps is three more than four times the number of 40¢ stamps. The total value of the stamps is $8.34. Find the number of 22¢ stamps in the collection.

25) A total of $27000 is invested, part of it at 12% and the rest at 13%. The total interest after one year is $3385. How much was invested at each rate?

26) A total of $50000 is invested, part of it at 5% and the rest at 7.5%. The total interest after one year is $3250. How much was invested at each rate?

27) A total of $9000 is invested, part of it at 10% and the rest at 12%. The total interest after one year is $1030. How much was invested at each rate?

28) A total of $18000 is invested, part of it at 6% and the rest at 9%. The total interest after one year is $1248. How much was invested at each rate?

29) An inheritance of $10000 is invested in 2 ways, part at 9.5% and the remainder at 11%. The combined annual interest was $1038.50. How much was invested at each rate?

30) Kerry earned a total of $900 last year on his investments. If $7000 was invested at a certain rate of return and $9000 was invested in a fund with a rate that was 2% higher, find the two rates of interest.

31) Jason earned $256 interest last year on his investments. If $1600 was invested at a certain rate of return and $2400 was invested in a fund with a rate that was double the rate of the first fund, find the two rates of interest.

32) Millicent earned $435 last year in interest. If $3000 was invested at a certain rate of return and $4500 was invested in a fund with a rate that was 2% lower, find the two rates of interest.

33) A total of $8500 is invested, part of it at 6% and the rest at 3.5%. The total interest after one year is $385. How much was invested at each rate?

34) A total of $12000 was invested, part of it at 9% and the rest at 7.5%. The total interest after one year is $1005. How much was invested at each rate?

35) A total of $15000 is invested, part of it at 8% and the rest at 11%. The total interest after one year is $1455. How much was invested at each rate?

36) A total of $17500 is invested, part of it at 7.25% and the rest at 6.5%. The total interest after one year is $1227.50. How much was invested at each rate?

37) A total of $6000 is invested, part of it at 4.25% and the rest at 5.75%. The total interest after one year is $300. How much was invested at each rate?

38) A total of $14000 is invested, part of it at 5.5% and the rest at 9%. The total interest after one year is $910. How much was invested at each rate?

39) A total of $11000 is invested, part of it at 6.8% and the rest at 8.2%. The total interest after one year is $797. How much was invested at each rate?

40) An investment portfolio earned $2010 in interest last year. If $3000 was invested at a certain rate of return and $24000 was invested in a fund with a rate that was 4% lower, find the two rates of interest.

41) Samantha earned $1480 in interest last year on her investments. If $5000 was invested at a certain rate of return and $11000 was invested in a fund with a rate that was two-thirds the rate of the first fund, find the two rates of interest.

2.4 Answers

1) 33 quarters, 70 dimes
2) 26 half dollars, 8 nickels
3) 236 adult, 342 child
4) 9 dimes, 12 quarters
5) 9 nickels, 18 dimes
6) 7 quarters, 4 half dollars
7) 9 nickels, 18 dimes
8) 25 nickels, 20 dimes
9) 203 adults, 226 child
10) 130 adults, 70 students
11) 128 card, 75 non-card
12) 73 hot dogs, 58 hamburgers
13) 135 students, 97 non-students
14) 12 dimes, 15 quarters
15) 13 nickels, 5 dimes
16) 8 20¢ stamps, 32 25¢ stamps
17) 6 15¢ stamps, 9 25¢ stamps
18) 5 15¢ stamps
19) 13 dimes, 19 quarters
20) 28 quarters
21) 15 nickels, 20 dimes
22) 20 $1 bills, 6 $5 bills
23) 8 $20 bills, 4 $10 bills
24) 27 22¢ stamps
25) $12500 @ 12% , $14500 @ 13%
26) $20000 @ 5%, $30000 @ 7.5%
27) $2500 @ 10%, $6500 @ 12%
28) $12400 @ 6%, $5600 @ 9%
29) $4100 @ 9.5%, $5900 @ 11%
30) $7000 @ 4.5%, $9000 @ 6.5%
31) $1600 @ 4%; $2400 @ 8%
32) $3000 @ 7%, $4500 @ 5%
33) $3500 @ 6%; $5000 @ 3.5%
34) $7000 @ 9%, $5000 @ 7.5%
35) $6500 @ 8%; $8500 @ 11%
36) $12000 @ 7.25%, $5500 @ 6.5%
37) $3000 @ 4.25%; $3000 @ 5.75%
38) $10000 @ 5.5%, $4000 @ 9%
39) $7500 @ 6.8%; $3500 @ 8.2%
40) $3000 @ 11%; $24000 @ 7%
41) $5000 @ 12%, $11000 @ 8%