Chapter 3
ALGEBRA

Overview

Algebra

3.1 Linear Equations and Applications
3.2 More Linear Equations
3.3 Equations with Exponents

Section 3.1
LinearEquations

3+ “what” = 7? If you have come through arithmetic, the answer is fairly obvious: 4.

However, if I were to ask something like:

2 times “what” plus 5 all divided by 7, then minus 6 = 5? There tends to be a little more difficulty in popping out the answer. The beauty of math is that it allows us to write down all of that stuff and then systematically make it simpler and simpler until we have only the number left. Wonderful.

We start with the easy ones to find out all of the rules and then we will build up to the big ones.

3+ “what” = 7

First, we need to adjust the fact that we are going to be writing “what” all the time. A very common thing is to put a letter in that place that could represent any number. We call that a **variable**. We replace the word “what” with “x” (or you could use p, q, r, f, m, l . . .) So our equation becomes:

$$3 + x = 7$$

The whole goal of math is to find the number that makes that statement true. We already know that the number is 4. We would write:

$$x = 4$$

Now, look at what happened to our original equation. Do you see that the right side is missing a 3 and the left side is now 3 lower as well. This gives us some insight into what we can do to equations! Try another one:

$$x + 8 = 10$$

What number would make that statement true? If x were equal to 2, it would work. We write:

$$x = 2$$

Notice how we get the number that would work by subtracting that 8 from both sides of the equation.

Let’s see if it works with some other equations:

$x - 7 = 2$ $\qquad\qquad$ $x - 3 = 10$

With these two equations, the answers are:

$x = 9$ $\qquad$ and $\qquad$ $x = 13$

We got the answers by adding the 7 and the 3 to the right hand sides. This brings up a good point. In the first couple of equations that we did, we subtracted when the equation was adding. In the next two equations, we added when the equation was using subtraction. Let’s look at what happens when we start doing multiplication:

$$4x = 20.$$

What number would work? That is right, 5.

$x = 5$

What would you do to 20 to get 5? Divide by 4. Holy smokes! That is the exact opposite of what the equation is doing. Here is another:

$\frac{x}{7} = 4$

What number divided by 7 equals 4? That's it, 28. We times 4 by 7 to get that answer. Multiplying by 7 is the exact opposite of dividing by 7.

This leads us to a couple of conclusions that form the basis for everything we will do in Algebra:

1) **When we want to get rid of numbers that are surrounding the variable, we need to do the opposite (technically called the inverse) of them.**

2) **We can add, subtract, multiply, or divide both sides of an equation by any number and still have the equation work.**

A great way to think about these concepts is as though you have a balance that is centered on the equal sign. As long as you put the same thing on both sides, you remain balanced.

Here is how it would work, one of each:

$x+7=11$	$4x=24$	$x-3=24$	$\frac{x}{5}=7$
$-7 \quad -7$	$/4 \quad /4$	$+3 \quad +3$	$(5) \quad (5)$
$x = 4$	$x = 6$	$x = 27$	$x = 35$

You may ask why we go through all of that when the answers are obvious. The answer is that these problems will not be so easy later on, and we need to practice these easy ones so that when we get the hard ones, they crumble before our abilities. Now to some which are a little tougher.

When we have one like this:

$2x - 7 = 11$

We could think about it long enough to find a number that works, and maybe you can do that, but I have to tell you that in just a little while we are going to have a problem that you won't be able to do that with too quickly. So, let's use what we learned to get rid of the 2 and the 7 so that x will be left by itself. If you remember the order of operations, you will remember that the 2 and the x are stuck together by multiplication, so we can't get rid of the 2 until the 7 has been taken care of like this:

$2x - 7 = 11$
$2x = 18$ (we added 7 to both sides)
$x = 9$ (divided both sides by 2)

To illustrate the idea of un-doing operations, I would like to try to stump you with math tricks.

We begin. I am thinking of a number, and it is your job to guess what the number is.

I am thinking of a number.
I times the number by two.
I get 10.

Not too hard to figure out, you say? You're right. The answer is 5 and you obtained that by taking the result and going backwards. Try the next one:

I am thinking of a number.
I times the number by 3.
Then I subtract 5.
Then I divide that number by 2.
Then I add 4 to that.
I get 18.
What was the number I started with?

Aha. A little tougher don't you think? Well, If you think about it just one step at a time, then the thing falls apart. What number would I add 4 to to get 18? 14 (notice that it is just 18 **subtract** 4). We can just follow **up** the line doing the exact opposite of what I did to my number. Here you go:

Start with 18
Subtract 4 = 14
Multiply by 2 = 28
Add 5 = 33
Divide by 3 = 11.

That's it! Most of Algebra is summed up in the concept of un-doing what was done.

I am thinking of a number.
I times it by 4.
Then I add 5.
Then I divide by 9.
Then I subtract 7.
I get -2. What did I start with?

This one is done the same way as the other one but I wanted to show you how you make that into an equation that will be useful in the rest of your math career. Instead of writing each step out, we construct an equation. We write it again but this time we will write the equation along with it:

I am thinking of a number.	*We call that x.*
I times it by 4.	$4x$
Then I add 5.	$4x+5$
Then I divide by 9.	$\frac{4x+5}{9}$
Then I subtract 7.	$\frac{4x+5}{9}-7$
I get -2. What did I start with?	$\frac{4x+5}{9}-7=-2$

That looks like a nasty equation, but it is done in exactly the same way. We just go backwards and un-do all of the things that were done to the original number. **We are using the rule that we can add, subtract, multiply or divide both sides of the equation by the same thing.**

I know you can do it when it is all written out, so I will show you what it looks like using the equation:

$\frac{4x+5}{9}-7=-2$	
$\frac{4x+5}{9}=5$	add 7 to both sides
$4x+5=45$	times both sides by 9
$4x=40$	subtract 5 from both sides
$x=10$	divide both sides by 4.

Notice here that we are still undoing in the opposite order of what was there.

10 is the number I started with! Go ahead and make sure by sticking it into the original problem, and you will see that we found the right number. We call that number a **solution**, because it is the only number that **solves** the equation.

Solving for a variable:

When given a formula, it is sometimes requested that you solve that formula for a specific variable. That simply means that you are to get that variable by itself.
An example:

Solve for t:
rt = d (Original equation of rate x time = distance)

We are supposed to get t by itself. How do we get rid of the “r”?
Divide both sides by r. It looks like this

$$rt = d$$

$$\frac{rt}{r}=\frac{d}{r}$$

$$t=\frac{d}{r}$$ Done. t is by itself.

Another example:

Solve for x:

$$y = bx + c$$

$y - c = bx$ subtract "c" from both sides

$\dfrac{y-c}{b} = x$ Divide both sides by "b".

Done. "x" is by itself.