

# **Chapter 4 Introduction to Valuation: The Time Value of Money**

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# Key Concepts and Skills

- After studying this chapter, you should be able to:
  - Determine the future value of an investment made today.
  - Determine the present value of cash to be received at a future date.
  - Calculate the return on an investment.
  - Predict how long it takes for an investment to reach a desired value.
- Be able to solve time value of money problems using:
  - Formulas
  - A financial calculator
  - A spreadsheet

# Chapter Outline

- 4.1 Future Value and Compounding
- 4.2 Present Value and Discounting
- 4.3 More on Present and Future Values

Solving for:

Implied interest rate

Number of periods

# Basic Definitions

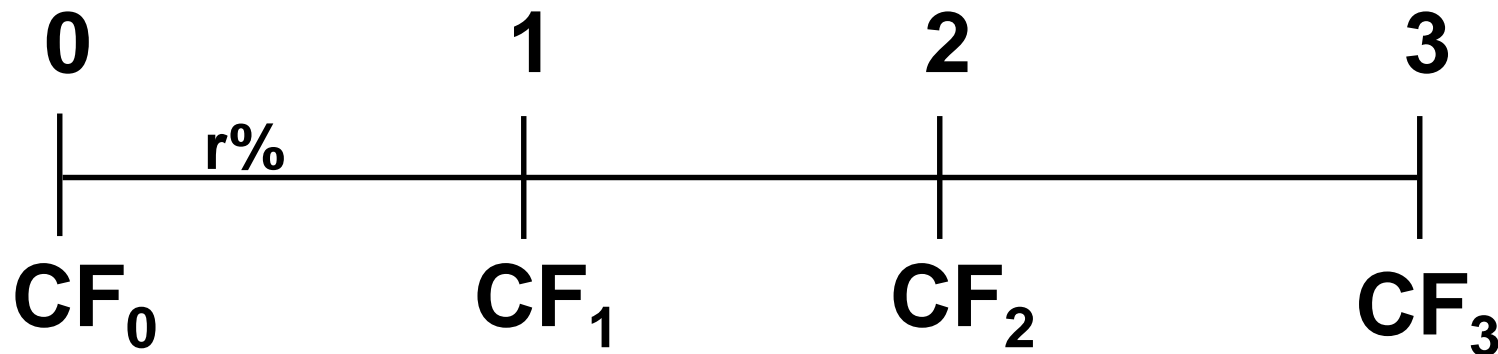
- Present Value (PV)
  - The current value of future cash flows discounted at the appropriate discount rate
  - Value at  $t=0$  on a time line
- Future Value (FV)
  - The amount an investment is worth after one or more periods.
  - “Later” money on a time line

# Basic Definitions

- Interest rate ( $r$ )
  - **Discount rate**
  - Cost of capital
  - Opportunity cost of capital
  - Required return
  - Terminology depends on usage

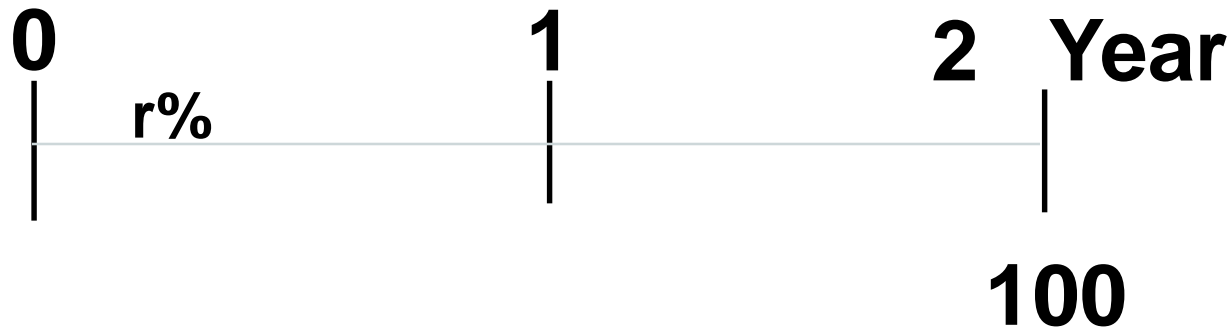
# Time Line of Cash Flows

- Tick marks at ends of periods
  - Time 0 is today;
  - Time 1 is the end of Period 1



**+CF = Cash INFLOW   -CF = Cash OUTFLOW   PMT = Constant CF**

# Time Line for a \$100 Lump Sum due at the End of Year 2



# Future Values: General Formula

$$FV = PV(1 + r)^t$$

FV = future value

PV = present value

r = period interest rate, expressed as a decimal

t = number of periods

- Future value interest factor =  $(1 + r)^t$

Note: “y<sup>x</sup>” key on your calculator



# Future Values: Example 1

Suppose you invest \$100 for one year at 10% per year.

What is the future value in one year?

- Interest =  $100(.10) = 10$

- Value in one year
  - = Principal + interest
  - =  $100 + 10 = 110$

- Future Value (FV)
  - =  $100(1 + .10) = 110$

# Future Values: Example 1

Suppose you leave the money in for another year. How much will you have two years from now?

$$\begin{aligned}FV &= 100(1.10)(1.10) \\ &= 100(1.10)^2 = 121.00\end{aligned}$$

# Effects of Compounding

- Simple interest
  - Interest earned only on the original principal
- Compound interest
  - Interest earned on principal and on interest received
  - “Interest on interest” – interest earned on reinvestment of previous interest payments



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# Effects of Compounding

- Consider the previous example
  - FV w/simple interest  
 $= 100 + 10 + 10 = 120$
  - FV w/compound interest  
 $= 100(1.10)^2 = 121.00$
  - The extra 1.00 comes from the interest of  $.10(10) = 1.00$  earned on the first interest payment

# Texas Instruments BA-II Plus

- ***FV*** = future value
  - ***PV*** = present value
  - ***I/Y*** = period interest rate ( $r$ )
  - ***N*** = number of periods
- } One of these **MUST** be negative

# Texas Instruments BA-II Plus

- $I/Y$  = period interest rate ( $r$ )
  - $C/Y$  must equal 1 for the  $I/Y$  to be the period rate ( $C/Y = 1$  = default on new BAII+)
  - Interest is entered as a percent, not a decimal
    - 5% interest = “5”, not “.05”
- $PMT = 0$  for this chapter only!
- Clear the registers before each problem
  - Press  $2^{nd}$  then **CLR TVM**
  - Or reenter each field

# Texas Instruments BA-II Plus

- Set number of decimal places to display
  - Press **2<sup>nd</sup>** key,
  - Press **Format** key (above “.”),
  - Enter desired decimal places (e.g., **4**).
  - Press **Enter** to set the displayed choice.

# Texas Instruments BA-II Plus

- Be sure “payment per period” or P/Y is set to “1”
  - Press **2<sup>nd</sup>** key,
  - Press **P/Y** (above **I/Y**),
  - Enter “**1**”,
  - Press **Enter**
  - Press **CE/C**



# TI BAI+ : Set Time Value Parameters

- Be sure calculator is set for cash flows at the END of each period
- To set **END** (for cash flows occurring at the end of the period),
  - Press **2<sup>nd</sup>** key,
  - Press **BGN** (above **PMT**).
    - This is a toggle switch. The default is **END**.
    - To change to **BEGIN**, hit **2<sup>nd</sup>** then **Set** (above **Enter**) to go back and forth.
  - Note: “BGN” will be displayed at the top right of the screen when the calculator is in **BEGIN** mode. When in **END** mode, this indicator will be blank.

# Future Values: Example 2

- Suppose you invest the \$100 from the previous example for 5 years. How much would you have?

## Formula Solution:

$$\begin{aligned} \mathbf{FV} &= \mathbf{PV(1+r)^t} \\ &= \mathbf{100(1.10)^5} \\ &= \mathbf{100(1.6105)} \\ &= \mathbf{161.05} \end{aligned}$$

# Table 4.1

**TABLE 4.1**

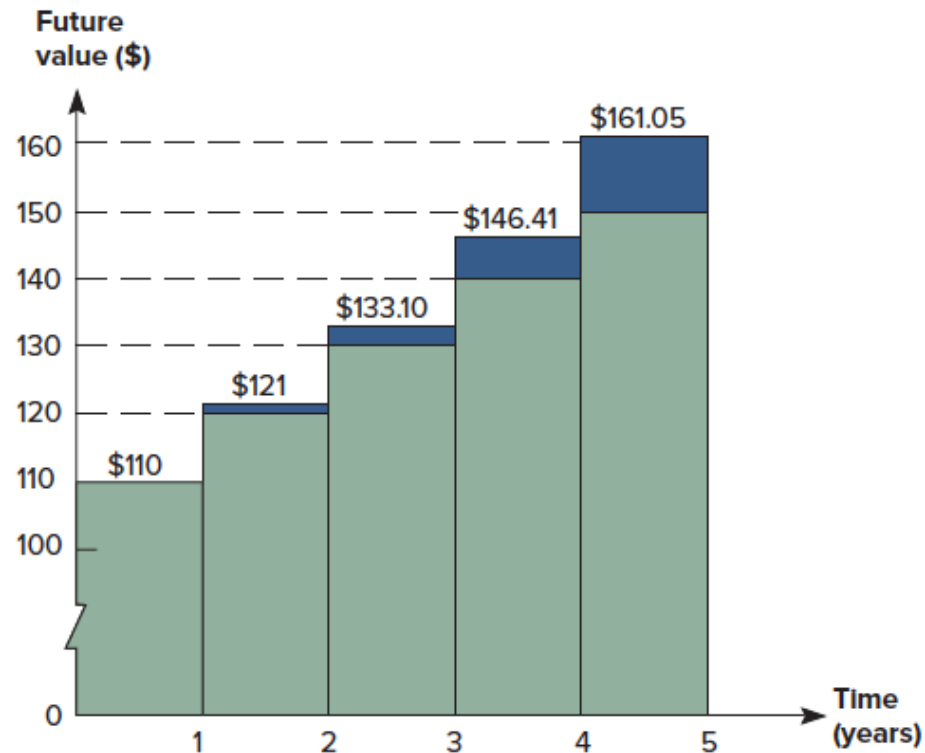
Future value of \$100  
at 10 percent

Year	Beginning Amount	Interest Earned	Ending Amount
1	\$100.00	\$10.00	\$110.00
2	110.00	11.00	121.00
3	121.00	12.10	133.10
4	133.10	13.31	146.41
5	146.41	14.64	161.05
		<b>Total interest \$61.05</b>	

# Figure 4.1

**FIGURE 4.1**

Future value, simple interest, and compound interest



Growth of \$100 original amount at 10% per year. Blue shaded area represents the portion of the total that results from compounding of interest.

# Texas Instruments BA-II Plus

- To calculate FV: 10% 5 years PV=\$100

<u>Key Entry</u>	<u>Display</u>
N	5.00
I/Y	10.00
PV	-100.00
PMT	0



161.05

CPT FV

# Excel Spreadsheet Functions

- Excel TVM functions:
  - =FV(rate,nper,pmt,pv)
  - =PV(rate,nper,pmt,fv)
  - =RATE(nper,pmt,pv,fv)
  - =NPER(rate,pmt,pv,fv)
- Use the formula icon ( $f_x$ ) when you can't remember the exact formula
- Click on the Excel icon to open a spreadsheet containing four different examples.



# Future Values: Example 3

Suppose you had a relative deposit \$10 at 5.5% interest 200 years ago. How much would the investment be worth today?

## Formula Solution:

$$\begin{aligned}FV &= PV(1+r)^t \\ &= 10(1.055)^{200} \\ &= 10(44718.984) \\ &= 447,189.84\end{aligned}$$

## Calculator Solution

200	N
5.5	I/Y
10	PV
0	PMT
CPT FV	= -447,189.84

Excel Solution: =FV(Rate, Nper, PMT, PV)

$$=FV(0.055, 200, 0, -10) = 447,189.84$$

**NOTE: Rate = decimal**

# Future Value: General Growth Formula

Suppose your company expects to increase unit sales of widgets by 15% per year for the next 5 years. If you currently sell 3 million widgets in one year, how many widgets do you expect to sell in 5 years?

## Formula Solution:

$$\begin{aligned} \text{FV} &= \text{PV}(1+r)^t \\ &= 3(1.15)^5 \\ &= 3(2.0114) \\ &= 6.0341 \text{ million} \end{aligned}$$

## Calculator Solution

5	N
15	I/Y
3	PV
0	PMT
CPT FV	= -6.0341

**Excel Solution:** =FV(Rate, Nper, PMT, PV)  
=FV(0.15,5,0,3) = -6.0341



# Future Value: Important Relationship I

For a given interest rate:

- The longer the time period,
- The higher the future value

$$FV = PV(1 + r)^t$$

**For a given  $r$ , as  $t$  increases,  $FV$  increases**

# Future Value:

## Important Relationship II

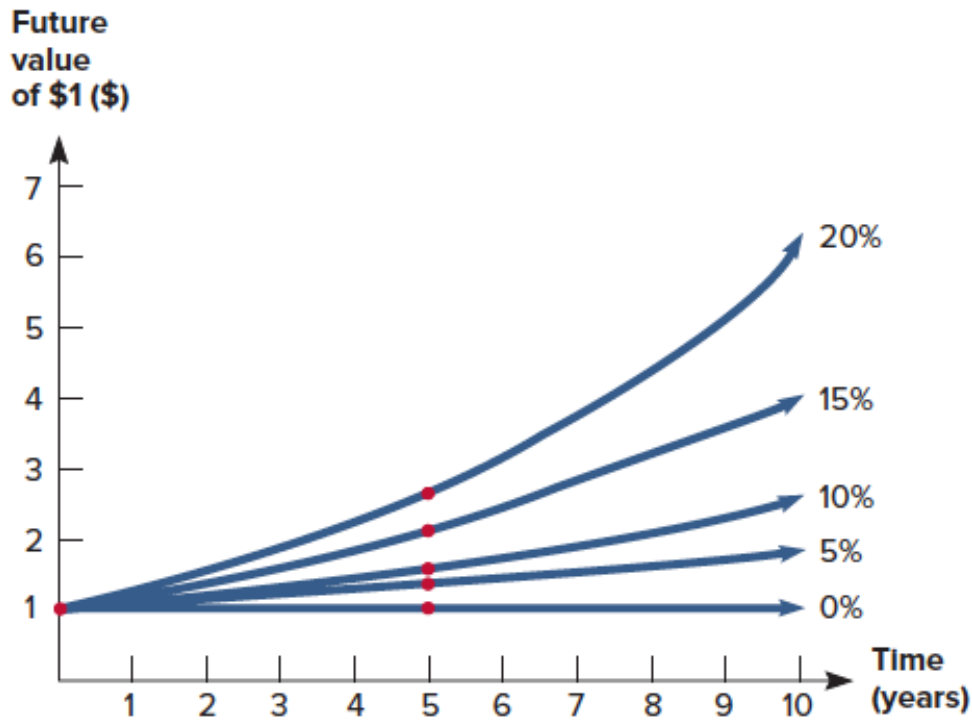
For a given time period:

- The higher the interest rate,
- The larger the future value

$$FV = PV(1 + r)^t$$

**For a given t, as r increases, FV increases**

# Figure 4.2



**FIGURE 4.2**

Future value of \$1 for different periods and rates

# Quick Quiz: Part 1

- What is the difference between simple interest and compound interest? ([Slide 4.11](#))
- Suppose you have \$500 to invest and you believe that you can earn 8% per year over the next 15 years. ([QQ1 Solution](#))
  - How much would you have at the end of 15 years using compound interest?
  - How much would you have using simple interest?

# Present Values

- The current value of future cash flows discounted at the appropriate discount rate
- Value at  $t=0$  on a time line
- Answers the questions:
  - How much do I have to invest today to have some amount in the future?
  - What is the current value of an amount to be received in the future?

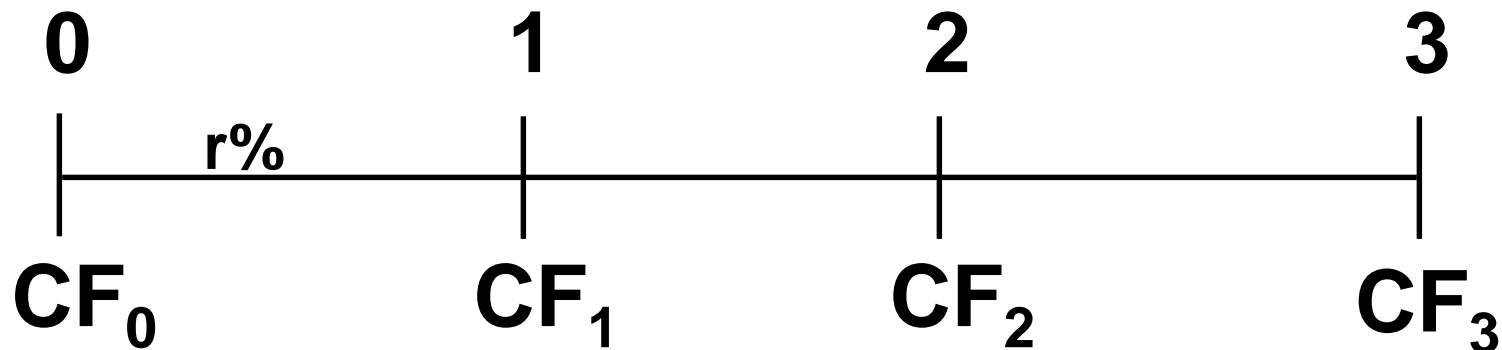
# Present Values

- *Present Value = the current value of an amount to be received in the future*
- Why is it worth less than face value?
  - Opportunity cost
  - Risk & Uncertainty

Discount Rate = f (time, risk)

# Time Line of Cash Flows

- Tick marks at ends of periods
  - Time 0 is today;
  - Time 1 is the end of Period 1



+CF = Cash INFLOW   -CF = Cash OUTFLOW   PMT = Constant CF

# Present Values

$$FV = PV(1 + r)^t$$

- Rearrange to solve for PV

$$PV = FV / (1+r)^t$$

$$PV = FV(1+r)^{-t}$$

- “Discounting” = finding the present value of one or more future amounts.

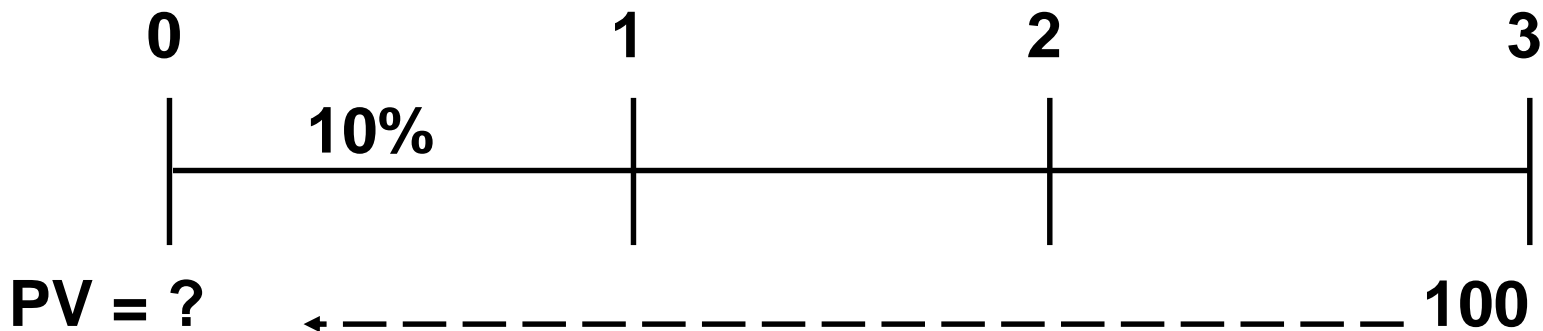


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# What's the PV of \$100 due in 3 Years if $r = 10\%$ ?

Finding PVs is discounting, and it's the reverse of compounding.



Formula:  $PV = FV(1+r)^{-t} = 100(1.10)^{-3} = \$75.13$

Calculator: 3 N    10 I/Y    0 PMT    100 FV  
CPT PV = -75.13

Excel: =PV(.10,3,0,100) = -75.13

# Present Value: Example 1

## Single Period

Suppose you need \$10,000 in one year for the down payment on a new car. If you can earn 7% annually, how much do you need to invest today?

### Formula Solution:

$$\begin{aligned} \text{PV} &= \text{FV}(1+r)^{-t} \\ &= 10,000(1.07)^{-1} \\ &= 10,000/1.07 \\ &= 9,345.79 \end{aligned}$$

### Calculator Solution

1      N  
7      I/Y  
0      PMT  
10000 FV  
CPT PV = -9345.79

**Excel Solution:** =PV(Rate, Nper, P, FV)  
=PV(0.07, 1, 0, 10000) = -9345.79

# Present Values: Example 2

## Multi-Periods

You want to begin saving for your daughter's college education and you estimate that she will need \$150,000 in 17 years. If you feel confident that you can earn 8% per year, how much do you need to invest today?

### Formula Solution:

$$\begin{aligned}PV &= FV(1+r)^{-t} \\ &= 150,000(1.08)^{-17} \\ &= 150,000/(1.08)^{17} \\ &= 40,540.34\end{aligned}$$

### Calculator Solution:

17	N
8	I/Y
0	PMT
150000	FV
CPT PV = -40,540.34	

### Excel Solution: =PV(Rate, Nper, PMT, FV)

$$=PV(0.08, 17, 0, 150000) = -40,540.34$$

# Present Values: Example 3

## Multi-Periods

Your parents set up a trust fund for you 10 years ago that is now worth \$19,671.51. If the fund earned 7% per year, how much did your parents invest?

### Formula Solution:

$$\begin{aligned} \text{PV} &= \text{FV}(1+r)^{-t} \\ &= 19,671.51(1.07)^{-10} \\ &= 19,671.51/(1.07)^{10} \\ &= -10,000 \end{aligned}$$

### Calculator Solution:

10	N
7	I/Y
0	PMT
19671.51	FV
CPT PV = -10000	

**Excel Solution:** =PV(Rate,Nper,Pmt,FV)

$$=PV(0.07,10,0,19671.51) = -10000$$

# Present Value: Important Relationship I

For a given interest rate:

- The longer the time period,
- The lower the present value

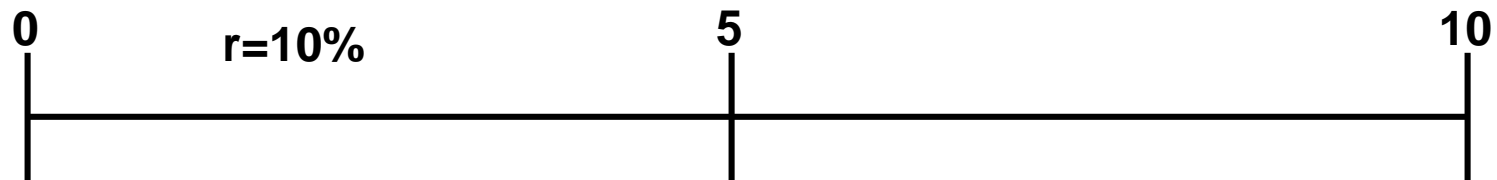
$$PV = \frac{FV}{(1+r)^t}$$

**For a given  $r$ , as  $t$  increases,  $PV$  decreases**

# Present Value:

## Important Relationship I

What is the present value of \$500 to be received in 5 years? 10 years? The discount rate is 10%



-310.46 PV? ← 500

-192.77 PV? ← 500

$$5 \text{ yrs: } PV = 500/(1.10)^5 = -310.46 \quad (1.10)^5 = 1.6105$$

$$10 \text{ yrs: } PV = 500/(1.10)^{10} = -192.77 \quad (1.10)^{10} = 2.5937$$

# Present Value: Important Relationship II

For a given time period:

- The higher the interest rate,
- The smaller the present value

$$PV = \frac{FV}{(1+r)^t}$$

**For a given t, as r increases, PV decreases**

# Present Value: Important Relationship II

What is the present value of \$500 received in 5 years if the interest rate is 10%? 15%?

**Rate = 10%**

**Calculator Solution:**

<b>5</b>	<b>N</b>
<b>10</b>	<b>I/Y</b>
<b>0</b>	<b>PMT</b>
<b>500</b>	<b>FV</b>
<b>CPT PV = -310.46</b>	

**Rate = 15%**

**Calculator Solution:**

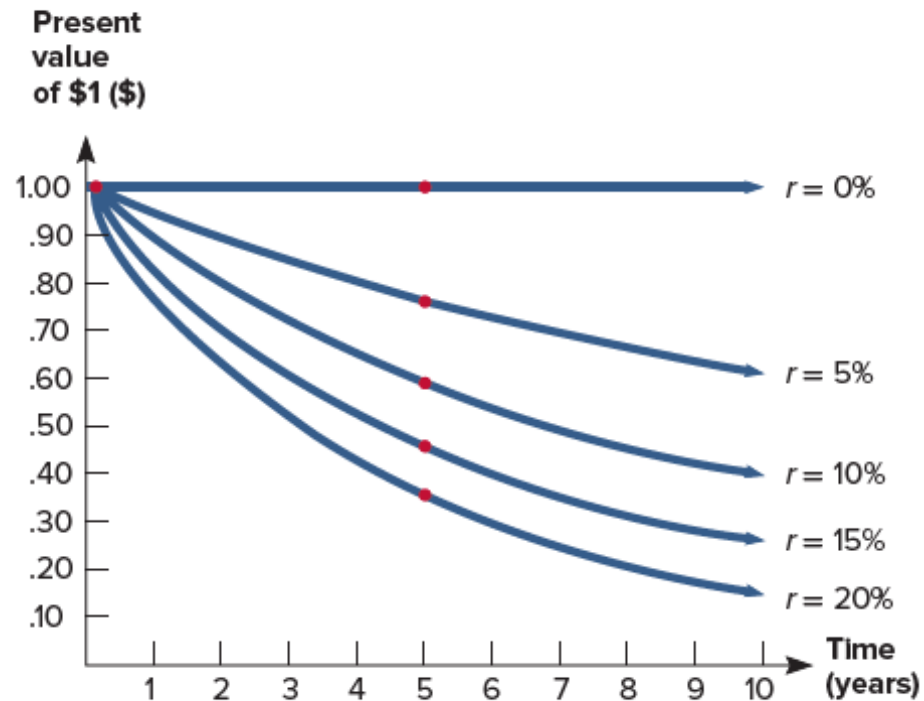
<b>5</b>	<b>N</b>
<b>15</b>	<b>I/Y</b>
<b>0</b>	<b>PMT</b>
<b>500</b>	<b>FV</b>
<b>CPT PV = -248.59</b>	



# Figure 4.3

**FIGURE 4.3**

Present value of \$1 for different periods and rates



# Quick Quiz: Part 2

- What is the relationship between present value and future value? (Slide 4.32)
- Suppose you need \$15,000 in 3 years. If you can earn 6% annually, how much do you need to invest today? (Solution)
- If you could invest the money at 8%, would you have to invest more or less than at 6%? How much? (Solution)

# The Basic PV Equation— Refresher

$$PV = FV / (1 + r)^t$$

There are four parts to this equation

- PV, FV, r and t
- Know any three, solve for the fourth
- Be sure and remember the sign convention

**+CF = Cash INFLOW   -CF = Cash OUTFLOW**

# Discount Rate

- To find the implied interest rate, rearrange the basic PV equation and solve for r:

$$FV = PV(1 + r)^t$$

$$r = (FV / PV)^{1/t} - 1$$

- If using formulas with a calculator, make use of both the  $y^x$  and the  $1/x$  keys

# Discount Rate: Example 1

You are looking at an investment that will pay \$1200 in 5 years if you invest \$1000 today. What is the implied rate of interest?

– Formula:

$$r = (1200 / 1000)^{1/5} - 1 = .03714 = 3.714\%$$

– Calculator – the sign convention matters!!!

5 N

-1000 PV (you pay \$1,000 today)

0 PMT

1200 FV (you receive \$1,200 in 5 years)

CPT I/Y = 3.714%

– Excel: =RATE(5,0,-1000,1200) = 0.03714

# Discount Rate: Example 2

Suppose you are offered an investment that will allow you to double your money in 6 years. You have \$10,000 to invest. What is the implied rate of interest?

<b>6</b>	<b>N</b>
<b>-10000</b>	<b>PV</b>
<b>0</b>	<b>PMT</b>
<b>20000</b>	<b>FV</b>

**CPT I/Y = 12.25%**

**Excel: =RATE(6,0,-10000,20000) = 0.1225**

# Discount Rate: Example 3

Suppose you have a 1-year old son and you want to provide \$75,000 in 17 years towards his college education. You currently have \$5,000 to invest. What interest rate must you earn to have the \$75,000 when you need it?

Calculator:

17 N, -5000 PV, 0 PMT, 75000 FV, CPT I/Y = 17.27%

Excel: =RATE(17,0,-5000,75000) = 0.1727

# Quick Quiz: Part 3

- What are some situations in which you might want to compute the implied interest rate?
- Suppose you are offered the following investment choices:
  - You can invest \$500 today and receive \$600 in 5 years. The investment is considered low risk.
  - You can invest the \$500 in a bank account paying 4% annually.
  - What is the implied interest rate for the first choice and which investment should you choose?

(Solution)



# Finding the Number of Periods

- Start with basic equation and solve for t:

$$FV = PV(1 + r)^t$$

$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1 + r)}$$

**Calculator:** CPT N

**Excel:** =NPER(Rate, Pmt, PV, FV)

# Number of Periods: Example

You want to purchase a new car and you are willing to pay \$20,000. If you can invest at 10% per year and you currently have \$15,000, how long will it be before you have enough money to pay cash for the car?

Calculator Solution:

10 I/Y; -15000 PV; 20000 FV;

CPT N = 3.02 years

Excel: =NPER(0.10,0,-15000,20000) = 3.02

# Number of Periods: Example

$$t = \frac{\ln\left(\frac{FV}{PV}\right)}{\ln(1+r)}$$

- Formula Solution:
  - $FV/PV = 20,000/15,000 = 1.333$
  - $\ln(1.333) = 0.2877$
  - $\ln(1.10) = 0.0953$
  - $t = 0.2877/0.0953 = 3.0189$

# Quick Quiz: Part 4

- When might you want to compute the number of periods?
- Suppose you want to buy some new furniture for your family room. You currently have \$500 and the furniture you want costs \$600. If you can earn 6%, how long will you have to wait if you don't add any additional money?

(Solution)

# Example: Work the Web

- Many financial calculators are available online
- [Click on this link](#) to go to the present value portion of the Moneychimp web site and work the following example:
  - You need \$40,000 in 15 years. If you can earn 9.8% interest, how much do you need to invest today?
  - You should get \$9,841

# Table 4.4

**TABLE 4.4**

Summary of time value of money calculations

**I. Symbols**

PV = Present value, what future cash flows are worth today

$FV_t$  = Future value, what cash flows are worth in the future

$r$  = Interest rate, rate of return, or discount rate per period—typically, but not always, one year

$t$  = Number of periods—typically, but not always, the number of years

$C$  = Cash amount

**II. Future value of  $C$  invested at  $r$  percent per period for  $t$  periods**

$$FV_t = C \times (1 + r)^t$$

The term  $(1 + r)^t$  is called the *future value factor*.

**III. Present value of  $C$  to be received in  $t$  periods at  $r$  percent per period**

$$PV = C / (1 + r)^t$$

The term  $1 / (1 + r)^t$  is called the *present value factor*.

**IV. The basic present value equation giving the relationship between present and future value is:**

$$PV = FV_t / (1 + r)^t$$

# Quick Quiz 1 Solution

Invest \$500 at 8% per year over 15 years.

How much would you have at the end of 15 years using compound interest?

- 15 N, 8 I/Y, -500 PV, 0 PMT, CPT FV 1586.08
- $500(1.08)^{15} = 1586.08$
- $=FV(.08, 15, 0, -500)$

How much would you have using simple interest?

- $500 + 15(500)(.08) = 1,100$



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# Quick Quiz 2 Solution

You need \$15,000 in 3 years. You can earn 6% annually, how much do you need to invest today?

3 N 6 I/Y 15000 FV 0 PMT

CPT PV = -12594.29

$$\begin{aligned} PV &= 15000 / (1.06)^3 = 15000 / (1.191016) = \\ &= 15000 \times 0.83962 = 12594.29 \end{aligned}$$

$$= PV(.06, 3, 0, 15000)$$



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# Quick Quiz 2 Solution

You need \$15,000 in 3 years. If you could invest the money at 8%, would you have to invest more or less than at 6%? How much?

3 N      8 I/Y      15000 FV      0 PMT

CPT PV = -11907.48

$$PV = 15000 / (1.08)^3 = 15000 / (1.125971)$$

$$= 15000 \times (0.79383) = 11907.48$$

$$= PV(.08, 3, 0, 15000)$$



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Difference = \$686.81

# Quick Quiz 3 Solution

## Investment choices:

- Invest \$500 today and receive \$600 in 5 years. The investment is considered low risk.
- Invest the \$500 in a bank account paying 4% annually.
- What is the implied interest rate for the first choice and which investment should you choose?

5 N -500 PV 0 PMT 600 FV

CPT I/Y 3.714%

$$r = (600/500)^{1/5} - 1 = 3.714\%$$

$$= \text{RATE}(5, 0, -500, 600)$$

The bank account pays a higher rate.



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# Quick Quiz 4 Solution

Suppose you want to buy some new furniture for your family room. You currently have \$500. And the furniture you want costs \$600. If you can earn 6%, how long will you have to wait if you don't add any additional money?

6 I/Y -500 PV 0 PMT 600 FV  
years

CPT N = 3.13

$t = \ln(600/500) / \ln(1.06) = 3.13$  years  
 $=\text{NPER}(.06, 0, -500, 600)$



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# Chapter 4

END