AQA IGCSE Further Maths Revision Notes

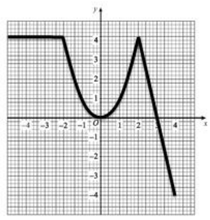
Formulas given in formula sheet:

- Volume of sphere: $\frac{4}{3}\pi r^3$ Surface area of sphere: $4\pi r^2$
- Volume of cone: $\frac{1}{3}\pi r^2 h$ Curve surface area: $\pi r l$
- Area of triangle: $\frac{1}{2}ab\sin C$
- Sine Rule: $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$ Cosine Rule: $a^2=b^2+c^2-2bc\cos A$
- Quadratic equation: $ax^2+bx+c=0 \rightarrow \quad x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
- Trigonometric Identities: $\tan\theta\equiv\frac{\sin\theta}{\cos\theta}$ $\sin^2\theta+\cos^2\theta\equiv 1$

1. Number

Specification	Notes	What can go ugly
1.1 Knowledge and use of numbers and the number system including fractions, decimals, percentages, ratio, proportion and order of operations are expected.	A few possibly helpful things: • If $a:b=c:d$ (i.e. the ratios are the same), then $\frac{a}{c}=\frac{b}{d}$ • "Find the value of a after it has been increased by $b\%$" If say b was 4, we'd want to $\times\ 1.04$ to get a a 4% increase. Can use $1+\frac{b}{100}$ as the multiplier, thus answer is: $a\left(1+\frac{b}{100}\right)$ "Show that $a\%$ of b is the same as $b\%$ of a" $\frac{a}{100}\times b=\frac{ab}{100}$ $\frac{b}{100}\times a=\frac{ab}{100}$	
1.2 Manipulation of surds, including rationalising the denominator.	GCSE recap: • Laws of surds: $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$ and $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$ But note that $a\times\sqrt{b}=a\sqrt{b}$ not $\sqrt{ab}$ Note also that $\sqrt{a}\times\sqrt{a}=a$ • To simplify surds, find the largest square factor and put this first: $\sqrt{12}=\sqrt{4}\sqrt{3}=2\sqrt{3}$ $\sqrt{75}=\sqrt{25}\sqrt{3}=5\sqrt{3}$ • $5\sqrt{2}\times 3\sqrt{2}$ Note everything is being multiplied here. Multiply surd-ey things and non surd-ey things separately. $=15\times 2=30$ • $\sqrt{8}+\sqrt{18}=2\sqrt{2}+3\sqrt{2}=5\sqrt{2}$ • To 'rationalise the denominator' means to make it a non-surd. Recall we just multiply top and bottom by that surd: $\frac{6}{\sqrt{3}}\rightarrow\quad\frac{6}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{6\sqrt{3}}{3}=2\sqrt{3}$ • The new thing at IGCSE FM level is where we have more complicated denominators. Just multiply by the 'conjugate': this just involves negating the sign between the two terms: $\frac{3}{\sqrt{6}-2}\rightarrow\quad\frac{3}{\sqrt{6}-2}\times\frac{\sqrt{6}+2}{\sqrt{6}+2}=\frac{3(\sqrt{6}+2)}{2}$ A trick to multiplying out the denominator is that we have the difference of two squares, thus $(\sqrt{6}-2)(\sqrt{6}+2)=6-4=2$ (remembering that $\sqrt{6}$ squared is 6, not 36!) • $\frac{2\sqrt{3}-1}{3\sqrt{3}+4}\rightarrow\quad\frac{2\sqrt{3}-1}{3\sqrt{3}+4}\times\frac{3\sqrt{3}-4}{3\sqrt{3}-4}=\frac{(2\sqrt{3}-1)(3\sqrt{3}-4)}{27-16}=\frac{18-3\sqrt{3}-8\sqrt{3}+4}{11}=\frac{22-11\sqrt{3}}{11}=2-\sqrt{3}$	I've seen students inexplicably reorder the terms in the denominator before they multiply by the conjugate, e.g. $(2+\sqrt{3})(\sqrt{3}-2)$ Just leave the terms in their original order! I've also seen students forget to negate the sign, just doing $(2+\sqrt{3})(2+\sqrt{3})$ in the denominator. The problem here is that it won't rationalise the denominator, as we'll still have surds! Silly error: $\frac{6\sqrt{6}}{2}\rightarrow 3\sqrt{3}$ (rather than $3\sqrt{6}$)

2. Algebra

Specification	Notes	What can go ugly
2.2 Definition of a function	A function is just something which takes an input and uses some rule to produce an output, i.e. $f(input) = output$ You need to recognise that when we replace the input x with some other expression, we need to replace every instance of it in the output, e.g. if $f(x) = 3x - 5$, then $f(x^2) = 3x^2 - 5$, whereas $f(x)^2 = (3x-5)^2$. See my Domain/Range slides. e.g. "If $f(x) = 2x + 1$, solve $f(x^2) = 51$ $2x^2 + 1 = 51 \rightarrow \; x = \pm 5$	
2.3 Domain and range of a function.	The domain of a function is the set of possible inputs. The range of a function is the set of possible outputs. Use x to refer to input and $f(x)$ to refer to output. Use "for all" if any value possible. Note that $<$ vs $\leq$ is important. • $f(x) = 2x$ Domain: for all x Range: for all $f(x)$ • $f(x) = x^2$ Domain: for all x Range: $f(x) \geq 0$ • $f(x) = \sqrt{x}$ Domain: $x \geq 0$ Range: $f(x) \geq 0$ • $f(x) = 2^x$ Domain: for all x Range: $f(x) > 0$ • $f(x) = \frac{1}{x}$ Domain: for all x except 0. Range: for all $f(x)$ except 0. • $f(x) = \frac{1}{x-2}$ Domain: for all x except 2 (since we'd be dividing by 0) Range: for all $f(x)$ except 0 (sketch to see it) • $f(x) = x^2 - 4x + 7$ Completing square we get $(x-2)^2 + 3$ The min point is (2,3). Thus range is $f(x) \geq 3$ You can work out all of these (and any variants) by a quick sketch and observing how x and y values vary. • Be careful if domain is 'restricted' in some way. Range if $f(x) = x^2 + 4x + 3, \; x \geq 1$ When $x = 1$, $f(1) = 8$, and since $f(x)$ is increasing after this value of x, $f(x) \geq 8$. • To find range of trigonometric functions, just use a suitable sketch, e.g. "$f(x) = \sin(x)$" $\rightarrow$ Range: $-1 \leq f(x) \leq 1$ However be careful if domain is restricted: "$f(x) = \sin(x), \; 180 \leq x < 360$". Range: $-1 \leq f(x) \leq 0$ (using a sketch) • For 'piecewise function', fully sketch it first to find range. "The function $f(x)$ is defined for all x: $f(x) = \begin{cases} 4 & x < -2 \\ x^2 & -2 \leq x \leq 2 \\ 12 - 4x & x > 2 \end{cases}$  Determine the range of $f(x)$." From the sketch it is clear $f(x) \leq 4$ • You may be asked to construct a function given information about its domain and range. e.g. "$y = f(x)$ is a straight line. Domain is $1 \leq x \leq 5$ and range is $3 \leq f(x) \leq 11$. Work out one possible expression for $f(x)$." We'd have this domain and range if line passed through points $(1,3)$ and $(5,11)$. This gives us $f(x) = 2x + 1$	Not understanding what $f(x^2)$ actually means. Not sketching the graph! (And hence not being able to visualise what the range should be). This is particularly important for 'piecewise' functions. Not being discerning between < and $\leq$ in the range. e.g. For quadratics you should have $\leq$ but for exponential graphs you should have <. Writing the range of a function in terms of x instead of the correct $f(x)$.
2.4 Expanding brackets and collecting like terms.	• **Deal with brackets with more than two things in them.** e.g. $(x + y + 1)(x + y) = x^2 + y^2 + 2xy + x + y$ Just do "each thing in first bracket times each in second" • **Deal with three (or more) brackets.** Just multiply out two brackets first, e.g. $(x+2)^3 = (x+2)(x+2)(x+2)$ $= (x+2)(x^2 + 4x + 4)$ $= x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$ $= x^3 + 6x^2 + 12x + 8$	Classic error of forgetting that two negatives multiply to give a positive. E.g. in $(y-4)^3$

2.5 Factorising	GCSE recap: You should know how to factorise difference of two squares, quadratics of form x^2+ax+b, form ax^2+bx+c and where you have a common factor. 1. **Sometimes multiple factorisation steps are required.** $x^4-25x^2=x^2(x^2-25)=x^2(x+5)(x-5)$ 2. **Sometimes use 'intelligent guessing' of brackets**, e.g. $15x^2-34xy-16y^2 \rightarrow (5x+2y)(3x-8y)$ 3. **If the expression is already partly factorised, identify common factors** rather than expanding out and starting factorising from scratch, e.g. "Factorise $(2x+3)^2-(2x-5)^2$" While we could expand, we might recognise we have the difference of two squares: $=\big((2x+3)+(2x-5)\big)\big((2x+3)-(2x-5)\big)$ $=(4x-2)(8)$ $=16(2x-1)$ Or "Factorise $(x+1)(x+3)+y(x+1)$" $=(x+1)(x+3+y)$	
2.6 Manipulation of rational expressions: Use of $+-\times\div$ for algebraic fractions with denominators being numeric, linear or quadratic.	"Simplify $\frac{x^2+3x-10}{x^2-9}\div\frac{x+5}{x^2+3x}$" Factorise everything first: $\frac{(x+5)(x-2)}{(x+3)(x-3)}\div\frac{x+5}{x(x+3)}$ Flip the second fraction and change $\div$ to $\times$: $=\frac{(x+5)(x-2)}{(x+3)(x-3)}\times\frac{x(x+3)}{x+5}$ $=\frac{x(x+5)(x-2)(x+3)}{(x+3)(x-3)(x+5)}$ $=\frac{x(x-2)}{x-3}$ "Simplify $\frac{x^3+2x^2+x}{x^2+x}$" $=\frac{x(x^2+2x+1)}{x(x+1)}=\frac{x(x+1)(x+1)}{x(x+1)}=x+1$	
2.7 Use and manipulation of formulae and expressions.	Same skills as GCSE. Isolate subject on one side of equation, and factorise it out if necessary. e.g. "Rearrange $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$ to make v the subject." Multiplying everything by fuv: $uv=fv+fu$ $uv-fv=fu$ $v(u-f)=fu$ $v=\frac{fu}{u-f}$	
2.8 Use of the factor theorem for integer values of the variable including cubics.	In: $7\div 3=2\ rem\ 1$, the 7 is the 'dividend', the 3 is the 'divisor', the 2 is the 'quotient' and the 1 is the 'remainder'. **Remainder Theorem**: For a polynomial $f(x)$, the remainder when $f(x)$ is divided by $(x-a)$ is $f(a)$. **Factor Theorem**: If $f(a)=0$, then by above, the remainder is 0. Thus $(x-a)$ is a factor of $f(x)$. e.g. When x^2-3x+2 is divided by $x-2$, remainder is $f(2)=2^2-3(2)+2=0$ (thus $x-2$ is a factor as remainder is 0). Note that we negated the -2 and subbed in 2 into the original equation. • "Show that $(x-2)$ is a factor of x^3+x^2-4x-4" $f(2)=2^3+2^2-4(2)-4=0$ • "If $(x-5)$ is a factor of $x^3-6x^2+ax-20$, determine the value of a" $f(5)=5^3-6(5^2)+5a-20=0$ $5a-45=0 \rightarrow a=9$ Harder questions might involve giving you two factors, and two unknowns – just use factor theorem to get two equations, then solve simultaneously. • "Fully factorise $x^3-3x^2-4x+12$" Step 1: Try a few values of x until you stumble upon a factor. $f(1)=1^3-3(1^2)-4(1)+12=6$ (so not a factor) $f(2)=2^3-3(2^2)-4(2)+12=0$ (so $(x-2)$ a factor)	If you find $f(2)=0$, then the factor is $(x-2)$ not $(x+2)$.

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<tr>
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<td>Step 2: Since a cubic, must factorise like:
$$(x-2)(x+?)(x+?)$$
Since all the constants must multiply to give +12 (as in the original cubic), the two missing numbers must multiply to give -6. This rounds down what we need to try with the remainder theorem:
$f(-2)=(-2)^3-3(-2)^2+\cdots=0$ (therefore a factor)
Since $(x+2)$ is a factor, last factor must be $(x-3)$
• "Solve $x^3+x^2-10x+8=0$"
Using above technique, factorisating gives:
$$(x-1)(x-2)(x+4)=0$$
$$x=1,2 \text{ or } -4$$</td>
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<td>2.9 Completing the square.</td>
<td>• Recap of GCSE: Halve number in front of x for number inside bracket. Square this and 'throw it away'.
e.g. $x^2+4x-3 \rightarrow (x+2)^2-4-3=(x+2)^2-7$
• If coefficient of x^2 (i.e. the constant on the front of the x^2 term) is not 1, always factorise this number out first, even if other numbers don't have this as a factor:
$$3x^2+12x-5$$
$$=3\left(x^2+4x-\frac{5}{3}\right)$$
$$=3\left((x+2)^2-4-\frac{5}{3}\right)$$
$$=3(x+2)^2-12-5$$
$$=3(x+2)^2-17$$
In the penultimate line we expanded out the outer $3(\dots)$ bracket. You can check your answer by expanding it back out and seeing if you get the original expression.
• Rearrange terms into form ax^2+bx+c first, e.g.
"Express $2-4x-2x^2$ in the form $a-b(x+c)^2$"
$$=-2x^2-4x+2$$
$$=-2(x^2+2x-1)$$
$$=-2((x+1)^2-1-1)$$
$$=-2(x+1)^2+4 = 4-2(x+1)^2$$</td>
<td>We always subtract when we 'throw away', even if number we halved was negative, e.g. $x^2-6x+1=(x-3)^2-9+1$
Often students add by mistake.

When coefficient of x^2 is not 1, be careful to maintain your outer bracket until the last step.</td>
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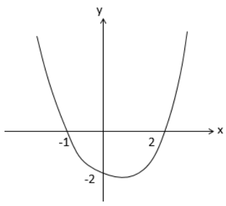
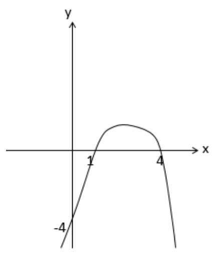
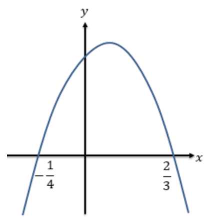
<td>2.10 Sketching of functions. Sketch graphs of linear and quadratic functions.</td>
<td>Quadratics:
• e.g. "Sketch $y=x^2-x-2$"
a. Factorise and set y to 0 to find x-intercepts (known as 'roots').
$(x+1)(x-2)=0$
$\rightarrow \quad (-1,0),(2,0)$
b. Set $x=0$ to find y-intercept.
$(0,-2)$
c. x^2 term is positive therefore 'smiley face' shape.


• Sketch $y=-x^2+5x-4$
Roots: $-x^2+5x-4=0$
$x^2-5x+4=0$
$(x-1)(x-4)=0$
$\rightarrow \quad (1,0),(4,0)$
But note frowney face shape as x^2 term is negative.




• You may have to go backwards: find the equation given the sketch.
In left example, brackets of
$$y=(4x+1)(3x-2)$$
would work as if $4x+1=0$ then $x=-\frac{1}{4}$ and similarly with $(3x-2)$
• GCSE recap: We can complete the square to find the minimum/maximum point (or use $\frac{dy}{dx}=0$!)
$$y=x^2-6x+10$$
$$=(x-3)^2+1$$
Min point is (3,1) (If $y=(x+a)^2+b$ then minimum point is $(-a,b)$)</td>
<td>Typical mistake is to do x-intercepts of say 2 and 3 if
$y=(x+2)(x+3)$

Similarly if $y=(x+2)^2+3$, an incorrect minimum point would be $(2,3)$

We might similarly make sign errors if doing the reverse: finding the equation from the graph. If one of the x-intercepts is $\frac{3}{4}$, then one of brackets is $(4x-3)$, not $(4x+3)$ or $(3x+4)$ or $(3x-4)$. Check by setting bracket to 0 and solving.</td>
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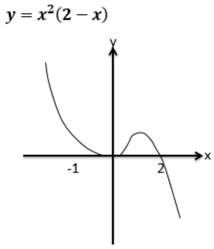
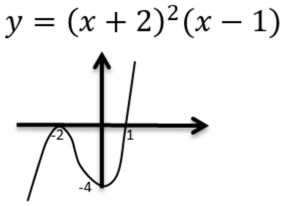
Cubics

- When cubic is in factorised form then:
  If $(x+a)$ appears once, curve crosses x-axis at $-a$
  If $(x+a)^2$ appears, curve touches at $x=-a$
  As before, can get y-intercept by setting $x=0$.
  If x^3 term is positive, uphill zig-zag shape, otherwise downhill.
  e.g.

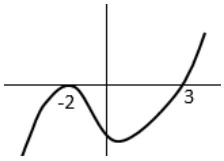




You may have to factorise yourself, e.g. $y = x^3 - 12x^2 \rightarrow$ $x^2(x-12)$ which crosses at $x = 12$ and touches at $x = 0$.

And again you may need to suggest an equation. Suitable equation for graph on right:

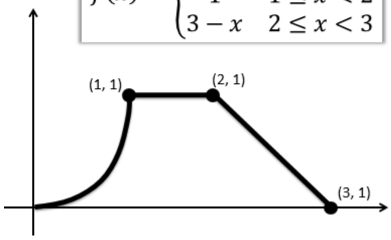
$$y = (x+2)^2(x-3)$$



- (Note that reciprocal graphs are in C1 only – you can find this in my IGCSEFM Sketching Graphs slides)

Piecewise Functions

$$f(x) = \begin{cases} x^2 & 0 \leq x < 1 \\ 1 & 1 \leq x < 2 \\ 3-x & 2 \leq x < 3 \end{cases}$$



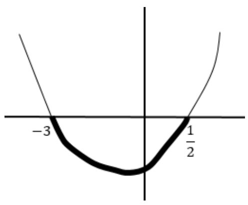
Just a function defined in 'pieces'. So in the above example, the function to draw between $0 \leq x < 1$ is $y = x^2$

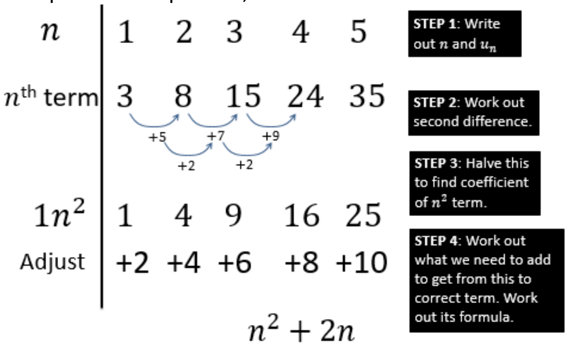
Note: You do <u>not</u> need to know about graph transforms for IGCSEFM (but do for C1).

2.11 Solution of linear and quadratic equations

GCSE recap. Solve $x^2 + 2x - 3 = 0$…

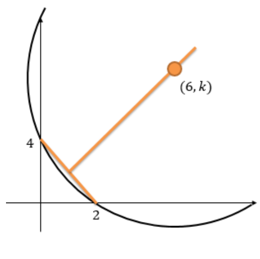
- …by factorisation. $(x+3)(x-1) = 0 \rightarrow x = -3, 1$
- …by completing the square
$$(x+1)^2 - 4 = 0$$
$$(x+1)^2 = 4$$
$$x + 1 = \pm 2$$
$$x = +2 - 1 = 1, \quad x = -2 - 1 = -3$$
- …by formula
$$a = 1, b = 2, c = -3$$
$$x = \frac{-2 \pm \sqrt{4 - (4 \times 1 \times -3)}}{2} = \cdots$$
- …by graph
  Sketch $y = x^2 + 2x - 3$. Comparing to original equation, we've substituted y for 0. So interested in values of x for which $y = 0$. We could similarly 'look up' values of x for other values of y, e.g. $x^2 + 2x - 3 = 5$.

2.12 Algebraic and graphical solution of simultaneous equations in two unknowns where the equations could both be linear or one linear and one second order.	• To 'graphically' solve simultaneous equations, sketch both lines, and look at the points where they intersect. • Equations might not be in the usual $ax + by = c$ form; if not, rearrange them! e.g. $2y + 3x + 4$ $2x = -3y - 7$ becomes: $3x - 2y = -4$ $2x + 3y = -7$ then solve in usual GCSE manner. Could also solve by substitution. Similarly: $\frac{x-1}{y-2} = 3 \quad \frac{x+6}{y-1} = 4$ becomes: $x - 1 = 3y - 6 \quad x + 6 = 4y - 4$ • For "one linear, one quadratic", solve as per GCSE method: rearrange linear equation to make x or y the subject, then sub into quadratic equation and solve, e.g. $x + y = 4$ $y^2 = 4x + 5$ Then: $y = 4 - x$ $(4 - x)^2 = 4x + 5$ $16 - 8x + x^2 = 4x + 5$ $x^2 - 12x + 11 = 0$ $(x - 11)(x - 1) = 0$ $x = 11 \rightarrow \ y = -7$ $x = 1 \rightarrow \ y = 3$	Suppose that $x = y + 4$ and $x^2 + y^2 = 20$ Common error is to accidentally drop the $+y^2$ after subbing in the x, i.e. $y^2 + 8x + 16 = 20$ (extra $+y^2$ has gone!) Other common error is squaring brackets. e.g. $(4 - x)^2 = 16 - 8x + x^2$ Incorrect variants: • $16 - x^2$ • $16 + x^2$ • $16 - 8x - x^2$ Don't forget to find the values of the other variable at the end, and make sure it's clear which x matches up to which y.
2.13 Solution of linear and quadratic inequalities	• When solving linear inequalities, just remember that dividing or multiplying by a negative number reverses the direction of the inequality. You can avoid this by putting x on the side which is positive. • To solve quadratic inequalities. $2x^2 + 5x \leq 3$ $2x^2 + 5x - 3 \leq 0$ Get 0 on one side. $(2x - 1)(x + 3) \leq 0$ Factorise This gives us 'critical values' of $x = \frac{1}{2}, x = -3$  Then you **MUST SKETCH**. Since on the left we sketched $y = (2x - 1)(x + 3)$ we're interested where $y \leq 0$ This is in indicated region on left, i.e. where $-3 \leq x \leq \frac{1}{2}$ Had we wanted $(2x - 1)(x + 3) \geq 0$, this would have given us the two 'tails' of the graph, and we'd write "$x < -3$ ***or*** $x \geq \frac{1}{2}$"	When solving quadratic inequalities, students usually get the 'critical values' right but stumble at the last hurdle because they don't sketch their quadratic, and therefore guess which way the inequality is supposed to go. Use the word 'or' when you want the two tails (and not 'and' or comma)
2.14 Index laws, including fractional and negative indices.	GCSE recap: $x^{-a} = \frac{1}{x^a} \quad x^0 = 1$ $8^{\frac{2}{3}} = 2^2 = 4 \quad 25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}} = \frac{1}{5}$ $\left(\frac{27}{8}\right)^{-\frac{2}{3}} = \left(\frac{8}{27}\right)^{\frac{2}{3}} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$ Example: "Solve $x^{\frac{3}{4}} = 27$" Just raise both sides to the reciprocal of the power to 'cancel it out'. $\left(x^{\frac{3}{4}}\right)^{\frac{4}{3}} = 27^{\frac{4}{3}}$ $x = 81$ Convert any mixed numbers to improper fractions first. "Solve $x^{-\frac{2}{3}} = 2\frac{7}{9}$" $\left(x^{-\frac{2}{3}}\right)^{-\frac{3}{2}} = \left(\frac{25}{9}\right)^{-\frac{3}{2}}$ $x = \left(\frac{9}{25}\right)^{\frac{3}{2}} = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$	You might get confused with straight line equations and raise both sides to the negative reciprocal rather than just the reciprocal?

	GCSE recap: To raise an algebraic term to a power, simply do each part of the term to that power, e.g. $(3x^2y^3)^2 \quad \rightarrow \quad 9x^4y^6$ $(9x^4y)^{\frac{1}{2}} \quad \rightarrow \quad 3x^2y^{\frac{1}{2}}$	
2.15 Algebraic Proof	*"Prove that the difference between the squares of two consecutive odd numbers is a multiple of 8."* Let two consecutive odd numbers be $2n+1$ and $2n+3$ $(2n+3)^2-(2n+1)^2$ $=4n^2+12n+9-4n^2-4n-1$ $=8n+8 \quad =8(n+1)$ which is divisible by 8. (The factoring out of 8 makes the divisibility explicit) *"Prove that $x^2-4x+7>0$ for all x"* (Just complete the square!) $(x-2)^2-4+7$ $=(x-2)^2+3$ $(x-2)^2 \geq 0$ thus $(x-2)^2+3>0$ for all x *"In this identity, h and k are integer constants.* $4(hx-1)-3(x+h) \equiv 5(x+k)$ *Work out the values of h and k"* The $\equiv$ means the left-hand-side and right-hand-side are equal for **all** values of x (known as an identity). Compare the coefficients of x and separately compare constants: $4hx-4-3x-3h=5x+5k$ Comparing x terms: $\quad 4h-3=5 \quad \rightarrow \quad h=2$ Comparing constant terms: $\quad -4-3h=5k \quad \rightarrow \quad k=-2$	
2.16 Sequences: nth terms of linear and quadratic sequences. Limiting value of a sequence as $n \rightarrow \infty$	Linear sequences recap: 4, 11, 18, 25… $\rightarrow$ nth term $7n-3$ For quadratic sequences, i.e. where second difference is constant:  **Limiting values:** *"Show that the limiting value of $\frac{3n+1}{6n-5}$ is $\frac{1}{2}$ as $n \rightarrow \infty$"* $n \rightarrow \infty$ means "as n tends towards infinity". Write "As n becomes large, $\frac{3n+1}{6n-5} \rightarrow \frac{3n}{6n} = \frac{1}{2}$" The idea is that as n becomes large, the +1 and -5 become inconsequential, e.g. if $n=1000$, then $\frac{3001}{5995} \approx \frac{3000}{6000} = \frac{1}{2}$	Make sure you check your formula against the first few terms of the sequence by using $n=1,2,3$.

3. Co-ordinate Geometry (2 dimensions only)

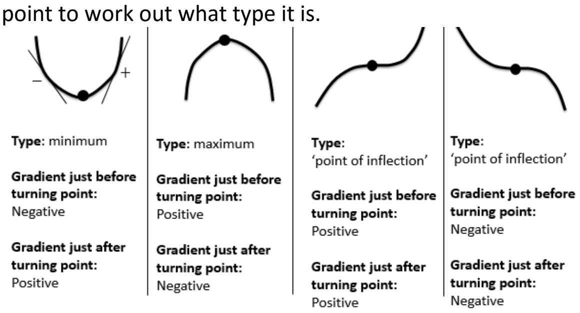
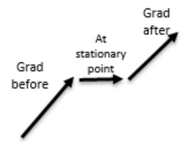
Specification	Notes	What can go ugly
3.1 Know and use the definition of gradient	Gradient is the change in y for each unit increase in x. $m=\frac{\Delta y}{\Delta x}$ (change in y over change in x) e.g. If a line goes through $(2,7)$ and $(6,5)$ $$m=-\frac{2}{4}=-\frac{1}{2}$$	Doing $\frac{\Delta x}{\Delta y}$ accidentally, or getting one of the two signs wrong.
3.2 Know the relationship between the gradients of parallel and perpendicular lines.	Parallel lines have the same gradient. For perpendicular lines: • One gradient is the negative reciprocal of the other. e.g. $2\ \rightarrow\ -\frac{1}{2}$ $\quad -4\rightarrow\frac{1}{4}$ $\frac{1}{5}\rightarrow -5 \quad \frac{2}{3}\rightarrow -\frac{3}{2}$ Remember that the reciprocal of a fraction flips it. • To show two lines are perpendicular, show the product of the gradients is -1: $$-\frac{1}{4}\times 4=-1$$ Example: "Show that $A(0,0), B(4,6), C(10,2)$ form a right-angled triangle." Gradients are: $m_{AB}=\frac{6}{4}=\frac{3}{2}\quad m_{AC}=\frac{2}{10}=\frac{1}{5},\quad m_{BC}=\frac{-4}{6}=-\frac{2}{3}$ Since $\frac{3}{2}\times-\frac{2}{3}=-1$, lines AB and BC are perpendicular so triangle is right-angled.	Doing just the reciprocal rather than the 'negative reciprocal'.
3.3 Use Pythagoras' Theorem to calculate the distance between two points.	$$d=\sqrt{\Delta x^2+\Delta y^2}$$ e.g. If points are $(3,2)$ and $(6,-2)$, then $$d=\sqrt{3^2+4^2}=5$$ Note that it doesn't matter if the 'change' is positive or negative as we're squaring these values anyway.	
3.4 Use ratio to find the coordinates of a point on a line given the coordinates of two other points.	"Two points $A(1,5)$ and $B(7,14)$ form a straight line. If a point $C(5,k)$ lies on the line, find k." Method 1 (implied by specification on left): On the x axis, 5 is 4 6ths of the way between 1 and 7. So "4 6ths" of the way between 5 and 14 is $$k=5+\frac{4}{6}\times 9=11$$ Method 2 (easier!): Find equation of straight line first. Using $y-y_1=m(x-x_1)$: $$m=\frac{9}{6}=\frac{3}{2}$$ $$y-5=\frac{3}{2}(x-1)$$ Thus if $x=5$ and $k=5$: $$k-5=\frac{3}{2}(5-1)$$ $$k=11$$	
3.5 The equation of a straight line in the forms $y=mx+c$ and $y-y_1=m(x-x_1)$	*"A line goes through the point $(4,5)$ and is perpendicular to the line with equation $y=2x+6$. Find the equation of the line. Put your answer in the form $y=mx+c$"* For all these types of questions, we need (a) the gradient and (b) a point, in order to use $y-y_1=m(x-x_1)$: $$m=-\frac{1}{2}$$ $$y-5=-\frac{1}{2}(x-4)$$ $$y-5=-\frac{1}{2}x+2$$ $$y=-\frac{1}{2}x+7$$ "Determine the coordinate of the point where this line crosses the x axis" $$0=-\frac{1}{2}x+7\ \rightarrow\ x=14\ \rightarrow\ (14,0)$$	Don't confuse x and x_1 in the straight line equation. x_1 and y_1 are constants, representing the point (x_1,y_1) the line goes through. x and y meanwhile are variables and must stay as variables. Be careful with negative values of x or y, e.g. if $m=3$ and $(-2,4)$ is the point, then: $y-4=3(x+2)$
3.6 Draw a straight line from given information.		

3.7 Understand the equation of a circle with any centre and radius.	Circle with centre (a,b) and radius r is: $$(x-a)^2+(y-b)^2=r^2$$ Examples: • *"A circle has equation* $(x+3)^2+y^2=25$. *What is its centre and radius?"* Centre: $(-3,0)$ $r=5$ • *"Does the circle with equation* $x^2+(y-1)^2=16$ *pass through the point* $(2,5)$*?"* In general a point is on a line if it satisfies its equation. $2^2+(5-1)^2=16$ $20=16$ So no, it is not on the circle. • *"A circle has centre* $(3,4)$ *and radius* 5. *Determine the coordinates of the points where the circle intercepts the* x *and* y *axis."* Firstly, equation of circle: $(x-3)^2+(y-4)^2=25$ On x-axis: $y=0$: $(x-3)^2+(0-4)^2=25$ $(x-3)^2=9$ $x-3=\pm 3 \rightarrow (0,0),(6,0)$ On y-axis, $x=0$: $(0-3)^2+(y-4)^2=25$ $(y-4)^2=16$ $y-4=\pm 4 \rightarrow (0,0),(0,8)$ • *"$A(4,7)$ and $B(10,15)$ are points on a circle and AB is the diameter of the circle. Determine the equation of the circle."* We need to find radius and centre. Centre is just midpoint of diameter: $(7,11)$ Radius using $(4,7)$ and $(7,11)$: $\sqrt{\Delta x^2+\Delta y^2}=\sqrt{3^2+4^2}=5$ Equation: $(x-7)^2+(y-11)^2=25$ See slides for harder questions of this type. • *"$x^2-2x+y^2-6y=0$ is the equation of a circle. Determine its centre and radius."* Need to complete the square to get in usual form. $(x-1)^2-1+(y-3)^2-9=0$ $(x-1)^2+(y-3)^2=10$ Centre: $(1,3)$ $r=\sqrt{10}$ **Using Circle Theorems** • Angle in semicircle is $90°$: which means that the two chords will be perpendicular to each other (i.e. gradients will multiply to give -1). • The perpendicular from the centre of the chord passes through the centre of the circle. *Example: "Two points on the circumference of a circle are (2,0) and (0,4). If the centre of the circle is $(6,k)$, determine k."*  Gradient of chord: $-\frac{4}{2}=-2$ Midpoint of chord: $(1,2)$ Gradient of radius $=\frac{1}{2}$ Equation of radius: $y-2=\frac{1}{2}(x-1)$ If $x=6$: $y-2=\frac{1}{2}(6-1)$ $y=4.5$	

	• The tangent to a circle is perpendicular to the radius. Example: "*The equation of this circle is* $x^2 + y^2 = 20$. $P(4,2)$ *is a point on the circle. Work out the equation of the tangent to the circle at* P*, in the form* $y = mx + c$" As always, to find an equation we need (i) a point and (ii) the gradient. Point: (4,2) Gradient of radius is $\frac{2}{4} = \frac{1}{2}$ ∴ Gradient of tangent $= -2$ $y - 2 = -2(x - 4)$ $y = -2x + 10$	

4. Calculus

Specification	Notes	What can go ugly
4.1 Know that the gradient function $\frac{dy}{dx}$ gives the gradient of the curve and measures the rate of change of y with respect to x. 4.2 Know that the gradient of a function is the gradient of the tangent at that point.	Whereas with say $y = 3x + 2$ the gradient is constant ($m = 3$), with curves, the gradient depends on the point. $\frac{dy}{dx}$ is the gradient function: it takes an x value and gives you the gradient at that point. e.g. If $\frac{dy}{dx} = 2x$, then at (5,12), the gradient is $2 \times 5 = 10$. Technically this the gradient of the *tangent* at this point. Another way of interpreting $\frac{dy}{dx}$ is "the rate of change of y with respect to x."	
4.3 Differentiation of kx^n where n is a positive integer or 0, and the sum of such functions.	Multiply by power and <u>then</u> reduce power by 1. $y = x^3 \rightarrow \frac{dy}{dx} = 3x^2$ $y = 5x^2 \rightarrow \frac{dy}{dx} = 10x$ $y = 7x \rightarrow \frac{dy}{dx} = 7$ $y = -3 \rightarrow \frac{dy}{dx} = 0$ Put expression in form kx^n first, and split up any fractions. *Then* differentiate. $y = (2x + 1)^2 = 4x^2 + 4x + 1$ $\frac{dy}{dx} = 8x + 4$ $y = \sqrt{x} = x^{\frac{1}{2}} \quad \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ $y = \frac{1 + x}{\sqrt{x}} = x^{-\frac{1}{2}} + x^{\frac{1}{2}}$ $\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$	Don't forget that constants disappear when differentiated. Common mistake is to reduce power by 1 then multiply by this new power. Don't forget that $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ with a negative power.
4.4 The equation of a tangent and normal at any point on a curve.	Use $\frac{dy}{dx}$ to find gradient at specific point (ensuring you use the negative reciprocal if we want the normal). You may need to use the original equation to also find y. Then use $y - y_1 = m(x - x_1)$ Example: "Work out the equation of the tangent to the curve $y = x^3 + 5x^2 + 1$ at the point where $x = -1$." $\frac{dy}{dx} = 3x^2 + 10x$ $m = 3(-1)^2 + 10(-1) = -7$ $y = (-1)^3 + 5(-1)^2 + 1 = 5$ Therefore: $y - 5 = -7(x + 1)$ *"Work out the equation of the normal to the curve* $y = x^3 + 5x^2 + 1$ *at the point where* $x = -1$*."* Exactly the same, except we use negative reciprocal for the gradient: $y - 5 = \frac{1}{7}(x + 1)$	Don't mix up the tangent to the curve and the normal to a curve (the latter which is perpendicular to the tangent).

4.5 Use of differentiation to find stationary points on a curve: maxima, minima and points of inflection.	At min/max points, the curve is flat, and the gradient therefore 0. Use gradient value just before and after turning point to work out what type it is.  Example: *"A curve has equation $y = 4x^3 + 6x^2 + 3x + 5$. Work out the coordinates of any stationary points on this curve and determine their nature."* $\frac{dy}{dx} = 12x^2 + 12x + 3 = 0$ $4x^2 + 4x + 1 = 0$ $(2x+1)^2 = 0$ $x = -\frac{1}{2}$ Find the y value of the stationary point: $y = 4\left(-\frac{1}{2}\right)^3 + 6\left(-\frac{1}{2}\right)^2 + 3\left(-\frac{1}{2}\right) + 5 = \frac{9}{2}$ So stationary point is $\left(-\frac{1}{2}, \frac{9}{2}\right)$. Look at gradient just before and after: When $x = -0.51, \frac{dy}{dx} = 0.0012$ When $x = -0.49, \frac{dy}{dx} = 0.0012$  Both positive, so a point of inflection.	Common error is to forget to find the y value of the stationary point when asked for the full coordinate
4.6 Sketch a curve with known stationary points.	Self-explanatory. Just plot the points and draw a nice curve to connect them.	

5. Matrix Transformations

Specification	**Notes**	**What can go ugly**
5.1 Multiplication of matrices	Do each row of the first matrix 'multiplied' by each column of the second. And by 'multiply', multiply each pair of number of numbers pairwise, and add these up. See my slides for suitable animation! e.g. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 1\times 5 + 2\times 6 \\ 3\times 5 + 4\times 6 \end{pmatrix} = \begin{pmatrix} 17 \\ 39 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 10 \end{pmatrix} = \begin{pmatrix} 5 & 20 \\ 11 & 40 \end{pmatrix}$ Important: When we multiply by a matrix, <u>it goes on the front</u>. So $\boldsymbol{A}$ multiplied by $\boldsymbol{B}$ is $\boldsymbol{BA}$, not $\boldsymbol{AB}$.	When multiplying matrices, doing each column in the first matrix multiplied by each row in the second, rather than the correct way.
5.2 The identity matrix, $\boldsymbol{I}$ (2×2 only).	$\boldsymbol{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Just as '1' is the identity in multiplication of numbers, as $a \times 1 = a$ and $1 \times a = a$ (i.e. multiplying by 1 has no effect), $\boldsymbol{I}$ is the same for matrices, i.e. $\boldsymbol{AI} = \boldsymbol{IA} = \boldsymbol{A}$.	
5.3 Transformations of the unit square in the $x - y$ plane.	Matrices allow us to represent transformations such as enlargements, rotations and reflections. Example: *"Find the matrix that represents the* $90°$ *clockwise rotation of a 2D point about the origin."* Easiest way to is to consider some arbitrary point, say $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$, and use a sketch to see where it would be after the transformation, in this case $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$. Thus more generically we're looking for a matrix such that: $\begin{pmatrix} \quad & \quad \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix}$ It is easy to see this will be $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	

<table>
<tr><td></td><td>Using the same technique we can find:
• Rotation $90°$ anticlockwise about the origin: $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$
• Reflection $180°$ about the origin: $\begin{pmatrix}-1 & 0\\0 & -1\end{pmatrix}$
• Reflection in the line $y=x$: $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$
• Reflection in the line $x=0$: $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$
• Reflection in the line $y=0$: $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$
• Enlargement scale factor 2 centre origin: $\begin{pmatrix}2 & 0\\0 & 2\end{pmatrix}$
Note that a rotation is <u>anticlockwise</u> if not specified.

The 'unit' square consists of the points $\begin{pmatrix}0\\0\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}$.
To find the effect of a transformation on a unit square, just transform each point in turn.
e.g. "On the grid, draw the image of the unit square after it is transformed using the matrix $\begin{pmatrix}3 & 0\\0 & 3\end{pmatrix}$."
Transforming the second point for example we get:
$$\begin{pmatrix}3 & 0\\0 & 3\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}3\\0\end{pmatrix}$$</td><td></td></tr>
<tr><td>5.4 Combination of transformations.</td><td>The matrix BA represents the combined transformation of A followed by B. Example:
"A point P is transformed using the matrix $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$, i.e. a reflection in the line $x=0$, followed by $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$, i.e. a reflection in the line $y=x$.
(a) Give a single matrix which represents the combined transformation.
(b) Describe geometrically the single transformation this matrix represents."
(a) $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}=\begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix}$
(b) Rotation $90°$ clockwise about the origin.</td><td>It is easy to accidentally multiply the matrices the wrong way round. It does matter which way you multiply them!</td></tr>
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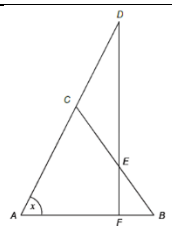
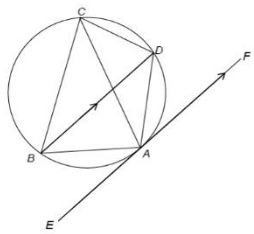
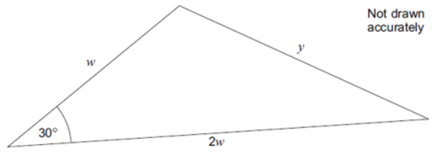
6. Geometry

Specification	Notes	What can go ugly
6.1 Perimeter and area of common shapes including area of triangle $\frac{1}{2}ab\sin C$ and volumes of solids. Circle Theorems.		
6.2 Geometric proof: Understand and construct geometric proofs using formal arguments.	Examples: *"Triangle ABC is isosceles with $AC = BC$. Triangle CDE is isosceles with $CD = CE$. ACD and DEF are straight lines.* *(a) Prove that angle $DCE = 2x$ and (b) Prove that DF is perpendicular to AB"*  Make clear at each point what the angle is you're calculating, with an appropriate reason. It may help to work out the angles on the diagram first, before writing out the steps. (a) $\angle CBA = x$ (base angles of isosceles triangle are equal) $\angle ACB = 180 - 2x$ (angles in ΔABC add to 180) $\angle DCE = 2x$ (angles on straight line add to 180) (b) $\angle DEC = \frac{180-2x}{2} = 90 - x$ (base angles of isosceles triangle are equal) $\angle DFA = 180 - (90 - x) - x = 90°$ $\therefore DF$ is perpendicular to AB. (In general with proofs it's good to end by restating the thing you're trying to prove) *"A, B, C and D are points on the circumference of a circle such that BD is parallel to the tangent to the circle at A. Prove that AC bisects angle BCD."*  $\angle BCA = \angle BAE$ (by Alternate Segment Theorem) $\angle BAE = \angle DBA$ (alternate angles are equal) $\angle DBA = \angle ACD$ (angles in the same segment are equal) So $\angle BCA = \angle ACD$. AC bisects $\angle BCD$.	Not given reasons for each angle. Angles (and their reasons) not being given in a logical sequence. Misremembering Circle Theorems! (learn the wording of these verbatim)
6.3 Sine and cosine rules in scalene triangles.	**Sine rule**: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (recall from GCSE that if have a missing angle, put sin's at top). **Cosine rule**: $a^2 = b^2 + c^2 - 2bc\cos A$ (use when missing side is opposite known angle, or all three sides known and angle required) Example: *"If area is 18cm², work out y."*  Area is given, so use area formula: $\frac{1}{2} \times w \times 2w \times \sin 30° = 18$ $\frac{1}{2}w^2 = 18 \rightarrow \ w = 6$ Then using cosine rule to find y: $y^2 = 6^2 + 12^2 - 2 \times 6 \times 12 \times \cos 30°$ $y = 7.44cm$	Forgetting to square root at the end when using cosine rule to find a side.

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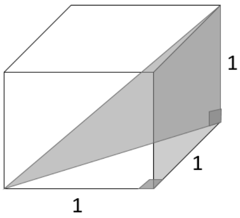
<tr>
<td>6.4 Use of Pythagoras' Theorem in 2D and 3D.</td>
<td>

We often have to form a 2D triangle 'floating' in 3D.
e.g. "Find the length of the diagonal joining opposite corners of a unit cube."
We want the hypotenuse of the indicated shaded triangle. First use Pythagoras on base of cube to get $\sqrt{2}$ bottom length of triangle. Then required length is $\sqrt{2+1^2}=\sqrt{3}$.</td>
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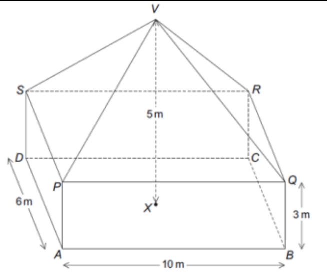
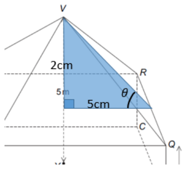
<tr>
<td>6.5 Find angle between a line and a plane, and the angle between two planes.</td>
<td>

"(a) Work out the angle between line VA and plane $ABCD$."
I use what I call the 'pen drop' strategy. If a pen was the line VA and I dropped it onto the plane (ABCD), it would fall to AX. Thus the angle we're after is between VA and AX. By using simple trig on the triangle VAX (and using Pythagoras on the square base to get $AX=\sqrt{34}$), we get $\angle VAX=\tan^{-1}\left(\frac{5}{\sqrt{34}}\right)$
(b) "Work out the angle between the planes VQR and $PQRS$."


When the angle is between planes, our 'pen' this time must be perpendicular to the line formed by the intersection of the two planes. Thus we put our 'pen' between V and the midpoint of RQ. We then drop the 'pen' onto the plane $PQSR$. We get the pictured triangle.</td>
<td>In (b) in the example, we might accidentally find the angle between VQ and the plane (this angle will be too steep).</td>
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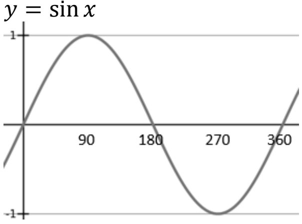
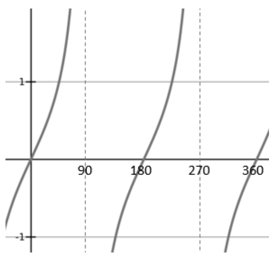
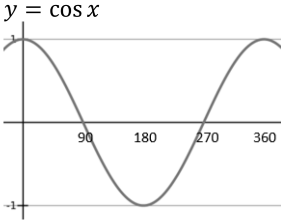
<tr>
<td>6.6 Graphs $y=\sin x$, $y=\cos x$, $y=\tan x$ for $0°\leq x\leq 360°$</td>
<td>$y=\sin x$


$y=\tan x$


$y=\cos x$


(Note that $y=\tan x$ has asymptotes at $x=90°, 270°$, etc. The result is that tan is undefined at these values)</td>
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<tr>
<td>6.7 Be able to use the definitions $\sin\theta$, $\cos\theta$ and $\tan\theta$ for any positive angle up to $360°$ (measured in degrees only)</td>
<td>The 4 rules of angles as I call them!
1. $\sin(x)=\sin(180°-x)$
2. $\cos(x)=\cos(360°-x)$
3. sin and cos repeat every $360°$
4. tan repeats every $180°$
We'll see this used in [6.10].</td>
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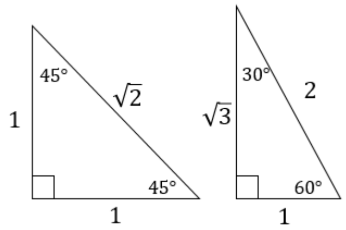
</table>

6.8 Knowledge and use of $30°, 60°, 90°$ triangles and $45°, 45°, 90°$.	We can use half a unit square (which has angles $45°, 45°, 90°$) and half an equilateral triangle originally with sides 2 (angles $30°, 60°, 90°$), as pictured below, to get exact values of $\sin 30°$, $\sin 45°$, etc. We use Pythagoras to obtain the remaining side length.  Then using simple trigonometry on these triangles: $\sin 30° = \frac{1}{2}$ $\sin 60° = \frac{\sqrt{3}}{2}$ $\sin 45° = \frac{1}{\sqrt{2}}$ Similarly $\cos 30° = \frac{\sqrt{3}}{2}$, $\cos 60° = \frac{1}{2}$, $\cos 45° = \frac{1}{\sqrt{2}}$ $\tan 30° = \frac{1}{\sqrt{3}}$, $\tan 60° = \sqrt{3}$, $\tan 45° = 1$ You don't need to memorise all these, just the two triangles!	
6.9 Trig identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \cos^2\theta = 1$	Remember that $\sin^2\theta$ just means $(\sin\theta)^2$ *"Prove that $1 - \tan\theta \sin\theta \cos\theta \equiv \cos^2\theta$"* Generally a good idea to replace $\tan\theta$ with $\frac{\sin\theta}{\cos\theta}$. $1 - \frac{\sin\theta}{\cos\theta}\sin\theta\cos\theta \equiv \cos^2\theta$ $1 - \frac{\sin^2\theta\cos\theta}{\cos\theta} \equiv \cos^2\theta$ $1 - \sin^2\theta \equiv \cos^2\theta$ $\cos^2\theta \equiv \cos^2\theta$ *"Prove that $\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta}$"* Generally a good idea to combine any fractions into one. $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \equiv \frac{1}{\sin\theta\cos\theta}$ $\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} \equiv \frac{1}{\sin\theta\cos\theta}$ $\frac{1}{\sin\theta\cos\theta} \equiv \frac{1}{\sin\theta\cos\theta}$ See my slides for more examples.	
6.10 Solution of simple trigonometric equations in given intervals.	***"Solve $\sin(x) = -0.3$ in the range $0° \leq x < 360°$"*** $x = \sin^{-1}(-0.3) = -17.46°$ At this point, we use the rules in [6.7] to get the solutions in the range provided. We usually get a pair of solutions for each 360° interval: $180 - -17.46 = 197.46°$ (since $\sin(x) = \sin(180° - x)$) $-17.46° + 360° = 342.54°$ (since sin repeats every 360°) ***"Solve $2\tan(x) = 1$ in the range $0° \leq x < 360°$"*** $\tan(x) = \frac{1}{2}$ $x = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$ $26.6° + 180° = 206.6°$ (tan repeats every 180°) ***"Solve $\sin x = 2\cos x$ in the range $0° \leq x < 360°$"*** When you have a mix of sin and cos (neither squared), divide both sides of the equation by cos: $\tan x = 2$ $x = \tan^{-1}(2) = 63.4°, 243.4°$ ***"Solve $\tan^2\theta + 3\tan\theta = 0$ in the range $0° \leq x < 360°$"*** Factorising: $\tan\theta(\tan\theta + 3) = 0$ $\tan\theta = 0 \quad or \quad \tan\theta = -3$ $\theta = 0°, 180°, \quad -71.6°, 108.4°, 288.4°$ (Cross out any solutions outside the range, i.e. $-71.6°$)	One of two main risks: (a) Missing out solutions, either because we haven't used all the applicable rules in 6.7, or we've forgotten the negative solution when square rooting both sides (where applicable). In $\tan^2\theta + 3\tan\theta = 0$, it would be wrong to divide by $\tan\theta$ because we lose the solution where $\tan\theta = 0$ (in general, never divide both sides of an equation by an expression involving a variable – always factorise!) (b) Mixing up the rules in 6.7, e.g. doing $180 -$ when you were supposed to $360 -$

	"***Solve*** $\boldsymbol{cos^2 \theta = \frac{1}{4}}$ ***in the range*** $\boldsymbol{0° \leq x < 360°}$" You get both $\cos\theta = \frac{1}{2}$ and $\cos\theta = -\frac{1}{2}$, so solve both! "***Solve*** $\boldsymbol{2\,sin^2 \theta - sin\,\theta - 1 = 0}$ ***in the range*** $\boldsymbol{0° \leq x < 360°}$" Again factorising: $(2\sin\theta + 1)(\sin\theta - 1) = 0$ $$\sin\theta = -\frac{1}{2} \quad or \sin\theta = 1$$ ... "***Solve*** $\boldsymbol{2cos^2 \theta + 3\,sin\,\theta = 3}$ ***in the range*** $\boldsymbol{0° \leq x < 360°}$" If you have a mix of sin and cos with one of them squared, use $\sin^2 x + \cos^2 x = 1$ to change the squared term. $2(1 - \sin^2\theta) + 3\sin\theta = 0$ $2 - 2\sin^2\theta + 3\sin\theta = 0$ $2\sin^2\theta - 3\sin\theta - 2 = 0$ $(2\sin\theta - 1)(\sin\theta - 1) = 0$ $\sin\theta = \frac{1}{2}\ or \sin\theta = 1$	