Edexcel AS Physics in 100 Pages

Yajun Wei

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-----an easy-to-understand textbook & exam preparation guide

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Preface

The purpose of this book is to offer a concise and easy-to-understand textbook and exam preparation guide of the Edexcel AS Physics.

This book is slim, with only 100 pages! But it covers almost all that you need to learn to crack the Edexcel AS Physics Unit 1 and Unit 2 exams. It can be used as a textbook for your AS physics courses. It is an excellent revision guide as well. The principles and terms are explained in very simple words and sentences. They are illustrated in the students' point of view, rather in the physicists or teachers' point of view. There are plenty of carefully designed diagrams and examples to illustrate difficult concepts.

This work is written based on my teaching experience. During my teaching, I always tried to find the ways of instruction that are easy for the students to understand. I decide to put down the contents in the ways that my students found easiest to understand and memorize. You will find it a bit easier to understand some difficult physical concepts with this book compared to some other books.

This book is written based on the specifications published by Edexcel and my study on the past papers. It is excellent for exam preparation. Contents of particular importance are highlighted in gray background. The examiners frequently request you to write down these sentences or use these formulae for calculations. You will find a great match between highlighted contents in this book and the exam paper questions!

Finally, good luck.

---------The author, Yajun Wei

About the author

The author is an A-level physics teacher. He has a B.S. in physics and M.S. in electrical engineering. He is interest in physics, engineering sciences and physics education. He began to teach A-level physics since 2007 and most of his students end up getting A in the exams. He also has experience in teaching International Baccalaureate physics and mathematics. He also do research in these fields and published some papers.

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Chapter 0 Physical Quantities and Units

A physical quantity is a physical property that can be measured quantitatively (in numbers). Mass, pressure, density, temperature and forces are some examples of physical quantities.

Physical quantities can be divided to **base quantities** and **derived quantities**. Base quantities are expressed in **base units**. There are 7 base quantities, length, mass, time, electric current, temperature, amount of substance and luminous intensity. All other quantities such as force, energy are derived quantities. Derived quantities are expressed in **derived units**. The following table summarizes all the 7 base quantities and their corresponding SI base units. All derived units can be derived from the 7 base units.

Physical quantity	SI unit	Symbol for the unit
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount of substance	mole	mol
luminous intensity	candela	cd

The seven base quantities and base units

To deal with the AS physics tests, you only need to memorize the five highlighted base quantities and units as the other two are not be covered in this quantification.

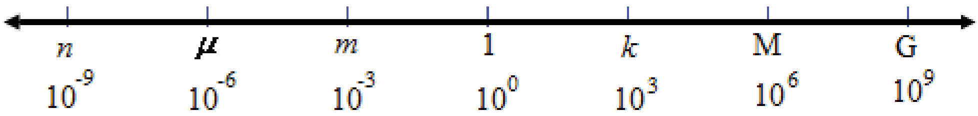
Example: (get back to this example after you finish the whole book)
(a) Express the unit N in terms of base units only.
(b) Express the unit of potential difference volt in terms of base units only.
Solution:
(a) $F = ma$. Since the unit of mass m is kg and the unit of acceleration a if ms^{-2}, the unit Force F is $kgms^{-2}$.
(b)Potential difference is given by

$$V = \frac{W}{Q} = \frac{Fs}{It} = \frac{mas}{It}$$

The unit of m is kg, unit of a is ms^{-2}, unit of s is m, unit of I is A and unit of t is s. So the unit of potential difference is then given by

$$V = \frac{kg \cdot ms^{-2} \cdot m}{A \cdot s} = kgm^2 s^{-3} A^{-1} .$$

Also, remember the following scale. It will make your life easier while doing unit conversions.



Chapter 1 Mechanics

1.1 Motion in one dimension

Speed, velocity, distance and displacement

We use speed to describe how fast dose an object moves. In physics, **speed** is defined as the distance traveled in unit time (one second). That is

$$speed = \frac{distance\ traveled}{time\ taken} \tag{1.1}$$

or

$$v = \frac{s}{t} \tag{1.2}$$

Equation(1.1) and Equation(1.2) describe the same thing, but Equation(1.1) is called **symbol equation** and Equation(1.2) is called **word equation**. The most commonly used unit for speed is ms^{-1} and kmh^{-1}. The conversion between these two units are given by

$$1kmh^{-1} = 1\times1000\text{m}/3600\text{s} = (1/3.6)\,ms^{-1}$$

and

$$1ms^{-1} = (1/1000)km/(1/3600)h = 3.6\,kmh^{-1}$$

For example, if a train travels 300 km in one and half hours, then its speed is 200 kmh^{-1}, or 55.6 ms^{-1}.

In most cases, the speed is not constant. For the motion of the train, it may go as slow as 20km/h a few seconds after it departures, but may increase its speed to 300km/h a few minutes later. The speed at a particular instant is called **instantaneous speed**. But in the case of train discussed above, we are more interested in the **average speed** which is defined as the total distance traveled during a given time interval.

Take a look at the Fig 1.1.1. A student walks from A to B along a curved path along a water pond (the blue line). Then the distance walked is the length of the path. The length of the direct line joining A and B is called the **displacement**.

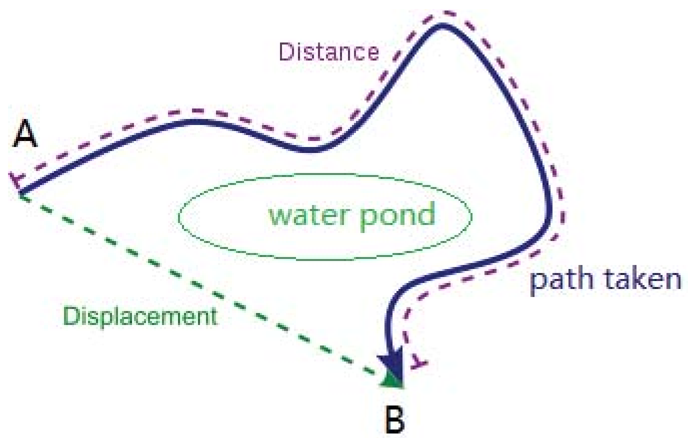


Fig 1.1.1 Distance and displacement

But actually, this is only part of displacement; it is the size of displacement. Displacement is a quantity that has both size (also called **magnitude**) and direction. This kind of physical quantity is called a **vector** quantity. Distance is a quantity that has only size but no direction. This kind of physical quantity is called a **scalar** quantity.

When an object is moving, it has a direction of its movement. The direction of **velocity** is the direction of movement. The size of the velocity is speed. Velocity is also a vector. We may say that the train is traveling at a velocity of 300km/h towards southeast or the train is traveling at a speed of 300km/m.

Acceleration

When an object increases its velocity, we say that it **accelerates. Acceleration** describes how fast velocity is increased. It is defined as the rate of change of velocity with time. That is

$$acceleration = \frac{final\ velocity - initial\ velocity}{time\,taken} \qquad or \qquad a = \frac{\Delta v}{\Delta t} = \frac{v-u}{\Delta t} \quad (1.3)$$

The SI unit of acceleration is ms^{-2}. Acceleration is a vector quantity. Sometimes even though the speed doesn't change, there is acceleration. An example is the uniform circular motion where the size of the velocity (speed) is constant but the direction of the velocity is changing all the time.

If the velocity is decreasing, we may say that it decelerates. Then the acceleration is a negative value, which is also called deceleration.

Accelerating uniformly along a straight line

This is a motion where the acceleration is a constant and it is a very important type of motion. This type of motion can be described by five quantities and three equations:

s ----displacement or distance

t ---- time taken

u ----initial velocity or speed

v ---final velocity or speed

a ---acceleration (for free fall motion under gravity, it is written as g and is equal to $9.81ms^{-2}$)

$$v = u + at \qquad (1.4)$$

$$s = ut + \frac{1}{2}at^2 \qquad (1.5)$$

$$v^2 - u^2 = 2as \qquad (1.6)$$

Any problems of uniform acceleration motion can be solved by using the above three equations. A wise first step to solve problems during exam is to list the known quantities and the quantity you are asked to calculate. Then you will easily see which of the above equation(s) could be used to find the unknown.

Example:
A car accelerates uniformly from stop with a rate of 6.1 ms^{-2}. Find the displacement and speed after 0.82 seconds.
Solution:
List the known quantities and unknown quantities as below:

Known: $u = 0$, $t = 0.82s$, $a = 6.1ms^{-2}$

Unknown: s , v

To find s , equation(1.5) can be used: $s = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 6.1 \times 0.82^2 = 2.05m$

To find v , equation(1.4) should be used: $v = u + at = 6.1 \times 0.82 = 5.0ms^{-1}$

Example:
A small stone is fired vertically upwards by a catapult with a speed of 10 ms^{-1}, find the maximum height the stone can reach.
Solution:
At maximum height, its speed is zero. List the known quantities and unknown quantities as below:

Known: $u = 10ms^{-1}$, $v = 0$, $a = g = 9.81ms^{-2}$

Unknown: s , t

To find the maximum height s with the knowledge of u , v and a , equation(1.6) should be used: $v^2 - u^2 = 2as$ gives $s = \frac{v^2 - u^2}{2a} = \frac{0 - 10^2}{2 \times 9.81} = -5.1m$

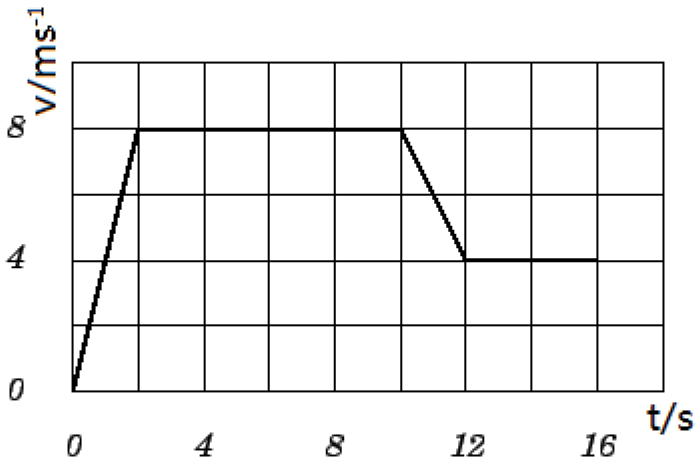
The minus sign indicates that the direction of displacement is opposite to the direction of the acceleration. The maximum height the stone can reach is 5.1m.

Motion graphs

Motion graphs show how displacement or velocity changes with time. It is a very useful and straightforward way of describing motion by graphs. In a **displacement/distance--time graph**, a straight line indicates that the object is moving at a constant velocity. If the graph is a curve, it means that the object is accelerating. The gradient of a displacement/distance—time graph is the change of displacement/distance over time, which is velocity/speed.

In a **velocity/speed--time graph**, a straight line parallel to the time axis indicates that the object is moving at a constant velocity. If the graph is a straight line with a slope, it means that the object is accelerating uniformly. The gradient of a velocity/speed—time graph is the change of velocity/speed over time, which is acceleration. The area under the velocity/speed—time graph represents the displacement/distance moved.

Example:
Consider the motion of the object whose velocity-time graph is given in the diagram below. (a)What is the acceleration of the object between times t=0s and t=2s? (b)What is the average speed between times t=8s and t=12s?



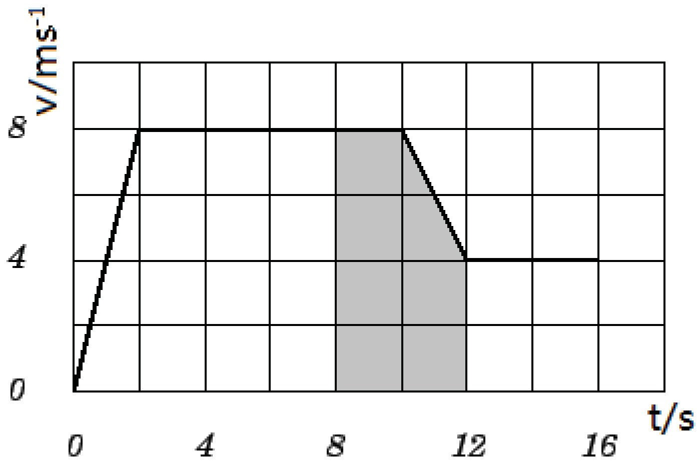
Solution:
(a) The acceleration is equal to the gradient of the line between t=0s and t=2s, which is

$$a = \frac{\Delta v}{\Delta t} = \frac{v-u}{\Delta t} = \frac{8-0}{2} ms^{-2} = 4ms^{-2}$$

(b)The average speed should be calculated using total distance divided by time taken. Time taken is 12-8=4s. Total distance traveled during this time interval is equal to the shaded area under the v—t graph, as shown in the figure below.

$$\text{total displacement} = \text{ shaded area} = 2\times 8 + (1/2)\times(4+8)\times 2 = 28\text{m}$$

Then the average speed=28m/4s=7ms^{-1}.



1.2 Combining and resolving vectors

We have already known that scalars have only magnitude while vectors have both magnitude and direction. In this section we will discuss vectors in more detail. Displacement, velocity, acceleration and force are four vector quantities we have already or will study in this book. All other quantities encountered in this book (therefore AS Unit1 and Unit2 tests) are scalars.

Combining/adding vectors

Mass is a scalar quantity, if you add masses of 5 kg and 3 kg, you simply get a total mass of 8 kg. But while combining vectors, you can't just simply add their magnitudes together. Directions need to be taken into account as well.

If you want to find the resultant of two vectors, the parallelogram rule or the triangle rule should be used.

Triangle rule:

Fig 1.2.1 illustrates how to add/combine vectors. Draw the two vectors to be combined (added) tip-to tail and the resultant vector is the third side of the triangle pointing from the tail of the first vector (A) to the tip of the second vector (B).

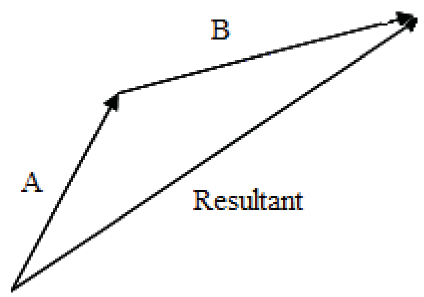


Fig 1.2.1 combining vectors using triangle rule

Example:

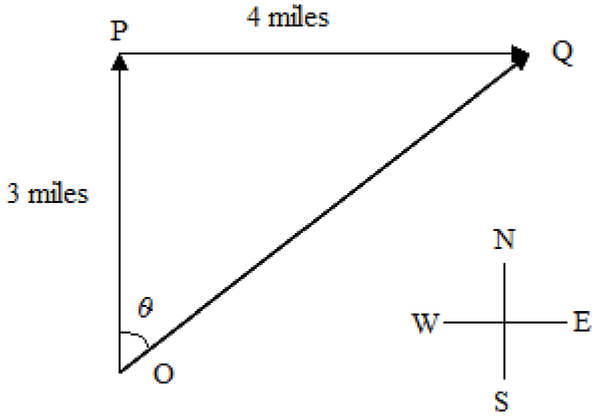
A student walks due north from point O to P for 3 miles and then walks 4 miles due east from P to Q, find the total displacement of the student OQ.

Solution:

As shown in the diagram below, the size of the total displacement is equal to the length of OQ, which is $\sqrt{3^2+4^2}$ =5 *miles*

The direction of the displacement is measure by the angle θ. Since $\tan\theta = \frac{4}{3}$,

$\theta = 53^o$.



Parallelogram rule:

Draw the two vectors to be combined tail-to-tail as two sides of a parallelogram, as shown in Fig 1.2.2. The resultant vector is simply the diagonal of the parallelogram.

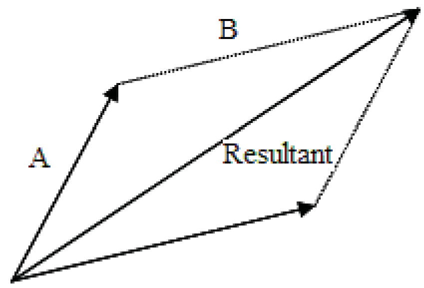
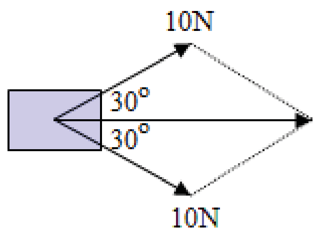


Fig 1.2.2 Combing vetors using parallelogram rule

Example:

Two forces F_1 and F_2 are acting on an object with a mass of 5kg, as shown in the graph below. Both forces are 10 N. Find the acceleration of the object.



Solution:

To find the acceleration, we need to find the resultant force first.

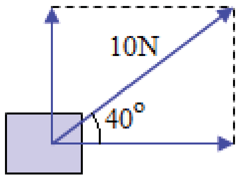
Using geometrical knowledge, it is easy to find the diagonal of the parallelogram. It is $2 \times 10 \times \cos 30^o = 17.3N$. So the size of the resultant force is 17.3N. The acceleration is given by

$$a = \frac{\sum F}{m} = \frac{17.3N}{5kg} = 3.46ms^{-2}$$

Resolving/decomposing vectors

Vector decomposition is the process of representing a vector as the sum of two vectors perpendicular to each other. These two vectors are called the **component**s. These two components have the same effect as the single vector to be decomposed. This process is also called resolving vectors.

Example:
A student is towing a small box on a table with a force of 10N at an angle of 40^o to the horizontal. Find the horizontal and vertical components of the force.
Solution:
Draw the force diagram as follows



The horizontal component of the force is $F_{horizontal} = 10N \times \cos 40^o = 7.7N$

The vertical component of the force is $F_{vertical} = 10N \times \sin 40^o = 6.4N$

1.3 Motion and Force

Newton's First Law

Newton's first law of motion states that:
If the resultant force on an object is zero, it will stay at rest or move at constant velocity.
When the resultant force on an object is zero and it stays at rest, then we say that the object is in stationary, or in **equilibrium**. The branch of physics that study bodies in equilibrium is called **statics**.
The Newton's first law is also called **inertia law. Inertia** describes any object's resistance to change of its state of motion. Inertia is measured by mass. The larger the mass, the larger the inertia.

Newton's Second Law

Newton's second law of motion states that:
Acceleration is proportional to the resultant force on it and inversely proportional to the mass.
In mathematical form, it is

$$\Sigma F = ma \quad (1.7)$$

where ΣF is the resultant force, m is the mass of the object and a is the acceleration. Newton's first law and second indicate that force is not the cause of motion, but it is the cause of change of motion, acceleration.

Example:
A driver is starting his car with a mass of 1000kg when the traffic light changes from red to green.
(a)When the driving force is 500N, the car is still at rest, find the friction at this time.
(b)When the driving force is increased to 6000N, the speed of the car increases from 0 to 10ms^{-1} in 2 seconds, what is the friction during this process?
Solution:
(a) Since the car is at rest, the resultant force should be zero. This means the driving force and the friction cancel each other. Then the friction is also 500N, with its direction opposite of the driving force.
(b) The acceleration of the car is given by

$$a = \frac{v-u}{t} = \frac{10-0}{2} = 5ms^{-1}$$

If f is the friction, according to Newton's second law, we have $F_{driving\ force} - f = ma$.

That is $6000N - f = 1000kg \times 5ms^{-2}$. Solving this gives $f = 1000N$

Newton's Third Law

Newton's third law of motion states that:

If body A exerts a force on body B, then body B exerts a force of the same type on body A that is equal in size but opposite in direction.

These two forces are called a **Newton third law pair**. Note that the Newton third law pair forces (action and reaction) are exerted on different bodies.

Example:

A man is standing still on the floor. List all the forces acting on him and find their Newton third law pair forces.

Solution:

Force	Body the force acting on	Newton third law pair force	Body the Newton third law pair force acting on	Type of the forces
weight	man	Attraction force on earth from the car	earth	Gravitational force
Push from the ground	man	Normal contact force from the feet of the man	floor surface	Normal contact force

Free-body diagram

A **free-body diagram** is a diagram that shows all forces of all types acting on a particular body that we are interested in. It is also called a **free-body force diagram**. The body of our interest is isolated. The free-body diagram dose not show forces acting on other objects. Fig 1.3.1 shows the free body diagram for a block on a ramp.

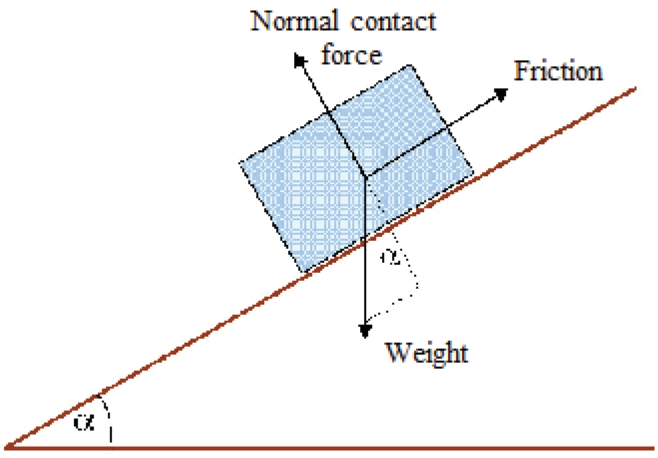


Fig 1.3.1 Free body diagram for a block on a ramp

Usually the first step to solve mechanics problems is to draw free body diagram. And then the forces are decomposed to two directions perpendicular to each other. Equations of Newton's second law in each direction can be written down. Finally the equations can be solved to find the unknown.

Centre of gravity & centre of mass

The gravitational force from the earth acts on every point on an object. But we often draw the weight of an object as acting on a single point. This point is the mean location of the gravitational force acting on the object. It is called the **centre of gravity**.

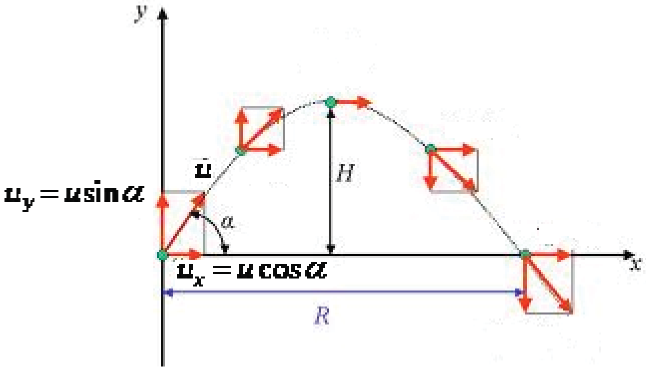
Centre of mass is the point at which all the object's mass are considered to concentrate. This point is the mean location of all the mass of the object.

The term center of mass is often used interchangeably with center of gravity. For a uniform object, centre of gravity and centre of mass are located in the middle of the object.

1.4 Motion in two dimensions—Projectile motion

Projectile motion

The object that is thrown out (projected) is called a **projectile**. The path this projectile follows is called **trajectory**. The trajectory of a projectile is a parabola (if projected vertically, it is a straight line).



In a projectile motion, the motion can be decomposed into two directions, the horizontal direction and the vertical direction. In horizontal direction, there is no force acting on the object (air resistance is neglected) and thus the object will move with a constant velocity in this direction.

$$v_x = u_x = u\cos\alpha \qquad (1.8)$$

$$s = v_x t = (u\cos\alpha)t \qquad (1.9)$$

In vertical direction, the acceleration is g=9.8ms^{-2} as weight is the only force acting on the object.

$$v_y = u_y - gt = u\sin\alpha - gt \qquad (1.10)$$

$$h = u_y t - \frac{1}{2}gt^2 = (u\sin\alpha)t - \frac{1}{2}gt^2 \qquad (1.11)$$

Using the above four equations enables you to find the velocity and displacement of the projectile at any given time.

Sometimes, it is useful to know how far a projectile can go if the initial velocity is given. This is called the **range** of the projection. Equation (1.9) should be used to find out the range. Let h=0 m in equation(1.11) to find out the time it takes for the projectile to land.

$$h = (u\sin\alpha)t - \frac{1}{2}gt^2 = 0 \qquad (1.12)$$

So,

$$t = \frac{2u\sin\alpha}{g} \qquad (1.13)$$

Substituting equation(1.13) into equation(1.9), the range of the projection R (horizontal displacement when it lands) is obtained as

$$\begin{aligned} R &= ut\cos\alpha \\ &= u\frac{2u\sin\alpha}{g}\cos\alpha \\ &= \frac{u^2\sin 2\alpha}{g} \end{aligned} \qquad (1.14)$$

We can see that the range depends both on the magnitude and direction of the initial velocity. When $\alpha = 45^o$, the maximum range is achieved. It is u^2 / g

Vertical projection

Vertical projection is a special case of a projectile motion when $\alpha = 90^o$. This means the motion is only along the vertical direction.

Horizontal projection

Horizontal projection is another special case of a projectile motion when $\alpha = 0^o$. This means that the initial velocity is completely in the horizontal direction. In this case, the object undergoes free fall in vertical direction and uniform motion in horizontal direction.

Letting the download direction as the positive direction, the motion equations can be simplified as follows.

In horizontal direction, it moves at uniform velocity.

$$v_x = u \qquad (1.15)$$

$$s = v_x t = ut \qquad (1.16)$$

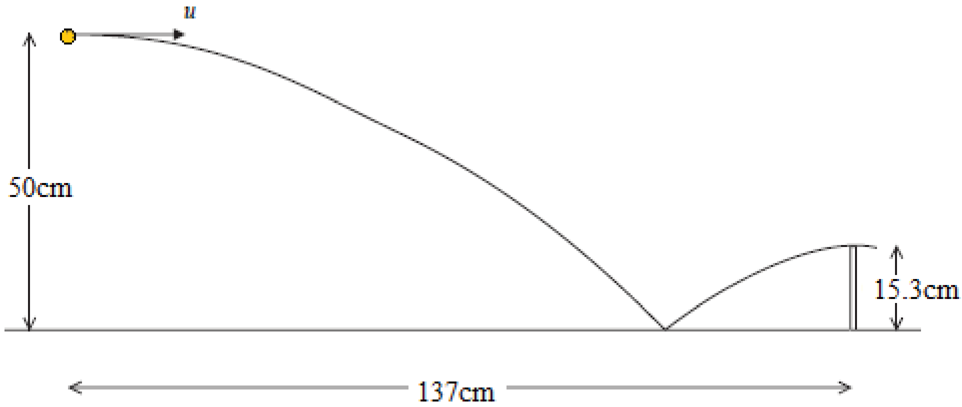
In vertical direction, the acceleration is g=9.8ms^{-2} as weight is the only force acting on the object.

$$v_y = gt \qquad (1.17)$$

$$h = \frac{1}{2}gt^2 \qquad (1.18)$$

Example:

A table tennis ball is thrown by a player with a height of 0.5 m from the table. The ball leaves the player's hand horizontally with an initial velocity *u*. After bouncing once, it passes just over the net at the top of its bounce. The height of a table tennis net is approximately 15.3cm. The length of the table is 274cm.



(a) How long dose it take the ball to reach the table.
(b) How long does it take the ball to rise 15.3cm after bouncing?
(c) Use your answers to parts (a) and (b) to calculate the initial horizontal velocity of the ball.

Solution:

(a)Known: $h = 0.5m$.

To find: t.

From equations(1.15)--(1.18) we can see that equation(1.18) $h = \frac{1}{2}gt^2$ can be used to find the t when the height is given.

The time taken to fall is $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 0.5}{9.81}} = 0.32s$.

(b)Since the ball passes just over the net at the top of its bounce, its speed at this point is zero and it reached its maximum height. Consider the motion in vertical direction.

Known: final velocity $v_y = 0$, displacement $h_{net} = 0.153m$, acceleration

$a = -g = -9.81ms^{-2}$

Unkown: initial velocity u_y , time t

In the five quantities of motion, you have already known three, so you know you can find the other two using the three equations. Now you should feel happy.

List the three equations. $v = u + at$ $s = ut + \frac{1}{2}at^2$ and $v^2 - u^2 = 2as$.

To find t, we need to use $v = u + at$ or $s = ut + \frac{1}{2}at^2$. But in either way, we need to known the initial velocity. The initial velocity can be find using $v^2 - u^2 = 2as$.

That is

$$v_y{}^2 - u_y{}^2 = -2gh_{net} .$$

Substituting the values into the above equation and solving for u_y yields

$$0^2 - u_y{}^2 = -2 \times 9.81 \times 0.153$$

$$u_y = 1.73 ms^{-1}$$

Now we can use $v = u + at$ to find the time t. For our case, it is $0 = 1.73 - 9.81t$ and thus the time taken is $t = 5.67s$

(c) Consider the motion in horizontal direction. The equation is $s = ut$. So the initial velocity is given by

$$u = \frac{0.137m}{0.32s + 5.67s} = 0.023ms^{-1}$$

Comment: To solve projectile motions problems the first step is to resolve the motion into horizontal and vertical directions. Then for the motion in each direction, just use the three equations to find the unkown. It is quite simple!

1.5 Work and Energy

Energy is the ability to perform work. There are many forms of energy. A few examples are kinetic energy, gravitational potential energy, elastic potential energy, electrical energy, solar energy, chemical energy.

Energy conservation

The **principle of conservation of energy** states that:

Energy cannot be created or destroyed.

This is a very important statement that you should memorize. Total energy in the universe is conserved, which means its amount doesn't change. However, energy can be transferred from one object to another or be transformed from one form to another form. For example, if billiard ball A hits billiard ball B, then some kinetic energy of ball A is transferred to ball B, but the total energy of the two doesn't change. A light bulb transforms electrical energy to light and heat energy.

There are two ways to transfer energy, by heating and working. By heating an object, heat energy (also called thermal energy) is transferred to it.

If there is a force acting on an object and there is a displacement along the direction of the force, the force dose work on the object and energy is transferred to the object.

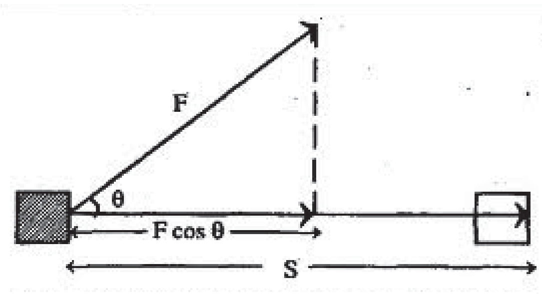


Fig 1.5.1 Work done by a force

The work done is equal to the displacement multiply the force component along the direction of the displacement, that is

$$W = Fs\cos\theta \qquad (1.19)$$

The SI unit for work is joule (J). 1 J of work is done if a 1N force is applied on an object and the object moves 1metre along the direction of the force. 1J=1Nm.

When the force is not a constant but changes with displacement, the work done can be estimated by the area under the force-displacement graph.

Work done on an object is equal to the increase of its energy. When 1 J work is done on an object by a force, the object increases its energy by 1 J. The unit of energy is also J.

Kinetic energy

Kinetic energy of an object is the energy that it possesses due to its motion. The amount of kinetic energy that an object with mass m moving in speed v possesses is

$$E_k = \frac{1}{2}mv^2 \qquad (1.20)$$

Example:
An airplane has a mass of 1500 kg. Calculate the kinetic energy of the airplane when it is flying at a speed of 500miles/h.
Solution:
The speed of the plane is

$$v = 500miles/h = (500\times1.64)km/h = 820km/h = 227.8ms^{-1}$$

The kinetic energy is then

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}\times1500\times227.8^2 = 3.89\times10^7\,J$$

Gravitational potential energy

When an object is lifted to a higher position, work is done against the gravitational force (weight). Thus, the **gravitational potential energy** is increased. The change of gravitational potential energy for an object is

$$\Delta E_{grav} = mg\Delta h \qquad (1.21)$$

Example:
A bucket of water together with the bucket has a total mass of 15kg. What is the change of gravitational potential energy of the bucket when it is raised from the 1st floor to the 2nd floor (the height difference is 3.0 metres)?
Solution:
The change of gravitational potential energy is

$$\Delta E_{grav} = mg\Delta h = 15\times9.81\times3.0\,J = 441J$$

Power

Power is the work done (or energy transferred) during unit time. That is

$$P = \frac{W}{t} = \frac{E}{t} \qquad (1.22)$$

The SI unit of power is Watt (W). 1W=1J/s. Another commonly used unit is Horsepower (HP). 1HP=750W.

From the definition of power, we can also derive the following expression

$$P = \frac{W}{t} = \frac{Fs}{t} = Fv \qquad (1.23)$$

This equation explains a phenomenon in our daily life. For a car to climb a steep slope, a bigger driving force is required. What the driver dose is to reduce the speed because by doing so, the driving force can be increased, as explained by equation(1.23).

Example:
What is the power of a car's engine if it is moving at a steady speed of $30ms^{-1}$ and the driving force is 1.5kN?
Solution:
The power is

$$P = Fv = 1.5kN \times 30ms^{-1} = 45kW$$

You might ask that since there is a force of 1.5kN exerted on the car, how could it move at a steady speed? This is because the force is not a resultant force. The 1.5kN driving force is cancelled out by the friction and air resistance. The resultant force is still zero. The power calculated here is the power of the driving force only.

Efficiency

Efficiency is defined as the useful energy output divided by total energy input.

$$efficiency = \frac{useful\ energy\ output}{total\ energy\ input} \times 100\% \qquad (1.24)$$

This can be also written as

$$efficiency = \frac{useful\ energy\ output}{total\ energy\ input} \times 100\% = \frac{useful\ \ power\ output \times time}{total\ \ power\ input \times time} \times 100\%$$

(1.25)

Cancelling the variable *time* in the above equation gives

$$efficiency = \frac{useful\ \ power\ output}{total\ \ power\ input} \times 100\% \qquad (1.26)$$

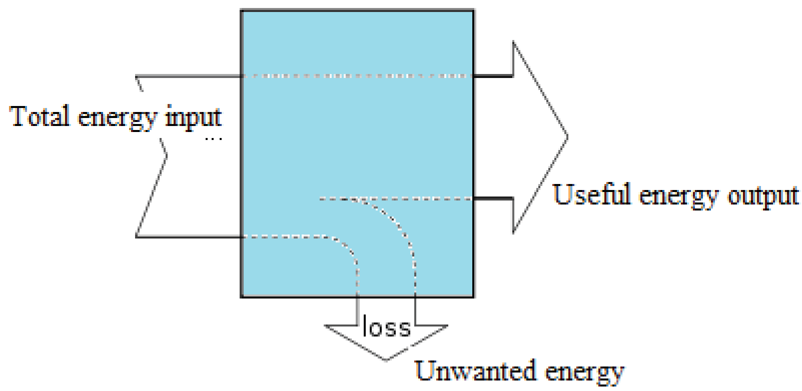


Fig 1.5.2 Efficiency

Example:

The input power of a car engine is 100kW. Its maximum speed is 120km/h if the drag force on it is 1500N. What is the efficiency of the car engine in this case? Where is the lost energy?

Solution:

The total power input is $P_{in} = 100kW = 10^5 W$

The useful power output is $P_{out} = Fv = 1500 \times (120 \times 10^3 / 3600)W = 50000W$

The efficiency of the car engine = P_{in} / P_{out} =50%

Some energy is lost due to heating.

Chapter 2 Materials

2.1 Statics of Fluid

Fluid is any substance that can flow. For example, water, milk and air are all fluid. Fluid includes gases and liquids.

Density

Density is an important property of materials. It is defined as the mass per unit volume (one cubic metre). That is

$$density = \frac{mass}{volume} \qquad or \qquad \rho = \frac{m}{V} \tag{2.1}$$

The above equation holds for all types of materials, including fluids and solids, pure substance and mixtures. The SI unit of density is kgm^{-3}. Another commonly used unit is gcm^{-3}. 1 kgm^{-3}=1000 gcm^{-3}. The density of water is 1000 kgm^{-3}. This means that $1m^3$ water has a mass of 1000 kg.

Example

A bottle of beer contains 250ml beer. The mass of the beer (excluding the bottle) is 0.238 kg. What is the density of the beer?

Solution:

The density of the beer is $\rho = \frac{m}{V} = \frac{0.228kg}{250\times10^{-6}m^3} = 9.52\times10^{-4}\,kgm^{-3}$

Example

One kilogram of salt is poured into 49.8L of water and makes 50L salt water. Find the density of the salt water.

Solution:

The mass of the salt water is equal to the sum of the mass of salt and water, that is

$$m_{salt\ water} = m_{salt} + m_{water} = m_{salt} + \rho_{water}V_{water} = 1kg + 1000\times(49.7\times10^{-3})kg = 50.7kg$$

The density of the salt water is $\rho = \frac{m_{salt\ water}}{V_{salt\ water}} = \frac{50.7kg}{50\times10^{-3}m^3} = 1.014\times10^3\,kgm^{-3}$

Upthrust

When an object is submerged in a fluid, it feels an upward force that is called **upthrust**. This is the force that keeps ships and boats floating. Greek scientist Archimedes found that **the size of the upthrust force is equal to the weight of the fluid that has been displaced by the object**. This is called the **Archimedes' Principle**. It can be written in mathematical form as

$$F = m_{displaced\ fluid}\, g = \rho_f V g \qquad (2.2)$$

Note that in the above equation ρ_f is the density of the fluid. Do not mistake it as the density of the object!

Example:
A brick with a size of $20\text{cm}\times 10\text{cm}\times 8\text{cm}$ has a density of 2000kgm^{-3}. This brick was thrown by a naughty boy to a swimming pool and it lies in the bottom of the pool. Find the normal contact force exerted by the brick on the floor of the swimming pool.
Solution:
To find the normal contact force from the brick to the floor, we may find its Newton third law pair force, the normal contact force from the floor to the brick first. To do this, a free body force diagram is draw as follows.

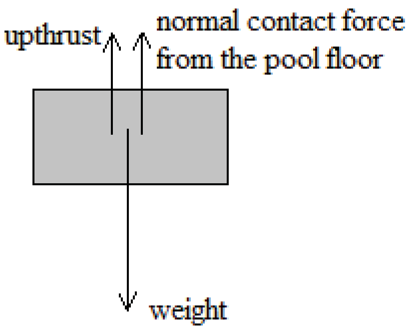


Fig 2.1.1 Free-body force diagram of the brick

As the brick is at rest in the bottom, we have

Upthrust+ Normal contact force form the floor= Weight

$$\rho_f V g + N = \rho_{brick} V g$$

Solving the equation for N gives

$$N = \rho_{brick} V g - \rho_f V g = (\rho_{brick} - \rho_f) V g$$

where $V = 20\text{cm}\times 10\text{cm}\times 8\text{cm} = 1600\text{cm}^3 = 1.6\times 10^{-3} m^3$ is the volume of the brick and $\rho_{brick} - \rho_f = 2000 - 1000 = 1000 kgm^{-3}$ is the density difference between the brick and water.

Substituting these values into the above equation, we get N=15.7N. So the normal contact force exerted on the floor of the swimming pool will also be 15.7N, but with its direction pointing downwards.

If you put a wood block into a water pool, it will float. If you put an iron block into a water pool, it will sink to the bottom of the pool. Whether an object released in a fluid will float or sink depends on the size of its weight and the upthrust. If the upthrust is smaller than its weight, then the object will **sink**. If the upthrust is just equal to its weight, then the object will be able to stay anywhere in the fluid, this state is called **suspension**. If the upthrust is larger than the object's weight, then the object will move upwards. When it gets to the surface of the fluid, part of its body will get out of the fluid and thus reduce the volume submerged in the fluid. So, the upthrust on the object will decrease to a value that is equal to the weight of the object. This state is called **floating**.

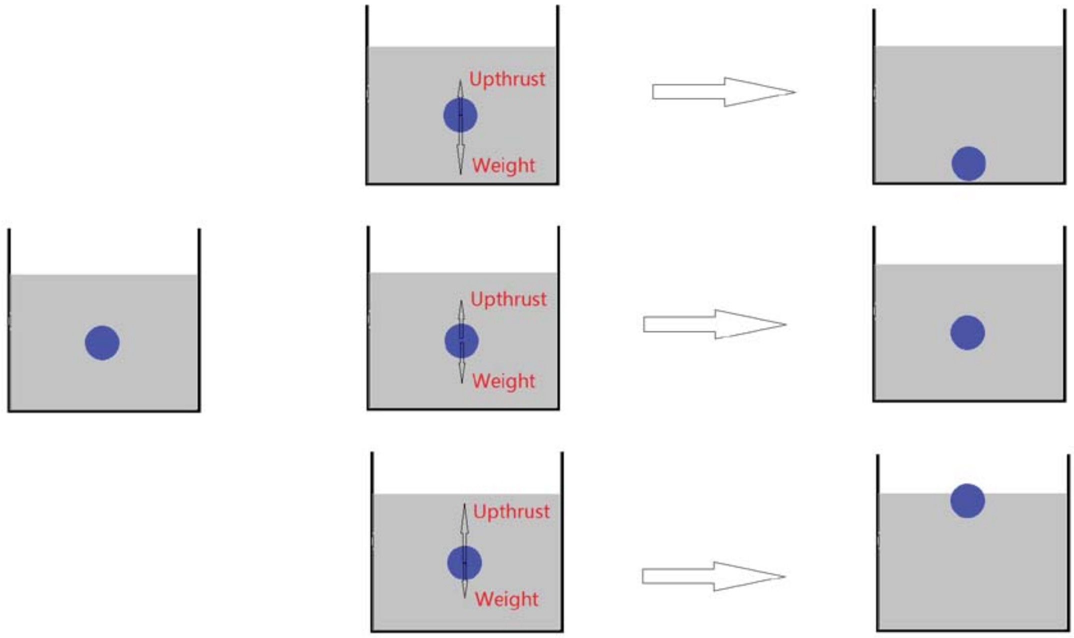


Fig 2.1.2 Sinking, suspension or floating

<u>Example:</u>

The hydrometer is a device that is used to determine the density of a liquid. It has a constant weight W. To balance the weight, you need to have a floating force with the same size. Thus, in liquids with different densities, the volumes (thus the depths) the hydrometer sink in the liquid are different.

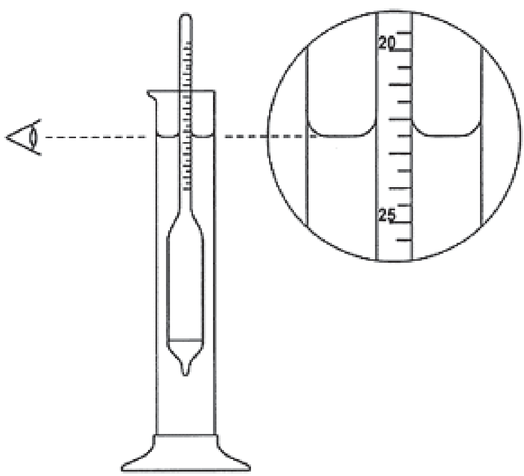


Fig 2.1.3 Hydrometer

If a hydrometer is put into pure water, 30% of the total volume of the hydrometer floats above the water surface. If the hydrometer is put into salt water with a density of 1.1 gm^{-3}, how much will it float above the surface of the salt water?

Solution:

According to Archimedes' Principle, the volume of the displaced liquid (also the volume that the hydrometer sinks) is inversely proportional to the density of the liquid. So, if we assume that x% of the hydrometer will float above the surface of the salt water, then

$$\frac{(1-30\%)V}{(1-x\%)V}=\frac{1.1gcm^{-3}}{1gcm^{-3}}$$

where V is the total volume of the hydrometer.

Solving the above equation gives $x\%=36\%$.

2.2 Dynamics of Fluid

Laminar and turbulent flows

In the section dynamics of fluid, we will focus on the flow of fluid. Fluid flow includes **laminar flow** and **turbulent flow**, as shown in Fig 2.2.1. Table 2.2.1 summarizes the most important features of these two types of flows.

Fig 2.2.1 Laminar and turbulent water flows

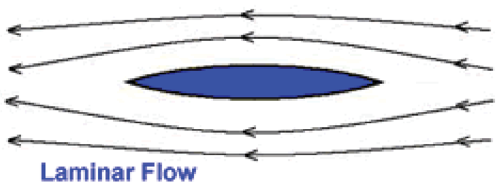
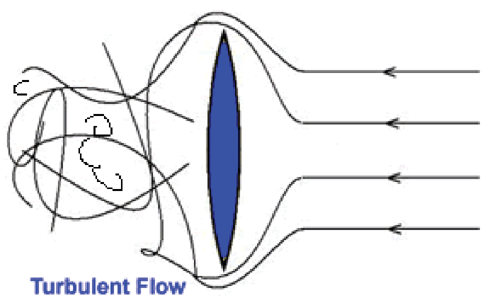
Laminar flow	**Turbulent flow**
● Velocity may be different at different places, but at any given place the velocity must be constant. ● Smooth, no sudden change of speed. ● Slow (occurs at low speed). ● Associated with small resistance	● Velocity at any given place changes with time. ● There are sudden changes of speed. There are eddies. ● Occurs at higher speed. ● Associated with big resistance
	
Example: slow water flow along a smooth pipe	Example: air flow in a storm

Table 2.2.1 Laminar flow and turbulent flow

Laminar flow is also called **streamlined flow**. The lines we use to represent the flow of laminar fluid flow is called **streamline**s, as shown in figure in Table 2.2.1
Objects moving in a turbulent flow experience a much bigger air resistance/drag force than moving in a laminar flow. Many vehicles are designed in streamlined shapes (plane, racing car) to reduce turbulent flow and thus reduce air resistance.

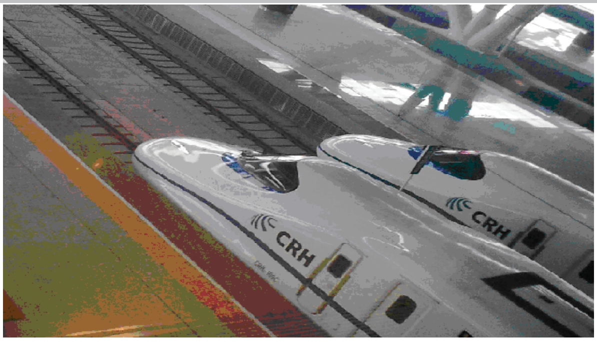


Fig 2.2.2 Streamlined design of high speed trains

Viscous drag

When an object is moving in a fluid, it will experience a drag force (resistance) that resists the motion of the object. If the object is moving downwards, this drag force is partially due to the upthrust. Another important source of the drag force is the **viscosity** of the fluid. The drag force caused by viscosity is called **viscous drag**. Viscosity is an intrinsic property of fluid. The factor **coefficient of viscosity** η (or simply viscosity) describes how large the viscosity of the fluid is. Or, in even simpler words, coefficient of viscosity describes how "sticky" a fluid is. The flow rate (volume of fluid passing through in unit time) of a fluid is inversely proportional to viscosity. The viscous drag is larger while moving in a fluid with larger coefficient of viscosity. Honey obviously has a much larger viscosity than water. A ball moving in honey will experience a much larger resistance than moving in water.

Viscosity, like density, is a property of material. Viscosity is dependent on temperature. In general, for gases, viscosity increases with temperature. For liquid, viscosity decreases with temperature. Table 2.2.2 lists viscosities for some common fluids.

Fluid	Temperature/°C	Viscosity/Pa s
Air	0	0.000017
	20	0.000018
	100	0.000022
Water	0	0.0018
	20	0.0010
	100	0.0003
Glycerine	20	1.5
	30	0.63

Table 2.2.2 viscosities for some common fluids.

The viscous drag on an arbitrary shaped object is difficult to calculate as it causes a lot of turbulent flow. Sir Stokes studied the viscous drag for a small sphere moving at low speed in a fluid. He found that in this case, the viscous drag is given by

$$F = 6\pi\eta rv \qquad (2.3)$$

where η is the viscosity of the fluid(Pa • s), r is the radius(m) of the sphere and v is the speed(ms^{-1}). This result is called **Stokes's law** and this viscous drag is also called **Stokes' force** in honor of Sir Stokes. You should be aware that Stokes' law is only valid for small sphere moving at low speed. In this case the fluid flow is laminar flow.

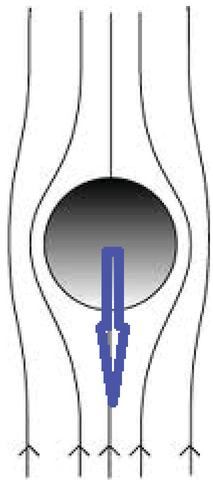


Fig 2.2.3 Streamlines around a sphere

Example:

Use Stokes' Law to calculate the viscous drag on a ball bearing with a diameter of 5mm, falling at a speed of $2mms^{-1}$ through glycerine at room temperature (viscosity of glycerine at 20°C is 1.5 Pas).

Solution:

According to the Stokes' Law, the viscous drag on the ball is given by

$$F = 6\pi\eta rv = 6\times3.14\times1.5\times\frac{5\times10^{-3}}{2}\times2\times10^{-3}N = 1.27\times10^{-4}N$$

Terminal velocity

In Chapter 1 we have discussed the free fall motion of an object due to gravity, where we neglected the air resistance and upthrust. In many cases, these two forces are comparable to the weight of the object and can not be neglected. If upthrust and viscous drag (air resistance) are also taken into account, the object will move under the influence of three forces. The weight and upthrust are constant forces, but the viscous drag increases with the increase of speed. At the moment the object is released, the speed is still zero; the viscous drag is also zero. The object will accelerate due to the gravitational force and upthrust. However, when the speed increases, the viscous drag increases accordingly, causing the net force to decrease. Although the acceleration is decreasing, the speed is increasing because there is acceleration. This process continues until when the speed reach a certain value so that the viscous drag is large enough to balance the weight and upthrust. Then the net force becomes zero and so do the acceleration. The speed will not change anymore. This velocity is called the **terminal velocity**.

The condition for terminal velocity is that the resultant force be 0. That is

$$\text{weight} = \text{upthrust} + \text{viscous drag} \tag{2.4}$$

For a small sphere, the above equation can be written as

$$m_s g = \rho_f V g + 6\pi\eta r v_{term} \tag{2.5}$$

where m_s, V, r and v_{term} are the mass, volume radius and terminal velocity of the sphere and ρ_f and η are the density and viscosity of the fluid respectively.

Substituting $V = \frac{4}{3}\pi r^3$ and $m_s = \rho_s V = \rho_s \frac{4}{3}\pi r^3$ (ρ_s is the density of the sphere) into the above equation gives

$$\rho_s \frac{4}{3}\pi r^3 g = \rho_f \frac{4}{3}\pi r^3 g + 6\pi\eta r v_{term} \tag{2.6}$$

Solving it for v_{term} gives

$$v_{term} = \frac{2r^2 g(\rho_s - \rho_f)}{9\eta} \tag{2.7}$$

We can see from the above expression that the terminal velocity is dependent on the radius of the sphere, the density difference between the sphere and the fluid, as well as the viscosity of the fluid.

Example:

A spherical meteorite, of radius 1m and made of pure iron whose density is 7800kgm^{-3}, falls into the Pacific Ocean, what is the final velocity the meteorite will move at? Suggest the assumptions you have made while doing the calculation and comment your result. (Viscosity of water can be found in Table 2.2.2)

Solution:

The meteorite will reach the terminal velocity after a sufficient time of travel in the sea.

$$v_{term} = \frac{2r^2 g(\rho_s - \rho_f)}{9\eta} = \frac{2\times 1^2 \times 9.81 \times (7800-1000)}{9\times 0.001} = 1.48\times 10^7\, ms^{-1}$$

The assumptions we made include that the temperature of the water is 20 °C, the ocean is deep enough for the meteorite to reach its terminal velocity and the velocity of the meteorite is slow to use the Stokes' formula. But actually from the result we see that this speed is too large to be true. The assumptions are not all valid. The speed of the meteorite gets very large and the Stokes' law doesn't hold anymore.

2.3 Strength of Solid Materials

Hooke's Law

In 1676 Robert Hooke discovered the famous relationship between the extension (or compression) of a spring and the force it exerts. This relationship is therefore called **Hooke's law** and it states that:

The force F exerted by a spring is proportional to the extension (or compression) Δx.

That is:

$$F = -k\Delta x \tag{2.8}$$

where k is a constant called spring constant, Hooke's constant or stiffness of the spring. The minus sign here only indicates that the force is in the opposite direction to the extension (or compression). You do not have to include the minus sign in your calculation.

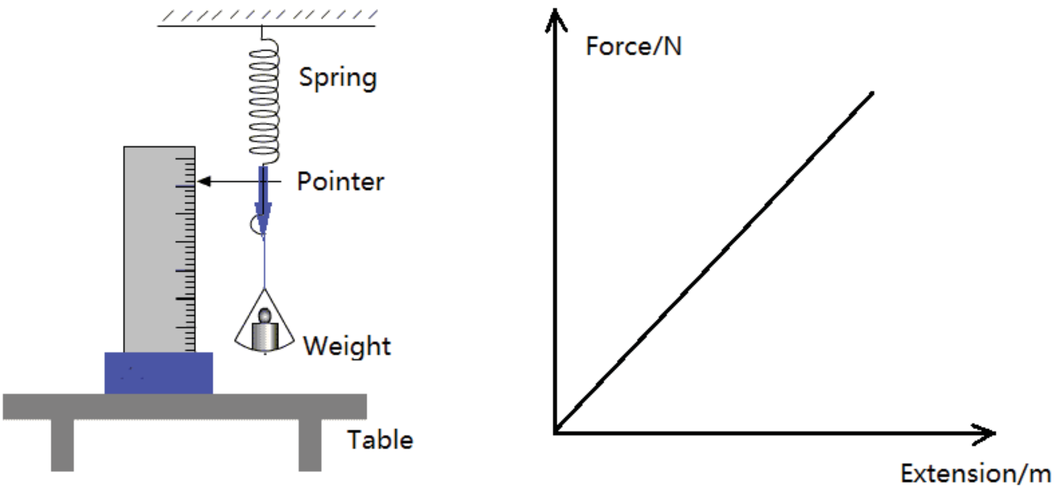


Fig 2.3.1 Hooke's Law experiment. The slope of the graph is the stiffness.

Elastic strain energy

When a spring is stretched or compressed, energy is stored in the spring because it has potential to do work. This form of energy is called **elastic potential energy** or **elastic strain energy**. The amount of elastic strain energy stored in a spring is equal to the work done by the force exerted on it. Thus, it can be obtained by calculating the area under the force-extension graph, which is

$$E_{el} = \frac{1}{2}F\Delta x = \frac{1}{2}k\Delta x^2 \tag{2.9}$$

Note that this equation is valid for springs and other elastic materials that obey Hooke's law. Some elastic materials do not follow Hooke's law, which means that the force is not proportional to extension. In this case, the elastic potential energy can still be determined by calculating the area under the force—extension graph.

Fig 2.3.2 Elastic strain energy stored in a catapult

Example:

A spring is compressed by 0.2m when a 28 Newton force is exerted on it. Find the spring constant and elastic strain energy stored in the spring.

Solution:

The spring constant is $k = \frac{F}{\Delta x} = \frac{28N}{0.2m} = 140Nm^{-1}$.

The elastic strain energy stored is $E_{el} = \frac{1}{2}k\Delta x^2 = \frac{1}{2}\times 140\times 0.2^2 J = 2.8J$

Force ~ extension graph

Some materials follow Hooke's law up to a certain point. The point beyond which force is no longer proportional to extension is called the **proportionality limit**. The region between the origin and the proportionality limit is called the **liner region**. For spring and many materials, there is a short further region where the behavior is still **elastic** (they return to their original lengths when the force is removed). This region ends at the **elastic limit**. After the elastic limit, further force produces **permanent deformation**, which means that after removing the force they can't return to their original lengths. This is also called the **plastic deformation**.

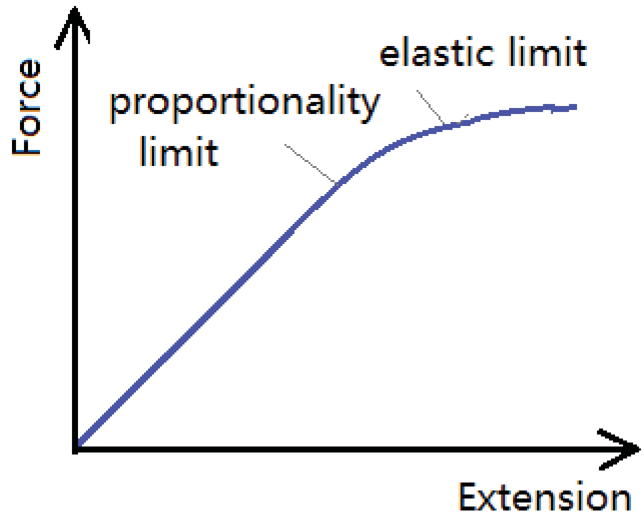


Fig 2.3.3 Force—extension graph

Stress, strain and Young Modulus

Stiffness of a spring measures how hard it is to produce extension on a spring. It is dependent on the material that the spring is made of and the dimensions of the spring. We now want to introduce a quantity that measures how hard it is to produce extension for a certain material. In other words, we want this quantity to be a property of the material and to be independent on its geometrical dimensions. To do this, I firstly wish you to know the two terms stress and strain. **Stress** σ is defined as the force divided by cross section area, that is

$$stress = \frac{force}{cross\ section\ area} \quad or \quad \sigma = \frac{F}{A} \qquad (2.10)$$

There are three types of stress, tensile stress (also called tensional stress), compressive stress (also called compressional stress) and shear stress. Fig 2.3.3 shows the difference between them.

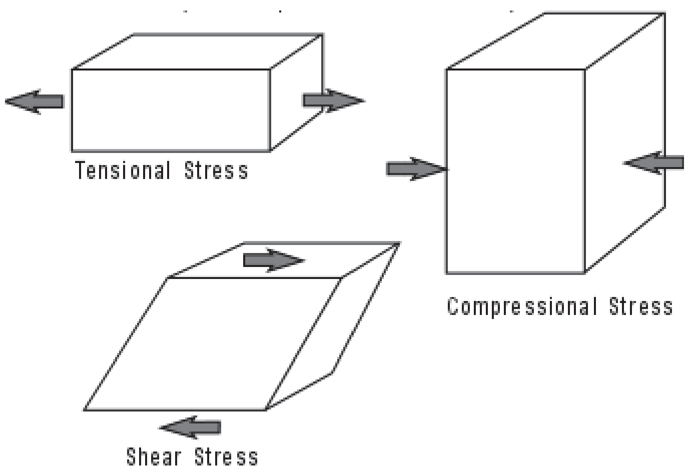


Fig 2.3.4 Different types of stresses

A material may break if the stress caused by a tension or compression is too big. The tensile stress at which a material fails is called **tensile strength**. The compressive stress at which a material fails is called **compressive strength** of the material.

Strain ε is the relative extension, that is

$$strain = \frac{extension}{original\ length} \quad or \quad \varepsilon = \frac{\Delta x}{x} \tag{2.11}$$

Similar to the Hooke's law which states that force is proportional to extension, in many materials, stress is proportional to strain. The proportionality constant is called **Young Modulus**. The letter E is used to denote Young Modulus. Written in mathematical form, it is

$$E = \frac{\sigma}{\varepsilon} \tag{2.12}$$

Young Modulus is a quantity that is similar to the spring constant (stiffness) of a spring as it also measures how difficult it is to produce an extension. The difference is that Young Modulus is a property of material and is not dependent on the dimensions of a particular sample. Anything made from the same material have the same Young Modulus, regardless of their shape or length. We can see this from the definition of Young Modulus. The purpose of dividing the force by cross section area while defining stress and dividing extension by original length while defining strain is to eliminate the influence of the dimensions of the sample.

Example:
A wire fence is made of steel wire of diameter 2.5 mm. A force of 1500 N is applied to tension a single length of this wire.
(a) Calculate the stress produced in the wire.
(b) If the Young modulus of steel is 210 GPa, calculate the extension, produced in a 33 m length of this wire when it is tensioned.
Solution:
(a) The stress in the wire is given by

$$\sigma = \frac{F}{A} = \frac{F}{\pi (d/2)^2} = \frac{1500}{\pi \times (2.5 \times 10^{-3}/2)^2} Pa = 3.1 \times 10^8 \text{ Pa}$$

(b) In the equation $E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\Delta x / x}$, the extension Δx is the quantity we need to calculate, while all the other quantities are given. Substituting the values into the equation and solve for Δx gives the value of extension to be 0.048 m

Stress ~ strain graph

A **stress—strain graph** is a very convenient tool to describe characteristics of a solid material. Figure 2.3.5 shows such a graph for a metal.

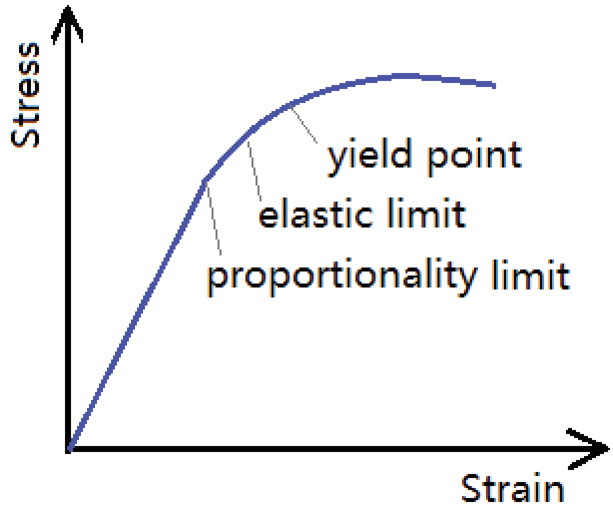


Fig 2.3.5 Stress—strain graph

The stress strain graph is similar to the force extension graph. The gradient of the linear part of the stress-strain graph is equal to Young Modulus of the material. Some important points and regions on the graph you should be familiar with are:

- **Proportionality limit**: before this point, stress $\propto$ strain; after this point, stress—strain graph is no longer linear.
- **Elastic limit**: before this point, the material behaviors elastically, which means that it will return to its original shape if the stress is removed; after this point, the material behaviors plastically, which means that it can't return to its original shape if the stress is removed.
- **Yield point**: after this point, it seems that the material gives up "fighting against" the stress. Even a very small increase of stress causes a big increase of strain. In other words, the gradient of the graph becomes much smaller after this point.
- **Linear region**: the region before the proportionality limit.
- **Elastic region**: the region before the elastic limit.
- **Plastic region**: the region after the elastic limit.

While solving exam problems, it is often difficult to locate the elastic limit on the stress strain graph. But do not worry about this, as long as you label the elastic limit somewhere between proportionality limit and yield point, you will get the mark.

Describing materials

Different materials behavior differently under external forces. You should be familiar with the following words that are used to describe materials.

- **Brittle** materials break/shatter/snap with little or no plastic deformation. Eg.ceramics, biscuit
- **Malleable** materials can be beaten into sheets and show a large plastic deformation under compression. Eg.iron, gold
- **Ductile** materials can be pulled into wires or threads; these materials show plastic

deformation before failure under tension. Eg.copper wire

- **Hard** materials resist plastic deformation. Eg.diamond
- **Tough** materials can withstand impact forces and absorb a lot of energy before breaking; large forces produce a moderate deformation. Eg. rubber, Kevlar(a type of material for fabricating bulletproof vest)

Example:

Boxes containing glass objects are often labeled “Be careful, fragile”. Which of the following words best matches the meaning of fragile?

A. plastic B. hard C. brittle D. tough

Solution:

The answer should be C because brittle materials are easy to break or shatter.

Chapter 3 Elementary of Waves

3.1 Introduction to Waves

When you throw a stone into a pond, a water wave will be formed. This is because the stone causes the water near it to oscillate (move up and down periodically). This oscillation will influence the water in the surrounding area and lead them to oscillate as well. This process continues and the oscillation travels from the point that the stone enters the pond. So, a wave is some sort of disturbance travelling through space.

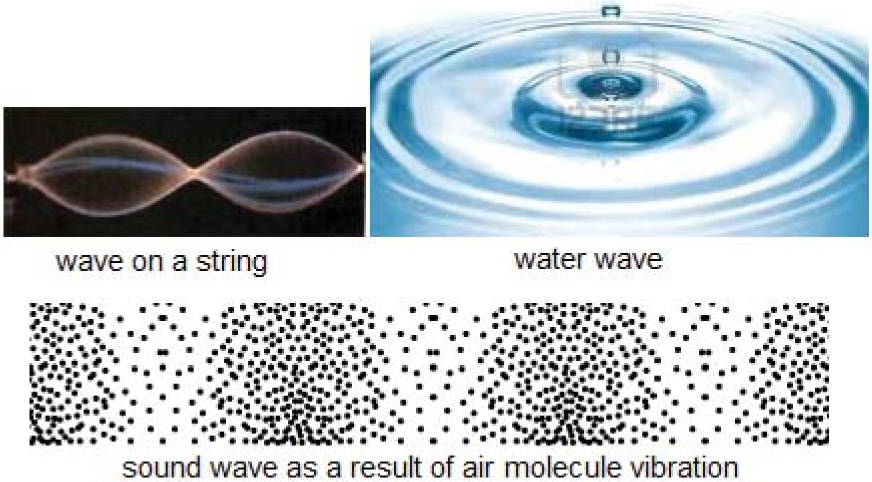


Fig 3.1.1 Various kinds of waves

Classifying waves

Waves can be classified as mechanical waves and electromagnetic waves depending on whether it requires a medium for the waves to travel (propagate) in. **Mechanical waves** require a medium to propagate. For example, sound wave is mechanical wave and it can propagate in air, water or other media. It can't propagate in vacuum. **Electromagnetic waves** require no medium to propagate. This means that they can travel in vacuum. Light is an example of electromagnetic wave.

Another way to classify waves is according to the relationship between the direction of oscillation and the direction of propagation. If the direction of oscillation is perpendicular to the direction of propagation of the waves, they are called **transverse waves**. Figure 3.1.2(a) shows a transverse wave propagating along horizontal direction on a string. The wave may be produced by vibrating one end of a string vertically. All electromagnetic waves are transverse waves. Water waves are also transverse waves.

If the direction of oscillation is parallel to the direction of propagation of the waves, they are called **longitudinal waves**. Figure 3.1.2(b) shows a longitudinal wave propagating along horizontal direction on a spring. This wave may be produced by vibrating one end of the spring horizontally. Sound wave is a longitudinal wave.

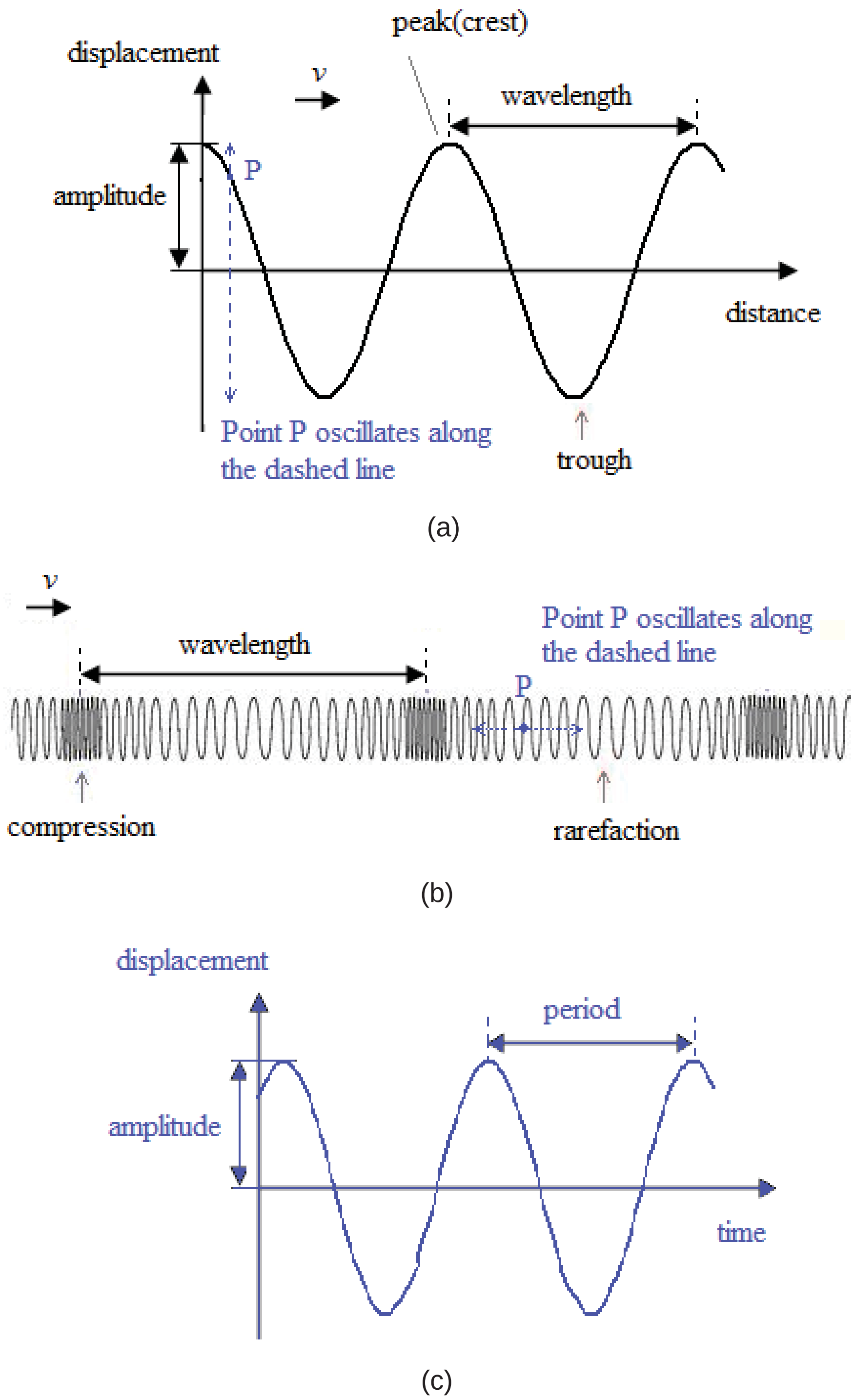


Fig 3.1.2 (a) A transverse waveform on a string (b)A longitudinal waveform on a spring (c) Displacement~time graph of point P on the string(spring).

Waves that have an infinite length are called **continuous waves**. Waves that have a beginning and end are called **wave trains**. Waves with a very short length(less than a couple of wavelengths) are called **pulses**. Fig 3.1.3 illustrates these concepts.

Actually, according to the definition of continuous wave, it has infinite length, which is not possible in reality. It is only a physics model. Usually, people regard long wave trains as continuous waves.

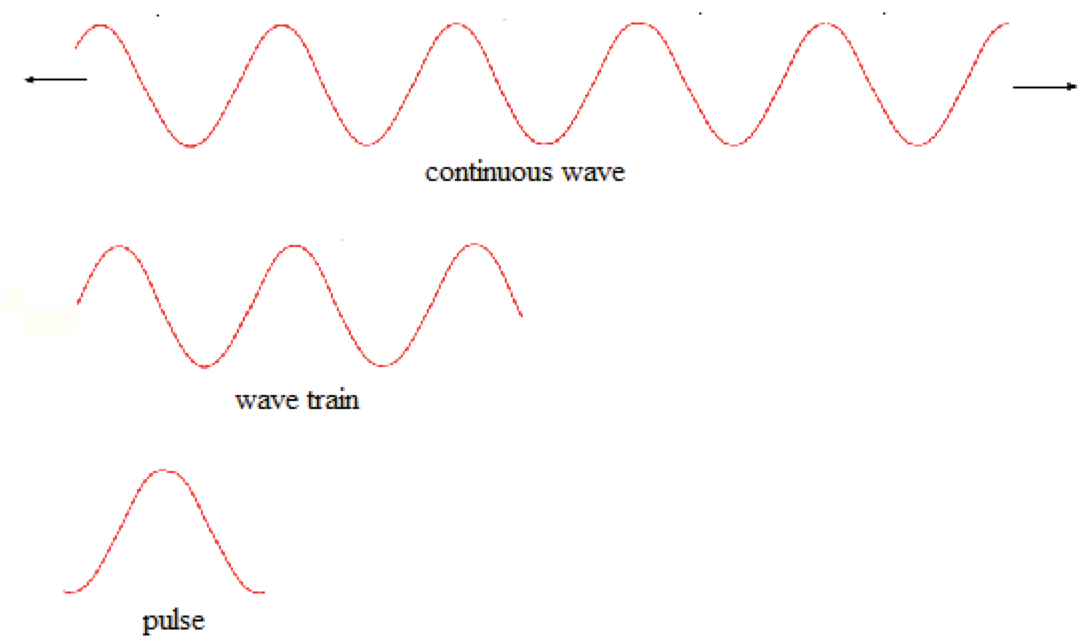


Fig 3.1.3 Continuous wave, Wave train, and pulse

Fig 3.1.2 and Fig 3.1.3 'view' the waves sideway. If we look from above, the pictures would look like those shown in Fig 3.1.4.

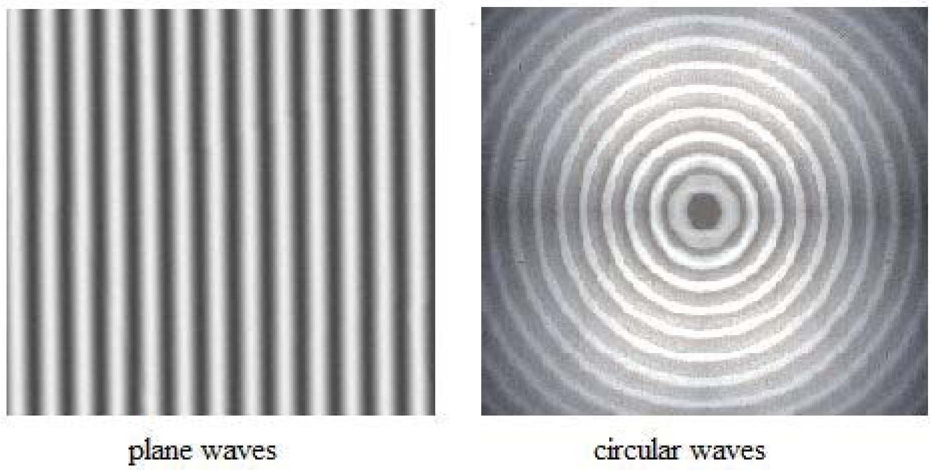


Fig 3.1.4 Plane and circular water waves in a pond viewed from above

Using lines or circles to represent crests, we can draw diagrams as shown in Fig 3.1.5 to describe waves. In such a diagram, the lines (or circles) representing crests are referred to as **wavefronts** and the **rays** tell the direction of propagation of the waves.

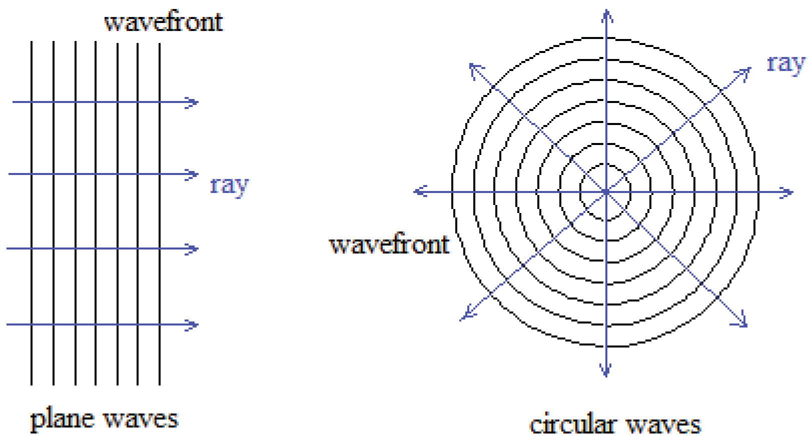


Fig 3.1.5 Wavefronts and rays to represent waves

Quantities describing waves

Four quantities are used to describe waves. Their definitions are

Amplitude: the maximum displacement from the equilibrium position.

Wavelength: distance between two adjacent peaks(for transverse waves) or distance between two adjacent compressions(for longitudinal waves).

Frequency: number of complete oscillations per second.

Period: time taken for one complete oscillation.

Figure 3.1.2 illustrates theses definitions.

If a certain point completes 5 oscillations in 1s, then its frequency is 5Hz. The time it takes to complete one oscillation is then 0.2s. So, its period is 0.2s. Period is the reciprocal of frequency. That is

$$T = \frac{1}{f} \tag{3.1}$$

or

$$fT = 1 \tag{3.2}$$

Speed of the wave is the distance traveled by the wave during unit time. In a period (the time interval that the wave source finish one complete oscillation), the wave propagates a distance of a wavelength. So,

$$v = \frac{\lambda}{T} = \lambda f \tag{3.3}$$

The speed of all electromagnetic waves(including light) in vacuum is $3.0 \times 10^8 ms^{-1}$. You should memorize this speed.

Points that are a whole wavelength apart oscillate in exactly the same way. We say that they oscillate **in phase**. Points that are a distance of half wavelength away oscillate in exactly the opposite way. We say that they oscillate **in antiphase** (or **out of phase**).

Example:
13.725 kHz is one of the frequencies that BBC broadcasts its East Asian programs. What is the wavelength of these radio waves?
Solution:

According to the equation $v = \lambda f$, the wavelength is given by

$$\lambda = \frac{v}{f} = \frac{3.0\times10^8}{13725} m = 21858m$$

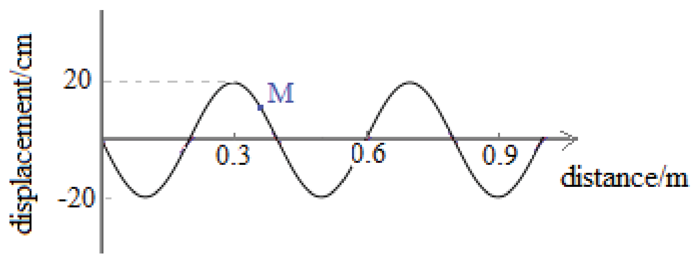
Using graphs to describe waves

Displacement-distance graph
Fig 3.1. (a) is a graph that shows the waveform on the string at a certain time. It's like a picture of the string. In this graph, the horizontal axis represents the distance from the source of the wave, the vertical axis represents the displacement of each point of the string at this time. This graph is called a **displacement-distance graph**. You can know the wavelength and amplitude from such a graph.

Two points that are one wavelength apart have a phase difference of 360^o. If two points are x wavelengths apart, then we say the phase difference between these two points is $360x$ degrees. Here, x can be either integer or fraction.

Displacement-time graph
Fig 3.1. (c) is a graph that shows the oscillation of a certain point on the string. It describes how the displacement of this point changes with time. In this graph, the horizontal axis represents the time and the vertical axis represents the displacement of this point at different times. This graph is called a **displacement-time** graph. You can know the period and amplitude from such a graph.

Example:
The diagram below shows a transverse wave travelling towards right at a speed of $2.0ms^{-1}$.



(a) From the diagram, determine the amplitude and wavelength of the wave.
(b) State whether point M is moving downwards or upwards at the moment this displacement-distance graph is obtained.
(c) Sketch the displacement-time graph of point M to describe its motion during next period.

(d) Mark a point on the waveform whose motion is exactly 180^o out of phase with the motion of point M. Label this point N.

Solution:

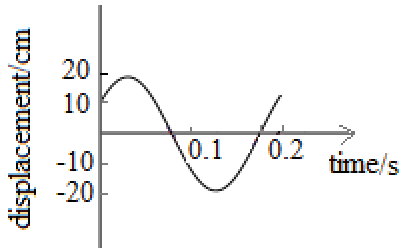
(a)The amplitude is 20cm. The wave length is 0.4m.

(b)Since the wave is travelling to the right. A point will duplicate the motion of its immediate left point. Point M's immediate left point is in the upper of M, so M will follow that point and travels to the upper first. Thus, the direction of the movement of point M at this time is upwards.

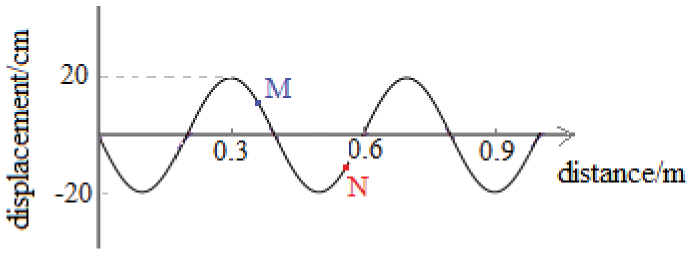
(c)To sketch the displacement-time graph of point M, we need to know the amplitude and period of its oscillation. The amplitude is 20cm, the period can be calculated using $v = \frac{\lambda}{T} = \lambda f$. The period is $T = \frac{\lambda}{v} = \frac{0.4m}{2.0ms^{-1}} = 0.2s$.

Also, we need to know the initial displacement of point M and its direction of movement at the moment. The initial displacement is about 10cm, according to the displacement-distance graph given. The direction of movement at the moment is upwards, which was worked out in (b).

Then the displacement-time graph of point M can be sketched as follows:



(d)We have known that in a displacement-distance graph, if two points are x wavelengths apart, then the phase difference between these two points is $360x$ degrees. Now the phase difference between M and N is 180^o. So, $360x = 180$. $x = 1/2$. Thus, N is any point that is 1/2 wavelength away from point M.



3.2 Reflection

Law of reflection

When a travelling wave reaches a boundary between two mediums, part of the wave will reflect back and part of the wave will transmit(go into the second medium). In wave reflection, the angle of incidence is equal to the angle of reflection. That is $\theta_i = \theta_r$, as shown in Figure 3.2.1.

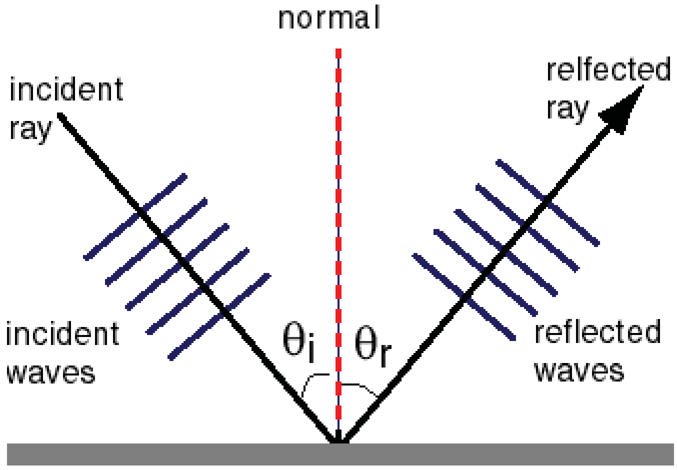


Fig 3.2.1 Reflection of wave

Pulse echo detection--imaging

For wave reflection, the greater the difference in density between the two materials, the stronger the reflection will be. This principle is used make imaging devices such as ultrasound scanning machine.

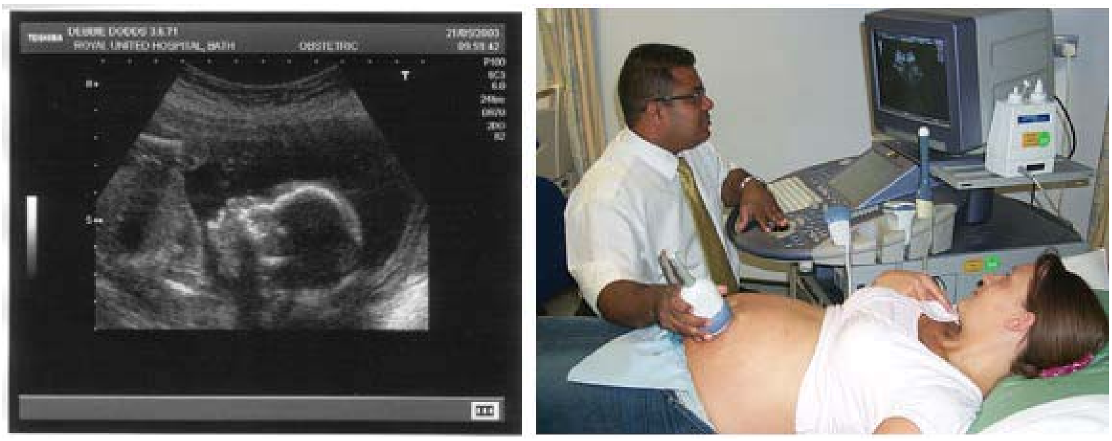


Fig 3.2.2 Ultrasound scanning

Ultrasound scanning technique is widely used in medical imaging. An ultrasound pulse is sent to the target, the pulse will partially reflect between the boundary of air and skin, the boundary between fat and muscle and the boundary between muscles and bones. Since these materials have different densities, the intensities of the reflected waves vary. From the intensities of the reflected waves, we can know the structure of the imaged objects.

In imaging technology, the smallest level of detail that can be seen is called the **resolution** of the image. Resolution of ultrasound scanning can be improved by reducing the wavelength of the sound waves used. There are two rules defining resolution. The first one defines resolution simply as the wavelength of the sound wave used. The second one defines resolution as half of the length of the pulse.

Pulse echo detection--echolocation

Pulse echo technique is also used to detect the position of an object.

Fig 3.2.3 Bats use echolocation to detect obstacles

A pulse is sent to the object and the time interval between the moment the device sends out the pulse and the moment it receives the reflected pulse is recorded. If the velocity of the wave is known, the distance between the sender and the obstacle can be decided. This is called **echolocation**. Bat uses this method to 'see' its surrounding environment at night. Radars use this method to locate airplanes as well.

Doppler effect

When waves are emitted from a moving source or detected by a moving receiver, the detected frequency differs from the emitted frequency. The shift in frequency is proportional to the relative speed of the motion. This phenomenon is called the **Doppler effect**. When the wave source moves relatively away from the detector (receiver), the detector will 'see' a stretched wavelength (or lower frequency); when the wave source moves relatively towards the detector, the detector will 'see' a compressed wavelength (or higher frequency).

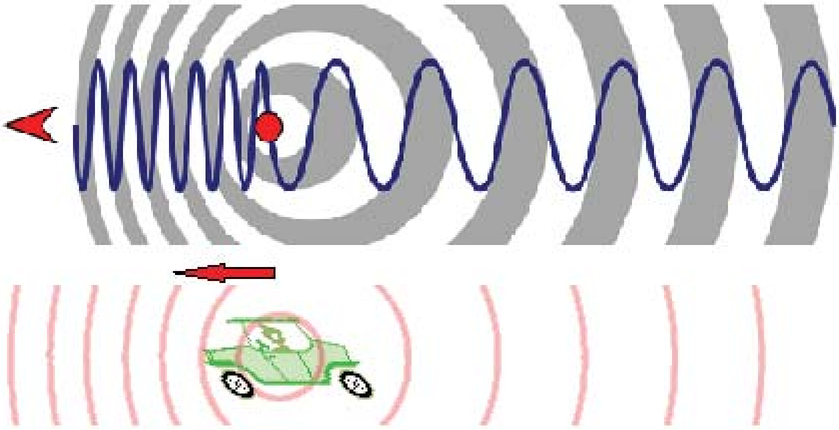


Fig 3.2.4 Doppler effect (From wikipedia)

A **Doppler speedometer** is a device that uses the Doppler effect to measure speed of moving vehicles. Its working principle is as follows:

A radio pulse of known frequency is sent from the speedometer to the targeting vehicle. The pulse is reflected from the moving vehicle back to the speedometer. The shift of frequency of reflected wave is proportional to the speed of the vehicle.

Fig 3.2.5 Doppler speedometer and traffic cameras

Example:

An air traffic control system sent out a radio pulse with frequency of 20GHz to an aircraft. After 0.003seconds the system received the pulse reflected back by the aircraft. The frequency of the received waves is now 20.00002GHz.

(a) What is the distance between the air control system and the aircraft?

(b) What other information about the aircraft can you get?

Solution:

(a) The distance travelled by the wave during 0.003s is

$$s = vt = 3.0\times10^{8}\times0.003m = 9\times10^{5}m = 900km$$

The radio waves travel to the aircraft and back for a total distance of 900km. So, the distance between the air control system and the aircraft is 450km.

(b)Since we are given that the frequency of the reflected waves is increased. We can know that the wavelength is compressed due to Doppler effect. So, the aircraft is moving towards the control system according.

Astronomers observed that the spectra from other galaxies far away from our galaxy are shifted towards the red end. This is called **red shift**. This phenomenon is due to Doppler effect. Red shift indicates that other galaxies are moving away from us. This means that our universe is expanding.

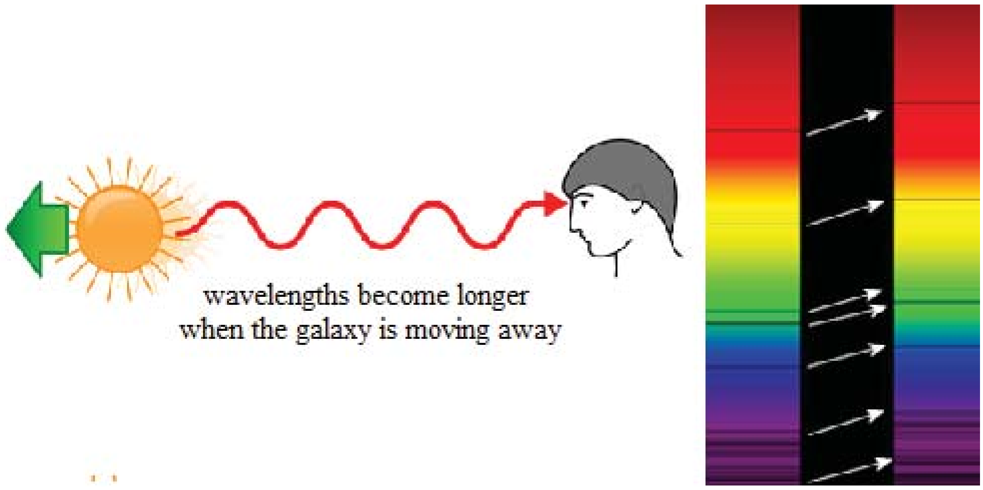


Fig 3.2.6 Red shift

Red shift dose not mean that the colour of the light from other galaxies is all red. It only means that the wavelengths of the light from other galaxies are stretched (become longer). It is named red shift because red light has the longest wavelength in the visible light range.

3.3 Refraction

Law of refraction

As shown in Fig 3.3.1, when a wave travels to the boundary of two mediums, part of the wave will go into the second medium and its direction of propagation will change. This is called refraction and the wave that gets into the second medium is called the refracted wave.

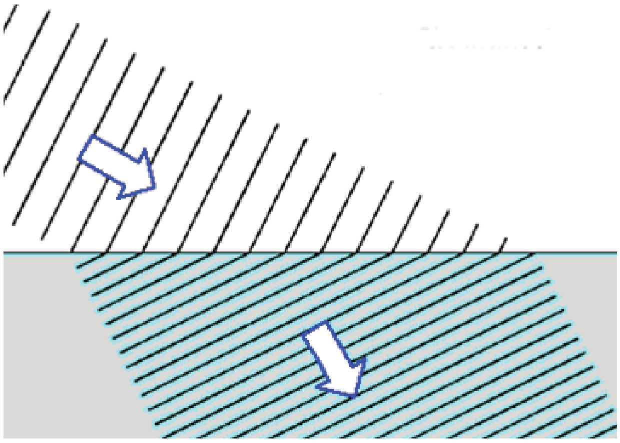


Fig 3.3.1 Refraction of wave

The direction of the propagation of the refracted wave is governed by the law of refraction (also called Snell's Law).

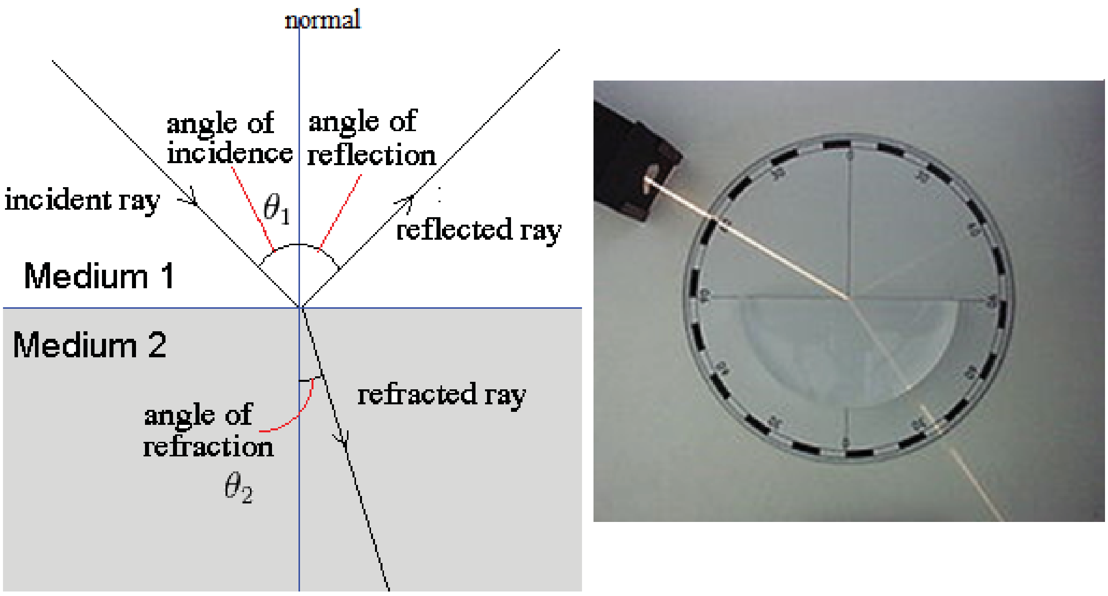


Fig 3.3.2 Snell's law

In symbol form, **Snell's Law** is

$$\frac{\sin\theta_1}{\sin\theta_2}=\frac{n_2}{n_1}=\frac{v_1}{v_2} \quad (3.4)$$

where:

θ_1 and θ_2 ----the angle of incidence and angle of refraction,

n_1 and n_2 ---- the refractive index of medium 1 and medium 2;

v_1 and v_2 ----speed of light in medium 1 and medium 2.

Refractive index is a quantity that measures the ability of a medium to deflect the direction of propagation of light when it travels from vacuum to the medium. The value of refractive index is related with the density of the medium. Denser medium has higher refractive index. Vacuum has a refractive index of 1. The refractive index of air is 1.003. In most calculations, we regard the refractive index of air as 1, unless otherwise indicated. The refractive indexes of water and glass are 1.33 and 1.5 respectively.

When light travels from one medium to another, its frequency doesn't change, but its speed and wavelength change. The speed of light in vacuum is $3.0\times10^8 ms^{-1}$. In air, the speed of light is only slight slower than the speed in vacuum. So we usually take the speed of light in air as $3.0\times10^8 ms^{-1}$ as well. In other media, its speed is much slower.

Example:
The refractive index of a type of glass for red light is 1.51. A ray of red light travels from air into the glass. If the incident angle of the ray is 61^o, calculate the speed of the ray in the glass and the angle of refraction.
Solution:

According to Snell's law, $\frac{\sin\theta_1}{\sin\theta_2}=\frac{n_2}{n_1}=\frac{v_1}{v_2}$.

So,

$$\frac{\sin 61^o}{\sin\theta_2}=\frac{1.51}{1}=\frac{3.0\times10^8 ms^{-1}}{v_2}$$

So, the speed in glass is

$$v_2=1.99\times10^8 ms^{-1}.$$

and angle of refraction is 41.5^o.

Total internal reflection

If a light ray passes from a denser to a less dense material and the incident angle is larger than a certain value called **critical angle**, refracted ray disappears and the entire incident light is reflected. This is called **total internal reflection**. The value of the critical angle C is decided by the equation

$$\sin C = \frac{n_2}{n_1} \qquad (3.5)$$

We can see that critical angle is only decided by the relative refractive index between the two media. For example, the critical angle of glass-air interface is 42^o. This means that if light travels from glass to air and its angle of incidence is larger than 42^o, then all the light will be reflected back and no light will refract into air.

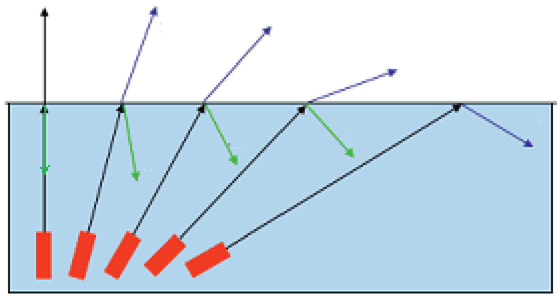


Fig 3.3.3 Total internal reflection

There are two conditions for total internal reflection to happen.

1. Light travels from a denser medium to a less dense medium ($n_1 > n_2$).
2. The incident angle is larger than the critical angle.

Example:
Calculate the critical angle of a water-air interface given the refractive index of water to be 1.33.
Solution:
The critical angle is calculated by using

$$\sin C = \frac{n_2}{n_1} = \frac{1}{1.33} = 0.752$$

The critical angle C is 48.8^o

3.4 Interference

Superposition

Where two waves meet, the total displacement at any point is the sum of the displacements that each individual wave would cause at that point.

This is called the **principle of superposition**. For example, as shown in Fig 3.4.1, there are two sound generators S_1 and S_2 oscillating. They cause the air molecules in the surrounding area to oscillate as well. Consider the oscillation of air molecules at point P at a particular time *t*, if only S_1 was in present and S_2 dose not exist, the wave generated by S_1 would cause a displacement of x_1 at point P. If only S_2 was in present and S_1 dose not exist, the wave generated by S_2 would cause a displacement of x_2 at point P. Now both S_1 and S_2 are in present, the actual displacement of air molecules at point P at this time should be $x_1 + x_2$.

Fig 3.4.1 An example of superposition

Path difference and interference

If two sets of waves have the same frequency and a constant phase difference, then we say that they are **coherent**. When two sets of coherent waves meet and superpose, the oscillation at any point will be the resultant vibration caused by both wave sources, we say that they **interfere**. And they can produce a stable **interference pattern**.

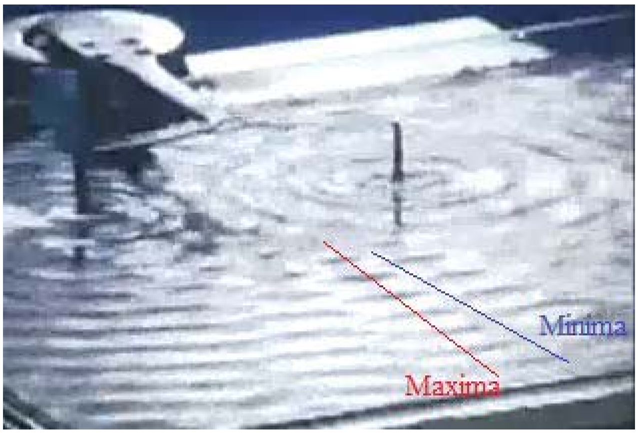


Fig 3.4.2 Interference pattern of water waves in a ripple tank

In some places, the displacements caused by the waves from the two sources are always the opposite and thus they cancel each other, so the vibration in these places are **minima**. We call this **destructive interference**. In some places, the displacements caused by the two sources are always the same and thus they add up, so the vibrations in these places are **maxima**. We call this **constructive interference**. Fig 3.4.2 shows an interference pattern of water waves.

If the distance from source S_1 to a point P is S_1P and the distance from source S_2 to the point P is S_2P, then S_1P-S_2P is called the **path difference**. For example, in Fig 3.4.1, if the distance between P and S_1 is 3m and the distance between P and S_2 is 4m, then the path difference at point P is 1m.

Consider the following case:

S_1 and S_2 are two wave sources oscillating in phase (oscillate exactly in the same way). Waveforms at a particular instant are shown in Fig 3.4.3. A solid semicircle represents a peak and a dashed semicircle represents a trough of the waves.

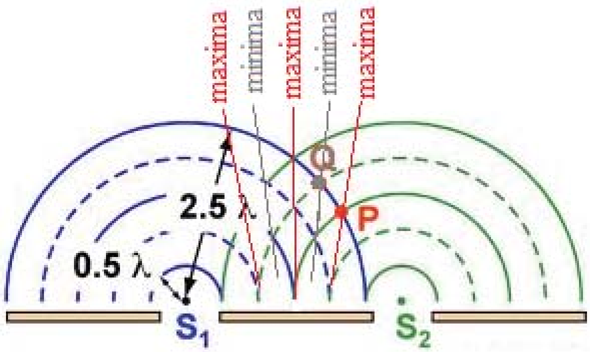


Fig 3.4.3 Path difference and interference

The path difference of point Q is $S_1Q - S_2Q = 2.5\lambda - 2\lambda = \frac{\lambda}{2}$. We can see from the figure that when the wave from source S_1 causes a peak at point Q, the wave from source S_2 causes a trough at point Q, so the resultant displacement at point Q is zero. The oscillation is at minima. Actually this result can be generalized. If the path difference is an odd number of half wavelength, then destructive interference(minima) takes place.

The path difference of point P is $S_1P - S_2P = 2.5\lambda - 1.5\lambda = \lambda$. We can see from the figure that when the wave from source S_1 causes a peak at point P, the wave from source S_2 causes a peak at point P as well, so the resultant displacement at point P is two times of the displacement caused by each source. The oscillation is at maxima. Acturally, if the path difference is an even number of half wavelength, then constructive interference (maxima) takes place.

During the exam, you might be asked to determine whether constructive or destructive interference takes at a point. To solve this kind of problems, you may follow these three simple steps:

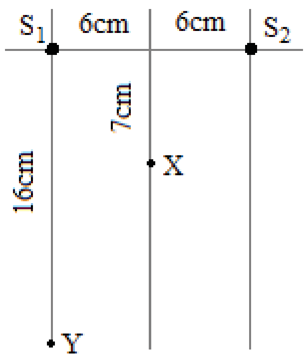
1. Calculate or measure the path difference.
2. Calculate the value of half wavelength $\frac{\lambda}{2}$.
3. Compare the path difference with half wavelength $\frac{\lambda}{2}$ to see whether the path difference is equal to an even number times of $\frac{\lambda}{2}$ or an odd number times of $\frac{\lambda}{2}$:

   If the path difference= an even number $\times \frac{\lambda}{2}$, constructive interference(maxima);

   If the path difference= an odd number $\times \frac{\lambda}{2}$, destructive interference(minima);

Example:

In a ripple tank, two dippers S_1 and S_2 vibrate in exactly the same way, producing coherent water waves with wavelength of 2cm. X and Y are two points in the ripple tank. Their distances from the two dippers are shown in the diagram below. Tell if constructive interference or destructive interference takes place at X and Y respectively.



Solution:

For point X, since its distances from the two dippers are the same, the path difference at point X is 0. Thus, constructive interference takes place at point X.

For point Y, the path difference is

$$YS_1 - YS_2 = 16 - \sqrt{16^2 + 12^2} = 16 - 20 = -4cm$$

The half wavelength is

$$\frac{\lambda}{2} = \frac{2cm}{2} = 1cm.$$

Since, $-4cm = 4 \times 1cm$ and 4 is an even number, constructive interference takes place at point Y.

Stationary wave

Waves can be classified as travelling waves and stationary waves. For a **travelling**

wave, its peaks and troughs are travelling. Travelling wave is also called **progressive wave**. For a **stationary wave**, its peaks and troughs do not move. Stationary wave is also called **standing wave**.

When two traveling waves with the same frequency and amplitude propagating in opposite direction meet, they interfere and can form a stationary wave. Fig 3.4.4 illustrates how a stationary wave is formed.

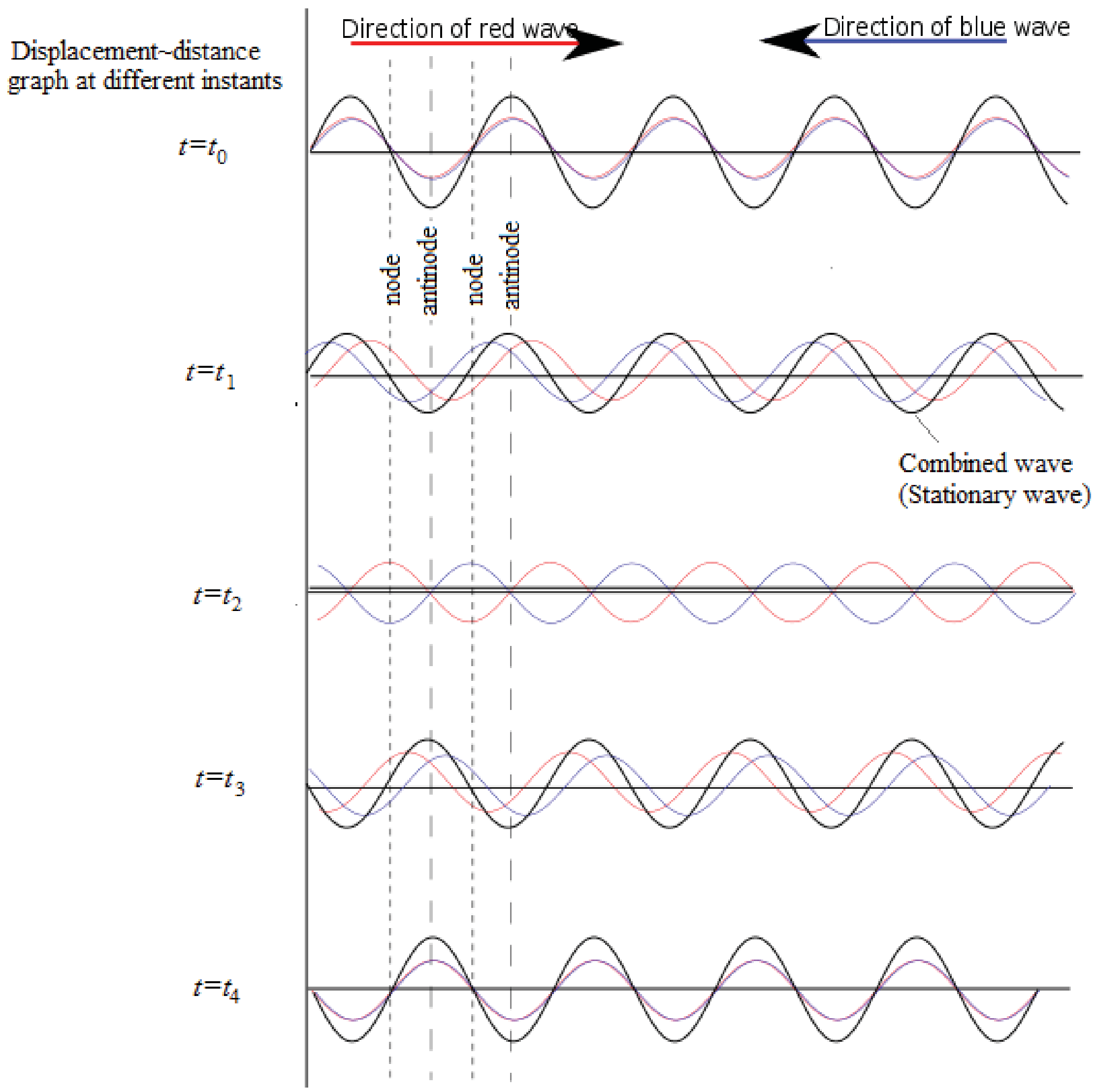


Fig 3.4.4 Two travelling waves(red and blue) form a stationary wave(black)

A stationary wave is basically an interference pattern. The minima are called **nodes** and maxima are called **antinodes**. Fig 3.4.5 shows the pattern of a stationary wave on a string fixed at both ends. The distance between two adjacent nodes (or antinodes) is equal to half wavelength.

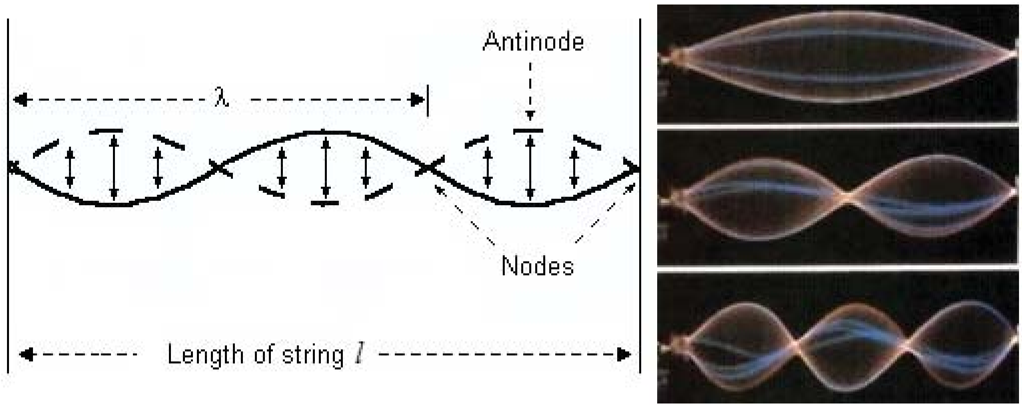


Fig 3.4.5 Stationary wave on a string

Since in such a string, the ends must be nodes, the following condition exists in order to form a stable stationary wave on a string with length l.

$$l = n \cdot \frac{\lambda}{2} \qquad (3.6)$$

where n must be an integer number.

That is to say, for a string with fixed length, only stationary waves with special wavelengths can be formed. These wavelengths are decided by equation(3.6), or $\lambda = 2l / n$.

When $n = 1$, $\lambda_1 = 2l / n = 2l$, $f_1 = \frac{v}{\lambda_1} = \frac{v}{2l}$. This frequency is called the **fundamental frequency** (sometimes called **first harmonic**).

When $n = 2$, $\lambda_2 = 2l / 2 = l$, $f_2 = \frac{v}{\lambda_2} = \frac{v}{l} = 2 f_1$. This frequency is called **second harmonic**.

When $n = 3$, $\lambda_3 = 2l / 3$, $f_3 = \frac{v}{\lambda_3} = \frac{3v}{2l} = 3 f_1$. This frequency is called **third harmonic**.

………

These frequencies, $f_1, 2 f_1, 3 f_1, \cdots$, are called **natural frequencies** of the string.

3.5 Diffraction

Diffraction is the apparent bending of waves around small obstacles and the spreading out of waves past small openings. It is an important phenomenon of all waves. Diffraction is obvious only if the size of obstacle or gap is similar to the wavelength, as shown in Figure 3.5.1.(These diagrams are important, you should know how to draw them)

Fig 3.5.1 Waveforms after passing a gap. Note how the width of the gap influences the shape of the wavefronts. After passing through the gap, the wavelength is not changed.

In 1927, two American physicists observed the diffraction of electrons from crystal of nickel in Bell Labs. The fact that electrons display the property of diffraction indicates that electron beams are also waves! Sounds strange? Aha. Yes, they are both waves and particles. It must be difficult for you to understand this without learning advanced quantum physics, but for now just remember the conclusion.

3.6 Polarisation

Light is caused by the oscillations of electric and magnetic fields.

In **unpolarised light**, these oscillations occur in all directions that are perpendicular to the direction of propagation.

In **plane polarised light**, the oscillations take place only in a single direction that is perpendicular to the direction of propagation. Plane polarised light is also called **linearly polarised light**.

Fig 3.6.1 shows unpolarised light and plane polarized light.

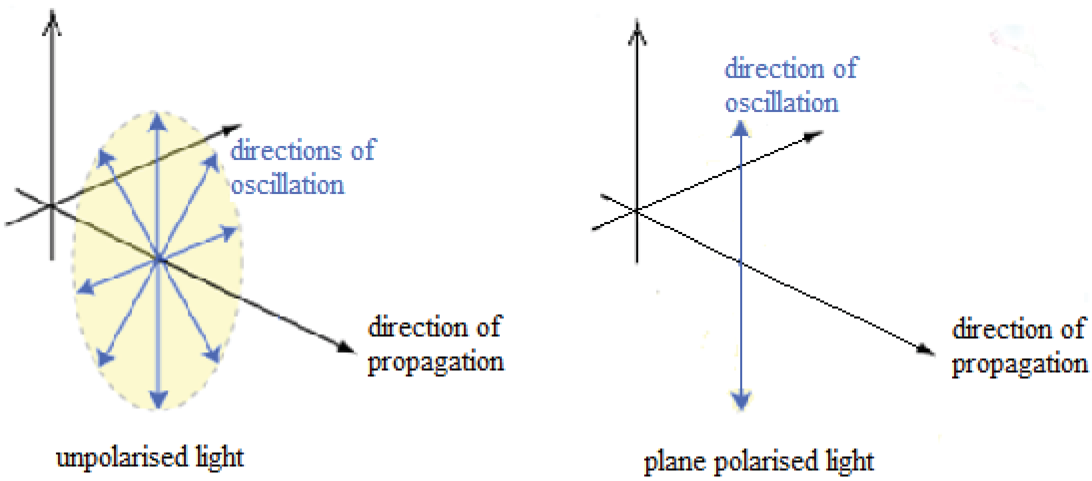


Fig 3.6.1 Unpolarised light and plane polarised light

Light directly from the sun and filament lamp are unpolarised light. Unpolarised light can be plane polarised by passing it through a **polariser** (also called **polaroid** or **polarising filter**). Fig 3.6.2(a) shows a real polariser. It looks like a piece of dark glass. In a polariser, molecules are lined up in a special way and only oscillations in a particular direction can pass through. All other oscillations will be filtered out (blocked). This direction is called the **axis** of the polariser. Fig 3.6.2(b) shows how a polariser works.

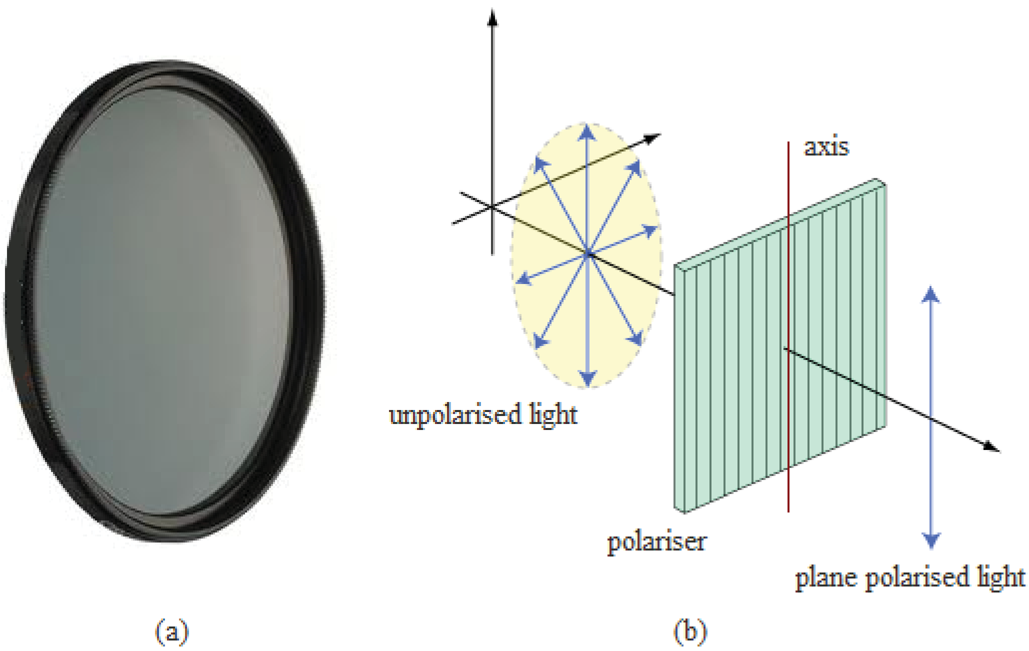


Fig 3.6.2 Polariser

What will you get if a plane polarised light passes through a polariser? The answer depends on the orientation of the polariser. Fig 3.6.3 shows the result.

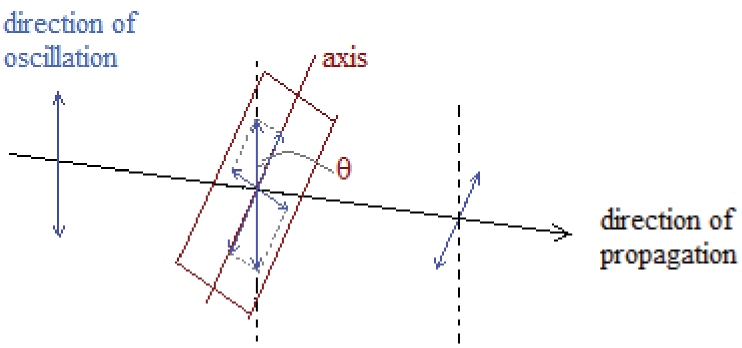


Fig 3.6.3 The effect of polariser on plane polarised light

The oscillation of the wave incident to a polariser can be decomposed to two components, the component parallel to the direction of the axis and the direction perpendicular to the direction of the axis. The component parallel to the direction of the axis will pass through the polariser. The component perpendicular to the direction of the axis will be blocked. So, the amplitude of the output wave will be $\cos\theta$ times of the input amplitude, as shown in Fig 3.6.3. Here θ is the angle between the axis of the polariser and the direction of the oscillation of the incident wave.

When the axis is parallel to the direction of oscillation ($\theta = 0$), the amplitude of the output light will be the same as that of the input light ($\cos 0 = 1$). Thus, the intensity of the output light will be the same as the intensity of the input light since intensity is proportional to amplitude squared.

When the axis is perpendicular to the direction of oscillation ($\theta = 90^o$), the amplitude of the output light will be zero ($\cos 90^{o} = 0$), the intensity of the output light will be zero as well. This means darkness.

Thus, if you rotate a polariser (polaroid) in front of a beam of plane polarised light, the brightness will change with the orientation of the polaroid. This is a method to identify plane polarised waves.

Polarisation is not a property that only light possesses. Other transverse waves (eg. microwave) have this property as well. But, longitudinal waves can not be polarised. This is because for longitudinal waves, the direction of oscillation is always parallel to the direction of the propagation of the waves (they are always the same).

Example:

Which of the following is not a property of sound waves?

A. reflection
B. refraction
C. diffraction
D. polarisation

Solution:

Sound waves are longitudinal waves. Polarisation is not a property of longitudinal waves. So the correct answer is D.

3.7 More about Electromagnetic Waves

Electromagnetic waves are transverse waves. They do not need a medium to propagate. Excerpt light, there are many other electromagnetic waves with different frequencies. The large family of all electromagnetic waves forms the **electromagnetic spectrum**. It includes radio waves, microwave, infrared (IR), visible light, ultraviolet (UV), X-ray and Gamma-ray.

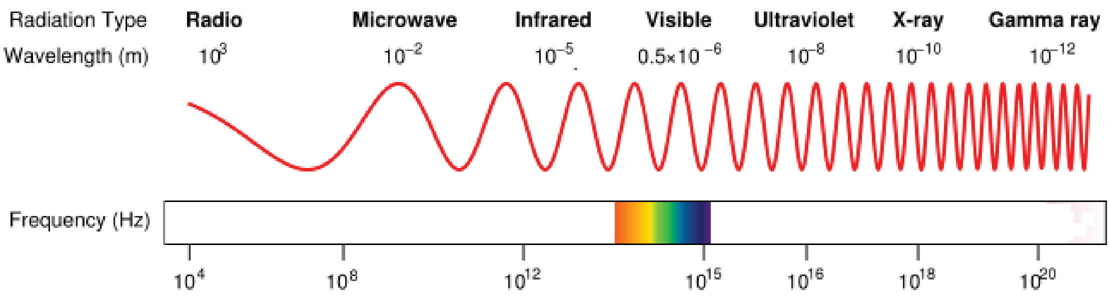


Fig 3.7.1 Electromagnetic spectrum

Examples of applications of each type of electromagnetic waves are given as below.

- Radio waves: TV and radio transmissions
- Mirowaves: mobile phone communications via satellite. Microwave oven.
- Infrared (IR): alarm system. Temperature detector. Fiber optical communications.
- Visible light: human eyes.
- Ultraviolet(UV): fluorescence
- X-ray: imaging objects inside a suitcase or a human body
- Gamma rays: used in hospitals to sterilise equipments.

We can see that visible light occupies only a very small portion of the electromagnetic spectrum. Fig 3.7.2 is a magnified visible spectrum from Fig 3.7.1. Wavelengths for each colour are added.

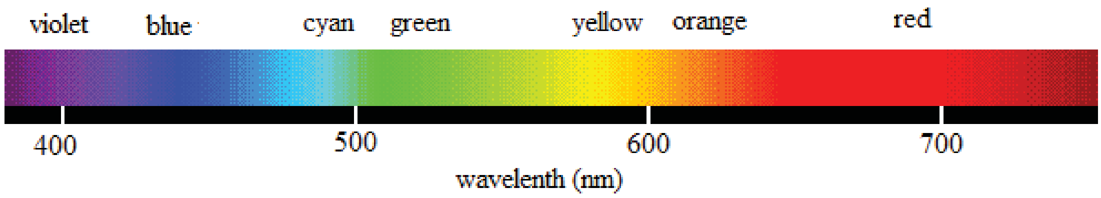


Fig 3.7.2 Spectrum of visible light

Example:

Describe the changes of frequency, wavelength and speed of light when it moves from red to violet in the visible spectrum.

Solution:

Frequency increases, wavelength decreases and speed in vacuum remains constant.

Chapter 4 Direct Current Electricity

4.1 Electrical Quantities

Electric charge and current

Matters are formed by atoms. Just as family is the unit of society, atom is the unit that forms matters.

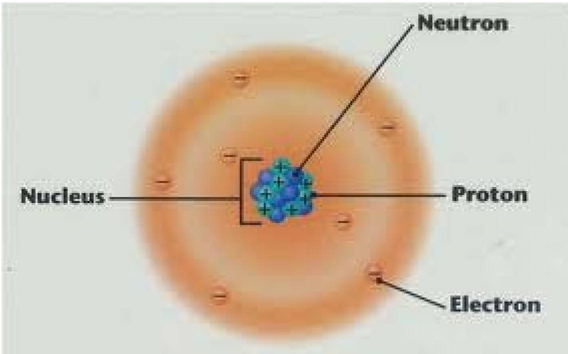


Fig 4.1.1 Structure of an atom

An atom consists of a nucleus with **positive charge** and surrounding electrons with **negative charge**. The amount of charge Q is measured in Coulomb(C). That means Coulomb is the unit of charge, like kg is the unit of mass. An electron carries − $1.6\text{x}10^{-19}$C charge. The negative sign only indicates that it is negative charge. This amount of charge is called the elementary charge.

An **electrical current** is the flow of charged particles (eg. electrons). If is also called current sometimes for simplicity. If during time interval t, the amount of charge passing through a given point is Q, then the electrical current I at this point is

$$I = \frac{Q}{t} \qquad (4.1)$$

In other words, electrical current is the rate of flow of charge. The SI unit of current is C/s, or ampere (A). Other common units for current are mA and μA. The direction of current flow is, by convention, the direction of movement of positive charge. Thus, the direction of current is the opposite direction of the movement of electrons as electrons are negatively charged.

Example:

If there are 5.6×10^{20} electrons passing through an electrical motor during one minute, calculate the average current flowing through the motor.

Solution:

The current is $I = \frac{Q}{t} = \frac{5.6\times10^{20}\times1.6\times10^{-19}C}{1\times60s} = 1.5A$

Current is classified as DC current and AC current:

Direct current (d.c. or DC) are current whose direction dose not change with time. E.g. electrical current in a torch.

Alternating current (a.c. or AC) are current whose direction alternates (changes) with time. Eg. Household electricity.

While describing current that changes with time (direction or magnitude), it is often more convenient to use current~time graph, similar to the velocity~time graph. In a current~time graph the area under the graph represents the charge that flows through. In a charge~time graph the gradient of the graph represents the current.

Not all materials can conduct electrical current. Those can conduct (or let current pass through) are called **conductor**s. Metal and water are typical conductors. Those who can't conduct current are called **insulator**s. Plastic, rubber and glass are typical insulators.

Potential difference

Potential difference, (also **p.d**. or **voltage**) is a measure of energy transfer between two points in an electric circuit. Potential difference is defined as the work done (energy supplied) per unit charge, or in symbol form

$$V = \frac{W}{Q} \qquad (4.2)$$

The SI unit of p.d. is J/C, or V(Volt). Other common units are mV and μV .

The analogy between water flow and electric current flow in Fig 4.1.2 may help you understand the concept of potential difference.

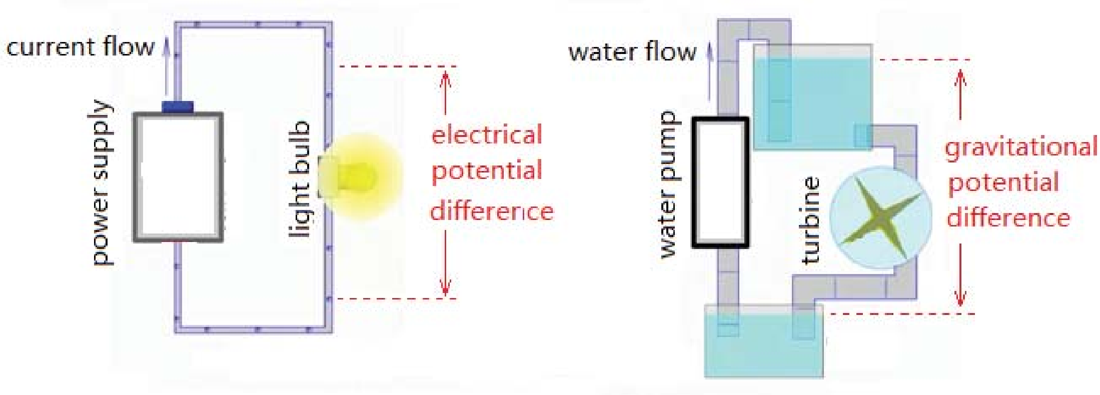


Fig 4.1.2 analogy between water flow and electric current flow

Example:

A potential difference of 4.5 V is measured across a toy electrical motor. The current passing through it is 1.0A. How much work is done on the motor if it is left operating for 3 minutes?

Solution:

As the p.d. V is given and we are asked to calculate work done W, we may think of using the equation $V = \frac{W}{Q}$. Hence, we need to know Q first. Since current and time is given, it is easy to find Q.

The charge is $Q = It = 1.0A \times (3 \times 60)s = 180C$

Work done is $W = VQ = 4.5 \times 180 = 810J$

Electronvolt

If the potential difference between two points A and B is V, an electron with charge e moves from Point A to Point B. Then the work done by electric force on the electron is $W = QV = eV$. The kinetic energy of the electron is changed by eV. The electric energy transferred to an electron when it is moved from a point to another with a p.d. of 1V is defined as 1 electronvolt, which is a new unit of energy. According to the definition,

$$1electronvolt = (1.6 \times 10^{-19}C) \times (1V) = 1.6 \times 10^{-19}J \quad (4.3)$$

Electronvolt is a non-SI unit used to measure very small amount of energy. It is often abbreviated as eV. Here, eV as a whole is a unit of energy and it should be differentiated from the eV in the equation $W = QV = eV$ which means the multiplication of the charge of an electron and the p.d.

If you feel that you can't understand the above two paragraphs, do not worry. It doesn't matter. Ignore them but just remember that electronvolt (abbreviated as eV) is a unit of energy which has the conversion relationship with Joule(J) as $1eV = 1.6 \times 10^{-19}J$

Ohm's Law and resistance

To measure the current passing through a point in a circuit, an **ammeter** is used, as show in Fig 4.1.3. Ammeter should be connected in series (we will explain this later). If the current to be measured is small, a **milliammeter**(mA) meter or **microammeter**(μA) is often used. To detect and measure very small current, the very sensitive ammeter galvanometer is used.

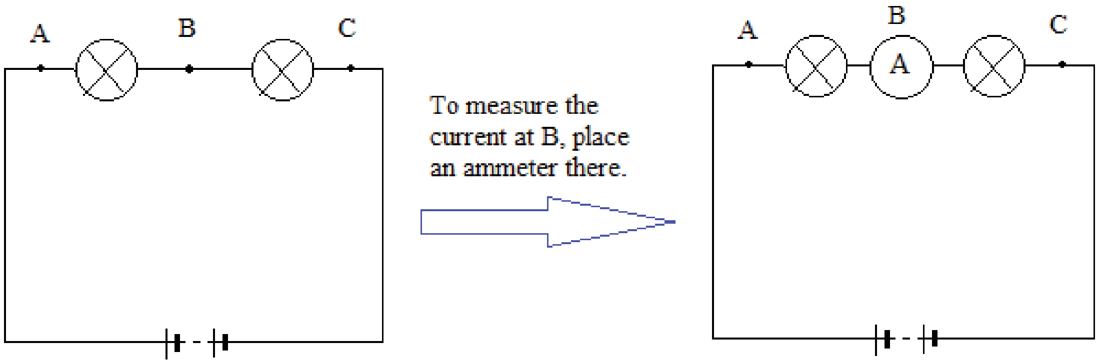


Fig 4.1.3 Measuring current

The measurement of potential difference between two points in a circuit is achieved by placing a **voltmeter** across the two points, as show in Fig 4.1.4. Voltmeter should be connected in series (we will explain this later). If the potential difference to be measured is small, a **millivoltmeter** (mV) meter or **microvoltmeter**(μV) is often used.

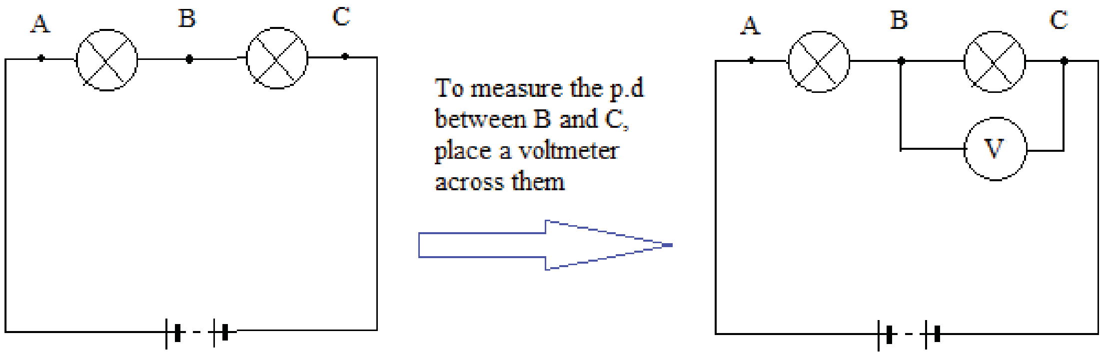


Fig 4.1.4 Measuring current

The quotient of the potential difference across an element (eg. a lamp) divided by the current flowing through it is called the **resistance** of the element. The resistance is denoted using the letter R. So writing in symbol form, the definition of resistance is

$$R = \frac{V}{I} \tag{4.4}$$

Resistance is a quantity that measures the ability to resist electric current flow. Its unit is Ω (or ohm). 1 Ω =1V/A. Equation (4.4) is a very important relationship in electricity, as important as F=ma in mechanics.

German scientist Ohm discovered that the resistance of a metal wire dose not change with the potential difference across it though the current changes. In other words, if you increase the potential difference, the current will also increase, keeping their quotient R constant. Ohm stated this as:

If the temperature and other physical quantities are kept constant, the current through a metal wire is proportional to the potential difference across it.

This is known as **Ohm's law**.

Example:
A lamp with a current 0.3A is connected to an AA cell with voltage 1.5V. Find the resistance of the lamp.
Solution:

The resistance is $R = \frac{V}{I} = \frac{1.5V}{0.3A} = 5.0\Omega$

For conductors such as metal wires, the current~voltage (p.d.) graph, or I-V graph, is a straight line because it obeys Ohm's law. This kind of conductor is called **ohmic conductor**.

There are also conductors that do not obey Ohm's law. That is to say, their I-V graphs are not straight lines (or their resistance is not constant be varies with p.d.). This type of conductor is called **non-ohmic conductor**.

Fig 4.1.5 shows four examples of ohmic conductor and non ohmic conductors.

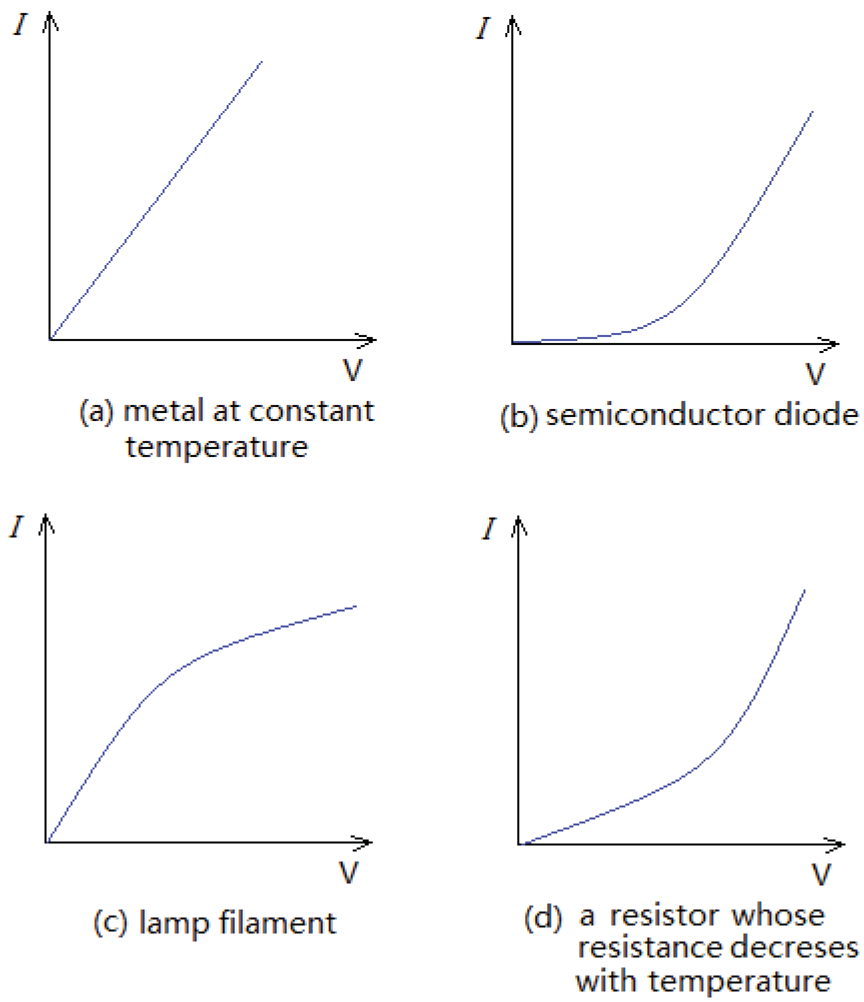


Fig 4.1.5 Ohmic conductor(a) and non ohmic conductors(b,c,d)

Resistivity

The resistance of an electrical component is a measure of its ability in resisting current flow. The bigger the resistance, the higher will the ability in resisting electrical current flow be. This means that it is more difficult for current to flow through. So we say that its ability in conducting electricity is low.

Resistance is decided by the intrinsic property of the material as well as the dimensions of the component. It is found that resistance of a metal wire is proportional to the length l but inversely proportional to the cross section area A, or

$$R = \rho \frac{l}{A} \tag{4.5}$$

where ρ is a constant that is decided by the material. ρ is called the **resistivity** of the material. It is a intrinsic characteristic of the material such as density, refractive index or Young Modulus. While R measures the metal wire's ability in resisting electricity, ρ measures the material's ability in resisting electricity. The unit for resistivity is $\Omega \cdot m$.

In general, for metals, resistivity increases with temperature. This is so because

temperature ↑ ⇨ vibration of atoms ↑ ⇨ collision between free electrons and atoms ↑ ⇨ resistivity ↑

Example:

The maximum allowable resistance for an undersea cylinder cable is 0.01Ω/m. If copper is the material used to make this cable. What condition should the diameter of the copper cable satisfy? (the resistivity of copper is 1.54 x 10^{-8} Ω m)

Solution:

We are given that for a length of $l = 1m$, the resistance should be smaller than $R = 0.01\Omega$. We also know the resistivity $\rho = 1.54 \times 10^{-8} \Omega\text{m}$. Obviously we can use equation (4.5) $R = \rho \frac{l}{A}$ to find cross section area A. Then diameter can be found.

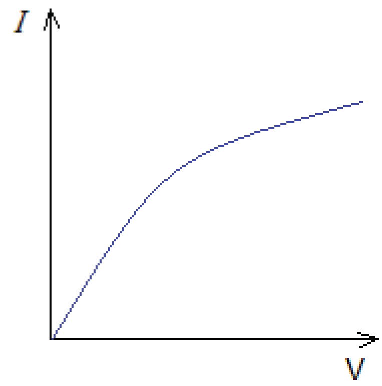
$$A = \rho \frac{l}{R} = 1.54 \times 10^{-8} \times \frac{1}{0.01} = 1.54 \times 10^{-6} m^2$$

$$A = \pi \left(\frac{d}{2}\right)^2 \Rightarrow d = 2\sqrt{A/\pi} = 2\sqrt{1.54 \times 10^{-6} / 3.14} = 1.4 \times 10^{-3} m$$

The diameter of the cable must be larger than $1.4 \times 10^{-3} m$ to ensure its resistance is smaller than 0.01Ω/m.

Example:

The following figure is a graph of current against potential difference for a filament lamp. Explain why the graph has such a shape.



Solution:

As the p.d increases, the current also increases, more heat is produced. The temperature of the filament increases, then the resistance of the filament lamp increases, which means *1/R* decreases. The gradient of the *I~V* graph therefore decreases.

Comment: Note that the gradient of *V~I* graph represents the resistance *R*. Therefore the gradient of *I~V* graph represents the reciprocal of *R*, *1/R*.

Thermistor and photoresistor

A resistor whose resistance changes with temperature is called a **thermistor.** If the resistance of the thermistor increases with the increase of temperature, it is called a **positive temperature coefficient (PTC)** thermistor. If the resistance of the thermistor decreases when temperature is increased, it is called a **negative temperature coefficient (NTC)** thermistor.

A resistor whose resistance is dependent on the intensity of light is called a **photoresistor(**or **light dependent resistor, LDR).**

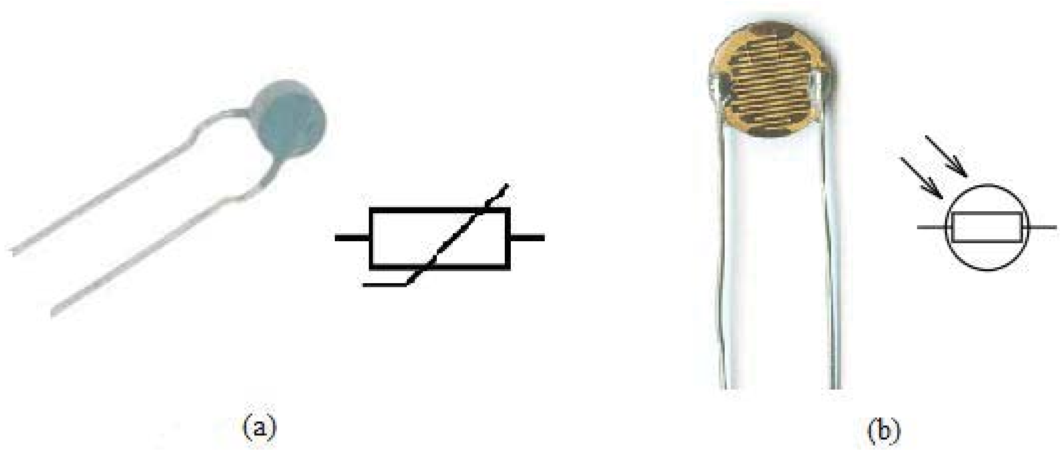


Fig 4.1.6 (a) Thermistor and its electronic symbol; (b) LDR and its electronic symbol;

Transport equation

Consider a piece of conductor as shown in Fig 4.1.7

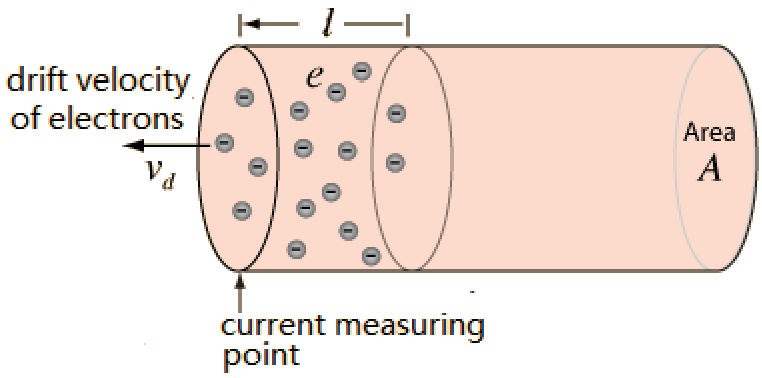


Fig 4.1.7 Transport equation and drift velocity

If n is the free **electron density** (or number of free electrons per unit volume) and v_d is the average traveling velocity of the free electrons (called **drift velocity**), we have:

Number of free electrons= $nV = nAl$

Total charge Q that will flow through=charge of an electron × total number of free electrons $= qnAl$

Time taken for this charge to flow through the current measuring point is $t = l / v_d$

The measured current should be $I = \dfrac{Q}{t} = \dfrac{qnAl}{l / v_d} = qnAv_d$

So, the current through a conductor is related to the microscopic quantities by

$$I = nAv_d q \qquad (4.6)$$

This is called the **transport equation**.

Example:

If the electron density in aluminum wire is 1.9×10^{35}m^{-3}, calculate the drift velocity of a piece of aluminum wire with diameter 3.8mm while carrying a current of 3A.

Solution:

According to $I = nAv_d q$, the drift velocity is given by

$$v_d = \frac{I}{nAe} = \frac{I}{n\pi\left(d/2\right)^2 q} = \frac{3}{1.9\times10^{35}\times3.14\times\left(3.8\times10^{-3}/2\right)^2\times1.6\times10^{-19}} = 8.7\times10^{-12}\,ms^{-1}$$

We can see from the result that drift velocity of electrons is very small.

Example:

A NTC thermistor is a resistor whose resistance decreases with temperature. It is made from semiconductor materials. With reference to the equation $I = nAv_d q$, explain why its resistance decreases if temperature is increased.

Solution:

When temperature is increased, more electrons can break free from the atoms and participate in conduction. That means that *n* increases, so current *I* increases, the resistance decreases.

Remarks: In fact, there are two effects if temperature increases:1) increase of vibration of atoms, which leads to the increase of resistance; 2) increase of free electron density *n*, which leads to the decrease of resistance. For metals, effect 2 is small and effect 1 is significant. So the overall result is the increase of resistance when temperature is increased. For semiconductor materials, *n* are very small. When temperature is increased, effect 1 is small and effect 2 is significant. So the overall result is the decrease of resistance when temperature is increased.

4.2 Electrical Circuits

Power and work in electrical circuit

Recall the definitions for current $I = \frac{Q}{t}$ and the definition for potential difference $V = \frac{W}{Q}$. If we multiply these two equations, we will get

$$VI = \frac{W}{Q} \times \frac{Q}{t} = \frac{W}{t} = P \quad (4.7)$$

The power dissipated by an electrical circuit (or a component) is

$$P = VI \quad (4.8)$$

The work done (or energy transferred) is then $W = Pt = VIt$.

Example:

In the UK, the mains voltage of household electricity is 230V. A light bulb is labeled 230V, 40W. (a)How much energy will it consume if kept open for 1 hour? (b) What is the current flowing through it?

Solution:

(a) Energy transferred is equal to work done,

$$W = Pt = 40W \times 3600s = 14400J = 14.4kJ$$

(b) Using the equation $P = VI$, the current is given by

$$I = \frac{P}{V} = \frac{40W}{230V} = 0.017A$$

Example:

(a) Express the power dissipated by a resistor in the form of I and R.
(b) Express the power dissipated by a resistor in the form of V and R.

Solution:

(a) The expression for power $P = VI$ includes both V and *I*. If V is not known but R is known, we can use Ohm's law formula $V = IR$ to eliminate V.

$$P = VI = IRI = I^2R$$

This is the expression for power that only contains I and R.

(b) If *I* is not known but R is known, we can use Ohm's law formula $I = V/R$ to eliminate V.

$$P = VI = V\frac{V}{R} = \frac{V^2}{R}$$

This is the expression for power that only contains V and R.

Series and parallel circuits

There are two basic types of circuit connections, **series circuit** and **parallel circuit**, as shown in Fig 4.2.1

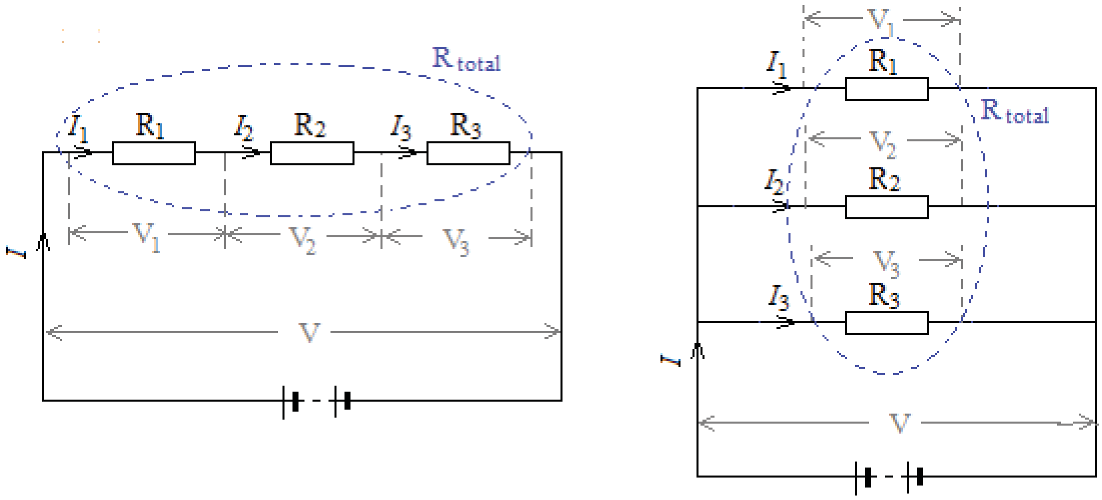


Fig 4.2.1 Series circuit and parallel circuit

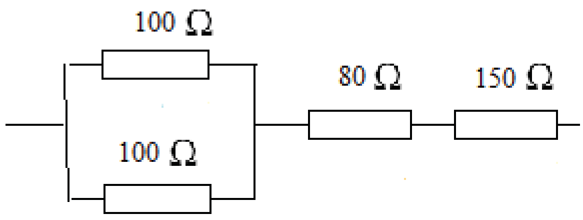
Table 4.2.1 gives a summary of characteristics of each type of the circuits.

	Series circuit	Parallel
Current	equal everywhere $$I = I_1 = I_2 = I_3 \quad (4.9)$$	$$I = I_1 + I_2 + I_3 \quad (4.10)$$
Voltage (p.d.)	$$V = V_1 + V_2 + V_3 \quad (4.11)$$	equal across each element $$V = V_1 = V_2 = V_3 \quad (4.12)$$
Resistance	$$R_{total} = R_1 + R_2 + R_3 \quad (4.13)$$	$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad (4.14)$$
Remarks	If you connect more resistors in series, the total resistance increases. This is equivalent to the increase of length in equation(4.5)	If you connect more resistors in parallel, the total resistance decreases. This is equivalent to the increase of cross section area in equation(4.5)

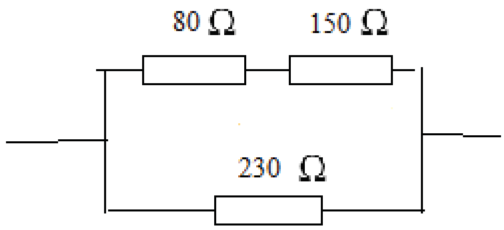
Table 4.2.1 Comparison of series and parallel circuits

Example:

Calculate the total resistance of the following resistor network.



(a)



(b)

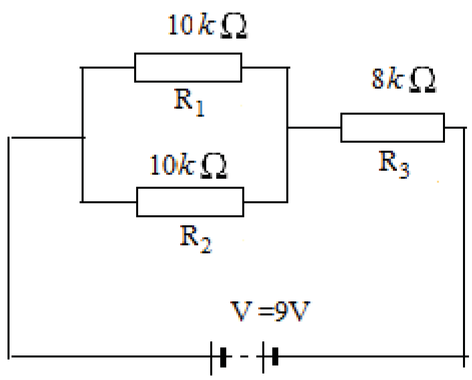
Solution:

(a) Two 100Ω resistors are connected in parallel, the combined resistance of these two resistors is 50Ω according to the formula for total resistance of parallel resistors(equation(4.14)equation reference goes here). The resistor network is then equivalent to a series connection of three resistors, the 50Ω, 80Ω and 150Ω resistors. The total resistance is then 280Ω according to the formula for total resistance of series resistors(equation(4.13)equation reference goes here).

(b) The combined resistance of the 80Ω and 150Ω resistors in series is 230Ω. This combined 230Ω resistor is connected in parallel with another 230Ω resistor. The total resistance is then 115Ω.

Example:

Find the current flowing through each resistor and the battery in the following circuit.



Solution:

The current flowing through R_3 and through the battery will be the same.

$$I_3 = I_{battery} = I_{total} = \frac{V}{R_{total}}$$

Where $R_{total} = R_{12} + R_3 = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} + R_3 = 13k\Omega$

Thus, $I_3 = I_{battery} = I_{total} = \dfrac{V}{R_{total}} = \dfrac{9V}{13k\Omega} = \dfrac{9V}{13\times10^3\Omega} = 0.69\times10^{-3}A = 0.69mA$

From the circuit diagram, we can see that $I_3 = I_1 + I_2$. Also, since R_1 and R_2 have the same resistance, so the current will be divided equally to the two resistors.
Then we have

$$I_1 = I_2 = 0.69mA/2 = 0.35mA$$

Example:
In a toy electric powered ambulance, an electric motor and a speaker (as an alarm) is connected in parallel. Two 1.5V AAA cells are used as a power source. When the ambulance is in operation, the electrical currents passing through the speaker and the motor are 0.1A and 1.1A respectively, what is the power of the toy ambulance and the resistance of the speaker?
Solution:
In parallel circuit, the total current is

$$I = I_{speaker} + I_{motor} = 0.1A + 1.1A = 1.2A$$

The power is

$$P = VI = 3.0V \times 1.2A = 3.6W$$

In parallel circuit, the p.d. across each element is the same. The resistance of the speaker is

$$R_{speaker} = \frac{V}{I_{speaker}} = \frac{3.0V}{0.1A} = 30\Omega$$

Potential divider

A **potential divider** is a very simple device that consists of two resistors connected in series, as shown in Fig 4.2.2.

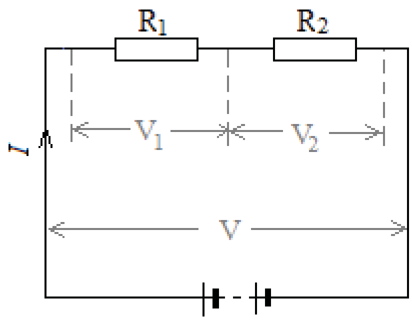


Fig 4.2.2 Two resistors connected in series form a potential divider

The current flow in this circuit can be written as

$$I = \frac{V}{R_{total}} = \frac{V_1}{R_1} = \frac{V_2}{R_2} \quad (4.15)$$

Then, the p.d. across R_1 is

$$V_1 = IR_1 = \frac{V}{R_1 + R_2} \cdot R_1 = \frac{R_1}{R_1 + R_2} \cdot V \quad (4.16)$$

and the p.d. across R_2 is

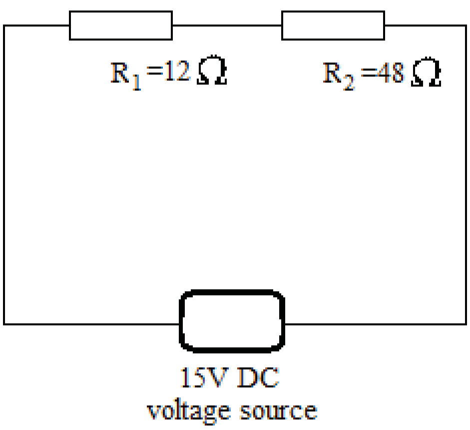
$$V_2 = IR_2 = \frac{V}{R_1 + R_2} \cdot R_2 = \frac{R_2}{R_1 + R_2} \cdot V \quad (4.17)$$

This means that the total potential difference is divided to two parts. The p.d. across each resistor increases with its resistance.

You do not need to memorize the above three equations, they can easily be derived from Ohm's law (V=IR) combined with the fact that in series circuit current is equal everywhere.

Example:

Two resistors are connected in series in the following circuit, find the p.d. across each resistor and compare the results.



Solution:

The current flow in this circuit is

$$I = \frac{V}{R_{total}} = \frac{V}{R_1 + R_2} = \frac{15}{12+48} A = 0.25A$$

Then, the p.d. across R_1 is

$$V_1 = IR_1 = 0.25 \times 12\ V = 3V$$

and the p.d. across R_2 is

$$V_2 = IR_2 = 0.25 \times 48\ V = 12V$$

Comment: We can see that resistor R_1 shares one fifth of the total voltage (as R_1 is 1/5 of the total resistance) and R_2 shares four fifth of the total voltage (as R_2 is 4/5 of the total resistance).

Based on the working principle of potential divider, **potentiometer** (also called **variable resistor**) is invented. A potentiometer is a resistor whose resistance can be changed, as shown in Fig 4.2.3. Sometimes, the terms potential divider, potentiometer and variable resistor are used interchangeably.

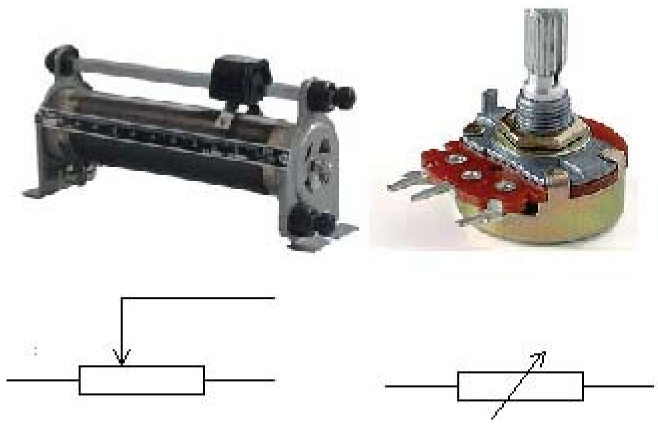


Fig 4.2.3 Potentiometer and its electronic symbols.

If a variable resistor is connected in the way shown in Fig 4.2.4, the output voltage V_{out} can be varied to be any value between 0 and V_{in}.

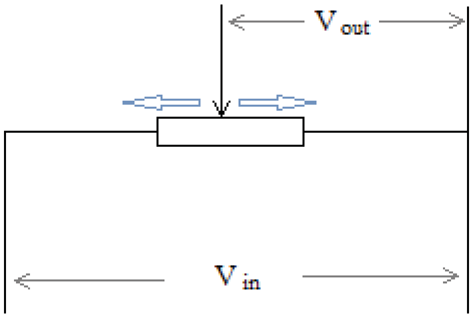
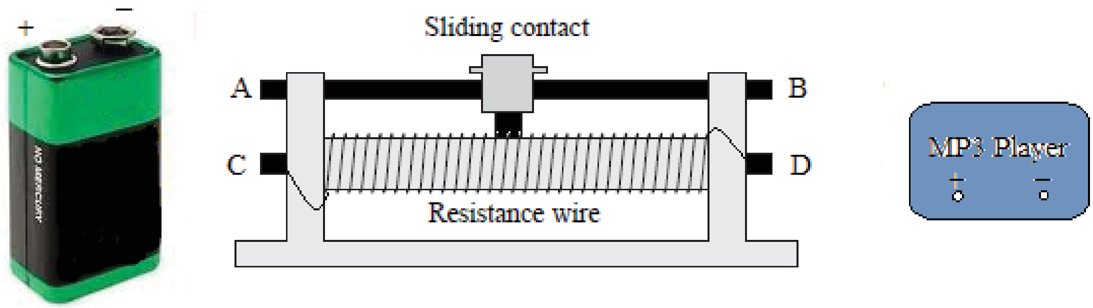


Fig 4.2.4 Using variable resistor to adjust voltage

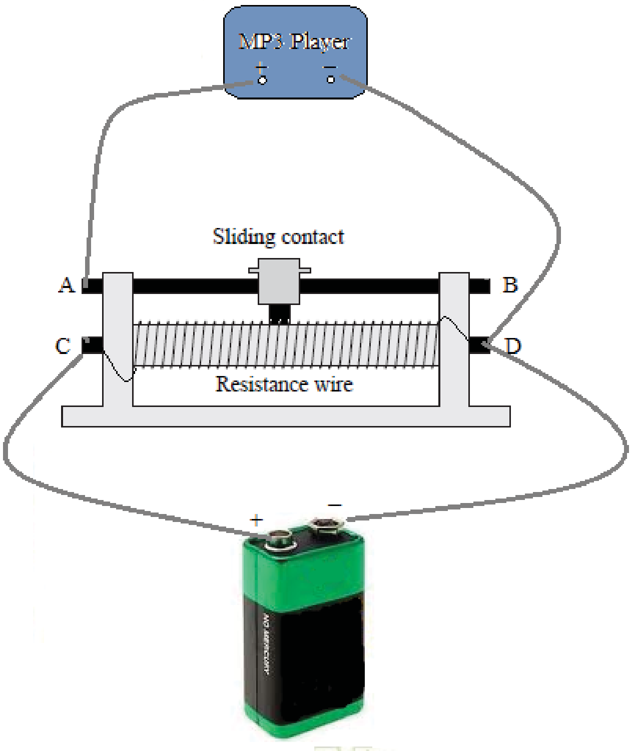
Example:

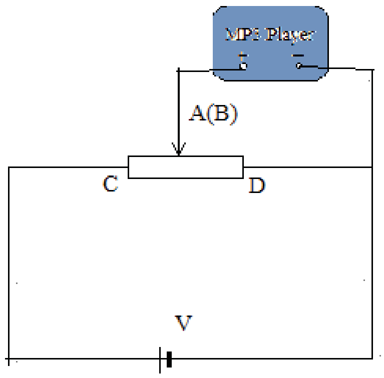
A 9V cell is available to supply 3. 6 V electrical power for a MP3 player with a very big resistance (a few Mega ohms). A variable resistor as shown below can be used. The resistance wire of the variable resistor has a maximum resistance of 100ohm is It is uniformly wrapped on a insulating ceramic cylinder of length 20cm. Draw a diagram to show how the three elements should be connected and then calculate the position that the sliding contact should be placed.



Solution:

The variable resistor should be connected in the way shown in Fig 4.2.2 so the output pd can be any value between 0 and 9 Volts. The following figure illustrates how they should be connected.





The sliding contact should be placed at a place to ensure that the p.d. across the MP3 Player is 3.6V. Let l denote the distance between the sliding contact point and D. Since p.d. is proportional to resistance and resistance is proportional to length, we have

$$\frac{3.6V}{9V}=\frac{pd\ across\ the\ MP3\ player}{pd\ across\ the\ cell}=\frac{R_{contacting\ point--D}}{R_{CD}}=\frac{l}{20cm}$$

$$l=\frac{3.6V}{9V}\times 20cm=8cm$$

The sliding contact should be placed at a place that is 8 cm away from point D.

EMF and Internal resistance

Electromotive force (emf) is a measure applied to a source of electrical power such as a battery (one or multiple cells). Although it is named 'force', it is actually not a force. Electromotive force of a battery is the total energy it supplies to each coulomb of charge. The SI unit of emf is also volt(V).

Comparing emf with p.d., we see that both quantities measure energy transferred (and work done) on unit charge. But they are not identical. Emf is used to measure a battery's ability to raise the energy of charges. It is similar to the ability of a pump to raise the gravitational potential energy of water. P.d describes the amount of work can be done by current flow if one coulomb of charge flows from one point in the circuit to another. It is similar to the difference in height which can be a measure of the amount of work that can be done by water flow if an amount of water weighted one Newton flows from one place to another.

Batteries have some small resistance to electrical current within itself. This is called the **internal resistance** of the battery, as shown in Fig 4.2.3.

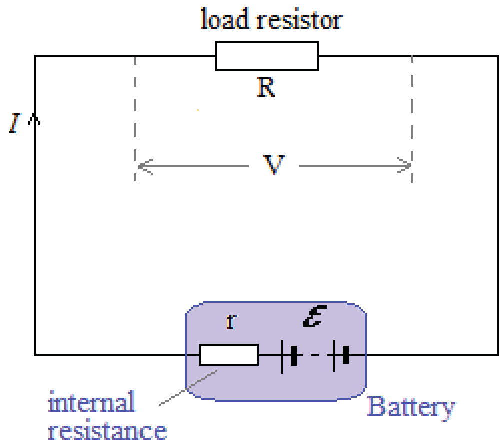


Fig 4.2.5 Emf and internal resistance

Some of the energy supplied by the battery will be lost due to the internal resistance. In a complete circuit (Fig 4.2.5), as a result of energy conservation,

total energy supplied by battery = energy consumed by load resistor + energy lost due to internal resistance

Written in equation, it is

$$\mathcal{E} Q = I^2Rt + I^2rt \quad (4.18)$$

$$\mathcal{E} Q = IRQ + IrQ \quad (4.19)$$

$$\mathcal{E} = IR + Ir \quad (4.20)$$

$$\mathcal{E} = V + Ir = IR + Ir \quad (4.21)$$

As shown in equation(4.21), the voltage V across the load resistor R will be a bit smaller than the emf of the battery. The term Ir is sometimes referred as '**lost volts**' and voltage V as '**terminal p.d**'. To make it simple, $\mathcal{E} = V + Ir$ means that the loader resistor of the circuit and the internal resistor of the battery share the emf. If the internal resistance is negligible (we have always neglected the internal resistance until now), then the voltage is the same as the emf.

Example:

An alkali cell with emf 1.5V and internal resistance $0.1\,\Omega$ is used to supply power for a toy electric motor car. The resistance of the motor is $1.4\,\Omega$. Find the current in the circuit and the potential difference across the motor.

Solution:

According to $\mathcal{E} = V + Ir = IR + Ir$, the current of the circuit is

$$I = \frac{\mathcal{E}}{R + r} = \frac{1.5}{1.4 + 0.1} = 1.0A$$

The potential difference across the motor is

$$V = \mathcal{E} - Ir = 1.5 - 0.1 \times 1.0 = 1.4\ V$$

For equation(4.21), there are two special occasions which correspond to two special circuits, open circuit and short circuit (Fig 4.2.6).

An **open circuit** is a circuit that is open. So there is a gap and no current can be formed. Thus, I=0, $V=\mathcal{E}$.

A **short circuit** means that the power supply (e.g. battery) terminals are joined directly by a connecting wire with no resistance. Thus V=0 and $I = \mathcal{E}/r$ which can be very large and dangerous!

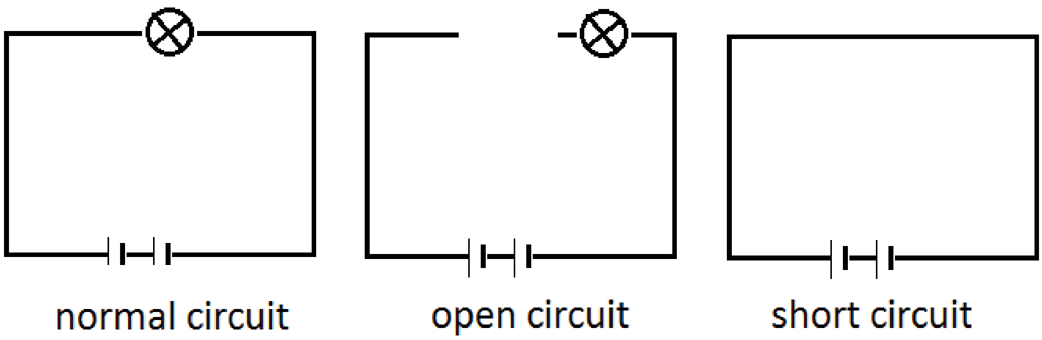


Fig 4.2.6 open circuit and short circuit

If a battery is connected in a complete normal circuit, you are not able to get the battery's emf $\mathcal{E}$ by placing a voltmeter across it. Rather, in this way you get the value of V. Only if you do so in an open circuit, you can directly measure emf. Fig 4.2.7. illustrates this.

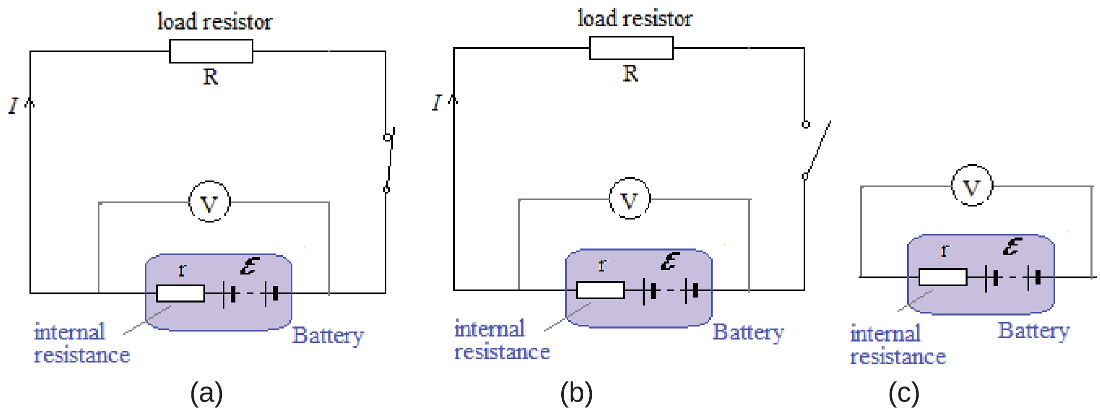


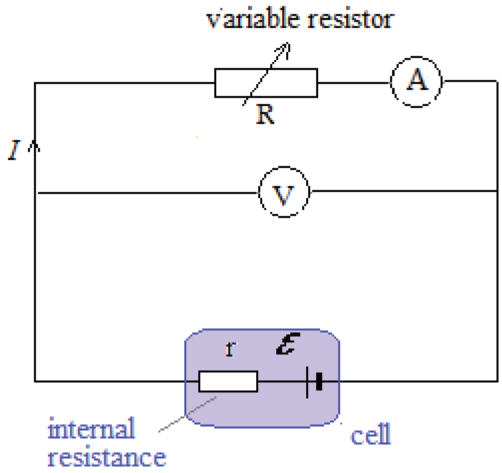
Fig 4.2.7. The voltmeter in circuit (a) measures the voltage across the load resistor; the voltmeters in circuit (b) and (c) measure the emf of the battery.

Example:

You are given a voltmeter, an ammeter, a variable resistor and some leads. You are asked to use these apparatus to measure the emf and internal resistance of a cell. Explain how you can do this.

Solution:

A circuit as shown below can be set up to measure the emf and internal resistance of the cell.



Since $\mathcal{E} = V + Ir$, $V = -rI + \mathcal{E}$. By changing the resistance of the variable resistor, the current of the circuit will be changed. Then the p.d. *V* will change. Record a few pairs of (*I*, *V*) readings and plot the *V*~*I* graph. Then the gradient of the *V*~*I* graph is the negative of internal resistance –*r* and the intercept of the graph on *V* axis is emf $\mathcal{E}$.

Chapter 5 Nature of Light

5.1 Photoelectric Effect

In 1887, scientist Heinrich Hertz observed a very interesting phenomenon about light. When a metal surface is exposed to light radiation above a certain threshold frequency the light is absorbed and electrons are emitted from the metal surface. This phenomenon is named **photoelectric effect** and the process of emitting the electrons is called **photoelectric emission**. An experiment setup as shown in Fig 5.1.1 was used to observe the photoelectric effect. The electrons emitted from photoelectric effect are called photoelectrons. These photoelectrons can form a current called **photoelectric current** if a closed circuit is formed.

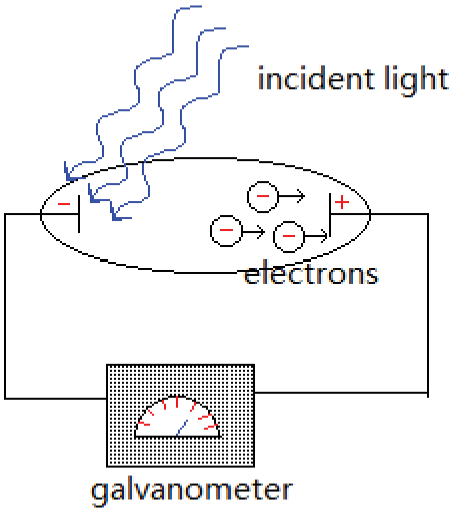
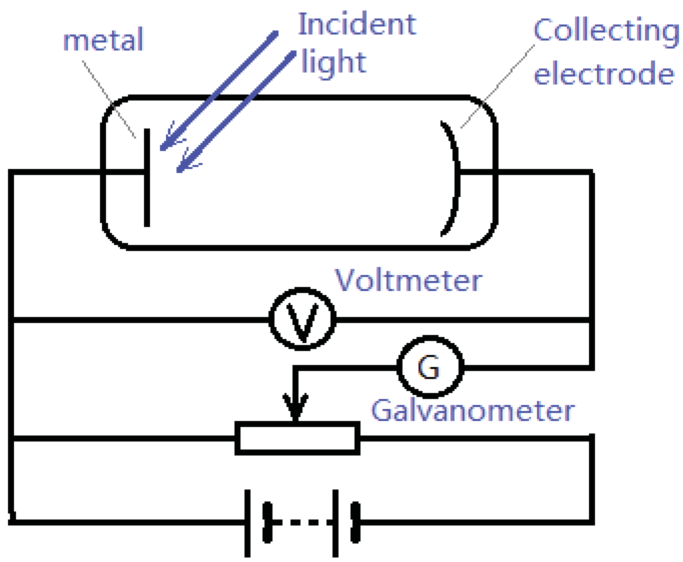


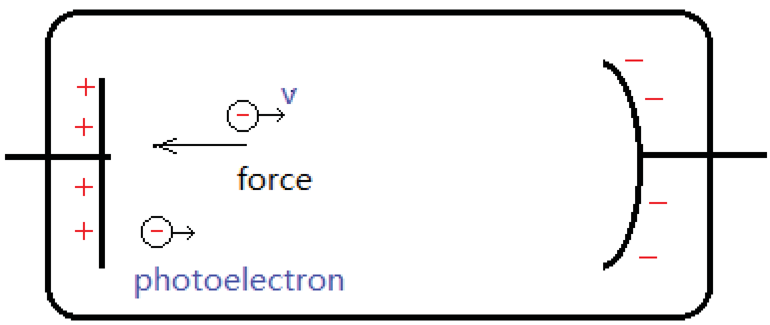
Fig 5.1.1 Experiment setup to observe photoelectric effect

Investigating photoelectric effect by experiment

A more detailed investigation into the photoelectric effect may be carried out by using the following setup (Fig 5.1.2(a)).



(a)



(b)

Fig 5.1.2 Photoelectric experiment

Light radiation of a known frequency is shone on the metal surface. The photoelectrons emitted from the metal move towards right to reach the collecting electrode and then form photoelectric current. The current is measured by the galvanometer.

A potential difference is applied between the metal surface and the collecting electrode to measure the kinetic energy of the emitted electrons. As a potential difference is applied (connected to the battery), positive charges will gather on the metal and negative charges will gather on the collecting electrode, as shown in Fig 5.1.2(b). Thus an electrical force will be exerted on photoelectrons emitted from the metal. The force is in the opposite direction of the movement of the photoelectrons. Thus it dose work to reduce the kinetic energy of the photoelectrons. Recall the definition of p.d. in Chapter 4. P.d. is the work done on unit charge. So, the work done is equal to p.d. times charge. Here, if the electron moves from the metal surface to the collecting electrode, the work done on the electron by electrical force is eV where e is the charge carried by an electron and V is the p.d. between the metal surface and the collecting electrode. If the potential difference V is increased, more work is done on a photoelectron to reduce its kinetic energy. As a result, those photoelectrons with relative lower kinetic energy will not be able to reach the collecting electrode. So, by increasing the potential difference, the photoelectric current will decrease as fewer photoelectrons can reach the collecting electrode to form electrical current. When the potential difference is increased to a certain value V_s, the reading on the galvanometer will be zero. This indicates that no photoelectron can reach the collecting electrode (even electrons with the maximum kinetic energy are stopped). This V_s is called the **stopping potential difference** or **stopping voltage**. As, work done on the photoelectron is equal to the decrease of its kinetic energy, we have

$$eV_s = \frac{1}{2}mv_{max}{}^2 \quad (5.1)$$

The value of V_s is given by the voltmeter when the reading on the galvanometer is just zero. The maximum kinetic energy of photoelectrons is measure in such a way.

Measuring the maximum kinetic energy of photoelectrons with various incident light (different frequencies) and plotting $\frac{1}{2}mv_{max}{}^2 \sim f$ graph, you should get a result similar to the one shown in Fig 5.1.3.

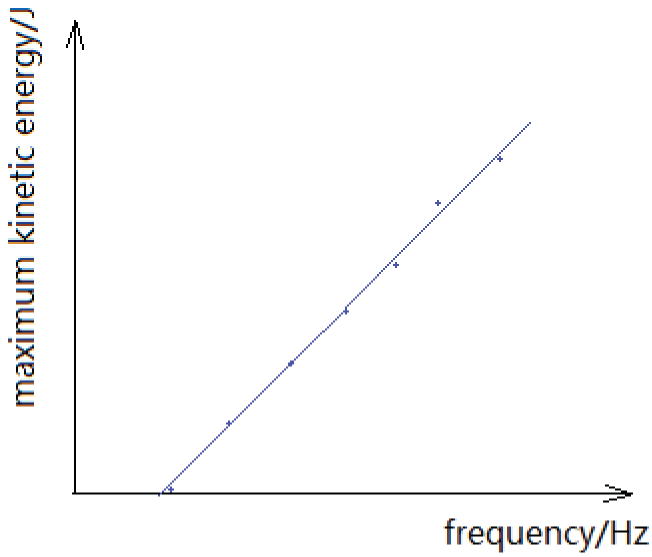


Fig 5.1.3 Maximum kinetic energy vs frequency graph

The following four important features are obtained from the photoelectric experiment:

1. When the light is shone on the metal, photoelectron is immediately emitted.
2. For each metal, there exists a minimum frequency for the incident light under which no photoelectric emission can happen. This frequency is called the **threshold frequency**. Only light whose frequency is higher than the threshold frequency of the metal can cause photoelectric effect.
3. The maximum kinetic energy of photoelectrons is proportional to the frequency of

incident light.
4. The photoelectric current increases with the increase of the intensity of the incident light.

Inconsistence of the experiment results with wave theory of light

These experiment results made physicist at that time a headache as they are not consistent with the theory.

We learned in Chapter 3 that light is a wave. According to the wave theory, wave is continuous and the electron can continuously absorb energy from the light wave until it gets enough energy to escape from the metal, so there should not be a threshold frequency. Also, it will take some time for the electron to absorb enough energy and thus photoelectron can not be emitted immediately. That is to say, features 1 and 2 can not be explained using wave theory of light.

Also, according to wave theory, energy is measured by intensity and is not related to frequency at all. So, the maximum kinetic energy should increase as the increase of intensity rather than frequency. That is to say, features 3 and 4 can not be explained using wave theory of light either.

Particle theory of light

To resolve the above paradox Einstein proposed a theory viewing light as stream of particles. This is called the particle theory of light. Einstein's theory successfully explained the experiment results of photoelectric effect and led him to the 1921 Nobel Prize in physics.

In this theory, it is stated that light consists of particles called **photon**s. Each photon has energy given by

$$E_{photon} = hf \tag{5.2}$$

where h is constant called Plank constant with a value of $6.63\times10^{-34}\,J\cdot s$ and f is the frequency of the light.

The **intensity** of light (also called **brightness**) is decided by the **flux** radiation, which is defined as the amount of energy landing on a unit area in a unit time.

$$F = \frac{E}{At} = \frac{P}{A} \tag{5.3}$$

If n is the number of photons, the above equation can also be written as

$$F = \frac{E}{At} = \frac{nE_{photon}}{At} = \frac{n}{At}\cdot E_{photon} \tag{5.4}$$

where E_{photon} is the energy carried by a single photon and $\frac{n}{At}$ is the rate of number of photons arrive on unit area.

Example:

(a) Calculate the energy of a photon for UV light with a frequency of $2\times10^{15}\,Hz$.

(b) Calculate the energy of a photon for gamma ray with a wavelength of $8.4\times10^{-13}\,m$. Compare the result with the one you obtained in part (a).

Solution:

(a) For the UV light, $E_{photon}=hf=6.63\times10^{-34}\times2\times10^{15}=1.3\times10^{-18}\,J$

(b) For the gamma ray, $E_{photon}=hf=\dfrac{hc}{\lambda}=6.63\times10^{-34}\times\dfrac{3.0\times10^{8}}{8.4\times10^{-13}}=2.4\times10^{-13}\,J$

It is easy to see that the photons of gamma ray have much higher energy than photons of UV light.

When a light beam is shone on a metal surface, an electron of the metal can only absorb one single photon. If the photon's energy is very small, it is not enough for the electron to escape from the attraction of nucleus and there will be no photoelectric effect. The minimum energy required to release the photoelectron from the metal surface is called the **work function**, Φ, of the metal. The corresponding frequency required for the incident light is called the **threshold frequency** f_0, which is decided by

$$\Phi=hf_0 \tag{5.5}$$

Work function and threshold frequency, like resistivity and density, are properties of materials.

If the photon's energy is just equal to the work function, the emitted electron uses all the absorbed energy to escape from the metal and has a kinetic energy of 0 when it just gets out of the metal.

If the energy of the photon is greater than the work function, then the energy absorbed by the electron from the photon is not all used and the rest become kinetic energy of the photoelectron. According to energy conservation,

Energy of incident photon = Work done for the photoelectron to escape from the metal + Kinetic energy of the photoelectron

Writing in symbol form, it is

$$hf=\Phi+\frac{1}{2}mv_{\max}^{2} \tag{5.6}$$

or

$$hf=hf_0+\frac{1}{2}mv_{\max}^{2} \tag{5.7}$$

Equation(5.6) or (5.7) is the famous **Einstein's photoelectric equation** (or simply called photoelectric equation). Rearranging the terms in equation(5.6), we can get

$$\frac{1}{2}mv_{max}{}^{2} = hf - \Phi \qquad (5.8)$$

From the above equation, we see that the maximum kinetic energy of the photoelectron is proportional to the frequency of incident light with the proportionality constant equaling Plank constant h. This why the experimental results plotted in Fig 5.1.3 is a straight line. We can also see that on the $\frac{1}{2}mv_{max}{}^{2} \sim f$ graph, the gradient is always h and the intercept on the vertical axis is the negative of the work function of the metal. $\frac{1}{2}mv_{max}{}^{2} \sim f$ graphs for different metals would have the same slope but different intercepts.

Example:

Explain how the experiment results of the photoelectric effect can be explained using particle theory of light.

Solution:

Each of the four experiment results can be well explained using particle theory of light as follows:

- According to particle theory, one photon is absorbed by one electron, immediately giving it kinetic energy and photoelectron is released without any delay.
- low frequency of incident light ⇒ energy of a photon lower than work function ⇒ electron: not enough energy to escape ⇒ no photoelectric effect
- frequency of the incident light ↑ ⇒ energy of a single photon ↑ ⇒ maximum kinetic energy of emitted photoelectron ↑
- intensity (or flux, or brightness) ↑ ⇒ number of photons per unit time ↑ ⇒ number of photonelectrons per unit time ↑ ⇒ photoelectric current ↑

Example:

Antimony-caesium's work function is 2.84×10^{-19}J. Whether yellow light of 590nm can cause photoelectric emission for antimony-caesium?

Solution:

To answer the question, we need to see if the frequency of the yellow light is above the threshold frequency of antimony-caesium or not.

The threshold frequency of antimony-caesium is given by

$$f_0 = \phi / h = (2.84\times10^{-19} / 6.63\times10^{-34})Hz = 4.31\times10^{-16}\,Hz$$

The frequency of the yellow light with wavelength 590nm is given by

$$f = c / \lambda = 3.00 \times 10^8 ms^{-1} / 590 \times 10^{-9} m = 5.08 \times 10^{-14} Hz$$

Since the frequency of the incident yellow light is higher than the threshold frequency, there will be photoelectric effect.

Example:

Light with a frequency of 7.06×10^{-19} Hz is shone on metal sodium whose work function is 2.28 eV, what will be the maximum kinetic energy of the photoelectrons emitted?

Solution:

According to the photoelectric equation $hf = \Phi + \frac{1}{2} m v_{\max}{}^2$, maximum kinetic energy of the photoelectrons is given by

$$\frac{1}{2} m v_{\max}{}^2 = hf - \phi = 6.63 \times 10^{-34} \times 7.06 \times 10^{14} - 2.28 \times 1.6 \times 10^{-19} = 9.01 \times 10^{-20} J$$

Wave-particle duality

Historically, there was a long "fighting" between scientists regarding the nature of light (whether light is wave or particle). Some scientists such as Newton said that light is a bunch of particles. They said that reflection of light and shades under the sun are good evidences to support the saying that light is made of particles.

Some scientists, such as Huygens, insisted that light is a wave as the diffraction and interference of light are only behaviors of waves. They argued that even the reflection and shades under the sun examples can be explained using wave theory, taking into account that the wavelengths of light are very small.

When the photoelectric effect was discovered, physicists found that only a particle theory can explain the phenomenon. Finally in early 20^{th} century, physicists agreed that light is both particle and wave. Light sometimes behaves like a wave (eg. diffraction, interference) and sometimes behaves like a particle (eg. photoelectric effect). Thus we call it **wave-particle** (or **wavicle**). This is known as the **wave-particle duality** of light.

5.2 Spectra

Bohr model of atom and energy levels

We know that atom is a basic unit of matter that consists of a dense, central nucleus with positive charge and surrounding electrons with negative charge.

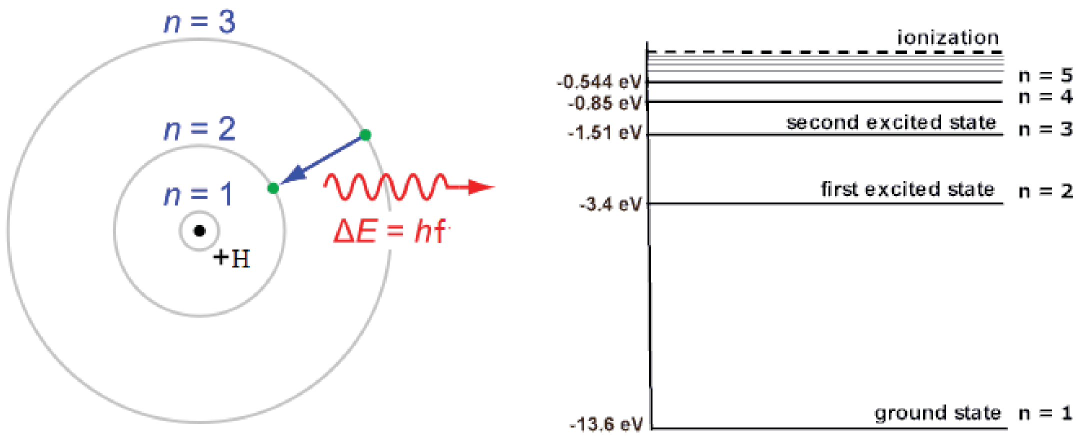


Fig 5.2.1 (a) Orbits of electron in hydrogen atom (b)Energy levels of hydrogen

The Danish physicist Bohr constructed a model of atom which states:

- The electrons of an atom revolve around the nucleus in circular orbits. The electrons can only travel in some special orbits, as shown in the orbits diagram in Fig 5.2.1(a).
- These orbits are associated with specific energies called **energy levels**, as shown in the energy level diagram in Fig 5.2.1(b).
- The electrons can gain or lose energy by jumping from one allowed orbit to another, in the form of absorbing or emitting a photon. This is called **electron transition**.

For an electron transition to happen, the energy of the incident photon should be exactly equal to the energy difference between the two energy levels. That is,

$$\Delta E = E_{photon} = hf \quad (5.9)$$

When the electron revolves around the nucleus in the 1st orbit (with smallest radius), we say that the atom is in **ground state**. It has the lowest energy in this state. In ground state, n=1.

When the electron absorbs a photon to jump to a higher energy orbit, we say that the atom is excited by the photon. This state is therefore called an **excited state**. In excited states, n=2,3,…Atoms in excited states are usually not stable and are likely to drop to ground state.

When the electron absorbs a photon with very high energy and jumps to far away from the nucleus, we say that the atom is ionized (or in **ionized state**). In this state, $n=\infty$.This process of removing the electron from the atom to form an ion is called **ionization**.

Example:

Calculate the frequency of light emitted through electron transition by hydrogen atom from level E_3 to ground level (the energy level diagram is shown in Fig 5.2.1(b)).

Solution:

The energy difference between the level E_3 and ground level is

$$\Delta E = E_3 - E_1 = -1.51 - (-13.6) = 12.1eV = 1.93\times10^{-18} J$$

As the energy of the emitted photon must be equal to the energy difference, or $\Delta E = hf$, the frequency of emitted light is

$$f = \Delta E/h = (1.93\times10^{-18} / 6.63\times10^{-34}) Hz = 2.9\times10^{15} Hz$$

Spectra

A **spectrum** is a family of different wavelengths. For visible light, wavelengths correspond to colors. A **spectrometer** is a device that can separate the wavelengths in a beam of radiation, to show those wavelengths that are present. A prism is a simple spectrometer.

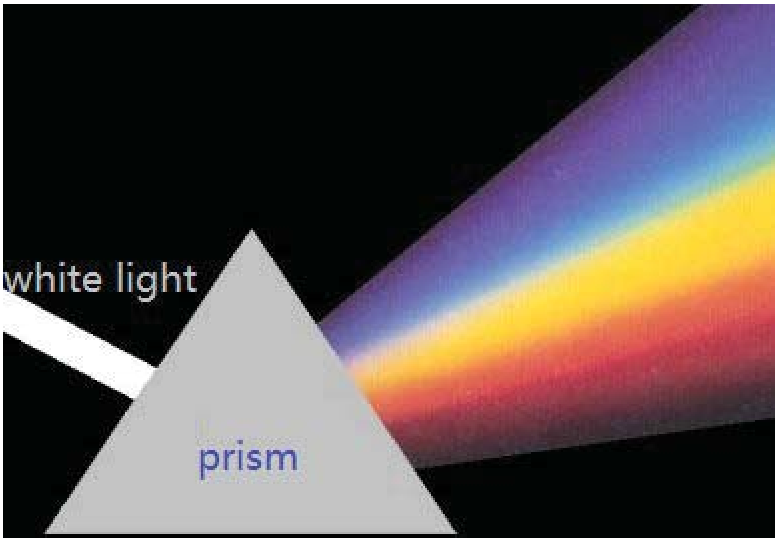


Fig 2.2.2 Prism can separate the colors (frequencies)

Continuous spectrum: consist of continuous range of wavelengths. Spectra of light directly from the sun and from incandescent lamps are continuous spectra.

Line spectrum: consists of discrete wavelengths. There are two types of line spectra, emission spectrum and absorption spectrum. **Emission spectrum** is a collection of wavelengths of the light emitted by an atom when its electrons transit from higher to lower energy levels. **Absorption spectrum** is a collection of wavelengths of the light absorbed by an atom when its electrons transit from lower to higher energy levels. Atoms of a certain element have their unique emission and absorption spectra because their atomic structure is unique. So analyzing emission or absorption spectrum is a useful method to identify elements.

- line spectrum
  - emission spectrum
  - absorption spectrum
- continuous spectrum

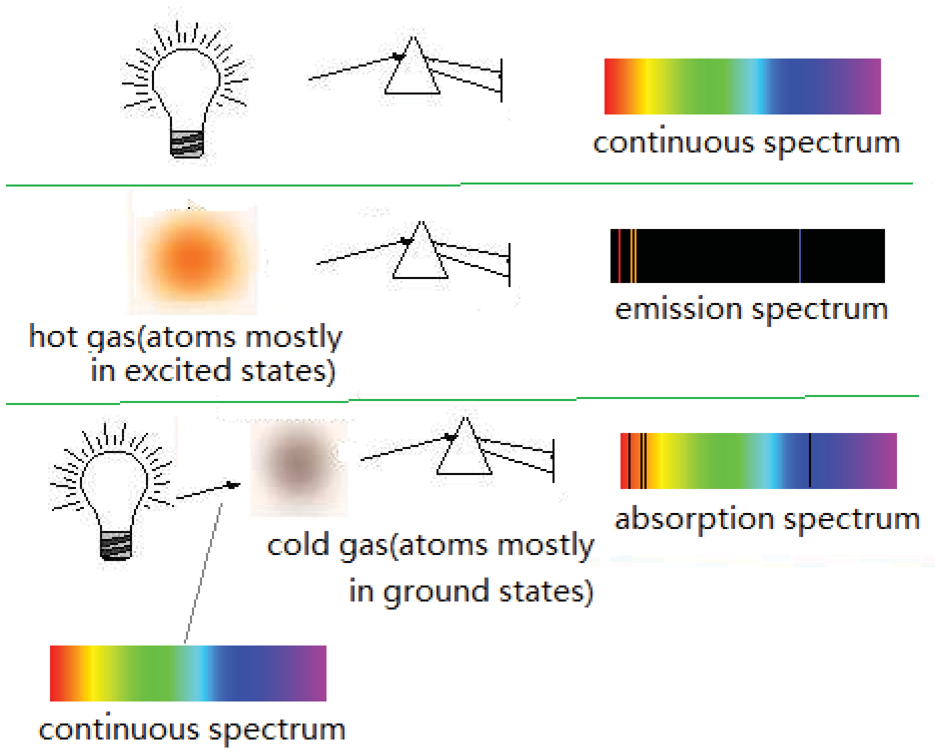


Fig 5.2.3 Continuous, emission and absorption spectra

5.3 Solar cells

The **photovoltaic effect** is the creation of a voltage (or a corresponding electric current) in a material upon exposure to light. This effect is similar to the photoelectric effect but the material is not metal but usually semiconductor. Solar cells use this to produce electricity.

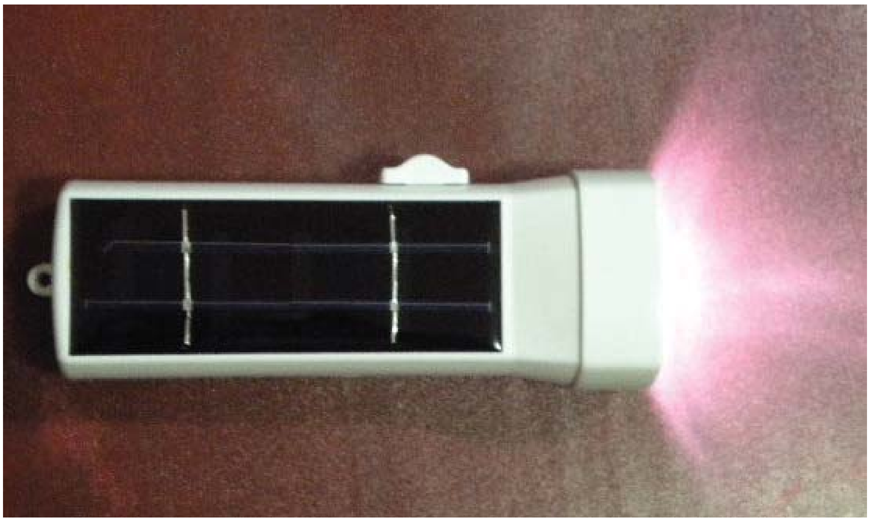


Fig 5.3.1 Torch powered by solar cells

The efficiency of solar cell is equal to the useful electrical energy output divided by the total solar energy input. The efficiencies of commercially available solar cells are currently between 5%-20%. Solar energy is a renewable green energy source. Low efficiency and high cost are major reasons why solar power is not very widely implemented.

Example:

A solar cell array used to power a small fan is made from 10 photovoltaic tiles. Each tile is a square with a sidelength of 2cm and has an efficiency of 10%. Calculate the output power of the array if it is illuminated by sunlight with a flux of 1000Wm^{-2}.

Solution:

The efficiency is known. To find the output power, we need to calculate the total power input. Using the equation $F = \frac{E}{At} = \frac{P}{A}$, the solar power input is

$$P_{in} = FA = 1000 \times (0.02^2 \times 10)W = 4W$$

Thus, the output electrical power is given by

$$P_{out} = 10\% P_{in} = 0.4W$$

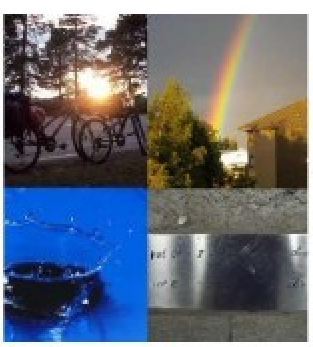
"If physics is dull, then the teacher is bad."
------ Walter Lewin

The purpose of this book is to offer a concise and easy-to-understand textbook for Edexcel AS Physics. The book tries its best to avoid frustrating students and explains physics in easy-to-understand ways.

You must have already noticed that this book is slim, with only 100 pages! But it covers almost all that you need to learn to crack the Edexcel AS Physics Unit 1 and Unit 2 exams. It can be used as a textbook for your AS physics courses. The principles and terms are explained in very simple words and sentences. They are illustrated in the students' point of view, rather in the physicists or teachers' point of view. There are plenty of carefully designed diagrams and examples to illustrate difficult concepts. It is an excellent revision guide as well.

This work is written based on my teaching experience. During my teaching, I always tried to find the ways of instruction that are easy for the students to understand. I decide to put down the contents in the ways that my students found easiest to understand and memorize. You will find it a bit easier to understand some difficult physical concepts with this book compared to some other books.

This book is written based on the specifications published by Edexcel and my study on the past papers. It is excellent for exam preparation. Contents of particular importance are highlighted in gray background. The examiners frequently request you to write down these sentences or use these formulae for calculations. You will find a great match between highlighted contents in this book and the exam paper questions!



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