# Discounted Cashflow Valuation: Equity and Firm Models 

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## Summarizing the Inputs

- In summary, at this stage in the process, we should have an estimate of the
- the current cash flows on the investment, either to equity investors (dividends or free cash flows to equity) or to the firm (cash flow to the firm)
- the current cost of equity and/or capital on the investment
- the expected growth rate in earnings, based upon historical growth, analysts forecasts and/or fundamentals
- The next step in the process is deciding
- which cash flow to discount, which should indicate
- which discount rate needs to be estimated and
- what pattern we will assume growth to follow


## Which cash flow should I discount?

- Use Equity Valuation
(a) for firms which have stable leverage, whether high or not, and
(b) if equity (stock) is being valued
- Use Firm Valuation
(a) for firms which have leverage which is too high or too low, and expect to change the leverage over time, because debt payments and issues do not have to be factored in the cash flows and the discount rate (cost of capital) does not change dramatically over time.
(b) for firms for which you have partial information on leverage (eg: interest expenses are missing..)
(c) in all other cases, where you are more interested in valuing the firm than the equity. (Value Consulting?)


## Given cash flows to equity, should I discount dividends or FCFE?

- Use the Dividend Discount Model
- (a) For firms which pay dividends (and repurchase stock) which are close to the Free Cash Flow to Equity (over a extended period)
- (b)For firms where FCFE are difficult to estimate (Example: Banks and Financial Service companies)
- Use the FCFE Model
- (a) For firms which pay dividends which are significantly higher or lower than the Free Cash Flow to Equity. (What is significant? ... As a rule of thumb, if dividends are less than $80 \%$ of FCFE or dividends are greater than $110 \%$ of FCFE over a 5 year period, use the FCFE model)
- (b) For firms where dividends are not available (Example: Private Companies, IPOs)


## What discount rate should I use?

- Cost of Equity versus Cost of Capital
- If discounting cash flows to equity $->$ Cost of Equity
- If discounting cash flows to the firm $\quad->$ Cost of Capital
- What currency should the discount rate (risk free rate) be in?
- Match the currency in which you estimate the risk free rate to the currency of your cash flows
- Should I use real or nominal cash flows?
- If discounting real cash flows -> real cost of capital
- If nominal cash flows -> nominal cost of capital
- If inflation is low ( $<10 \%$ ), stick with nominal cash flows since taxes are based upon nominal income
- If inflation is high ( $>10 \%$ ) switch to real cash flows


## Which Growth Pattern Should I use?

- If your firm is
- large and growing at a rate close to or less than growth rate of the economy, or
- constrained by regulation from growing at rate faster than the economy
- has the characteristics of a stable firm (average risk \& reinvestment rates)


## Use a Stable Growth Model

- If your firm
- is large \& growing at a moderate rate ( $\leq$ Overall growth rate $+10 \%$ ) or
- has a single product \& barriers to entry with a finite life (e.g. patents)


## Use a 2-Stage Growth Model

- If your firm
- is small and growing at a very high rate ( $>$ Overall growth rate $+10 \%$ ) or
- has significant barriers to entry into the business
- has firm characteristics that are very different from the norm

Use a 3-Stage or n-stage Model

## The Building Blocks of Valuation

| Choose a | Dividends  <br> Expected Dividends to Cashflows to Equity <br> Stockholders Net Income <br>  $-(1-\delta)($ Capital Exp. - Deprec'n) <br>  $-(1-\delta)$ Change in Work. Capital <br>  $=$ Free Cash flow to Equity (FCFE) <br>  $[\delta=$ Debt Ratio $]$ |  |  |
| :---: | :---: | :---: | :---: |
| Cash Flow |  |  | $\quad$ Cashflows to Firm EBIT (1- tax rate) - (Capital Exp. - Deprec'n) - Change in Work. Capital = Free Cash flow to Firm (FCFF) |
| \& A Discount Rate | Cost of <br> - Basis: The riskier the investment, <br> - Models: <br> CAPM: Riskfree Rate + Beta <br> APM: Riskfree Rate $+\Sigma$ Beta $_{\mathrm{j}}$ | Equity the greater is the cost of equity. <br> (Risk Premium) <br> (Risk Premium ${ }_{\mathrm{j}}$ ): $n$ factors | $\begin{aligned} & \text { Cost of Capital } \\ & \begin{aligned} \text { WACC } & =k_{\mathrm{e}}(\mathrm{E} /(\mathrm{D}+\mathrm{E})) \\ & +\mathrm{k}_{\mathrm{d}}(\mathrm{D} /(\mathrm{D}+\mathrm{E})) \end{aligned} \\ & \mathrm{k}_{\mathrm{d}}=\text { Current Borrowing Rate (1-t) } \\ & \text { E,D: Mkt Val of Equity and Debt } \end{aligned}$ |
| \& a growth pattern | Stable Growth |  | Three-Stage Growth |

## Classifying DCF Models



Damodaran

## Companies Valued

| Company | Model Used | Remarks |
| :--- | :--- | :--- |
| Con Ed | Stable DDM | Dividends=FCFE, Stable D/E, Low g |
| ABN Amro | 2-Stage DDM | FCFE=?, Regulated D/E, g>Stable |
| S\&P 500 | 2-Stage DDM | Collectively, market is an investment |
| Nestle | 2-Stage FCFE | Dividends $\neq F C F E$, Stable D/E, High g |
| Tsingtao | 3-Stage FCFE | Dividends $\neq F C F E$, Stable D/E,High g |
| DaimlerChrysler | Stable FCFF | Normalized Earnings; Stable Sector |
| Tube Investments | 2-stage FCFF | The value of growth? |
| Embraer | 2-stage FCFF | Emerging Market company (not...) |
| Global Crossing | 2-stage FCFF | Dealing with Distress |
| Amazon.com | n-stage FCFF | Varying margins over time |
|  |  |  |

## General Information

- The risk premium that I will be using in the latest valuations for mature equity markets is $4 \%$. This is the average implied equity risk premium from 1960 to 2003 as well as the average historical premium across the top 15 equity markets in the twentieth century.
- For the valuations from 1998 and earlier, I use a risk premium of 5.5\%.


## Con Ed: Rationale for Model

- The firm is in stable growth; based upon size and the area that it serves. Its rates are also regulated; It is unlikely that the regulators will allow profits to grow at extraordinary rates.
- Firm Characteristics are consistent with stable, DDM model firm
- The beta is 0.80 and has been stable over time.
- The firm is in stable leverage.
- The firm pays out dividends that are roughly equal to FCFE.
- Average Annual FCFE between 1999 and $2004=\$ 635$ million
- Average Annual Dividends between 1999 and 2004 = $\$ 624$ million
- Dividends as \% of FCFE $=98 \%$


## Con Ed: A Stable Growth DDM: December 31, 2004

- Earnings per share for $2004=\$ 2.72$ (Fourth quarter estimate used)
- Dividend Payout Ratio over $2004=83.06 \%$
- Dividends per share for $2004=\$ 2.26$
- Expected Growth Rate in Earnings and Dividends $=2 \%$
- Con Ed Beta $=0.80$ (Bottom-up beta estimate)

■ Cost of Equity $=4.22 \%+0.80 * 4 \%=7.42 \%$
Value of Equity per Share $=\mathbf{\$ 2 . 2 6 * 1 . 0 2 / ( . 0 7 4 2 - . 0 2 )}=\mathbf{\$ 4 2 . 5 3}$
The stock was trading at $\mathbf{\$ 3 . 4 2}$ on December 31, 2004

## Con Ed: Break Even Growth Rates




## Estimating Implied Growth Rate

- To estimate the implied growth rate in Con Ed's current stock price, we set the market price equal to the value, and solve for the growth rate:
- Price per share $=\$ 43.42=\$ 2.26^{*}(1+\mathrm{g}) /(.0742-\mathrm{g})$
- Implied growth rate $=2.11 \%$
- Given its retention ratio of $16.94 \%$ and its return on equity in 2003 of $10 \%$, the fundamental growth rate for Con Ed is:

Fundamental growth rate $=\left(.1694^{*} .10\right)=1.69 \%$

- You could also frame the question in terms of a break-even return on equity.
- Break even Return on equity $=\mathrm{g} /$ Retention ratio $=.0211 / .1694=12.45 \%$


## Implied Growth Rates and Valuation Judgments

- When you do any valuation, there are three possibilities. The first is that you are right and the market is wrong. The second is that the market is right and that you are wrong. The third is that you are both wrong. In an efficient market, which is the most likely scenario?
- Assume that you invest in a misvalued firm, and that you are right and the market is wrong. Will you definitely profit from your investment?
- Yes
- No


## Con Ed: A Look Back



## ABN Amro: Rationale for 2-Stage DDM in December 2003

- As a financial service institution, estimating FCFE or FCFF is very difficult.
- The expected growth rate based upon the current return on equity of $16 \%$ and a retention ratio of $51 \%$ is $8.2 \%$. This is higher than what would be a stable growth rate (roughly $4 \%$ in Euros)


## ABN Amro: Summarizing the Inputs

- Market Inputs
- Long Term Riskfree Rate (in Euros) $=4.35 \%$
- Risk Premium $=4 \%$ (U.S. premium : Netherlands is AAA rated)
- Current Earnings Per Share $=1.85$ Eur; Current DPS $=0.90$ Eur;

Variable
Length
Return on Equity $16.00 \%$
Payout Ratio 48.65\%
Retention Ratio 51.35\%
Expected growth
Beta
Cost of Equity
5 years
0.95

High Growth Phase Stable Growth Phase
.16*. $5135=. .0822$
$4.35 \%+0.95(4 \%)$
$=8.15 \%=8.35 \%$

Forever after yr 5
8.35\% (Set = Cost of equity)
52.10\% (1-4/8.35)
$47.90 \%(\mathrm{~b}=\mathrm{g} / \mathrm{ROE}=4 / 8.35)$
4\% (Assumed)
1.00
$4.35 \%+1.00(4 \%)$

## ABN Amro: Valuation

| Year | EPS | DPS | PV of DPS $($ at $8.15 \%)$ |
| :--- | :--- | :--- | :--- |
| 1 | 2.00 | 0.97 | 0.90 |
| 2 | 2.17 | 1.05 | 0.90 |
| 3 | 2.34 | 1.14 | 0.90 |
| 4 | 2.54 | 1.23 | 0.90 |
| 5 | 2.75 | 1.34 | 0.90 |
| Expected EPS in year $6=2.75(1.04)=2.86$ Eur |  |  |  |
| Expected DPS in year $6=2.86 * 0.5210=1.49$ Eur |  |  |  |
| Terminal Price (in year 5$)=1.49 /(.0835-.04)=34.20$ Eur |  |  |  |
| PV of Terminal Price $=34.20 /(1.0815)^{5}=23.11$ Eur |  |  |  |
| Value Per Share $=\mathbf{0 . 9 0}+\mathbf{0 . 9 0}+\mathbf{0 . 9 0}+\mathbf{0 . 9 0}+\mathbf{0 . 9 0}+\mathbf{2 3 . 1 1}=\mathbf{2 7 . 6 2}$ Eur |  |  |  |
| The stock was trading at $\mathbf{1 8 . 5 5}$ Euros on December 31, 2003 |  |  |  |



## The Value of Growth

- In any valuation model, it is possible to extract the portion of the value that can be attributed to growth, and to break this down further into that portion attributable to "high growth" and the portion attributable to "stable growth". In the case of the 2-stage DDM, this can be accomplished as follows:

$\mathrm{DPS}_{\mathrm{t}}=$ Expected dividends per share in year t
$r=$ Cost of Equity
$P_{n}=$ Price at the end of year $n$
$\mathrm{g}_{\mathrm{n}}=$ Growth rate forever after year n


## ABN Amro: Decomposing Value

- Value of Assets in Place $=$ Current DPS/Cost of Equity

$$
\begin{aligned}
& =0.90 \text { Euros } / .0835 \\
& =10.78 \text { Euros }
\end{aligned}
$$

- Value of Stable Growth $=0.90(1.04) /(.0835-.04)-10.78$ Euros

$$
=10.74 \text { Euros }
$$

(A more precise estimate would have required us to use the stable growth payout ratio to re-estimate dividends)

- Value of High Growth $=$ Total Value $-(10.78+10.74)$

$$
=27.62-(10.78+10.74)=\text { 6.10 Euros }
$$

## S \& P 500: Rationale for Use of Model

- While markets overall generally do not grow faster than the economies in which they operate, there is reason to believe that the earnings at U.S. companies (which have outpaced nominal GNP growth over the last 5 years) will continue to do so in the next 5 years. The consensus estimate of growth in earnings (from Zacks) is roughly $8 \%$ (with top-down estimates)
- Though it is possible to estimate FCFE for many of the firms in the S\&P 500, it is not feasible for several (financial service firms). The dividends during the year should provide a reasonable (albeit conservative) estimate of the cash flows to equity investors from buying the index.


## S \&P 500: Inputs to the Model (12/31/04)

- General Inputs
- Long Term Government Bond Rate $=4.22 \%$
- Risk Premium for U.S. Equities $=4 \%$
- Current level of the Index $=1211.92$
- Inputs for the Valuation

|  | High Growth Phase | Stable Growth Phase |
| :--- | :--- | :--- |
| Length | 5 years | Forever after year 5 |
| Dividend Yield | $1.60 \%$ | $1.60 \%$ |
| Expected Growth | $8.5 \%$ | $4.22 \%$ (Nominal g) |
| Beta | 1.00 | 1.00 |

## S \& P 500: 2-Stage DDM Valuation

|  | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Expected Dividends $=$ | $\$ 21.06$ | $\$ 22.85$ | $\$ 24.79$ | $\$ 26.89$ | $\$ 29.18$ |
| Expected Terminal Value $=$ |  |  |  |  | $\$ 760.28$ |
| Present Value $=$ | $\$ 19.46$ | $\$ 19.51$ | $\$ 19.56$ | $\$ 19.61$ | $\$ 531.86$ |
| Intrinsic Value of Index $=$ | $\$ 609.98$ |  |  |  |  |

Cost of Equity $=4.22 \%+1(4 \%)=8.22 \%$
Terminal Value $=29.18 * 1.0422 /(.0822-.0422)=760.28$

## Explaining the Difference

- The index is at 1212 , while the model valuation comes in at 610 . This indicates that one or more of the following has to be true.
- The dividend discount model understates the value because dividends are less than FCFE.
- The expected growth in earnings over the next 5 years will be much higher than $8 \%$.
- The risk premium used in the valuation (4\%) is too high
- The market is overvalued.


## A More Realistic Valuation of the Index

We estimated the free cashflows to equity for each firm in the index and averaged the free cashflow to equity as a percent of market cap. The average FCFE yield for the index was about $2.90 \%$ in 2004.
-With these inputs in the model:

|  | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Expected Dividends \& Buybacks $=$ | $\$ 38.14$ | $\$ 41.38$ | $\$ 44.89$ | $\$ 48.71$ | $\$ 52.85$ |
| Expected Terminal Value $=$ |  |  |  |  | $\$ 1,377.02$ |
| Present Value $=$ | $\$ 35.24$ | $\$ 35.33$ | $\$ 35.42$ | $\$ 35.51$ | $\$ 963.29$ |
| Intrinsic Value of Index $=$ | $\$ 1,104.80$ |  |  |  |  |

At a level of 1112, the market is overvalued by about $10 \%$.

## Nestle: Rationale for Using Model - January 2001

- Earnings per share at the firm has grown about $5 \%$ a year for the last 5 years, but the fundamentals at the firm suggest growth in EPS of about $11 \%$.
(Analysts are also forecasting a growth rate of $12 \%$ a year for the next 5 years)
- Nestle has a debt to capital ratio of about $37.6 \%$ and is unlikely to change that leverage materially. (How do I know? I do not. I am just making an assumption.)
- Like many large European firms, Nestle has paid less in dividends than it has available in FCFE.


## Nestle: Summarizing the Inputs

- General Inputs
- Long Term Government Bond Rate $(\mathrm{Sfr})=4 \%$
- Current EPS $=108.88$ Sfr; Current Revenue/share $=1,820 \mathrm{Sfr}$
- Capital Expenditures/Share=114.2 Sfr; Depreciation/Share=73.8 Sfr

Length
Beta
Return on Equity
Retention Ratio
Expected Growth
WC/Revenues
Debt Ratio
Cap Ex/Deprecn

High Growth
5 years
0.85
23.63\%
65.10\% (Current)
23.63\%*.651= 15.38\%
9.30\% (Existing)
37.60\%

Current Ratio

Stable Growth
Forever after yr 5
0.85

16\%
NA
4.00\%
9.30\% (Grow with earnings)
37.60\%

150\%

Estimating the Risk Premium for Nestle

|  |  | Revenues | Weight | Risk Premium |
| :---: | :---: | :---: | :---: | :---: |
|  | North America | $\begin{aligned} & 17.5 \\ & 4.00 \% \end{aligned}$ | 24.82\% |  |
|  | South America | $\begin{aligned} & 4.3 \\ & 12.00 \% \end{aligned}$ | 6.10\% |  |
|  | Switzerland | $\begin{aligned} & 1.1 \\ & 4.00 \% \end{aligned}$ | 1.56\% |  |
|  | Germany/France/UK | $\begin{aligned} & 18.4 \\ & 4.00 \% \end{aligned}$ | 26.10\% |  |
|  | Italy/Spain | $\begin{aligned} & 6.4 \\ & 5.50 \% \end{aligned}$ | 9.08\% |  |
|  | Asia | $\begin{aligned} & 5.8 \\ & 9.00 \% \end{aligned}$ | 8.23\% |  |
| Aswath Damodaran | Rest of W. Europe | $\begin{aligned} & 13 \\ & 4.00 \% \end{aligned}$ | 18.44\% |  |

## Nestle: Valuation

|  | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Earnings | $\$ 125.63$ | $\$ 144.95$ | $\$ 167.25$ | $\$ 192.98$ | $\$ 222.66$ |
| $-($ Net CpEX)* |  |  |  |  |  |
| $-\Delta$ WC* $^{*}(1-\mathrm{DR})$ | $\$ 29.07$ | $\$ 33.54$ | $\$ 38.70$ | $\$ 44.65$ | $\$ 51.52$ |
| Free Cashflow to Equity | $\$ 16.25$ | $\$ 18.75$ | $\$ 21.63$ | $\$ 24.96$ | $\$ 28.79$ |
| Present Value | $\$ 80.31$ | $\$ 92.67$ | $\$ 106.92$ | $\$ 123.37$ | $\$ 142.35$ |
|  | $\$ 74.04$ | $\mathbf{\$ 7 8 . 7 6}$ | $\mathbf{\$ 8 3 . 7 8}$ | $\mathbf{\$ 8 9 . 1 2}$ | $\$ 94.7$ |

Earnings per Share in year $6=222.66(1.04)=231.57$
Net Capital Ex ${ }_{6}=$ Deprecn'n $_{6} * 0.50=73.8(1.1538)^{5}(1.04)(.5)=78.5 \mathrm{Sfr}$
Chg in $\mathrm{WC}_{6}=\left(\operatorname{Rev}_{6}-\operatorname{Rev}_{5}\right)(.093)=1820(1.1538)^{5}(.04)(.093)=13.85 \mathrm{Sfr}$
$\mathrm{FCFE}_{6}=231.57-78.5(1-.376)-13.85(1-.376)=173.93 \mathrm{Sfr}$
Terminal Value per Share $=173.93 /(.0847-.04)=3890.16 \mathrm{Sfr}$
Value $=\$ 74.04+\$ 78.76+\$ 83.78+\$ 89.12+\$ 94.7+3890 /(1.0847)^{5}=3011 \mathrm{Sf}$
The stock was trading 2906 Sfr on December 31, 1999

## Nestle: The Net Cap Ex Assumption

- In our valuation of Nestle, we assumed that cap ex would be $150 \%$ of depreciation in steady state. If, instead, we had assumed that net cap ex was zero, as many analysts do, the terminal value would have been:
$\mathrm{FCFE}_{6}=231.57-13.85(1-.376)=222.93 \mathrm{Sfr}$
Terminal Value per Share $=222.93 /(.0847-.04)=4986 \mathrm{Sfr}$
Value $=\$ 74.04+\$ 78.76+\$ 83.78+\$ 89.12+\$ 94.7+4986 /(1.0847)^{5}=3740.91 \mathrm{Sfr}$



## The Effects of New Information on Value

- No valuation is timeless. Each of the inputs to the model are susceptible to change as new information comes out about the firm, its competitors and the overall economy.
- Market Wide Information
- Interest Rates
- Risk Premiums
- Economic Growth
- Industry Wide Information
- Changes in laws and regulations
- Changes in technology
- Firm Specific Information
- New Earnings Reports
- Changes in the Fundamentals (Risk and Return characteristics)


## Nestle: Effects of an Earnings Announcement

- Assume that Nestle makes an earnings announcement which includes two pieces of news:
- The earnings per share come in lower than expected. The base year earnings per share will be 105.5 Sfr instead of 108.8 Sfr.
- Increased competition in its markets is putting downward pressure on the net profit margin. The after-tax margin, which was $5.98 \%$ in the previous analysis, is expected to shrink to $5.79 \%$.
$\square$ There are two effects on value:
- The drop in earnings will make the projected earnings and cash flows lower, even if the growth rate remains the same
- The drop in net margin will make the return on equity lower (assuming turnover ratios remain unchanged). This will reduce expected growth.



## Tsingtao Breweries: Rationale for Using Model: June 2001

- Why three stage? Tsingtao is a small firm serving a huge and growing market - China, in particular, and the rest of Asia, in general. The firm's current return on equity is low, and we anticipate that it will improve over the next 5 years. As it increases, earnings growth will be pushed up.
- Why FCFE? Corporate governance in China tends to be weak and dividends are unlikely to reflect free cash flow to equity. In addition, the firm consistently funds a portion of its reinvestment needs with new debt issues.


## Background Information

- In 2000, Tsingtao Breweries earned 72.36 million CY(Chinese Yuan) in net income on a book value of equity of 2,588 million CY , giving it a return on equity of $2.80 \%$.
- The firm had capital expenditures of 335 million CY and depreciation of 204 million CY during the year.
- The working capital changes over the last 4 years have been volatile, and we normalize the change using non-cash working capital as a percent of revenues in 2000:
Normalized change in non-cash working capital = (Non-cash working capital ${ }_{2000} /$ Revenues $\left._{2000}\right)\left(\right.$ Revenuess $_{2000}-$ Revenues $\left._{\text {1999 }}\right)=(180 / 2253) *(2253-1598)=52.3$ million CY
Normalized Reinvestment
= Capital expenditures - Depreciation + Normalized Change in non-cash working capital
$=335-204+52.3=183.3$ million CY
- As with working capital, debt issues have been volatile. We estimate the firm's book debt to capital ratio of $40.94 \%$ at the end of 1999 and use it to estimate the normalized equity reinvestment in 2000.


## Inputs for the 3 Stages

Length
Beta 0.75
Risk Premium
ROE
Equity Reinv.

Equity Reinv.

| High Growth | Transition Phase | Stable Growth |
| :--- | :--- | :--- |
| 5 years | 5 years | Forever after yr 10 |
| Moves to 0.80 | 0.80 |  |
| $4 \%+2.28 \%$ | $-->$ | $4+0.95 \%$ |
| $2.8 \%->12 \%$ | $12 \%->20 \%$ | $20 \%$ |
| $149.97 \%$ | Moves to $50 \%$ | $50 \%$ |
| $44.91 \%$ | Moves to $10 \%$ | $10 \%$ |

- We wil asssume that

Equity Reinvestment Ratio $=$ Reinvestment (1- Debt Ratio) / Net Income $==183.3(1-.4094) / 72.36=149.97 \%$
Expected growth rate- next 5 years
$=$ Equity reinvestment rate $* \mathrm{ROE}_{\text {New }}+\left[1+\left(\mathrm{ROE}_{5}-\mathrm{ROE}_{\text {today }}\right) / \mathrm{ROE}_{\text {today }}\right]^{1 / 5-1}$
$=1.4997 * .12+\left[(1+(.12-.028) / .028)^{1 / 5}-1\right]=44.91 \%$

## Tsingtao: Projected Cash Flows

| Year | Expected Growth | Net Income | Reinvestment Rate | FCFE | Cost of Equity | Present Value |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Current |  | CY72.36 | $149.97 \%$ |  |  |  |
| 1 | $44.91 \%$ | CY104.85 | $149.97 \%$ | (CY52.40) | $14.71 \%$ | (CY45.68) |
| 2 | $44.91 \%$ | CY151.93 | $149.97 \%$ | $($ CY75.92) | $14.71 \%$ | (CY57.70) |
| 3 | $44.91 \%$ | CY220.16 | $149.97 \%$ | (CY110.02) | $14.71 \%$ | (CY72.89) |
| 4 | $44.91 \%$ | CY319.03 | $149.97 \%$ | (CY159.43) | $14.71 \%$ | (CY92.08) |
| 5 | $44.91 \%$ | CY462.29 | $149.97 \%$ | (CY231.02) | $14.71 \%$ | (CY116.32) |
| 6 | $37.93 \%$ | CY637.61 | $129.98 \%$ | (CY191.14) | $14.56 \%$ | (CY84.01) |
| 7 | $30.94 \%$ | CY834.92 | $109.98 \%$ | (CY83.35) | $14.41 \%$ | (CY32.02) |
| 8 | $23.96 \%$ | CY1,034.98 | $89.99 \%$ | CY103.61 | $14.26 \%$ | CY34.83 |
| 9 | $16.98 \%$ | CY1,210.74 | $69.99 \%$ | CY363.29 | $14.11 \%$ | CY107.04 |
| 10 | $10.00 \%$ | CY1,331.81 | $50.00 \%$ | CY665.91 | $13.96 \%$ | CY172.16 |
| Sum of the present values of FCFE during high growth $=$ |  |  |  |  |  |  |

## Tsingtao: Terminal Value

- Expected stable growth rate $=10 \%$
- Equity reinvestment rate in stable growth $=50 \%$
- Cost of equity in stable growth $=13.96 \%$
- Expected FCFE in year 11
$=$ Net Income $_{11}{ }^{*}(1-$ Stable period equity reinvestment rate $)$
$=$ CY 1331.81 (1.10)(1-.5) $=$ CY 732.50 million
- Terminal Value of equity in Tsingtao Breweries
$=\mathrm{FCFE}_{11} /($ Stable period cost of equity - Stable growth rate $)$
$=732.5 /(.1396-.10)=$ CY 18,497 million


## Tsingtao: Valuation

- Value of Equity
$=\mathrm{PV}$ of FCFE during the high growth period +PV of terminal value
=-CY $186.65+$ CY18,497/(1.14715*1.1456*1.1441*1.1426*1.1411*1.1396)
$=$ CY 4,596 million
- Value of Equity per share = Value of Equity/ Number of Shares

$$
=\text { CY 4,596/653.15 = CY } 7.04 \text { per share }
$$

- The stock was trading at 10.10 Yuan per share, which would make it overvalued, based upon this valuation.


## DaimlerChrysler: Rationale for Model June 2000

- DaimlerChrysler is a mature firm in a mature industry. We will therefore assume that the firm is in stable growth.
- Since this is a relatively new organization, with two different cultures on the use of debt (Daimler has traditionally been more conservative and bankoriented in its use of debt than Chrysler), the debt ratio will probably change over time. Hence, we will use the FCFF model.


## Daimler Chrysler: Inputs to the Model

- In 1999, Daimler Chrysler had earnings before interest and taxes of 9,324 million DM and had an effective tax rate of $46.94 \%$.
- Based upon this operating income and the book values of debt and equity as of 1998, DaimlerChrysler had an after-tax return on capital of $7.15 \%$.
- The market value of equity is 62.3 billion DM, while the estimated market value of debt is 64.5 billion
- The bottom-up unlevered beta for automobile firms is 0.61 , and Daimler is AAA rated.
- The long term German bond rate is $4.87 \%$ (in DM ) and the mature market premium of $4 \%$ is used.
- We will assume that the firm will maintain a long term growth rate of $3 \%$.


## Daimler/Chrysler: Analyzing the Inputs

- Expected Reinvestment Rate $=\mathrm{g} / \mathrm{ROC}=3 \% / 7.15 \%=41.98 \%$
- Cost of Capital
- Bottom-up Levered Beta $=0.61(1+(1-.4694)(64.5 / 62.3))=0.945$
- Cost of Equity $=4.87 \%+0.945(4 \%)=8.65 \%$
- After-tax Cost of Debt $=(4.87 \%+0.20 \%)(1-.4694)=2.69 \%$
- Cost of Capital $=8.65 \%(62.3 /(62.3+64.5))+2.69 \%(64.5 /(62.3+64.5))=5.62 \%$


## Daimler Chrysler Valuation

- Estimating FCFF

Expected EBIT $(1-\mathrm{t})=9324(1.03)(1-.4694)=5,096 \mathrm{mil} \mathrm{DM}$
Expected Reinvestment needs $=5,096(.42)=2,139 \mathrm{mil}$ DM
Expected FCFF next year = 2,957 mil DM

- Valuation of Firm

Value of operating assets $=2957 /(.056-.03)=112,847 \mathrm{mil} \mathrm{DM}$

+ Cash + Marketable Securities $=$
Value of Firm =
$130,915 \mathrm{mil}$ DM
- Debt Outstanding =

Value of Equity =

64,488 mil DM
66,427 mil DM

Value per Share $=72.7$ DM per share
Stock was trading at 62.2 DM per share on June 1, 2000

## Circular Reasoning in FCFF Valuation

- In discounting FCFF, we use the cost of capital, which is calculated using the market values of equity and debt. We then use the present value of the FCFF as our value for the firm and derive an estimated value for equity. Is there circular reasoning here?
- Yes
- No
- If there is, can you think of a way around this problem?


## Tube Investment: Rationale for Using 2-Stage FCFF Model June 2000

- Tube Investments is a diversified manufacturing firm in India. While its growth rate has been anemic, there is potential for high growth over the next 5 years.
- The firm's financing policy is also in a state of flux as the family running the firm reassesses its policy of funding the firm.



## Stable Growth Rate and Value

- In estimating terminal value for Tube Investments, I used a stable growth rate of $5 \%$. If I used a $7 \%$ stable growth rate instead, what would my terminal value be? (Assume that the cost of capital and return on capital remain unchanged.)


## The Effects of Return Improvements on Value

- The firm is considering changes in the way in which it invests, which management believes will increase the return on capital to $12.20 \%$ on just new investments (and not on existing investments) over the next 5 years.
- The value of the firm will be higher, because of higher expected growth.



## Return Improvements on Existing Assets

- If Tube Investments is also able to increase the return on capital on existing assets to $12.20 \%$ from $9.20 \%$, its value will increase even more.
- The expected growth rate over the next 5 years will then have a second component arising from improving returns on existing assets:
- Expected Growth Rate $=.122^{*} .60+\left\{(1+(.122-.092) / .092)^{1 / 5}-1\right\}$

$$
=.1313 \text { or } 13.13 \%
$$



# Tube Investments and Tsingtao: Should there be a corporate governance discount? 

- Stockholders in Asian, Latin American and many European companies have little or no power over the managers of the firm. In many cases, insiders own voting shares and control the firm and the potential for conflict of interests is huge. Would you discount the value that you estimated to allow for this absence of stockholder power?
- Yes
- No.


## Embraer: An Emerging Market Company? A Valuation in

 October 2003- We will use a 2-stage FCFF model to value Embraer to allow for maximum flexibility.

|  | High Growth |  |
| :--- | :--- | :--- |
| Beta | 1.07 | 1.00 |
| Lambda | 0.27 | 0.27 |
| Counry risk premium | $7.67 \%$ | $5.00 \%$ |
| Debt Ratio | $15.93 \%$ | $15.93 \%$ |
| Return on Capital | $21.85 \%$ | $8.76 \%$ |
| Cost of Capital | $9.81 \%$ | $8.76 \%$ |
| Expected Growth Rate | $5.48 \%$ | $4.17 \%$ |
| Reinvestment Rate | $25.04 \%$ | $4.17 \% / 8.76 \%=47.62 \%$ |



## Embraer's Cash and Cross Holdings

- Embraer has a $60 \%$ interest in an equipment company and the financial statements of that company are consolidated with those of Embraer. The minority interests (representing the equity in the subsidiary that does not belong to Embraer) are shown on the balance sheet at 23 million BR.
- Estimated market value of minority interests $=$ Book value of minority interest $* \mathrm{P} / \mathrm{BV}$ of sector that subsidiary belongs to $=23.12 * 1.5=34.68$ million BR or $\$ 11.88$ million dollars.
Present Value of FCFF in high growth phase $=$
\$1,342.97
Present Value of Terminal Value of Firm = \$3,928.67
Value of operating assets of the firm =
+ Value of Cash, Marketable Securities = \$794.52
Value of Firm =
Market Value of outstanding debt =
- Minority Interest in consolidated holdings $=34.68 / 2.92=$

Market Value of Equity =
\$5,349.42

- Value of Equity in Options = \$27.98
Value of Equity in Common Stock $=$ \$5,321.44
Market Value of Equity/share =
Market Value of Equity/share in BR =


## Dealing with Distress

- A DCF valuation values a firm as a going concern. If there is a significant likelihood of the firm failing before it reaches stable growth and if the assets will then be sold for a value less than the present value of the expected cashflows (a distress sale value), DCF valuations will understate the value of the firm.
- Value of Equity= DCF value of equity (1-Probability of distress) + Distress sale value of equity (Probability of distress)
- There are three ways in which we can estimate the probability of distress:
- Use the bond rating to estimate the cumulative probability of distress over 10 years
- Estimate the probability of distress with a probit
- Estimate the probability of distress by looking at market value of bonds..
- The distress sale value of equity is usually best estimated as a percent of book value (and this value will be lower if the economy is doing badly and there are other firms in the same business also in distress).



## Valuing Global Crossing with Distress

- Probability of distress
- Price of 8 . $653=\sum_{t=1}^{t=8} \frac{120\left(1-\pi_{\text {Distress }}\right)^{t}}{(1.05)^{\mathrm{t}}}+\frac{1000\left(1-\pi_{\text {Distress }}\right)^{8}}{(1.05)^{8}}$
- Probability of distress $=13.53 \%$ a year
- Cumulative probability of survival over 10 years $=(1-.1353)^{10}=23.37 \%$
- Distress sale value of equity
- Book value of capital $=\$ 14,531$ million
- Distress sale value $=15 \%$ of book value $=.15 * 14531=\$ 2,180$ million
- Book value of debt $=\$ 7,647$ million
- Distress sale value of equity $=\$ 0$
- Distress adjusted value of equity
- Value of Global Crossing $=\$ 3.22(.2337)+\$ 0.00(.7663)=\$ 0.75$


## More than one way to skin a cat...

- In the conventional approach to firm valuation, we discount the cash flows back at a risk adjusted discount rate to arrive at value.
- There are frequent claims from both academics and practitioners of better ways of doing discounted cash flow valuation. In particular, there are two alternatives offered to the classic discounted cash flow model
- The adjusted present value model, where we value the firm as if it were all equity funded and then add on the effects of debt (good and bad) to this value
- The excess return model, where we compute the present value of expected excess returns that the firm will earn and add it to the capital invested in the firm



## Adjusted Present Value Model

- In the adjusted present value approach, the value of the firm is written as the sum of the value of the firm without debt (the unlevered firm) and the effect of debt on firm value
- Firm Value $=$ Unlevered Firm Value $+($ Tax Benefits of Debt - Expected Bankruptcy Cost from the Debt)
- The unlevered firm value can be estimated by discounting the free cashflows to the firm at the unlevered cost of equity
- The tax benefit of debt reflects the present value of the expected tax benefits. In its simplest form,

$$
\text { Tax Benefit }=\text { Tax rate } * \text { Debt }
$$

- The expected bankruptcy cost is a function of the probability of bankruptcy and the cost of bankruptcy (direct as well as indirect) as a percent of firm value.


## An APV Valuation of Titan Cement Step 1: Unlevered firm value

- In the conventional approach, we valued Titan using the levered beta for the company of 0.93 and the debt to capital ratio of $17.6 \%$ to estimate a cost of capital for discounting the free cash flows to the firm.
- the APV approach, we use the unlevered beta of 0.80 to estimate the unlevered cost of equity, For the first 5 years, with a riskfree rate of $3.41 \%$ and a risk premium of $4.46 \%$, this yields a cost of equity of $6.98 \%$.
- Unlevered cost of equity $=3.41 \%+0.80(4.46 \%)=6.98 \%$

■ Beyond year 5, we will use an unlevered beta of 0.875 to correspond with the levered beta of 1 used in illustration 6.2. With the market risk premium reduced to $4 \%$, this yields a cost of equity of $6.91 \%$.

- The levered beta used in illustration 6.2 was 1 , the debt to equity ratio assumed for the stable growth period was $21.36 \%$ and the tax rate was $33 \%$.Unlevered beta $=$ $1.00 /(1+(1-.33)(.2136))=0.875$
- Unlevered stable period cost of equity $=3.41 \%+0.875(4 \%)=6.91 \%$


## The Unlevered Firm Value

| Year | Current |  | 1 |  | 2 |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |

## The Tax Benefits of Debt

- The tax benefits from debt are computed based upon Titan's existing dollar debt of 414 million Euros and a tax rate of $25.47 \%$ :
- Expected tax benefits in perpetuity $=$ Tax rate $(\mathrm{Debt})=0.2547$ ( 414 million) $=$ 105.45 million Euros
- This captures the tax benefit on the dollar debt outstanding today and does not factor in future debt issues (or increases in the debt ratio) and the tax benefits that will accrue from that additional debt.


## The Expected Bankruptcy Costs

- To estimate this, we made two assumptions.
- First, based upon its existing synthetic rating of AA, the probability of default at the existing debt level is very small ( $0.28 \%$ ).
- Second, we estimate the cost of bankruptcy is $30 \%$ of unlevered firm value.
- Expected bankruptcy cost =Probability of bankruptcy * Cost of bankruptcy * (Unlevered firm value + Tax benefits from debt) $=0.0028 * 0.30 *(2,759+105)$ $=2.41$ million Euros


## The APV Value of Titan Cements

- The value of the operating assets can now be computed

Value of the operating assets
$=$ Unlevered firm value + PV of tax benefits - Expected Bankruptcy Costs
$=2,759+105.45-2.41=2,862$ million Euros

- In contrast, we valued the operating assets at 2,897 million Euros with the cost of capital approach. The difference between the two approaches can be attributed to the tax benefits built into each one.
- The APV model considers the tax benefits only on existing debt whereas the cost of capital approach adds in the tax benefits from future debt issues.


## Excess Return Models

- You can present any discounted cashflow model in terms of excess returns, with the value being written as:
- Value $=$ Capital Invested + Present value of excess returns on current investments + Present value of excess returns on future investments
- This model can be stated in terms of firm value (EVA) or equity value.


## An EVA Valuation of Titan Cement

$\left.\begin{array}{|l|r|r|r|r|r|l|}\hline \text { Year } & & 1 & 2 & & 3 & 4\end{array}\right)$

# The Dark Side of Valuation 

Aswath Damodaran<br>http://www.stern.nyu.edu/~adamodar

## To make our estimates, we draw our information from..

- The firm's current financial statement
- How much did the firm sell?
- How much did it earn?
- The firm's financial history, usually summarized in its financial statements.
- How fast have the firm's revenues and earnings grown over time? What can we learn about cost structure and profitability from these trends?
- Susceptibility to macro-economic factors (recessions and cyclical firms)
- The industry and comparable firm data
- What happens to firms as they mature? (Margins.. Revenue growth... Reinvestment needs... Risk)
- We often substitute one type of information for another; for instance, in valuing Ford, we have 70 years+ of historical data, but not too many comparable firms; in valuing a software firm, we might not have too much historical data but we have lots of comparable firms.


## The Dark Side...

- Valuation is most difficult when a company
- Has negative earnings and low revenues in its current financial statements
- No history
- No comparables ( or even if they exist, they are all at the same stage of the life cycle as the firm being valued)



## Amazon's Bottom-up Beta

$$
\text { Unlevered beta for firms in internet retailing }=\quad 1.60
$$

Unlevered beta for firms in specialty retailing $=\quad 1.00$

- Amazon is a specialty retailer, but its risk currently seems to be determined by the fact that it is an online retailer. Hence we will use the beta of internet companies to begin the valuation but move the beta, after the first five years, towards the beta of the retailing business.


## Estimating Synthetic Ratings and cost of debt

- The rating for a firm can be estimated using the financial characteristics of the firm. In its simplest form, the rating can be estimated from the interest coverage ratio

$$
\text { Interest Coverage Ratio }=\text { EBIT } / \text { Interest Expenses }
$$

- Amazon.com has negative operating income; this yields a negative interest coverage ratio, which should suggest a low rating. We computed an average interest coverage ratio of 2.82 over the next 5 years. This yields an average rating of BBB for Amazon.com for the first 5 years. (In effect, the rating will be lower in the earlier years and higher in the later years than BBB)


## Estimating the cost of debt

- The synthetic rating for Amazon.com is BBB. The default spread for BBB rated bonds is $1.50 \%$
- Pre-tax cost of debt $=$ Riskfree Rate + Default spread

$$
=6.50 \%+1.50 \%=8.00 \%
$$

- After-tax cost of debt right now $=8.00 \%(1-0)=8.00 \%$ : The firm is paying no taxes currently. As the firm's tax rate changes and its cost of debt changes, the after tax cost of debt will change as well.

|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Pre-tax | $8.00 \%$ | $8.00 \%$ | $8.00 \%$ | $8.00 \%$ | $8.00 \%$ | $7.80 \%$ | $7.75 \%$ | $7.67 \%$ | $7.50 \%$ | $7.00 \%$ |
| Tax rate | $0 \%$ | $0 \%$ | $0 \%$ | $16.1 \%$ | $35 \%$ | $35 \%$ | $35 \%$ | $35 \%$ | $35 \%$ | $35 \%$ |
| After-tax | $8.00 \%$ | $8.00 \%$ | $8.00 \%$ | $6.71 \%$ | $5.20 \%$ | $5.07 \%$ | $5.04 \%$ | $4.98 \%$ | $4.88 \%$ | $4.55 \%$ |

## Estimating Cost of Capital: Amazon.com

- Equity
- Cost of Equity $=6.50 \%+1.60(4.00 \%)=12.90 \%$
- Market Value of Equity = \$ 84/share* 340.79 mil shs $=\$ 28,626 \mathrm{mil}(98.8 \%)$
- Debt
- Cost of debt $=6.50 \%+1.50 \%($ default spread $)=8.00 \%$
- Market Value of Debt = \$ 349 mil ( $1.2 \%$ )
- Cost of Capital

Cost of Capital $=12.9 \%(.988)+8.00 \%(1-0)(.012))=12.84 \%$

- Amazon.com has a book value of equity of \$ 138 million and a book value of debt of $\$ 349$ million. Shows you how irrelevant book value is in this process.


## Calendar Years, Financial Years and Updated Information

- The operating income and revenue that we use in valuation should be updated numbers. One of the problems with using financial statements is that they are dated.
- As a general rule, it is better to use 12 -month trailing estimates for earnings and revenues than numbers for the most recent financial year. This rule becomes even more critical when valuing companies that are evolving and growing rapidly.

Revenues
EBIT

Last 10-K
\$ 610 million

- \$125 million

Trailing 12-month
\$1,117 million

- \$ 410 million


## Are S, G \& A expenses capital expenditures?

- Many internet companies are arguing that selling and G\&A expenses are the equivalent of R\&D expenses for a high-technology firms and should be treated as capital expenditures.
- If we adopt this rationale, we should be computing earnings before these expenses, which will make many of these firms profitable. It will also mean that they are reinvesting far more than we think they are. It will, however, make not their cash flows less negative.
- Should Amazon.com's selling expenses be treated as cap ex?


## Amazon.com's Tax Rate

| Year | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| EBIT | $-\$ 373$ | $-\$ 94$ | $\$ 407$ | $\$ 1,038$ | $\$ 1,628$ |
| Taxes | $\$ 0$ | $\$ 0$ | $\$ 0$ | $\$ 167$ | $\$ 570$ |
| EBIT $(1-\mathrm{t})$ | $-\$ 373$ | $-\$ 94$ | $\$ 407$ | $\$ 871$ | $\$ 1,058$ |
| Tax rate | $0 \%$ | $0 \%$ | $0 \%$ | $16.13 \%$ | $35 \%$ |
| NOL | $\$ 500$ | $\$ 873$ | $\$ 967$ | $\$ 560$ | $\$ 0$ |

After year 5, the tax rate becomes $35 \%$.

## Estimating FCFF: Amazon.com

- EBIT (Trailing 1999) $=-\$ 410$ million
- Tax rate used $=0 \%$ (Assumed Effective $=$ Marginal)
- Capital spending (Trailing 1999) = \$ 243 million (includes acquisitions)
- Depreciation (Trailing 1999) = \$ 31 million
- Non-cash Working capital Change (1999) $=-80$ million
- Estimating FCFF (1999)

Current EBIT * $(1-$ tax rate $)=-410(1-0) \quad=-\$ 410$ million

- (Capital Spending - Depreciation) $=\$ 212$ million
- Change in Working Capital $=-\$ 80$ million

Current FCFF $=-\$ 542$ million

## Growth in Revenues, Earnings and Reinvestment: Amazon

|  | Year | Revenue Growth Investment | Chg in <br> Revenue | New | Sales/Capital | ROC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | $\begin{gathered} 150.00 \% \\ 3.00 \end{gathered}$ | $\begin{aligned} & \$ 1,676 \\ & -76.62 \% \end{aligned}$ | \$559 |  |  |
|  | 2 | $\begin{gathered} 100.00 \% \\ 3.00 \end{gathered}$ | $\begin{aligned} & \$ 2,793 \\ & -8.96 \% \end{aligned}$ | \$931 |  |  |
|  | 3 | $\begin{aligned} & 75.00 \% \\ & 20.59 \% \end{aligned}$ | \$4,189 | \$1,396 | 3.00 |  |
|  | 45 | $\begin{aligned} & 50.00 \% \\ & 25.82 \% \end{aligned}$ | \$4,887 | \$1,629 | 3.00 |  |
|  | 5 | $\begin{aligned} & 30.00 \% \\ & 21.16 \% \end{aligned}$ | \$4,398 | \$1,466 | 3.00 |  |
|  | 6 | $\begin{aligned} & 25.20 \% \\ & 22.23 \% \end{aligned}$ | \$4,803 | \$1,601 | 3.00 |  |
|  | 7 | $\begin{aligned} & 20.40 \% \\ & 22.30 \% \end{aligned}$ | \$4,868 | \$1,623 | 3.00 |  |
|  | 8 | $15.60 \%$ | \$4,482 | \$1,494 | 3.00 |  |
| Aswath Damodaran |  | 21.87\% |  |  |  |  |

## Amazon.com: Stable Growth Inputs

- Beta
- Debt Ratio
- Return on Capital
- Expected Growth Rate
- Reinvestment Rate

|  | High Growth | Stable Growth |
| :--- | :--- | :--- |
| 1.60 | 1.00 |  |
| $1.20 \%$ | $15 \%$ |  |
| Negative | $20 \%$ |  |
| NMF | $6 \%$ |  |
| $>100 \%$ | $6 \% / 20 \%=30 \%$ |  |

## Estimating the Value of Equity Options

- Details of options outstanding
- Average strike price of options outstanding $=\$ 13.375$
- Average maturity of options outstanding $=8.4$ years
- Standard deviation in $\ln ($ stock price $)=50.00 \%$
- Annualized dividend yield on stock $=\quad 0.00 \%$
- Treasury bond rate =
6.50\%
- Number of options outstanding =

38 million

- Number of shares outstanding =
340.79 million

■ Value of options outstanding (using dilution-adjusted Black-Scholes model)

- Value of equity options $=\$ 2,892$ million


What do you need to break-even at $\$ 84$ ?

|  | $6 \%$ |  | $8 \%$ |  | $10 \%$ |  | $12 \%$ |  | $14 \%$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| $30 \%$ | $\$$ | $(1.94)$ | $\$$ | 2.95 | $\$$ | 7.84 | $\$$ | 12.71 | $\$$ |
| $35 \%$ | $\$$ | 1.41 | $\$$ | 8.37 | $\$$ | 15.33 | $\$$ | 22.27 | $\$$ |
| $40 \%$ | $\$$ | 6.10 | $\$$ | 15.93 | $\$$ | 25.74 | $\$$ | 35.54 | $\$$ |
| $45 \%$ | $\$$ | 12.59 | $\$$ | 26.34 | $\$$ | 40.05 | $\$$ | 53.77 | $\$$ |
| $50 \%$ | $\$$ | 21.47 | $\$$ | 40.50 | $\$$ | 59.52 | $\$$ | 78.53 | $\$$ |
| $55 \%$ | $\$$ | 33.47 | $\$$ | 59.60 | $\$$ | 85.72 | $\$$ | 111.84 | $\$$ |
| $60 \%$ | $\$$ | 49.53 | $\$$ | 85.10 | $\$$ | 120.66 | $\$$ | 156.22 | $\$$ |




## Amazon over time...



Aswath
Damodaran

