Lecture 4: Techniques for Integration

II. part II: Integral of the $\tan x, \sec x$

Recall

$$(\tan x)' = \sec^2 x \qquad 1 + \tan^2 x = \sec^2 x$$

$$(\sec x)' = \sec x \tan x$$

$$\int \tan x \, dx = \ln|\sec x| + c = -\ln|\cos x| + c$$

ex. $\displaystyle\int \sec^2 x \; dx$ $\quad , \displaystyle\int \sec^2 x \tan^3 x \; dx$

$=$ $\qquad\qquad =$

ex. $\displaystyle\int \sec x \tan x \; dx$ $\quad , \displaystyle\int \sec^3 x \tan x \; dx$

$=$ $\qquad\qquad =$

$$\int \tan^2 x \, dx, \int \tan^3 x \sec x \, dx, \int \sec x \, dx, \int \sec^3 x \, dx$$

$=?$

$$\int \tan^m x \sec^n x dx$$

(1) n is **even**: Isolate a $\boxed{\sec^2 x}$,rewrite the rest $\sec x$ in terms of $\tan x$, if needed.

Then use $u-$sub: let $u = \tan x$.

(2) m is **odd**, $m > 1, n \neq 0$: Isolate a $\boxed{\tan x \sec x}$, set the rest $\tan x$ in terms of $\sec x$, if needed.

Then use $u-$sub: let $u = \sec x$.

(3) m is **even**: use

$$\tan^2 x = \sec^2 x - 1,$$

to convert the $\tan^m x$ in terms of $\sec^2 x$.

NOTE: Similar rules apply to $\int \cot^m x \csc^n x dx$

Evaluate:

ex. 1. $\int \tan^6 x \sec^4 x dx$ $\quad (\frac{\tan^7 x}{7} + \frac{\tan^9 x}{9} + c)$

(EVEN power of $\sec x$)

ex. 2. $\int \tan^3 x \sec x dx$ $\quad (\frac{\sec^3 x}{3} - \sec x + c)$

(ODD power of $\tan x$)

ex. 3. $\int \tan^2 x dx$ (EVEN power of $\tan x$)

$(\tan x - x + c)$

ex. 4. $\int \sec x dx$ $(\ln|\sec x + \tan x| + c)$

ex. 5. $\int \sec^3 x dx$ $(\frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + c)$

NOTE: In this class, we will learn how to evaluate the integrals of the $\sec x$, $\tan x$ and $\csc x$, $\cot x$ **without** using a table of integrals.

NYTI Integration gymnastics:

1. Similarly, you can apply the same technique here to evaluate the integrals of the cosecant and cotangent functions.

$$(\cot x)' = -\csc^2 x, \qquad 1+\cot^2 x = \csc^2 x$$

$$(\csc x)' = -\csc x \cot x$$

$$\int \cot x \; dx = \ln|\sin x| + c$$

ex. Evaluate $\displaystyle\int \csc^2 x \, dx$, $\displaystyle\int \csc^2 x \cot^3 x \, dx$

$=$ $=$

ex. Evaluate $\displaystyle\int \csc x \cot x \, dx$, $\displaystyle\int \csc^3 x \cot x \, dx$

$=$ $=$

ex. Evaluate $\displaystyle\int \csc x dx$ $(\ln|\csc x - \cot x| + c)$

ex. Evaluate $\displaystyle\int \csc^3 x dx$

2. $\int \tan^4 x dx$ $\quad (\frac{\tan^3 x}{3} - \tan x + x + c)$

3. $\int \tan^3 x dx$ $\quad$ Idea:

$(\frac{1}{2}\tan^2 x + \ln|\cos x| + c)$

4. $\int \tan^5 x dx$ $\quad \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} - ln|\cos x| + c$

5. $\int \frac{\cos^5 x}{\sqrt{\sin x}} dx$ $\quad 2\sqrt{\sin x} - \frac{4\sin^{5/2} x}{5} + \frac{2\sin^{9/2} x}{9} + c$

6. $\int x \sec x \tan x \ dx$ $\quad x\sec x - ln|\sec x + \tan x| + c$

7. $\int \frac{\cos x + \sin x}{\sin 2x} \ dx$

8. $$\int \frac{1-\tan^2 x}{\sec^2 x}\ dx$$

9. $$\int x\cos^2 x\ dx$$

10. $$\int \cos x \cos^5(\sin x)\ dx$$

11. Show $$\int \frac{\tan^3 x}{\cos^4 x}dx = \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + c$$

12. $$\int \frac{dx}{\cos x - 1}$$

13. $$\int x\sec^2(x^2)\tan^4(x^2)\ dx$$