Managerial Economics & Business Strategy

Chapter 5
The Production Process and Costs

McGraw-Hill/Irwin

Overview

I. Production Analysis
- Total Product, Marginal Product, Average Product.
- Isoquants.
- Isocosts.
- Cost Minimization

II. Cost Analysis
- Total Cost, Variable Cost, Fixed Costs.
- Cubic Cost Function.
- Cost Relations.

III. Multi-Product Cost Functions

Production Analysis

- Production Function
  - Q = F(K,L)
    - Q is quantity of output produced.
    - K is capital input.
    - L is labor input.
    - F is a functional form relating the inputs to output.
  - The maximum amount of output that can be produced with K units of capital and L units of labor.
- Short-Run vs. Long-Run Decisions
- Fixed vs. Variable Inputs

Production Function Algebraic Forms

- Linear production function: inputs are perfect substitutes.

$$Q = F(K, L) = aK + bL$$

- Leontief production function: inputs are used in fixed proportions.

$$Q = F(K, L) = \min\{bK, cL\}$$

- Cobb-Douglas production function: inputs have a degree of substitutability.

$$Q = F(K, L) = K^a L^b$$

Productivity Measures: Total Product

- Total Product (TP): maximum output produced with given amounts of inputs.
- Example: Cobb-Douglas Production Function:

$$Q = F(K,L) = K^{.5} L^{.5}$$

  - K is fixed at 16 units.
  - Short run Cobb-Douglass production function:

$$Q = (16)^{.5} L^{.5} = 4 L^{.5}$$

  - Total Product when 100 units of labor are used?

$$Q = 4 (100)^{.5} = 4(10) = 40 \text{ units}$$

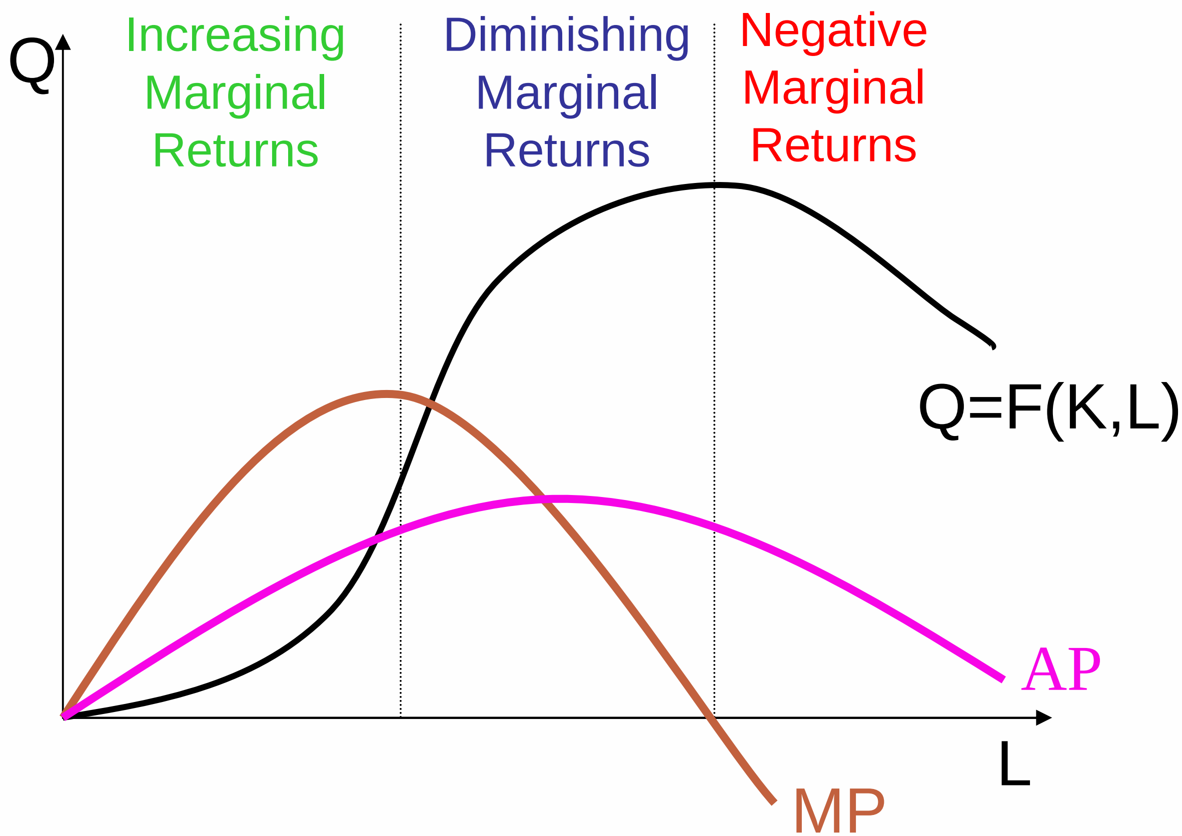
Productivity Measures: Average Product of an Input

- Average Product of an Input: measure of output produced per unit of input.
  - Average Product of Labor: $AP_L = Q/L$.
    - Measures the output of an "average" worker.
    - Example: $Q = F(K,L) = K^{.5} L^{.5}$
      - If the inputs are K = 16 and L = 16, then the average product of labor is $AP_L = [(16)^{0.5}(16)^{0.5}]/16 = 1$.
  - Average Product of Capital: $AP_K = Q/K$.
    - Measures the output of an "average" unit of capital.
    - Example: $Q = F(K,L) = K^{.5} L^{.5}$
      - If the inputs are K = 16 and L = 16, then the average product of capital is $AP_K = [(16)^{0.5}(16)^{0.5}]/16 = 1$.

Productivity Measures: Marginal Product of an Input

- Marginal Product on an Input: change in total output attributable to the last unit of an input.
  - Marginal Product of Labor: $MP_L = \Delta Q/\Delta L$
    - Measures the output produced by the last worker.
    - Slope of the short-run production function (with respect to labor).
  - Marginal Product of Capital: $MP_K = \Delta Q/\Delta K$
    - Measures the output produced by the last unit of capital.
    - When capital is allowed to vary in the short run, MP_K is the slope of the production function (with respect to capital).

Increasing, Diminishing and Negative Marginal Returns



Guiding the Production Process

- Producing on the production function
  - Aligning incentives to induce maximum worker effort.
- Employing the right level of inputs
  - When labor or capital vary in the short run, to maximize profit a manager will hire:
    - labor until the value of marginal product of labor equals the wage: $VMP_L = w$, where $VMP_L = P \times MP_L$.
    - capital until the value of marginal product of capital equals the rental rate: $VMP_K = r$, where $VMP_K = P \times MP_K$.

Isoquant

- Illustrates the long-run combinations of inputs (K, L) that yield the producer the same level of output.
- The shape of an isoquant reflects the ease with which a producer can substitute among inputs while maintaining the same level of output.

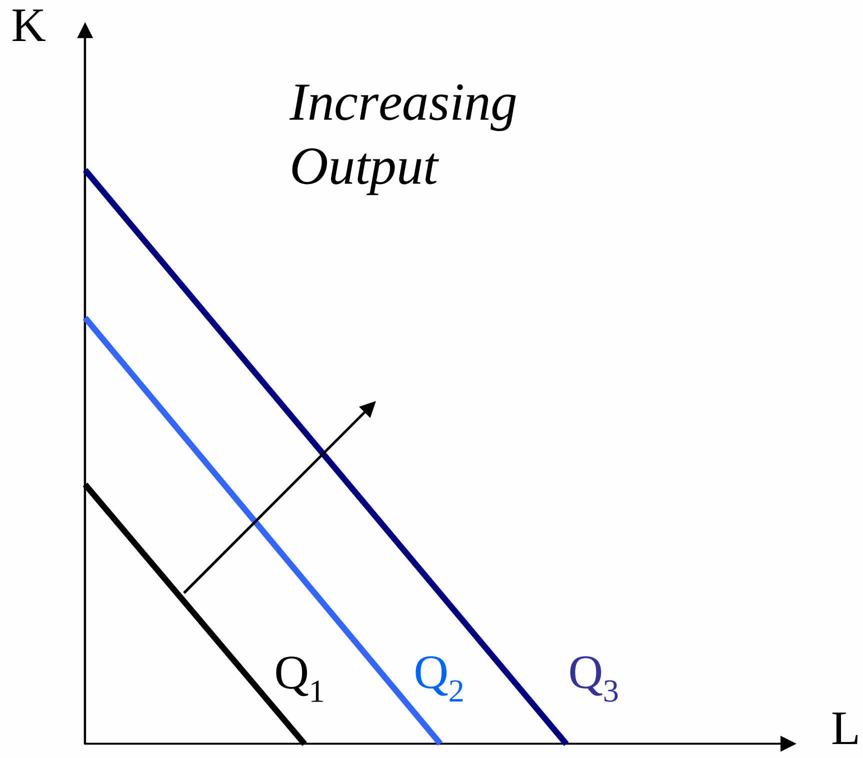
Marginal Rate of Technical Substitution (MRTS)

- The rate at which two inputs are substituted while maintaining the same output level.

$$MRTS_{KL} = \frac{MP_L}{MP_K}$$

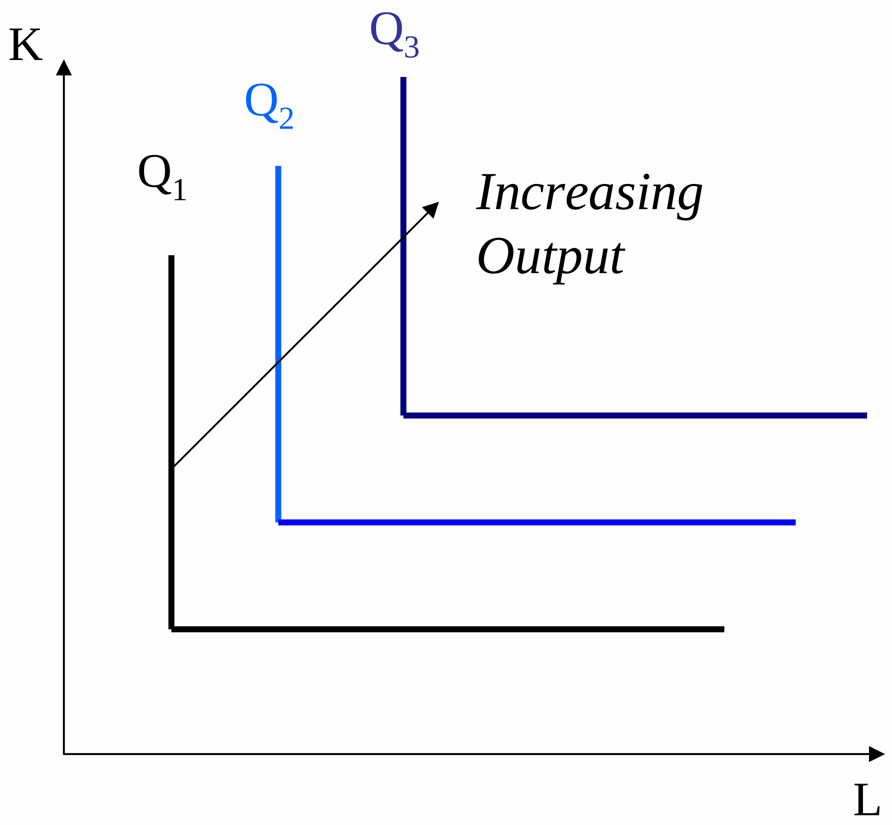
Linear Isoquants

- Capital and labor are perfect substitutes
  - $Q = aK + bL$
  - $MRTS_{KL} = b/a$
  - Linear isoquants imply that inputs are substituted at a constant rate, independent of the input levels employed.



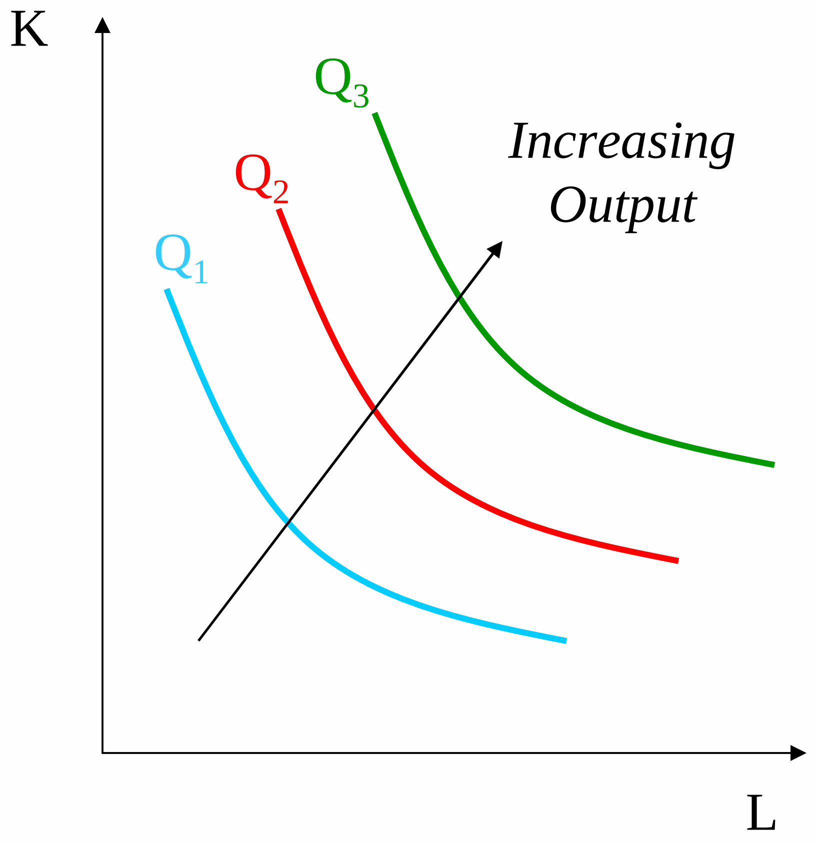
Leontief Isoquants

- Capital and labor are perfect complements.
- Capital and labor are used in fixed-proportions.
- Q = min {bK, cL}
- Since capital and labor are consumed in fixed proportions there is no input substitution along isoquants (hence, no $MRTS_{KL}$).



Cobb-Douglas Isoquants

- Inputs are not perfectly substitutable.
- Diminishing marginal rate of technical substitution.
  - As less of one input is used in the production process, increasingly more of the other input must be employed to produce the same output level.
- $Q = K^a L^b$
- $MRTS_{KL} = MP_L/MP_K$



Isocost

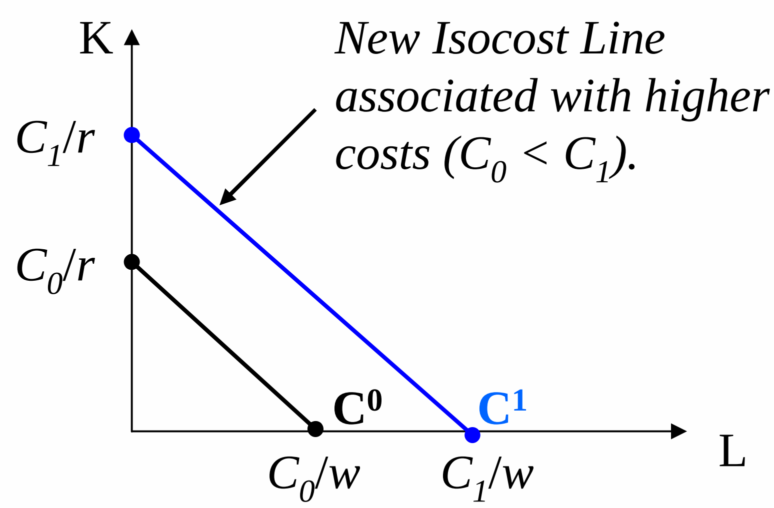
- The combinations of inputs that produce a given level of output at the same cost:

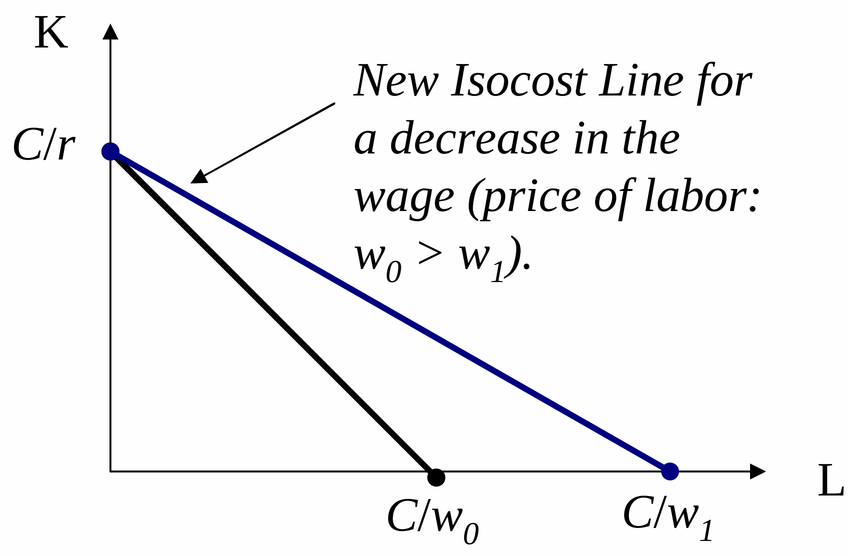
$$wL + rK = C$$

- Rearranging,

$$K = (1/r)C - (w/r)L$$

- For given input prices, isocosts farther from the origin are associated with higher costs.
- Changes in input prices change the slope of the isocost line.





Cost Minimization

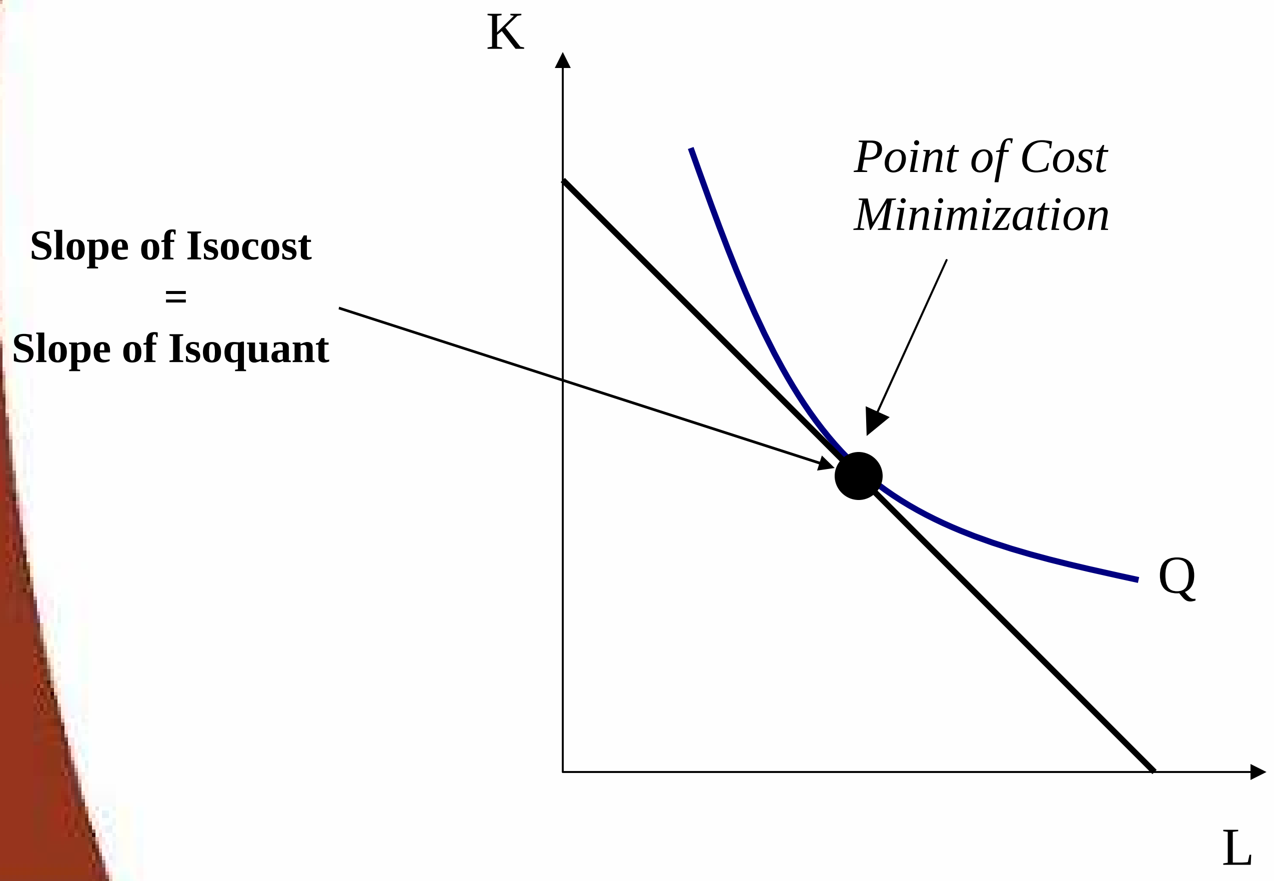
- Marginal product per dollar spent should be equal for all inputs:

$$\frac{MP_L}{w} = \frac{MP_K}{r} \Leftrightarrow \frac{MP_L}{MP_K} = \frac{w}{r}$$

- But, this is just

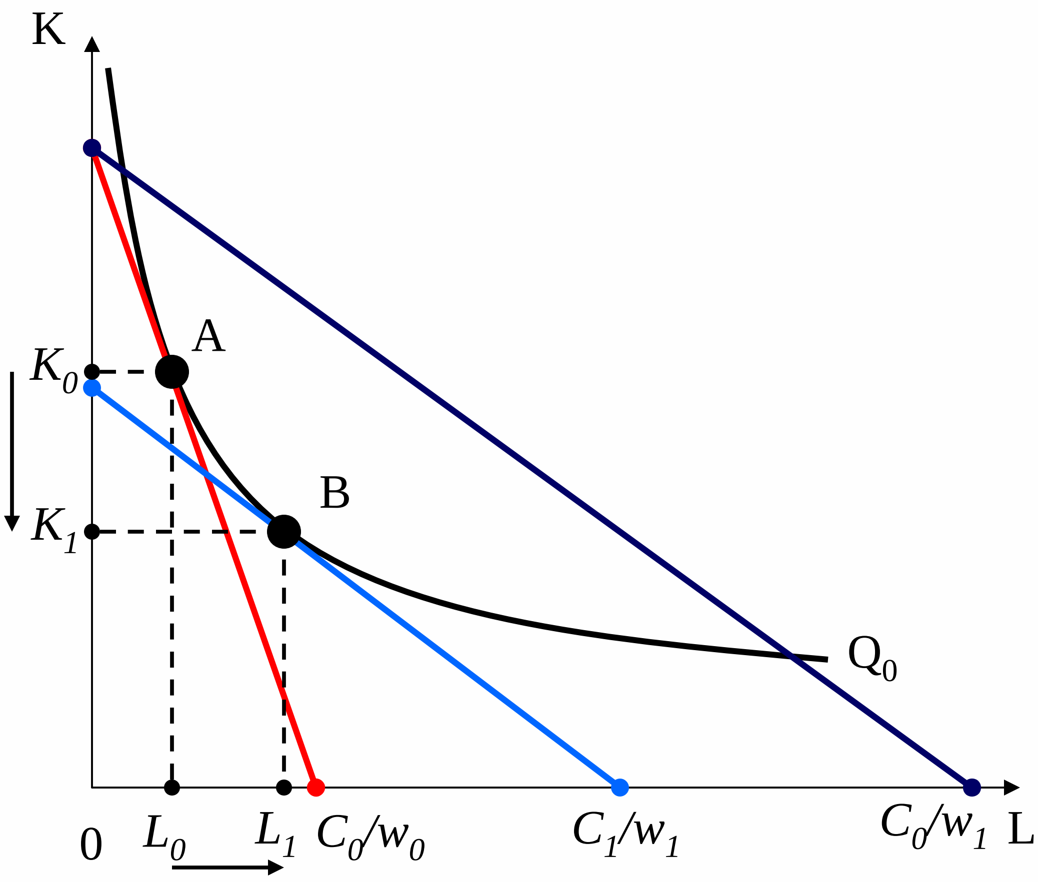
$$MRTS_{KL} = \frac{w}{r}$$

Cost Minimization



Optimal Input Substitution

- A firm initially produces Q_0 by employing the combination of inputs represented by point A at a cost of C_0.
- Suppose w_0 falls to w_1.
  - The isocost curve rotates counterclockwise; which represents the same cost level prior to the wage change.
  - To produce the same level of output, Q_0, the firm will produce on a lower isocost line (C_1) at a point B.
  - The slope of the new isocost line represents the lower wage relative to the rental rate of capital.



Cost Analysis

- Types of Costs
  - Short-Run
    - Fixed costs (FC)
    - Sunk costs
    - Short-run variable costs (VC)
    - Short-run total costs (TC)
  - Long-Run
    - All costs are variable
    - No fixed costs

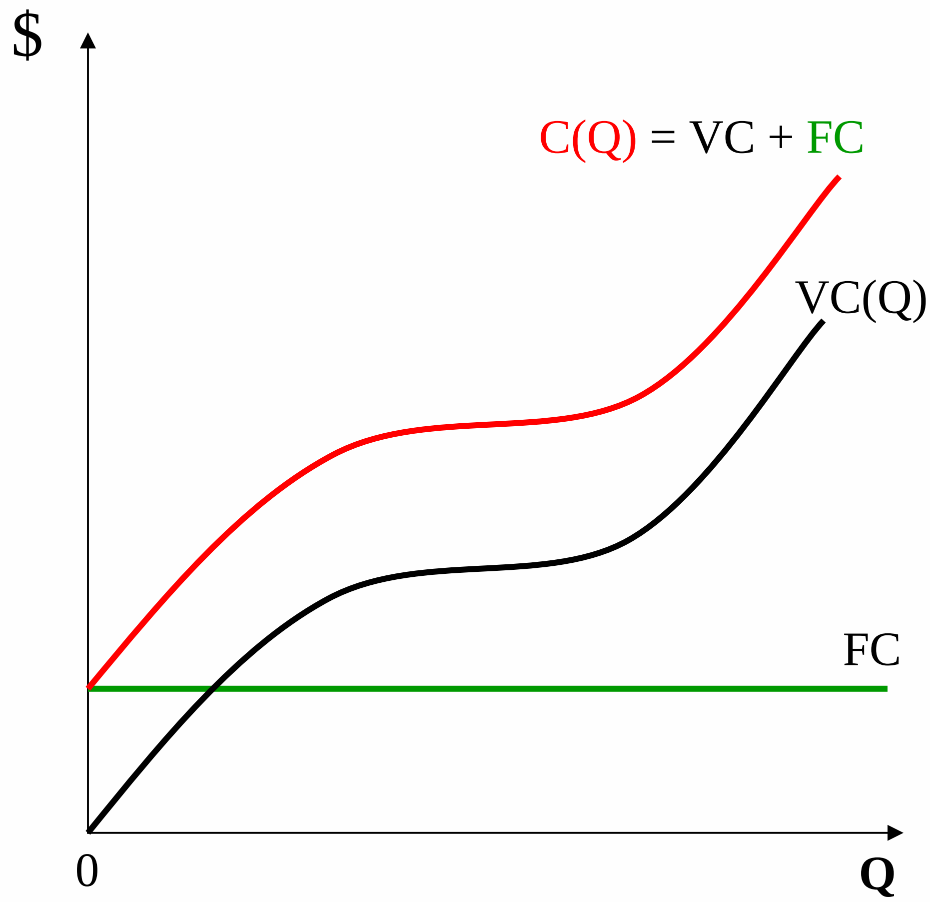
Total and Variable Costs

C(Q): Minimum total cost of producing alternative levels of output:

$$C(Q) = VC(Q) + FC$$

VC(Q): Costs that vary with output.

FC: Costs that do not vary with output.

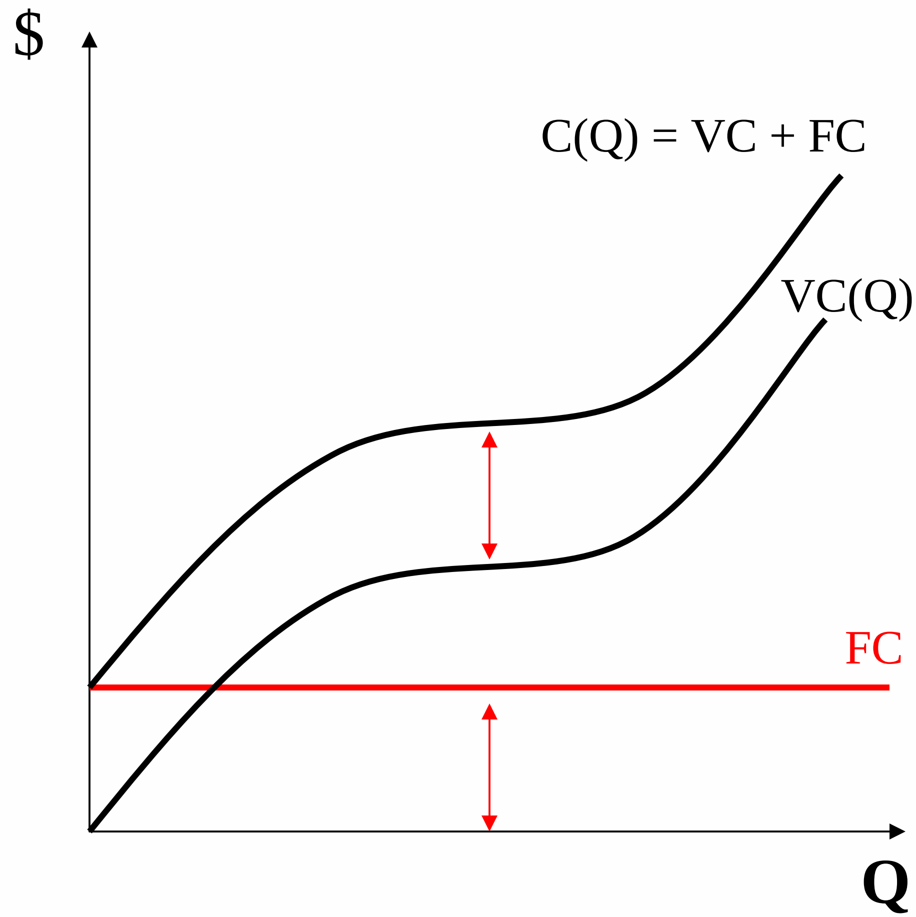


Fixed and Sunk Costs

FC: Costs that do not change as output changes.

Sunk Cost: A cost that is forever lost after it has been paid.

Decision makers should ignore sunk costs to maximize profit or minimize losses



Some Definitions

Average Total Cost

$ATC = AVC + AFC$

$ATC = C(Q)/Q$

Average Variable Cost

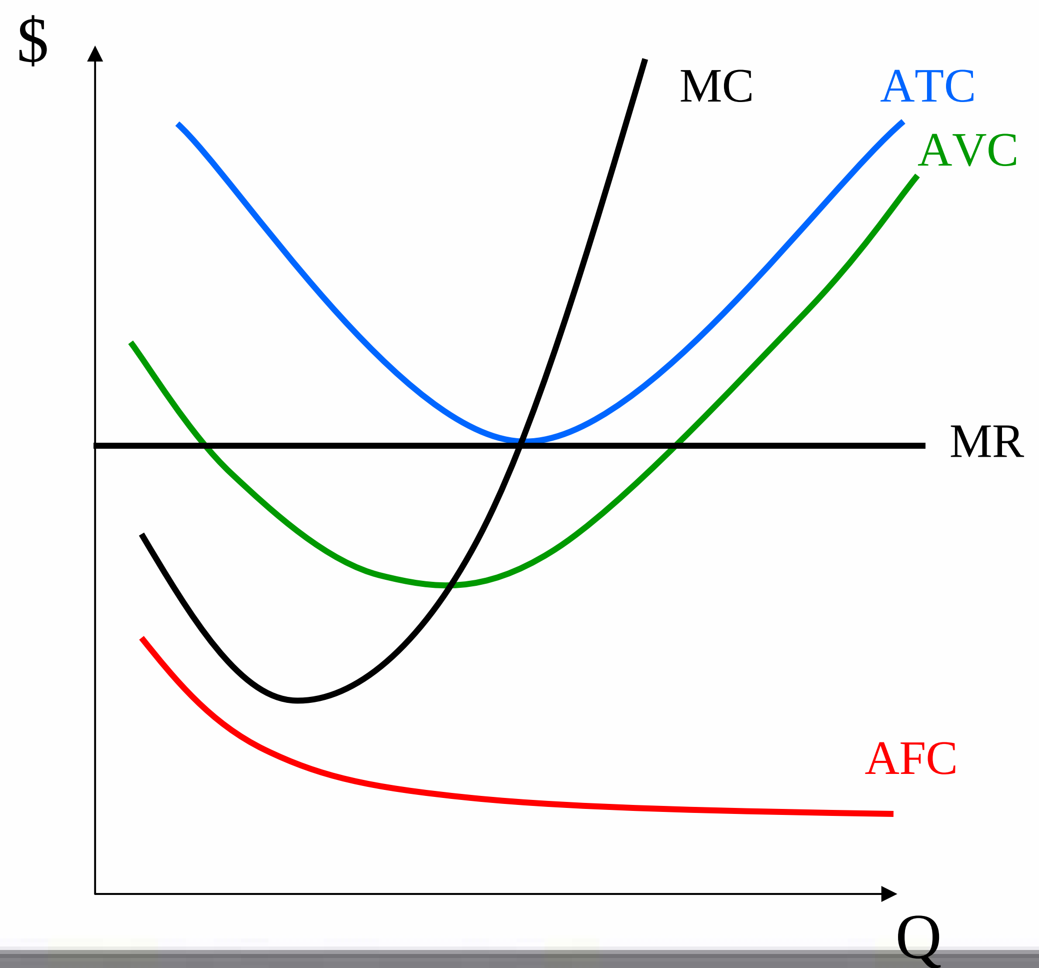
$AVC = VC(Q)/Q$

Average Fixed Cost

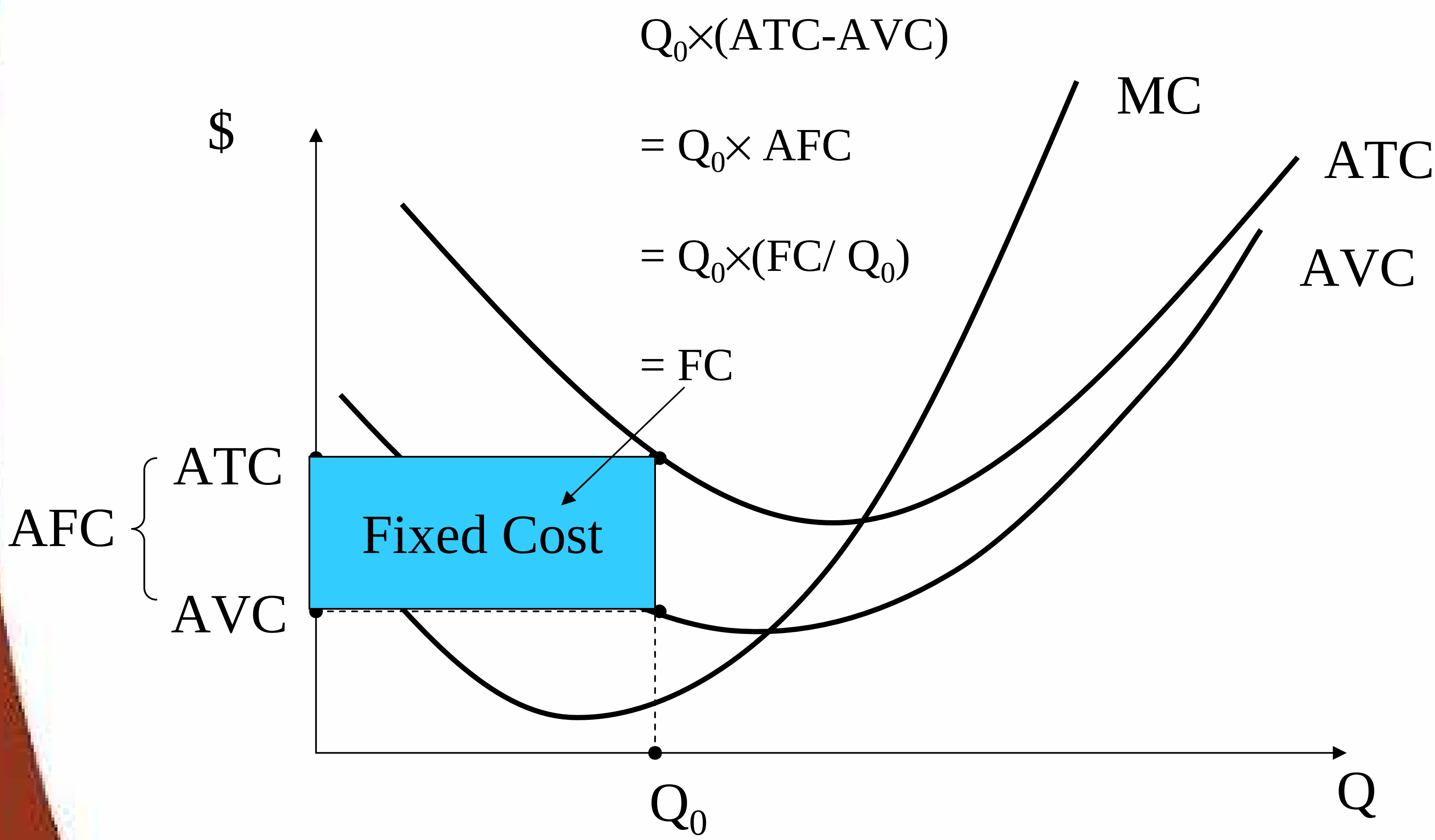
$AFC = FC/Q$

Marginal Cost

$MC = DC/DQ$

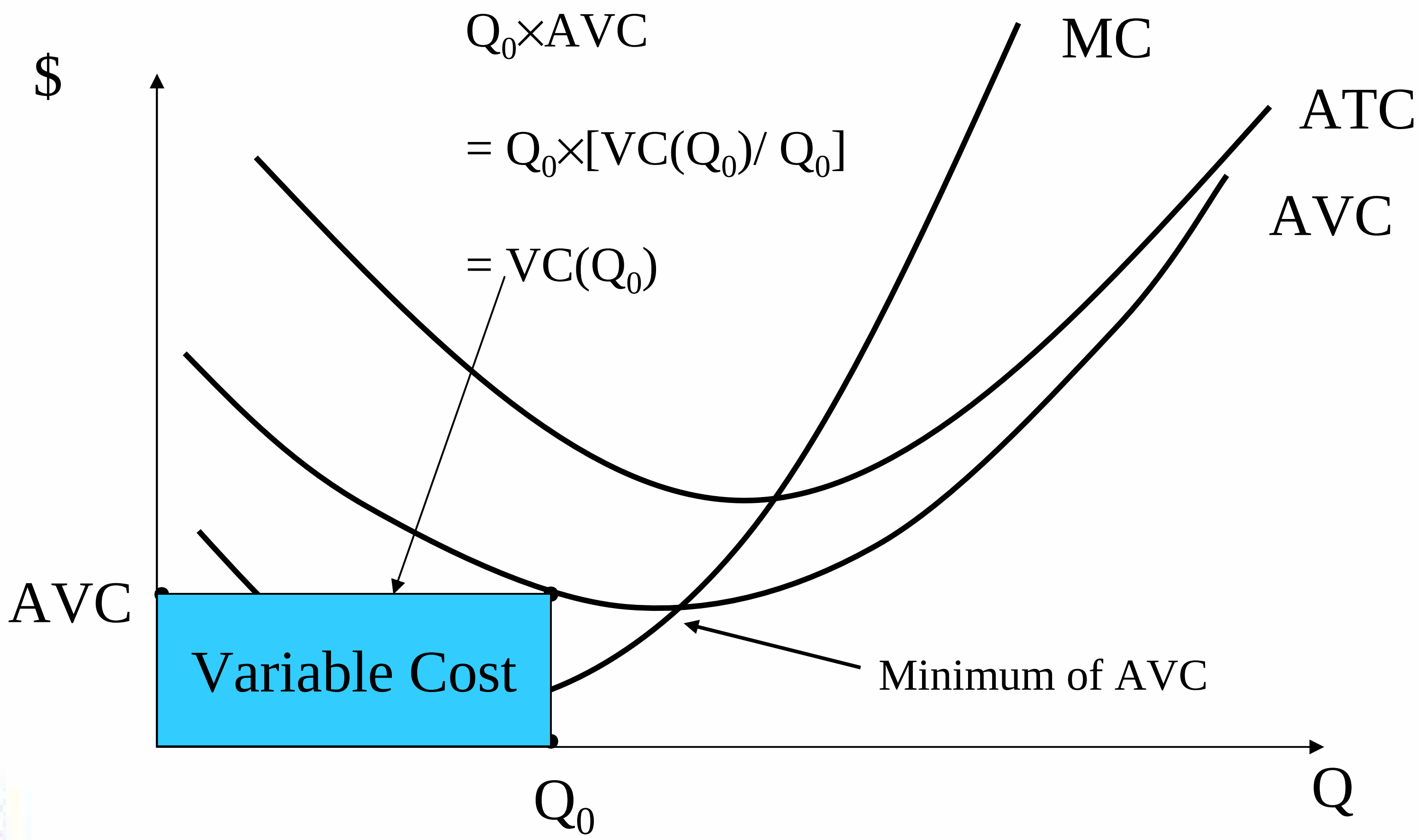


Fixed Cost

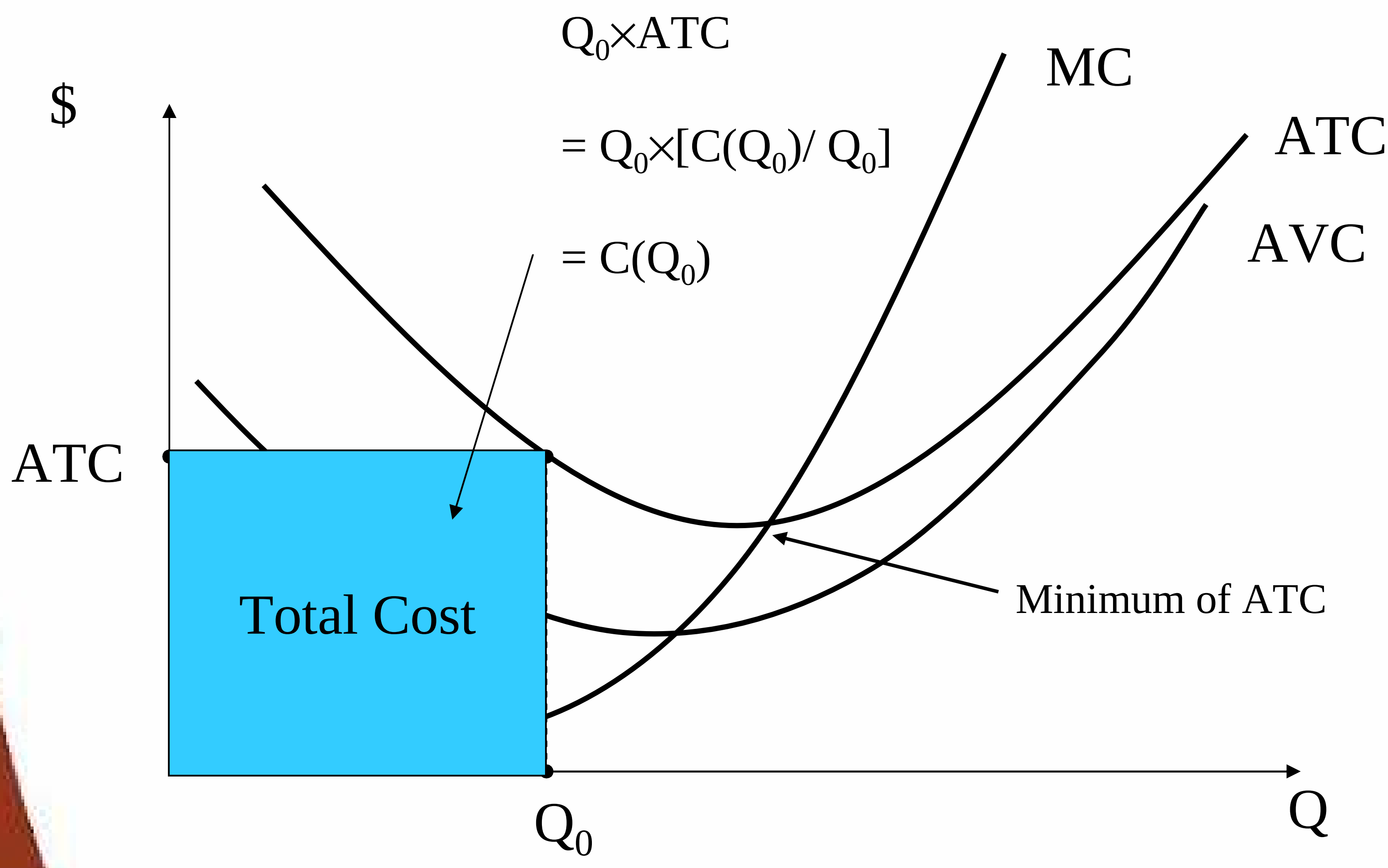


$Q_0 \times$(ATC-AVC)
$= Q_0 \times$ AFC
$= Q_0 \times (FC/ Q_0)$
= FC
MC
ATC
AVC
$
ATC
AFC
AVC
Fixed Cost
Q_0
Q

Variable Cost



Total Cost


$Q_0 \times ATC$
$= Q_0 \times [C(Q_0)/ Q_0]$
$= C(Q_0)$
MC
ATC
AVC
$
ATC
Total Cost
Minimum of ATC
Q_0
Q

Cubic Cost Function

- $C(Q) = f + a\,Q + b\,Q^2 + cQ^3$
- Marginal Cost?
  – Memorize:

$$MC(Q) = a + 2bQ + 3cQ^2$$

  – Calculus:

$$dC/dQ = a + 2bQ + 3cQ^2$$

An Example

- Total Cost: $C(Q) = 10 + Q + Q^2$
- Variable cost function:

$$VC(Q) = Q + Q^2$$

- Variable cost of producing 2 units:

$$VC(2) = 2 + (2)^2 = 6$$

- Fixed costs:

$$FC = 10$$

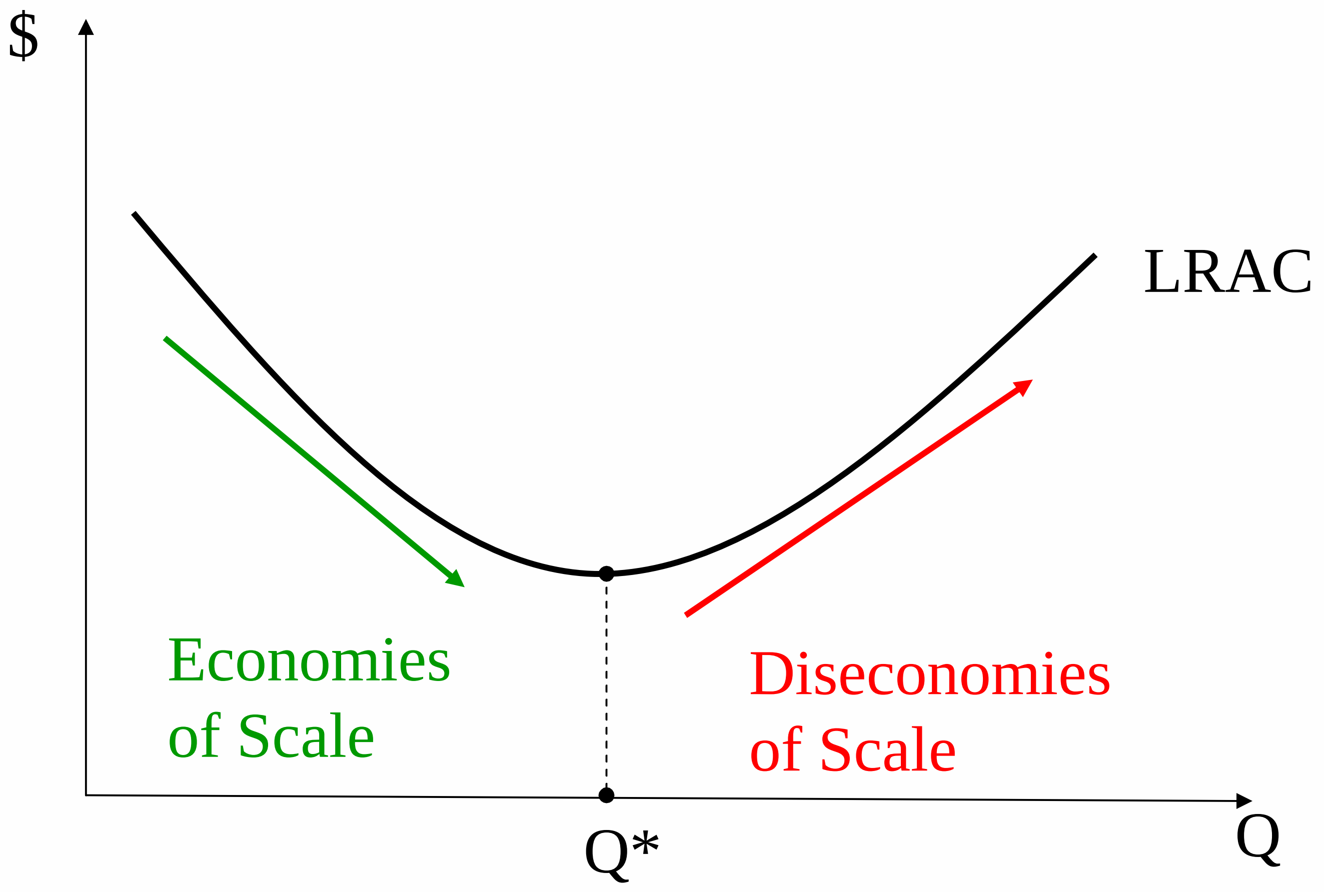
- Marginal cost function:

$$MC(Q) = 1 + 2Q$$

- Marginal cost of producing 2 units:

$$MC(2) = 1 + 2(2) = 5$$

Long-Run Average Costs



Multi-Product Cost Function

- $C(Q_1, Q_2)$: Cost of jointly producing two outputs.
- General function form:

$$C(Q_1, Q_2) = f + aQ_1Q_2 + bQ_1^2 + cQ_2^2$$

Economies of Scope

- $C(Q_1, 0) + C(0, Q_2) > C(Q_1, Q_2)$.
  - It is cheaper to produce the two outputs jointly instead of separately.
- Example:
  - It is cheaper for Time-Warner to produce Internet connections and Instant Messaging services jointly than separately.

Cost Complementarity

- The marginal cost of producing good 1 declines as more of good two is produced:

$$\Delta MC_1(Q_1, Q_2) / \Delta Q_2 < 0.$$

- Example:
  – Cow hides and steaks.

Quadratic Multi-Product Cost Function

- $C(Q_1, Q_2) = f + aQ_1Q_2 + (Q_1)^2 + (Q_2)^2$
- $MC_1(Q_1, Q_2) = aQ_2 + 2Q_1$
- $MC_2(Q_1, Q_2) = aQ_1 + 2Q_2$
- Cost complementarity: $a < 0$
- Economies of scope: $f > aQ_1Q_2$

$C(Q_1, 0) + C(0, Q_2) = f + (Q_1)^2 + f + (Q_2)^2$

$C(Q_1, Q_2) = f + aQ_1Q_2 + (Q_1)^2 + (Q_2)^2$

$f > aQ_1Q_2$: Joint production is cheaper

A Numerical Example:

- $C(Q_1, Q_2) = 90 - 2Q_1Q_2 + (Q_1)^2 + (Q_2)^2$
- Cost Complementarity?

  Yes, since $a = -2 < 0$

  $MC_1(Q_1, Q_2) = -2Q_2 + 2Q_1$
- Economies of Scope?

  Yes, since $90 > -2Q_1Q_2$

Conclusion

- To maximize profits (minimize costs) managers must use inputs such that the value of marginal of each input reflects price the firm must pay to employ the input.
- The optimal mix of inputs is achieved when the $MRTS_{KL} = (w/r)$.
- Cost functions are the foundation for helping to determine profit-maximizing behavior in future chapters.