Basic properties of the logarithm and exponential functions

- When I write "log(x)", I mean the natural logarithm (you may be used to seeing "ln(x)").

  If I specifically want the logarithm to the base 10, I'll write $\log_{10}$.
- If $0 < X < \infty$, then $-\infty < \log(X) < \infty$. You can't take the log of a negative number.
- If $-\infty < X < \infty$, then $0 < \exp(X) < \infty$. The exponential of any number is positive.
- $\log(XY) = \log(X) + \log(Y)$
- $\log(X/Y) = \log(X) - \log(Y)$
- $\log(X^b) = b*\log(X)$
- $\log(1) = 0$
- $\exp(X+Y) = \exp(X)*\exp(Y)$
- $\exp(X-Y) = \exp(X)/\exp(Y)$
- $\exp(-X) = 1/\exp(X)$
- $\exp(0) = 1$
- $\log(\exp(X)) = \exp(\log(X)) = X$

Problems:

1. Simplify the following expressions

   a) exp(4)/exp(2)

   b) log(3X) - log(X)

c) exp(X+Y)/exp(X)

d) exp(X + 3*Y +2*Z)/exp(X - 2*Y +2*Z)

e) $\log(3X^2Y) - \log(X) + \log(Z/3)$

2. Suppose log(p/(1-p)) = r. Show that p = exp(r)/(1 + exp(r)).

3. In 2 (above) suppose $-\infty < r < \infty$. What is the range of possible values of p?

4. Suppose h = a*exp(b). Find an expression for log(h).

5. Suppose $S = X^{\exp(b)}$ where $0 < S < 1$. Find an expression for log(-log(S)).

Solutions

1.
   a) $\exp(2)$
   b) $\log(3)$
   c) $\exp(Y)$
   d) $\exp(5Y)$
   e) $\log(XYZ)$

2. $\log(p/(1-p)) = r$

   $p/(1-p) = \exp(r)$

   $(1-p)/p = 1/\exp(r)$

   $1/p - 1 = 1/\exp(r)$

   $1/p = 1 + 1/\exp(r) = (1 + \exp(r))/\exp(r)$

   $p = \exp(r)/(1+\exp(r))$

3. $0 < p < 1$

4. $h = a*\exp(b)$

   $\log(h) = \log(a) + b$

5. $S = X^{\exp(b)}$

   $\log(-\log(S)) = \log(-\log(X^{\exp(b)})) = \log(-\exp(b)\log(X)) = \log(-\log(X)) + b$