Logic and Conditional Statements

Reporting Category Reasoning, Lines, and Transformations

Topic Investigating symbolic form while working with conditional statements

Primary SOL G.1 The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
  a) identifying the converse, inverse, and contrapositive of a conditional statement;
  b) translating a short verbal argument into symbolic form;
  c) using Venn diagrams to represent set relationships; and
  d) using deductive reasoning.

Related SOL G.2a, G.6

Materials
- Activity Sheets 1, 2, and 3 (attached)
- Flash cards (attached)
- Logic and Conditional Statements handout (attached)

Vocabulary
intersection, union, Venn diagram (earlier grades)

conditional statement, hypothesis, conclusion, negation, negate, converse, inverse, contrapositive, biconditional, Law of the Contrapositive, Law of Detachment, Law of Syllogism, valid argument, logical argument, conjecture, verify, proof, prove, disprove, counterexample, undefined term, postulate, theorem (G.1)

Student/Teacher Actions (what students and teachers should be doing to facilitate learning)
1. Distribute copies of Activity Sheet 1, and review the basic vocabulary included.
2. Distribute copies of Activity Sheet 2, and work through the examples with students.
3. Distribute copies of Activity Sheet 3, and have students work in pairs or small groups to complete the problems.
4. Have students discuss findings with their partners.
5. Discuss findings as a whole group. Distribute copies of the attached Logic and Conditional Statements handout, and review it with students. Have students use it for quick reference.
6. Use flash cards as a quick review and a check for understanding.

Assessment
- **Questions**
  - What is the inverse of the converse of $p \rightarrow q$? Use symbols p, q, $\rightarrow$, and ~ and math vocabulary to answer this question.
  - What conclusion can you draw, using all of the following statements?
    $$\sim q \rightarrow s \qquad t \rightarrow \sim r \qquad q \rightarrow t \qquad u \rightarrow \sim s$$

- **Journal/Writing Prompts**
    - Draw a Venn diagram to represent the two statements, "No reptiles have fur" and "All snakes are reptiles." Then, draw a logical conclusion, if possible.
    - Write a converse-inverse-contrapositive poem by writing an "if…, then" statement followed by its converse, inverse, and contrapositive. Create additional stanzas using related "if…, then" statements.
    - Read one of Laura Numeroff's books, such as *If You Give a Mouse a Cookie,* to the class. Discuss it as an extended syllogism or logical chain, and have students write a story that is a logical chain of syllogisms.
- **Other**
    - Have students work in pairs to evaluate strategies.
    - Use activity sheets to help assess student understanding.

Extensions and Connections (for all students)
- Have students investigate Lewis Carroll's logic puzzles.
- Have students solve the logic puzzle from J. K. Rowling's *Harry Potter and the Sorcerer's Stone,* 1998, p. 285.
- Invite a politician or political analyst to visit the class. Ask the guest speaker to explain the relationships among facts, trends, and educated guesses.

Strategies for Differentiation
- Have advanced students explore truth tables.
- Have students write their own conditional statement in "if…, then" form. Take four slips of paper. Write "If" on one slip, "then" on another, the hypothesis on the third, and the conclusion on the fourth. Flip the hypothesis (top to bottom), and write the negation of the hypothesis on the back. Do the same for the conclusion. Use these slips to illustrate converse, inverse, and contrapositive. This can also be done in a larger format with students holding the slips in front of the class. When introducing symbols, label the hypothesis, conclusion, and negation statements with p, $\sim p$, q, and $\sim q$.
- Use three slips of paper, as above, labeled with p, $\rightarrow$, and q to illustrate converse, inverse, and contrapositive using symbols. (Write $\sim p$ on the back of p and $\sim q$ on the back of q, flipping from top to bottom.)
- Have students write an "if…, then" statement of their choosing on an index card. Write the converse, inverse, and contrapositive on the back of the card. Check that they are correct. During the next class, use cards to quiz each other in pairs, and then trade with another pair of students.

Activity Sheet 1: Vocabulary and Symbols

Name _____ **Date** _____

Define each of the following vocabulary terms.

Term	Definition
Conditional Statement	S_____ that can be w_____ in "if…, then" form
Hypothesis	Part of a c_____ statement that f_____ "if"
Conclusion	Part of a conditional s_____ that follows "then"
Negation	D_____ of a s_____ formed by adding or removing the word _____ from a statement
Negate	To add or remove the word _____ from a statement to change its truth value from true to _____ or from false to _____
Converse	S_____ formed from a c_____ statement by s_____ the h_____ and c_____
Inverse	S_____ formed from a c_____ statement by n_____ the h_____ and c_____
Contrapositive	S_____ formed from a c_____ statement by s_____ AND n_____ the h_____ and c_____
Biconditional	Statement combining a c_____ statement and its c_____, using the phrase "if and o_____ if"

Fill in the meaning of each of the following symbols.

p, q, r, s, t, etc.	Meaning:		
→	Meaning:	∨	Meaning:
~	Meaning:	∴	Meaning:
∧	Meaning:	↔	Meaning:

Activity Sheet 1: Vocabulary and Symbols
Instructor's Reference

Define each of the following vocabulary terms.

Term	Definition
Conditional Statement	Statement that can be written in "if…, then" form
Hypothesis	Part of a conditional statement that follows "if"
Conclusion	Part of a conditional statement that follows "then"
Negation	Denial of a statement formed by adding or removing the word not from a statement
Negate	To add or remove the word not from a statement to change its truth value from true to false or from false to true
Converse	Statement formed from a conditional statement by switching the hypothesis and conclusion
Inverse	Statement formed from a conditional statement by negating the hypothesis and conclusion
Contrapositive	Statement formed from a conditional statement by switching AND negating the hypothesis and conclusion
Biconditional	Statement combining a conditional statement and its converse, using the phrase "if and only if"

Fill in the meaning of each of the following symbols.

p, q, r, s, t, etc.	Meaning: Symbols used to represent statements such as hypotheses and conclusions		
$\rightarrow$	Meaning: if…, then (implies)	$\vee$	Meaning: or
$\sim$	Meaning: not	$\therefore$	Meaning: therefore
$\wedge$	Meaning: and	$\leftrightarrow$	Meaning: if and only if (iff)

Activity Sheet 2: Logic and Conditional Statements

Name _____ **Date** _____

Use the following conditional statement to answer the problems: "If I win, then you don't lose."

1. Write the hypothesis. _____

2. Write the conclusion. _____

3. Negate the hypothesis. _____

4. Negate the conclusion. _____

5. Write the converse. _____

6. Write the inverse. _____

7. Write the contrapositive. _____

8. Write the biconditional. _____

Use the following conditional statement to answer the problems: "If elephants fly, then fish don't swim." Each answer should be a complete sentence, *not* symbols.

1. p is the hypothesis. Write p. _____

2. q is the conclusion. Write q. _____

3. $\sim p$ means "the negation of p." Write $\sim p$. _____

4. $\sim q$ means "the negation of q." Write $\sim q$. _____

5. (converse) $q \rightarrow p$ means "q implies p" or "If q, then p." Write $q \rightarrow p$.

    _____

6. (inverse) $\sim p \rightarrow \sim q$ means "Not p implies not q" or "If not p, then not q." Write $\sim p \rightarrow \sim q$.

    _____

7. (contrapositive) $\sim q \rightarrow \sim p$ means "Not q implies not p" or "If not q, then not p." Write $\sim q \rightarrow \sim p$. _____

8. $p \wedge q$ means "p and q." Write $p \wedge q$. _____

9. $p \vee q$ means "p or q." Write $p \vee q$. _____

10. $\therefore p$ means "therefore p." Write $\therefore p$. _____

11. $p \leftrightarrow q$ means "p if and only if q." Write $p \leftrightarrow q$. _____

Activity Sheet 3: Logic and Conditional Statements

Name _____ **Date** _____

1. Write each of the following statements as a conditional statement. Then, circle the hypothesis, and underline the conclusion.
   a. Mark Twain wrote, "If you tell the truth, you don't have to remember anything."

      _____

   b. Helen Keller wrote, "One can never consent to creep when one feels the impulse to soar."

      _____

   c. Mahatma Ghandi wrote, "Freedom is not worth having if it does not include the freedom to make mistakes."

      _____

   d. Benjamin Franklin wrote, "Early to bed and early to rise makes a man healthy, wealthy, and wise."

      _____

2. Write the converse, inverse, and contrapositive for each of the following conditional statements. Determine whether each is true or false.
   a. "If I win, then you don't lose."

      Converse: _____

      Inverse: _____

      Contrapositive: _____

      True or false: _____

   b. "If two segments are congruent, then they have the same length."

      Converse: _____

      Inverse: _____

      Contrapositive: _____

      True or false: _____

3. Use the Law of Detachment to reach a logical conclusion about the following statement: "If it is raining, then Sam and Sarah will not go to the football game." This is a true conditional, and it is raining.

   _____

4. <u>Statement 1</u>: "If two adjacent angles form a linear pair, then the sum of the measures of the angles is 180°."
   <u>Statement 2</u>: "If the sum of the measures of two angles is 180°, then the angles are supplementary."
   By the Law of Syllogism, which statement below follows from Statements 1 and 2?_____
   a. If the sum of the measures of two angles is 180°, then the angles form a linear pair.
   b. If two adjacent angles form a linear pair, then the sum of the measures of the angles is 180°.
   c. If two adjacent angles form a linear pair, then the angles are supplementary.
   d. If two angles are supplementary, then the sum of the measures of the angles is 180°.

5. Let p: you see lightning and q: you hear thunder. Write each of the following statements in symbolic notation:
   a. If you see lightning, then you hear thunder._____

   b. If you hear thunder, then you see lightning._____

   c. If you don't see lightning, then you don't hear thunder._____

   d. If you don't hear thunder, then you don't see lightning._____

6. Let p: two planes intersect and q: the intersection is a line. Write each of the following statements in symbolic notation:
   a. If two planes don't intersect, then the intersection is a line._____

   b. If the intersection is not a line, then two planes do not intersect._____

7. Draw a Venn Diagram below for each of the following statements:
   a. All squares are rhombi.
   b. Some rectangles are squares.
   c. No trapezoids are parallelograms.

Flash Cards

Conditional Statement	**p implies q**
Always	
Sometimes	
Never	
If…, then	$\rightarrow$
Not	**~**
Converse	**"Switch"**
Inverse	**"Negate"**
Contrapositive	**"Switch and Negate"**
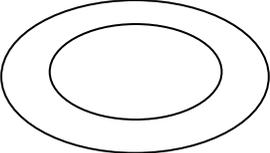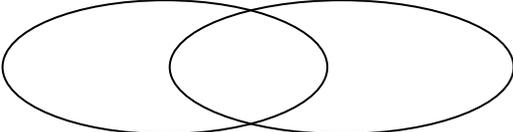
Logic and Conditional Statements

Conditional Statement

If   Hypothesis   then   Conclusion

If   *p,*   then   *q*

$$p \longrightarrow q$$

<u>or</u>   *p* implies *q*

$\sim p$ is read   "not *p*"   and means   the opposite of *p*

Converse
$p \rightarrow q$

"Switch"
$q \rightarrow p$

Inverse
$p \rightarrow q$

"Negate"
$\sim p \rightarrow \sim q$

Contrapositive
$p \rightarrow q$

"Switch and Negate"
$\sim q \rightarrow \sim p$

Always   Sometimes   Never