# **S&P Dow Jones** Indices

A Division of S&P Global

# Fixed Income Index Mathematics Methodology

# **Table of Contents**

Introduction		2
	Highlights	2
	Different Varieties of Indices	2
Index Calculations		3
	Daily Index Values	3
	Daily Index Returns	3
	Total Return	4
	Interest Return	4
	Price Return	5
	Price Return (Loans)	5
	Unhedged Return	5
	Hedged Return	6
	Hedge Size	6
	Market Value	6
	Additional Weight Factor	7
	Weight	7
	Yield to Maturity Adjustment for Inflation-linked Securities	7
	Hedge Adjusted Yield	8
S&P Dow Jones Indices' Contact Information		9
	Client Services	9
Disclaimer		10

## Introduction

#### **Highlights**

This document covers the mathematics of fixed income index calculations, including how they are calculated, how coupon payments are handled, and how other adjustments are made to the index. For details on a specific index, please refer to that index's methodology document available at <a href="https://www.spdii.com">www.spdii.com</a>.

#### **Different Varieties of Indices**

A majority of S&P Dow Jones fixed income indices are market value weighted, where each bond's weight in the index is proportional to its market value. Sometimes an index has capping requirements which set the target weights for the index securities. An additional weight factor (AWF) is used to make market value adjustments to the index securities in order to satisfy the capping rules.

### Index Calculations

#### **Daily Index Values**

Index values are calculated each day by applying the current day's index return to the previous day's index value, as follows:

$$TRIV_{\ell} = TRIV_{\ell \cdot 1} * (1 + IndexTR_{\ell}) \tag{1}$$

$$PRIV_{t} = PRIV_{t+1} * (1 + IndexPR_{t})$$
 (2)

$$IRIV_t = IRIV_{t-1} * (1 + IndexIR_t)$$
 (3)

where:

 $TRIV_t$  = Total return index value on day t.  $PRIV_t$  = Price return index value on day t.  $IRIV_t$  = Interest return index value on day t.

 $IndexTR_t$  = Index total return on day t.  $IndexPR_t$  = Index price return on day t.  $IndexIR_t$  = Index interest return on day t.

#### **Daily Index Returns**

The individual index security returns are aggregated to calculate returns for the index. Specifically, on a given day, the total return, interest return and price return for the index are equal to a weighted average of the returns of the securities that constitute the index. The weight of each index security return is equal to the relative weight of that security in the index as of the <u>previous</u> trading day (adjusted for principal prepayments, etc.). Each cash security has a foreign exchange return related to the index currency as its price return and zero as its interest return. The formulae are as follows:

$$IndexTR_t = \sum_i SecurityWeight_{i,t-1} * tr_{i,t}$$
 (4)

$$IndexPR_t = \sum_{i} SecurityWeight_{i,t-1} * pr_{i,t}$$
 (5)

$$IndexIR_t = \sum_i SecurityWeight_{i,t-1} * ir_{i,t}$$
 (6)

where:

 $IndexTR_t$  = Index total return on day t.  $IndexPR_t$  = Index price return on day t.  $IndexIR_t$  = Index interest return on day t.

 $tr_{i,t}$  = Total return of index security *i* on day  $t(ir_{i,t} + pr_{i,t})$ .

 $pr_{i,t}$  = Price return of index security i on day t.  $ir_{i,t}$  = Interest return of index security i on day t.

Security Weight, t=1 = Adjusted market value weight of index security i at the close of day t-1.

#### **Total Return**

The total return, *TR*, for a given security on day *t* is the sum of the market price, interest, and FX return on day *t*:

$$TR_t = IR_t + PR_t \tag{7}$$

where:

 $IR_t$  = Interest return on day t.  $PR_t$  = Price return on day t.

Price return measures the return due to the change in the market price of the security. Interest return (or coupon return) includes the return due to the interest earned on that security. In the case of zero coupon bonds, the accretion in price due to interest return is reported as price return.

#### Interest Return

The formula for the interest return on an individual index security on day t is as follows:

$$IR_{t} = (AI_{t} - AI_{t+1} + Cpn_{t}) / DirtyPrice_{t+1}$$
(8)

where:

 $IR_t$  = Daily interest return for the security on day t.

 $AI_t$  = Accrued interest, up to and including day t.

 $DirtyPrice_{t-1}$  = Dirty price of the security on day t-1.

 $Cpn_t$  = Coupon payment\* on day t.

#### **Interest Return (Loans)**

In the following formula, PAR should be treated as  $(AWP^*PAR)$ . The formula for the interest return on an individual index loan on day t is as follows:

$$IRt = \frac{(PARt * Rt)/360}{MV_{Beq}}$$
 (9)

where:

 $IR_t$  = Interest return on day t.

 $PAR_t$  = Par amount of the index loan as of the last weekly rebalancing, adjusted for principal prepayments, etc., up to and including day t.

 $R_t$  = Interest rate on day t.

 $MV_{Beg}$  = Market value, at the beginning of day t.

#### **Index Interest Rate**

The index interest rate is determined by the weighted average spread to LIBOR/EURIBOR.

<sup>\*</sup> For securities trading ex-dividend, the coupon is recognized on ex-dividend date. Securities in default do not accrue interest.

<sup>&</sup>lt;sup>1</sup> The dirty price of a security is defined as the sum of the market quoted price and the interest deemed to be earned on that security, but not yet paid to the investor. The clean price is the market quoted price without accrued interest.

#### **Price Return**

The formula for the price return for an index security on day t is as follows:

$$PR_t = (CleanPrice_t - CleanPrice_{t-1} - FX return) / DirtyPrice_{t-1}$$
 (10)

where:

 $PR_t$  = Price return for the security on day t.

 $CleanPrice_t = Market quoted price for the security without accrued interest on day t.$ 

 $DirtyPrice_{t-1}$  = Market quoted price for the security with accrued interest on day t-1.

#### Price Return (Loans)

The formula for the price return for an index loan on day *t* is as follows:

$$PRt = \frac{PARt * \left(\frac{Pt - Pt - 1}{100}\right) + Print * \frac{RP - Pt - 1}{100}}{MV_{Beg}}$$

$$(11)$$

where:

 $PR_t$  = Price return on day t.

 $PAR_t$  = Par amount of the index loan as of the last weekly rebalancing, adjusted for principal prepayments, etc., up to and including day t.

 $P_t$  = Loan price on day t.

 $P_{t-1}$  = Loan price on the previous day.

 $Prin_t$  = Principal pre-payments, etc., on day t.

 $MV_{Beg}$  = Market value, beginning of day t.

RP = Redemption price.

Note that the formula for the Price Return (11) itself has two components. The first term, in the numerator on the left side, represents the <u>unrealized</u> return due to any change in the price, while the second term (on the right) represents the <u>realized</u> return due to receiving a principal repayment at the Redemption Price (which could differ from par) rather than at the current end of day price.

#### **Unhedged Return**

Total unhedged return measures the return by converting the local currency return of the underlying securities to the index currency. The formula for the unhedged daily return for a security is composed of its local currency return and the foreign currency return.

$$TR_{UH,t} = \left(1 + R_{L,t}\right) * \left(1 + \frac{FX_t - FX_{t-1}}{FX_{t-1}}\right) - 1 \tag{12}$$

where:

 $TR_{UH,t}$  = Total unhedged return on day t.

 $R_{L,t}$  = Total local currency return on day t.

 $FX_t$  = Foreign exchange spot rate on day t.

 $FX_{t-1}$  = Foreign exchange spot rate on day t-1.

#### **Hedged Return**

Hedged return measures the return by hedging currency risk through a one-month forward currency contract. The formula for the hedged return for an index is shown as follows:

$$HedgedMTD_{t} = H_{t} + H_{t} * \left(\frac{S_{0}}{F_{0,30}} - \frac{S_{t}}{F_{t,30-t}}\right) + (1 + BaseMTD_{t} - H_{t}) * \frac{S_{0}}{S_{t}}$$

$$TR_{H,t} = \left(\frac{HedgedMTD_{t}}{HedgedMTD_{t-1}}\right) - 1$$
(13)

$$TR_{H,t} = \left(\frac{\text{HedgedMTD}_t}{\text{HedgedMTD}_{t-1}}\right) - 1 \tag{14}$$

where:

= Total hedged return on day t

HedgedMTD<sub>t</sub> = Month to date hedged return on day t

BaseMTD<sub>t</sub> = Month to date return of the local currency index on day t

= Foreign exchange spot rate on day t

= Forward rate on the rebalancing date 0 with 30 days remaining in the contract, F<sub>0.30</sub>

assuming a 30/360 day count convention.

= Interpolated forward rate on day t with 30-t days remaining in the contract

= Hedge size on day t

#### **Hedge Size**

The hedge size is determined by using the index weighted yield of the pro forma index on the rebalancing day. Assuming the yield is an annual return proxy, the size of the monthly hedge is a projection of the monthly return of the local currency index. The hedging size used for a MTD calculation on day t is shown

$$HedgeSize = 1 + \left(\frac{yield}{2}\right)^{1/6} \tag{15}$$

$$H_t = HedgeSize^{t/30} ag{16}$$

#### **Market Value**

The market value of the security as represented in the index as of the close on day t is calculated as follows:

$$MV_{t} = PAR_{t} * \frac{(P_{t} + AI_{t})}{100} * FX_{t}$$

$$\tag{17}$$

The adjusted market value is applied as follows:

$$AMV_t = AWF^*MV_t \tag{18}$$

where:

 $MV_t$ = Market value of the index security on day t.

 $AMV_t$ = Adjusted market value of the index security on day t.

 $PAR_t$ = Par amount of the index security as of the last rebalancing, adjusted for principal

pre-payments, etc., up to and including day t.

 $P_t$ = Clean price of the index security on day t.

 $AI^2$ = Accrued interest on the index security, up to and including day t.

 $FX_t$ = Foreign exchange rate used to convert to the target currency on day t.

<sup>&</sup>lt;sup>2</sup> For loans, Alt is calculated on a 360-day basis. Accrued interest is reduced to zero every 90 days after a loan enters the index.

AWF = Additional weight factor.

If the valuation date is not a trading day, the market value is based on the price as of the immediate prior trading day, plus interest accrued to the valuation date. On the cash payout day, the accrued interest is set to the coupon cash value.

#### **Additional Weight Factor**

The Additional Weight Factor (AWF) is used to tilt the original market value weight of a security within the index. It is calculated on the rebalancing reference date and is implemented on the rebalancing effective date. The AWF remains static until the subsequent rebalancing.

The AWF for all market value-weighted indices is equal to 1. For target weighted indices, the AWF is calculated as follows:

$$AWF_{i} = TW_{i} * \frac{\sum_{1}^{N} MV_{i}}{MV_{i}}$$

$$\tag{19}$$

where:

 $AWF_i$  = Additional weight factor of the index security *i*.

 $TW_i$  = Target weight for the index security *i*.

 $MV_i$  = Market value of the index security i in the underlying index.

 $\sum_{i=1}^{N} MV_{i}$  = Aggregate market value of all N securities in the underlying index.

In equal-weight indices TW = 1/N, where N is the number of securities included in the index.

#### Weight

The relative weight of an index security *i* is defined as the adjusted market value of that security expressed as a percentage of the aggregate adjusted market value of all the securities in the index portfolio, as follows:

$$weight_i = \frac{AMV_i}{\sum_i AMV_i}$$
 (20)

#### Yield to Maturity Adjustment for Inflation-linked Securities<sup>3</sup>

To calculate the yield to maturity adjustment, one has to project the inflation rate for future cash flows of the bond. The projected inflation rate is an approximation of future inflation and is calculated using the following formula:

Projected inflation Rate 
$$t = 100 * (\frac{CPI_t}{CPI_{t-1 year}} - 1)$$
 (21)

where:

*CPI*<sup>t</sup> = The reference national CPI for day *t* is calculated through a linear interpolation of the two most recent CPI levels available as of the start of the month.

CPI<sub>t-1year</sub> = The reference national CPI as of one year before the valuation date *t* with the same interpolation as CPI<sub>t</sub>.

<sup>&</sup>lt;sup>3</sup> Nominal yield is applied to inflation/index-linked securities only.

Using the projected inflation rate, the current index ratio and the nominal cash flows, we generate the expected inflated cash flows, which are then used to calculate the nominal yield.

#### **Hedge Adjusted Yield**

The hedge adjusted yield represents the yield of an index after accounting for the cost of hedging foreign currency exposure in the forward market. At each rebalance, a projected market value using the proforma index yield is hedged using the forward rate. The hedge adjusted yield accounts for the premium or discount of the forward rate and applies it to the yield used to project the future market value as follows:

Hedge adjusted yield 
$$i = \text{Yield } i, t + (\frac{FF_{0,30}}{FX_t} - 1) * 12$$
 (22)

Where:

Yield i, t = The index weighted yield found in the rebalancing day's pro-forma universe

 $FF_{0,30}$  = Forward rate on the last rebalancing date 0 with 30 days remaining in the

contract, assuming a 30/360 day count convention.

 $FX_t$  = Foreign exchange spot rate on day t.

## S&P Dow Jones Indices' Contact Information

**Client Services** 

index services@spglobal.com

### Disclaimer

Copyright © 2021 S&P Dow Jones Indices LLC. All rights reserved. STANDARD & POOR'S, S&P, S&P 500, S&P 500 LOW VOLATILITY INDEX, S&P 100, S&P COMPOSITE 1500, S&P MIDCAP 400, S&P SMALLCAP 600, S&P GIVI, GLOBAL TITANS, DIVIDEND ARISTOCRATS, S&P TARGET DATE INDICES, GICS, SPIVA, SPDR and INDEXOLOGY are registered trademarks of Standard & Poor's Financial Services LLC ("S&P"). DOW JONES, DJ, DJIA and DOW JONES INDUSTRIAL AVERAGE are registered trademarks of Dow Jones Trademark Holdings LLC ("Dow Jones"). These trademarks together with others have been licensed to S&P Dow Jones Indices LLC. Redistribution or reproduction in whole or in part are prohibited without written permission of S&P Dow Jones Indices LLC. This document does not constitute an offer of services in jurisdictions where S&P Dow Jones Indices LLC, S&P, S&P Trucost Limited, SAM (part of S&P Global), Dow Jones or their respective affiliates (collectively "S&P Dow Jones Indices") do not have the necessary licenses. Except for certain custom index calculation services, all information provided by S&P Dow Jones Indices is impersonal and not tailored to the needs of any person, entity or group of persons. S&P Dow Jones Indices receives compensation in connection with licensing its indices to third parties and providing custom calculation services. Past performance of an index is not an indication or guarantee of future results.

It is not possible to invest directly in an index. Exposure to an asset class represented by an index may be available through investable instruments based on that index. S&P Dow Jones Indices does not sponsor, endorse, sell, promote or manage any investment fund or other investment vehicle that is offered by third parties and that seeks to provide an investment return based on the performance of any index. S&P Dow Jones Indices makes no assurance that investment products based on the index will accurately track index performance or provide positive investment returns. S&P Dow Jones Indices LLC is not an investment advisor, and S&P Dow Jones Indices makes no representation regarding the advisability of investing in any such investment fund or other investment vehicle. A decision to invest in any such investment fund or other investment vehicle should not be made in reliance on any of the statements set forth in this document. Prospective investors are advised to make an investment in any such fund or other vehicle only after carefully considering the risks associated with investing in such funds, as detailed in an offering memorandum or similar document that is prepared by or on behalf of the issuer of the investment fund or other investment product or vehicle. S&P Dow Jones Indices LLC is not a tax advisor. A tax advisor should be consulted to evaluate the impact of any tax-exempt securities on portfolios and the tax consequences of making any particular investment decision. Inclusion of a security within an index is not a recommendation by S&P Dow Jones Indices to buy, sell, or hold such security, nor is it considered to be investment advice.

These materials have been prepared solely for informational purposes based upon information generally available to the public and from sources believed to be reliable. No content contained in these materials (including index data, ratings, credit-related analyses and data, research, valuations, model, software or other application or output therefrom) or any part thereof ("Content") may be modified, reverseengineered, reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of S&P Dow Jones Indices. The Content shall not be used for any unlawful or unauthorized purposes. S&P Dow Jones Indices and its third-party data providers and licensors (collectively "S&P Dow Jones Indices Parties") do not guarantee the accuracy, completeness, timeliness or availability of the Content. S&P Dow Jones Indices Parties are not responsible for any errors or omissions, regardless of the cause, for the results obtained from the use of the Content. THE CONTENT IS PROVIDED ON AN "AS IS" BASIS. S&P DOW JONES INDICES PARTIES DISCLAIM ANY AND ALL EXPRESS OR IMPLIED WARRANTIES, INCLUDING, BUT NOT LIMITED TO, ANY WARRANTIES OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE OR USE. FREEDOM FROM BUGS, SOFTWARE ERRORS OR DEFECTS, THAT THE CONTENT'S FUNCTIONING WILL BE UNINTERRUPTED OR THAT THE CONTENT WILL OPERATE WITH ANY SOFTWARE OR HARDWARE CONFIGURATION. In no event shall S&P Dow Jones Indices Parties be

liable to any party for any direct, indirect, incidental, exemplary, compensatory, punitive, special or consequential damages, costs, expenses, legal fees, or losses (including, without limitation, lost income or lost profits and opportunity costs) in connection with any use of the Content even if advised of the possibility of such damages.

Credit-related information and other analyses, including ratings, research and valuations are generally provided by licensors and/or affiliates of S&P Dow Jones Indices, including but not limited to S&P Global's other divisions such as S&P Global Market Intelligence. Any credit-related information and other related analyses and statements in the Content are statements of opinion as of the date they are expressed and not statements of fact. Any opinion, analyses and rating acknowledgement decisions are not recommendations to purchase, hold, or sell any securities or to make any investment decisions, and do not address the suitability of any security. S&P Dow Jones Indices does not assume any obligation to update the Content following publication in any form or format. The Content should not be relied on and is not a substitute for the skill, judgment and experience of the user, its management, employees, advisors and/or clients when making investment and other business decisions. S&P Dow Jones Indices LLC does not act as a fiduciary or an investment advisor. While S&P Dow Jones Indices has obtained information from sources they believe to be reliable, S&P Dow Jones Indices does not perform an audit or undertake any duty of due diligence or independent verification of any information it receives.

To the extent that regulatory authorities allow a rating agency to acknowledge in one jurisdiction a rating issued in another jurisdiction for certain regulatory purposes, S&P Global Ratings reserves the right to assign, withdraw or suspend such acknowledgement at any time and in its sole discretion. S&P Dow Jones Indices, including S&P Global Ratings, disclaim any duty whatsoever arising out of the assignment, withdrawal or suspension of an acknowledgement as well as any liability for any damage alleged to have been suffered on account thereof. Affiliates of S&P Dow Jones Indices LLC, including S&P Global Ratings, may receive compensation for its ratings and certain credit-related analyses, normally from issuers or underwriters of securities or from obligors. Such affiliates of S&P Dow Jones Indices LLC, including S&P Global Ratings, reserve the right to disseminate its opinions and analyses. Public ratings and analyses from S&P Global Ratings are made available on its Web sites, <a href="https://www.standardandpoors.com">www.standardandpoors.com</a> (free of charge), and <a href="https://www.standardandpoors.com">www.ratingsdirect.com</a> and <a href="https://www.standardandpoors.com">www.globalcreditportal.com</a> (subscription), and may be distributed through other means, including via S&P Global Ratings publications and third-party redistributors. Additional information about our ratings fees is available at <a href="https://www.standardandpoors.com/usratingsfees">www.standardandpoors.com/usratingsfees</a>.

S&P Global keeps certain activities of its various divisions and business units separate from each other in order to preserve the independence and objectivity of their respective activities. As a result, certain divisions and business units of S&P Global may have information that is not available to other business units. S&P Global has established policies and procedures to maintain the confidentiality of certain non-public information received in connection with each analytical process.

In addition, S&P Dow Jones Indices provides a wide range of services to, or relating to, many organizations, including issuers of securities, investment advisers, broker-dealers, investment banks, other financial institutions and financial intermediaries, and accordingly may receive fees or other economic benefits from those organizations, including organizations whose securities or services they may recommend, rate, include in model portfolios, evaluate or otherwise address.